Assignment VI Periodic Connected Slot Array Equivalent Circuit

EE4620 Spectral Domain Methods in Electromagnetics

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Abstract

In this assignment, the equivalent circuit for a periodic connected slot array is derived based on the active impedance and periodic longitudinal spectral Green's function of the array.

I. Derivation Procedures and High Abstraction Circuit

The active input impedance of the periodic connected slot array is already derived as

$$Z_{a,slot} = -\frac{1}{d_x}\sum_{m_x=-\infty}^{\infty}\frac{sinc^2(\frac{k_{xm}\delta_s}{2})}{4D_{\infty}^{HM}(k_{xm})}, \tag{1}$$

where the period longitudinal spectral Green's function $D_{\infty}^{HM}(k_x)$ is derived as

$$D_{\infty}^{HM}(k_x) = \frac{1}{d_y}\sum_{m_y=-\infty}^{\infty} G_{XX}^{HM}(k_x, k_{ym}) J_0(\frac{k_{ym}w_s}{2}). \tag{2}$$

A. Derivation Steps

To derive the equivalent circuit of a periodic connected slot array, the steps are as follows:

1) determine a high abstraction circuit (Thevenin or Norton equivalent circuit) depending on the impressed current at the delta-gap sources;
2) decompose the slot active impedance $Z_{a,slot}$ in $m_x = 0$ Floquet mode and higher $m_x \neq 0$ modes components, $Z_{m_x=0}$ and $Z_{m_x\neq 0}$ respectively;
3) decompose $Z_{m_x=0}$ in fundamental $(m_x = 0, m_y = 0)$ mode and higher $(m_x = 0, m_y \neq 0)$ modes components, Z_{00} and $Z_{m_x=0,m_y\neq 0}$ respectively;
4) represent Z_{00} as the impedance seen into a transformer and define the turns ratio n;
5) deconstruct the impedance connected to the secondary of the previously defined transformer into transverse electric (TE) and transverse magnetic (TM) components.

B. High Abstraction Circuit

The impressed current on the delta-gap sources for a periodic connected slot array is defined as

$$j_{inc,y} = \sum_{n_x=-\infty}^{\infty} I_0 \frac{rect_{\delta_s}(x - n_x d_x)}{\delta_s} e^{-jk_{x0}n_x d_x}; \tag{3}$$

and it is in terms of the total current I_0 on the delta-gap source. Therefore, the source is interpreted as a current source, and Norton equivalent circuit is used as a high abstraction. The Norton equivalent circuit of the array with the feed's TL impedance Z_L and the input impedance $Z_{a,slot}$ is shown in Fig.1.

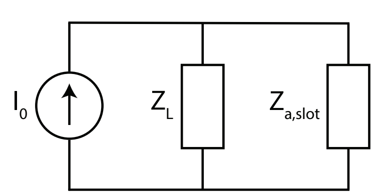

Fig. 1. Norton equivalent circuit of the periodic connected slot array.

II. Decomposition of x Floquet Modes in the Active Input Impedance

Decomposing $Z_{a,slot}$ into $m_x = 0$ and $m_x \neq 0$ mode components gives

$$\begin{aligned} Z_{a,slot} &= Z_{m_x=0} + Z_{m_x\neq 0} \\ &= -\frac{1}{d_x}\frac{sinc^2(\frac{k_{x0}\delta_s}{2})}{4D_\infty^{HM}(k_{x0})} - \frac{1}{d_x}\sum_{m_x\neq 0}\frac{sinc^2(\frac{k_{xm}\delta_s}{2})}{4D_\infty^{HM}(k_{xm})}, \end{aligned} \tag{4}$$

where the $m_x = 0$ and $m_x \neq 0$ components are

$$Z_{m_x=0} = -\frac{1}{d_x}\frac{sinc^2(\frac{k_{x0}\delta_s}{2})}{4D_\infty^{HM}(k_{x0})}, \tag{5a}$$

$$Z_{m_x\neq 0} = -\frac{1}{d_x}\sum_{m_x\neq 0}\frac{sinc^2(\frac{k_{xm}\delta_s}{2})}{4D_\infty^{HM}(k_{xm})}. \tag{5b}$$

The two impedance components are summed, therefore, they are connected in series.

For periodic connected dipole arrays, the active admittance has this equation form, instead of the impedance. Hence, for periodic connected dipole arrays, the sum is in the denominator of the active impedance, and results in purely imaginary and negative values, corresponding to capacitance. In contrast, for periodic connected slot arrays, the sum is in the nominator of the active impedance. The inverse of purely imaginary and negative number results in purely imaginary and positive number. Therefore, according to this argument, for periodic connected slot array, $Z_{m_x\neq 0}$ should be an inductance. Moreover, if $Z_{m_x\neq 0}$ is a capacitance, as $\omega \to 0$, $Z_{a,slot} \to \infty$, therefore the bandwidth would be limited and harder to match. However, connected slot arrays have infinite bandwidth, defined as f_{max}/f_{min}, similarly to connected dipole arrays, and are in fact preferred over connected dipole arrays due to being easier to match. The physical interpretation of this component is the inductance created by the impressed current on the delta-gap source. [1] The equivalent circuit after decomposition in $Z_{m_x\neq 0}$ and $Z_{m_x=0}$ is shown in Fig.2.

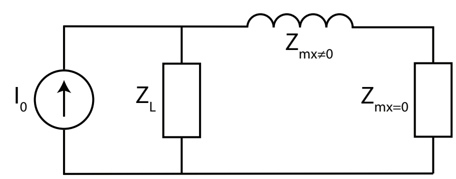

Fig. 2. Equivalent circuit after decomposition of $Z_{a,slot}$ in $Z_{m_x=0}$ and $Z_{m_x} \neq 0$.

III. Decomposition of y Floquet Modes

The periodic longitudinal spectral Green's function $D_\infty^{HM}(k_{x0})$ is in the denominator of $Z_{m_x=0}$ in Eq.5a; due to being harder to manipulate sums in the denominator, in order to transfer the sum in the nominator, the $m_x = 0$ admittance is taken by

$$\begin{aligned} Y_{m_x=0} &= \frac{1}{Z_{m_x=0}} \\ &= -d_x\frac{4}{sinc^2(\frac{k_{x0}\delta_s}{2})}D_\infty^{HM}(k_{x0}); \end{aligned} \tag{6}$$

substituting $D_\infty^{HM}(k_{x0})$, in Eq.2, in $Y_{m_x=0}$, in Eq.6, gives

$$Y_{m_x=0} = -d_x\frac{4}{sinc^2(\frac{k_{x0}\delta_s}{2})}\frac{1}{d_y}\sum_{m_y=-\infty}^{\infty}G_{XX}^{HM}(k_{x0},k_{ym})J_0(\frac{k_{ym}w_s}{2}). \tag{7}$$

Decomposing $Y_{m_x=0}$ into components with $(m_x = 0, m_y = 0)$ fundamental mode and $(m_x = 0, m_y \neq 0)$ higher-order modes gives

$$\begin{aligned} Y_{m_x=0} &= Y_{00} + Y_{m_x=0,m_y\neq 0} \\ &= -\frac{d_x}{d_y}\frac{4J_0(\frac{k_{y0}w_s}{2})}{sinc^2(\frac{k_{x0}\delta_s}{2})}G_{XX}^{HM}(k_{x0},k_{y0}) - \frac{d_x}{d_y}\frac{4}{sinc^2(\frac{k_{x0}\delta_s}{2})}\sum_{m_y\neq 0}G_{XX}^{HM}(k_{x0},k_{ym})J_0(\frac{k_{ym}w_s}{2}), \end{aligned} \tag{8}$$

[1] The component can be determined by calculating, plotting $Z_{m_x\neq 0}$, and checking whether the value is purely imaginary and negative or positive. However, analytical argument is determined to be satisfactory in this case, while numerical calculations would have consumed a lot of time in programming.

where the $(m_x = 0, m_y = 0)$ and $(m_x = 0, m_y \neq 0)$ components are

$$Y_{00} = -\frac{d_x}{d_y} \frac{4J_0(\frac{k_{y0}w_s}{2})}{sinc^2(\frac{k_{x0}\delta_s}{2})} G^{HM}_{XX}(k_{x0}, k_{y0}), \tag{9a}$$

$$Y_{m_x=0,m_y\neq 0} = -\frac{d_x}{d_y} \frac{4}{sinc^2(\frac{k_{x0}\delta_s}{2})} \sum_{m_y\neq 0} G^{HM}_{XX}(k_{x0}, k_{ym}) J_0(\frac{k_{ym}w_s}{2}). \tag{9b}$$

The sum of admittance represents parallel component, hence, Y_{00} and $Y_{m_x=0,m_y\neq 0}$ are parallel. Moreover, using similar arguments as for $Z_{m_x\neq 0}$ being an inductance, $Y_{m_x=0,m_y\neq 0}$ is a capacitance. The physical interpretation of the $Y_{m_x=0,m_y\neq 0}$ is the capacitive effect on the slot's width over its length. The equivalent circuit after decomposition in $Y_{m_x=0,m_x\neq 0}$ and Y_{00} is shown in Fig.3.

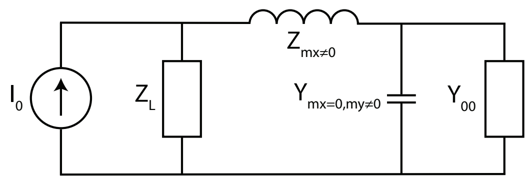

Fig. 3. Equivalent circuit after decomposition of $Y_{m_x=0}$ in Y_{00} and $Z_{m_x} = 0, m_y \neq 0$.

IV. Represent Admittance of Fundamental Mode as a Transformer

Assuming the transformer has one primary turn and n secondary turns, i.e. the turn ratio is $1 : n$. The admittance seen at the primary coil of the transformer is

$$Y_{00} = \frac{i_1}{v_1}, \tag{10}$$

while the admittance connected to the secondary coil is

$$Y'_{00} = \frac{i_2}{v_2}. \tag{11}$$

The ratio between the currents and voltages at the primary and secondary is

$$i_1 = ni_2, \tag{12a}$$

$$v_1 = \frac{v_2}{n}, \tag{12b}$$

where n is the number of turns at the secondary. Therefore, the ratio between the impedance connected at the secondary coil and seen at the input of the transformer is

$$Y_{00} = \frac{ni_2}{\frac{v_2}{n}} = n^2 \frac{i_2}{v_2} = n^2 Y'_{00}. \tag{13}$$

The transformer is shown in Fig.4.

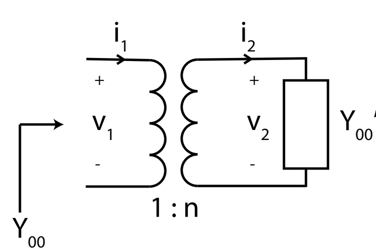

Fig. 4. Transformer used to describe Y_{00}.

The admittance connected to the secondary coil Y'_{00} is defined by

$$Y'_{00} = -G^{HM}_{XX}(k_{x0}, k_{y0}); \tag{14}$$

therefore, using Eq.9a, the number of turns is

$$n = \sqrt{\frac{d_x}{d_y}\frac{4J_0(\frac{k_{y0}w_s}{2})}{sinc^2(\frac{k_{x0}\delta_s}{2})}}. \tag{15}$$

The equivalent circuit after introducing the transformer is shown in Fig.5.

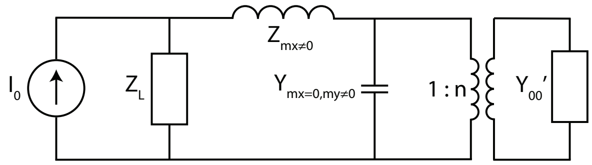

Fig. 5. Equivalent circuit after substituting the transformer in Y_{00}.

V. Decomposition of the Admittance at the Secondary into TE and TM Components

The admittance connected to the secondary coil of the transformer Y'_{00} is decomposed into TE and TM components by

$$Y'_{00} = -G^{HM}_{XX}(k_{x0}, k_{y0}) = \frac{i_{TE}k^2_{x0} + i_{TM}k^2_{y0}}{k^2_\rho}, \tag{16}$$

with propagation vector components

$$k_{x0} = k_0 \sin\theta \cos\phi, \tag{17a}$$

$$k_{y0} = k_0 \sin\theta \sin\phi, \tag{17b}$$

$$k_{z0} = k_0 \sin\theta; \tag{17c}$$

therefore, Y'_{00} becomes

$$Y'_{00} = i_{TE}\cos^2\phi + i_{TM}\sin^2\phi. \tag{18}$$

The TE and TM modes can be represented as transformers with number of turns at the secondary $n_{TE} = \cos\phi$ and $n_{TM} = \sin\phi$. The sum of the modes in the admittance indicates two two modes are parallel in the equivalent circuit. The equivalent circuit after decomposing the modes is shown in Fig.6.

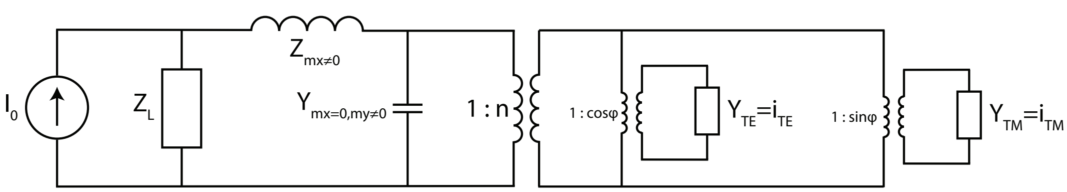

Fig. 6. Equivalent circuit after decomposition of TE and TM modes.

The slot array is located in free space, therefore, the TLs extend infinitely in the positive and negative z directions. Moreover, the spectral *HM* Green's function is described in terms of i_{TE} and i_{TM}, therefore, the secondary coils of the transformers are connected in series to the TLs. The final equivalent circuit with TLs is shown in Fig.7.

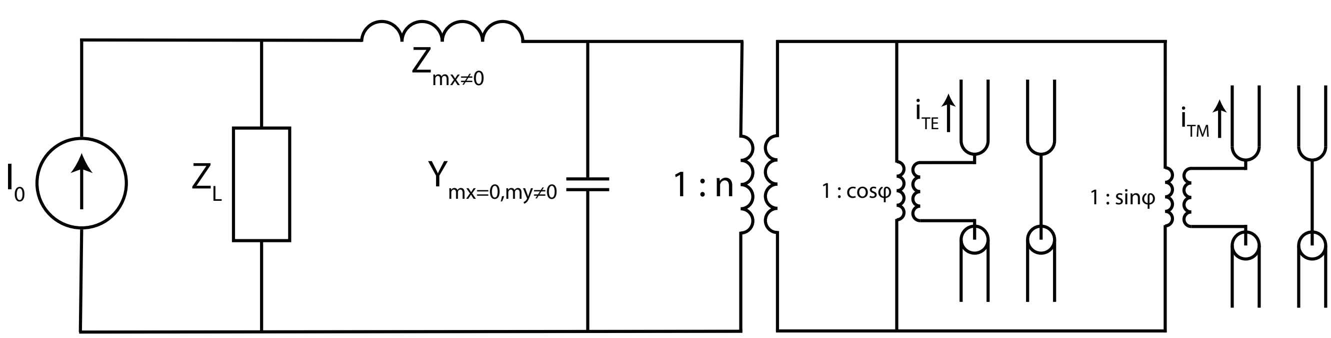

Fig. 7. Final equivalent circuit after placing TLs in the TE and TM modes for an array in free space.