Astronomy C10: Week 4

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Weekly Reminders

Reminders!

HW 5 due at 6pm on Friday via Gradescope

TALC 5-7pm on Wednesday and Thursday in Campbell 131 and on Zoom

My **office hours** 9am-10am on Thursday and 4-5pm on Monday in Campbell 355

Midterm Monday Oct 13th! MCQ and in lecture (more on this next week)

This weeks plan!

Newton's Laws and Practice [15 mins]

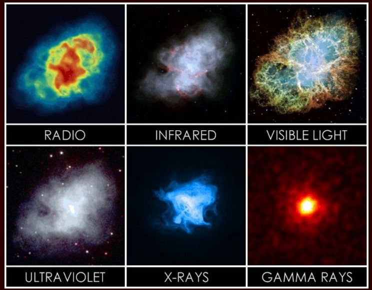
Kepler's laws and Practice [15 mins]

Return Quiz [~15 mins]

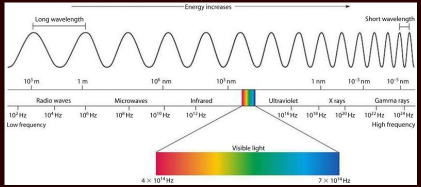
Questions? [Remaining time]

Recap!

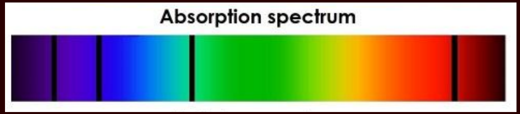
Light is super important in astronomy



Light can behaves as a ***wave*** and particle



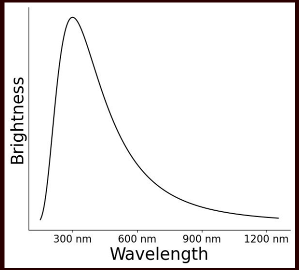
Emission/Absorption lines reveal chemical makeup.



Use these lines to measure relative velocity (**Doppler**)

$$\frac{\Delta\lambda}{\lambda_{rest}} = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}} = \frac{v}{c}$$

Objects shine as **blackbody**



We have all these tools to **“measure”** / **observe**, how do we make sense of the **physics**?

Newton’s Laws

Important quantities!

To describe any object we want to know……

Important quantities!

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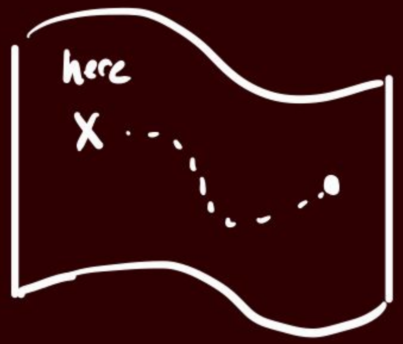
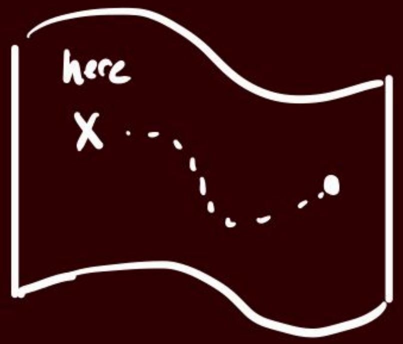


Where is it?

Position

Important quantities!

To describe any object we want to know……



Where is it?

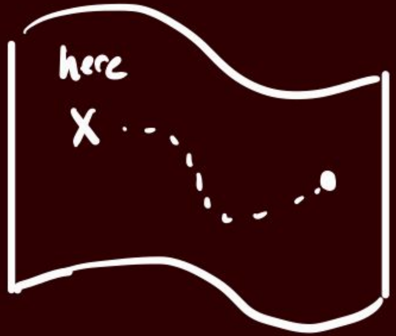
<u>Position</u>

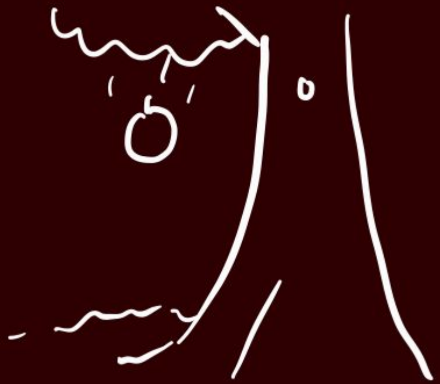
How fast is it moving?

<u>Velocity</u>

Important quantities!

To describe any object we want to know……





Where is it?

How fast is it moving?

How fast is speed changing?

Position

Velocity

Acceleration

Important quantities!

We also want to describe how much stuff there is!

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How much stuff?

Mass

Important quantities!

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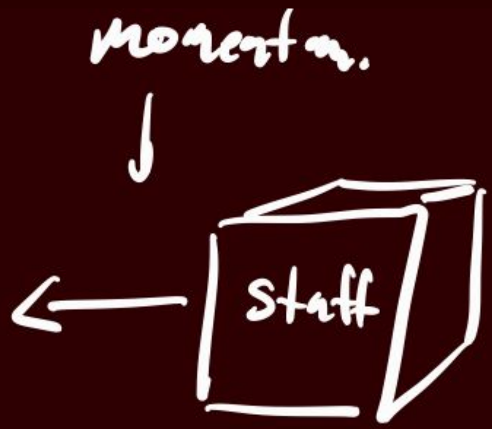


How much stuff?

How much stuff is moving?

Mass

Momentum

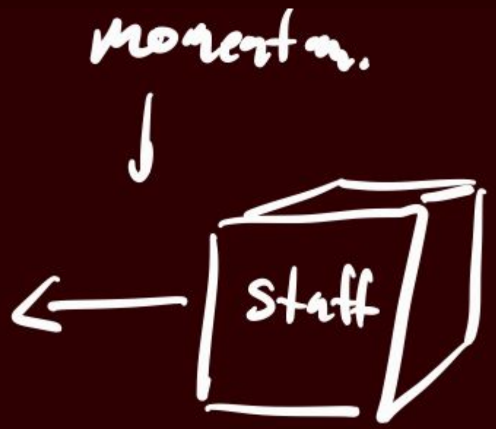
p = mv

Important quantities!

We also want to describe how much stuff there is!

How much stuff?

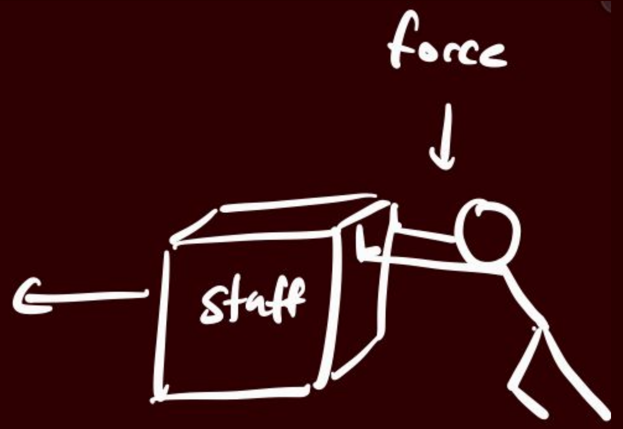
Mass



How much stuff is moving?

Momentum

p = mv



How much stuff is accelerated?

Force

F = ma

Newton’s Laws

If no forces act on a body, its speed and direction of motion remain constant

Force = ***mass*** x ***acceleration***

When two bodies interact, they exert equal and opposite forces on each other

Newton’s 1st Law

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an object at rest remains at rest

Newton's 1st Law

If no forces act on a body, its speed and direction of motion remain constant

If no force is acting…

an object at rest remains at rest

an object in motion remains in motion (with same velocity)

Newton’s 2nd + 3rd Law

2. Force = mass x acceleration

3. When two bodies touch, they exert equal and opposite forces on each other

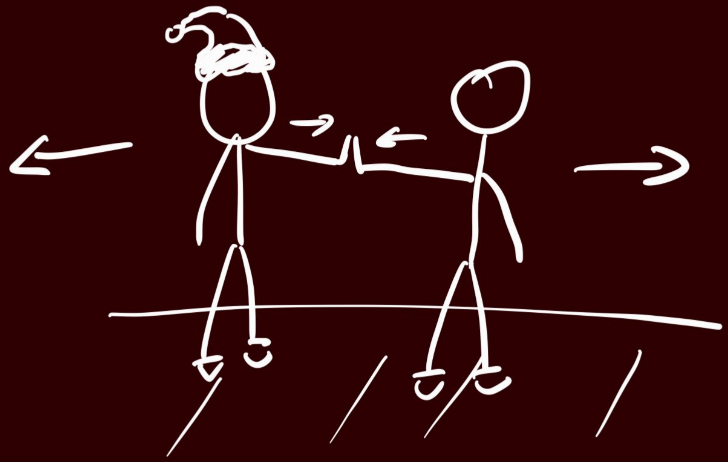
Newton's 2nd + 3rd Law

2. Force = mass x acceleration

3. When two bodies touch, they exert equal and opposite forces on each other

When two skaters push away from each other…

- Both will begin moving
- If they are roughly the same size, they will have roughly the same speeds and accelerations
- If one is much less massive than the other, the lighter skater will accelerate more



Newton’s law of universal gravitation

Gravitational force between two objects depends on their masses m_1 and m_2 and on the distance d between them:

$$F = Gm_1m_2/d^2$$

Important equations for this section

$$F = Gm_1m_2/d^2$$

$$F = ma$$

How are Newton’s Laws useful?

Demo [with skateboards]

Practice Problem 1

The Sun has a mass of 2.0 x 1030 kg and Saturn has a mass of 5.7 x 1026 kg. The average distance between Saturn and the Sun is 1.4 x 109 km.

a) What is the ratio of the gravitational force exerted on Saturn by the Sun to the gravitational force exerted on the Sun by Saturn?

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b) *What is the ratio of Saturn's acceleration to the Sun's acceleration?*

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c) *How would the gravitational force change if the Sun's mass were doubled? What if Saturn's mass were doubled?*
d) *How would the gravitational force change if the distance between Saturn and the Sun decreased by a factor of 4?*

Solutions

1) $M_{sun} = 2\times10^{30}$ kg
$M_{sat} = 5.7\times10^{26}$ kg
$d = 1.4\times10^{9}$ km

a) apply Newton's 3rd law
equal and opposite force!

b) $\boxed{F = ma}$ $\boxed{F = \frac{-Gm_1m_2}{d^2}}$

① part a) we know the forces are the same $\Rightarrow$ $F_{sun} = F_{sat}$

② $F = ma \Rightarrow a = \frac{F}{m}$

$$\frac{a_{sun}}{a_{sat}} = \frac{\left(\frac{F_{sun}}{M_{sun}}\right)}{\left(\frac{F_{sat}}{M_{sat}}\right)} = \frac{\left(\frac{\cancel{F_{sun}}}{M_{sun}}\right)}{\left(\frac{\cancel{F_{sun}}}{M_{sat}}\right)} = \boxed{\frac{M_{sat}\,1}{M_{sun}}} \quad \frac{\frac{1}{M_{sun}}}{\left(\frac{1}{M_{sat}}\right)} = \frac{1}{M_{sun}}\cdot M_{sat} = \frac{M_{sat}}{M_{sun}}$$

$F_{sun} = F_{sat}$

c) $F = \frac{-Gm_1m_2}{d^2}$

$F_1 = \frac{-G\,M_{sun}\,M_{sat}}{d^2}$

$2M_{sun} \Rightarrow F_2 = \frac{-G\,(2M_{sun})(M_{sat})}{d^2}$

$$\frac{F_2}{F_1} = \frac{\frac{-\cancel{G}\,(2\cancel{M_{sun}})(\cancel{M_{sat}})}{\cancel{d^2}}}{\frac{\cancel{-G\,M_{sun}\,M_{sat}}}{\cancel{d^2}}} = \boxed{2}$$

Same for Saturn.

d) $F = -\frac{Gm_1m_2}{d^2}$

$F_1 = \frac{-GM_{sun}\,M_{sat}}{d_1^2}$

$d_2 = \frac{1}{4}d_1 \Rightarrow F_2 = \frac{-GM_{sun}\,M_{sat}}{(\frac{1}{4}d_1)^2}$

$$\Rightarrow \frac{F_2}{F_1} = \frac{\frac{\cancel{-Gm_1m_2}}{(\frac{1}{4}d)^2}}{\frac{\cancel{-Gm_1m_2}}{d^2}} = \frac{\frac{1}{(\frac{1}{4}d)^2}}{\frac{1}{d^2}} = \frac{\frac{1}{\frac{1}{16}\cancel{d^2}}}{\frac{1}{\cancel{d^2}}} = \boxed{16}$$

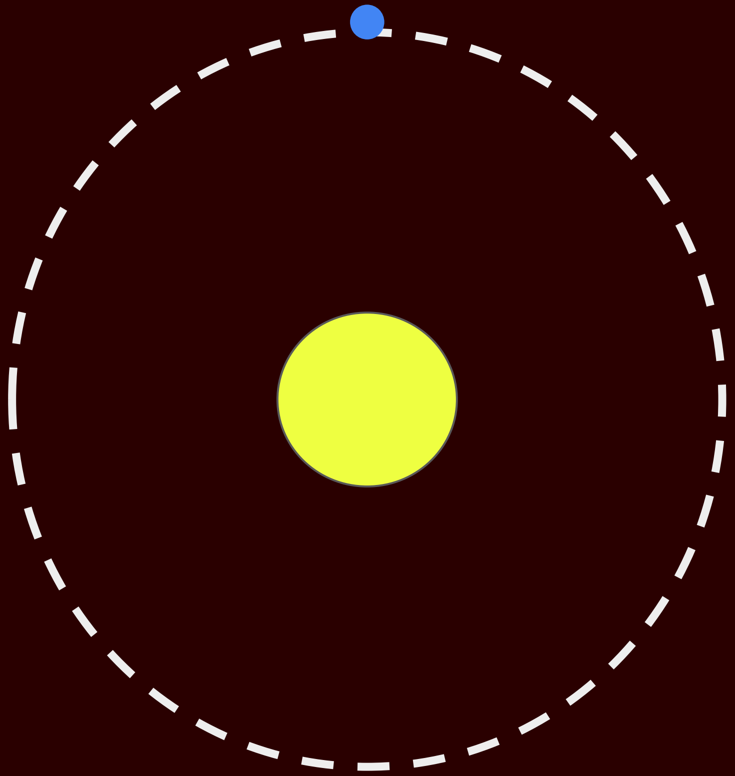
$$\frac{1}{\frac{1}{16}d^2} = \frac{16}{d^2}$$

Kepler’s Laws

Let's use ***Newton's laws*** *of motion to understand and simplify orbital mechanics ->* ***Kepler's laws*** *.*

Orbits

1) Gravity pulls object inward



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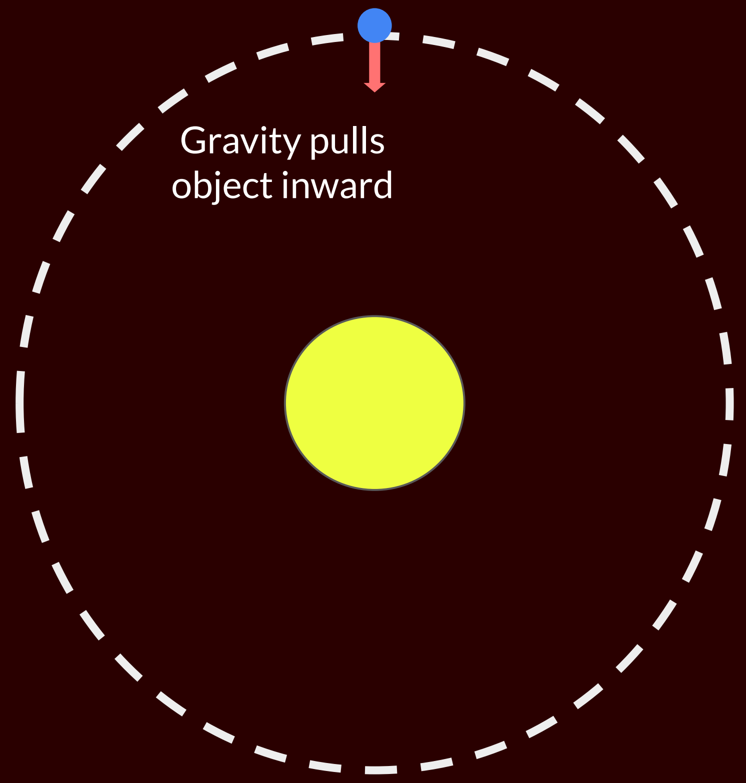
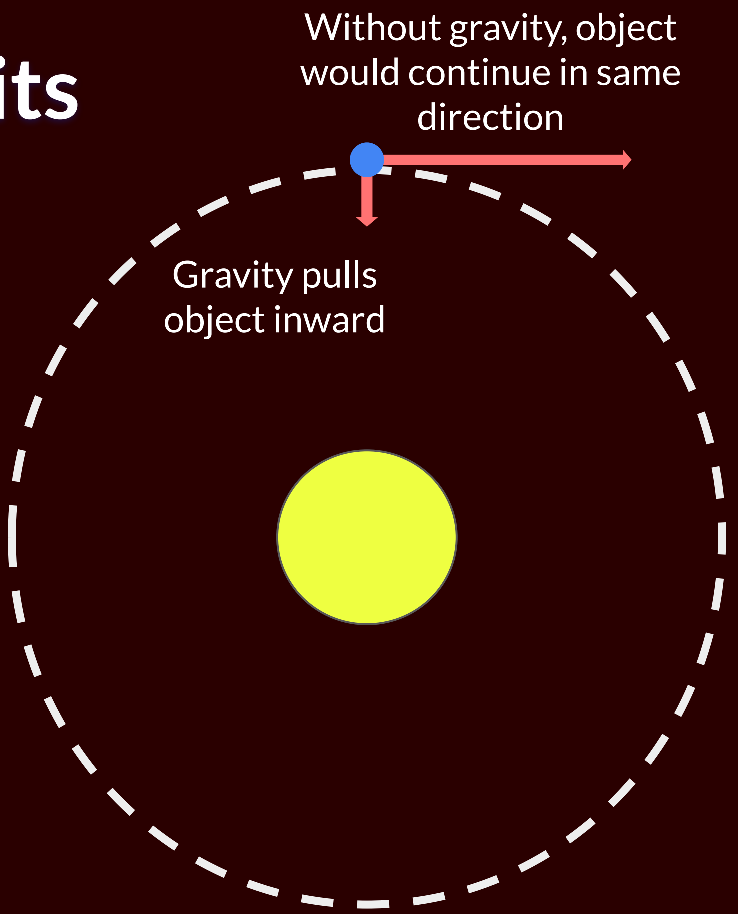


Orbits

1) Gravity pulls object inward

2) Without gravity, object tends to continue moving in same direction and with same speed



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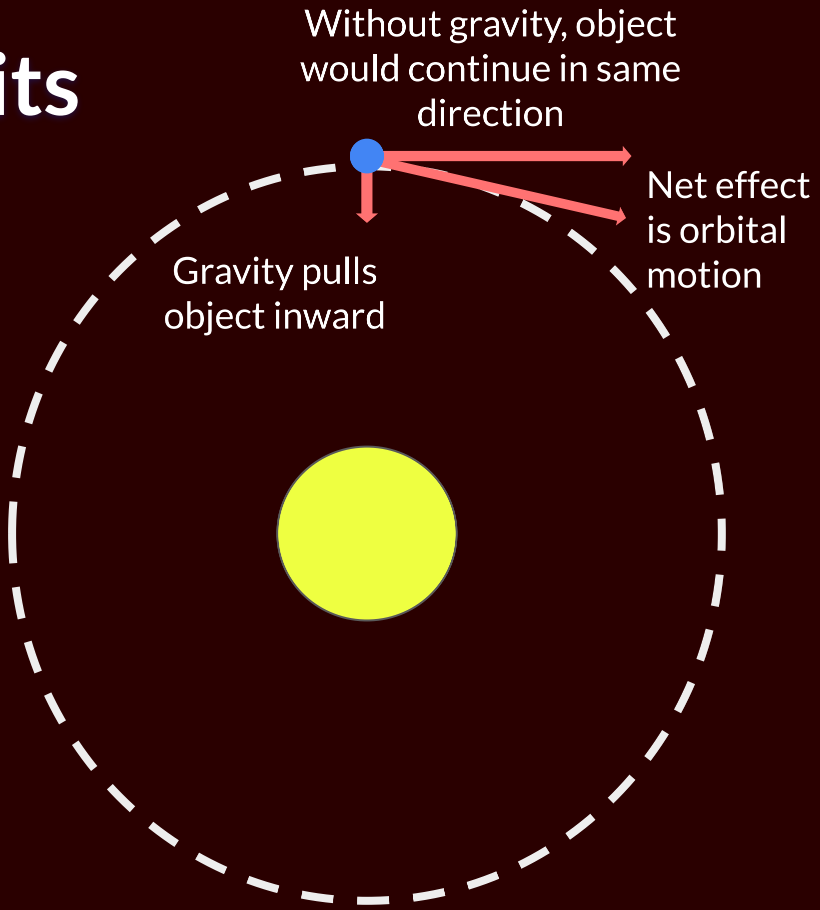
3) Together these two effects produce an orbit!



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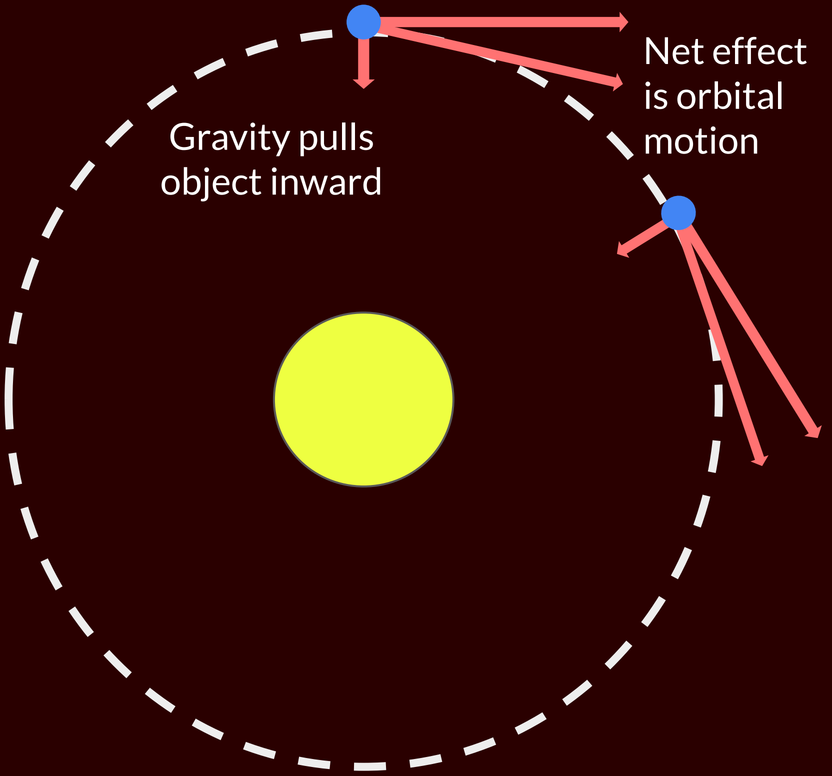

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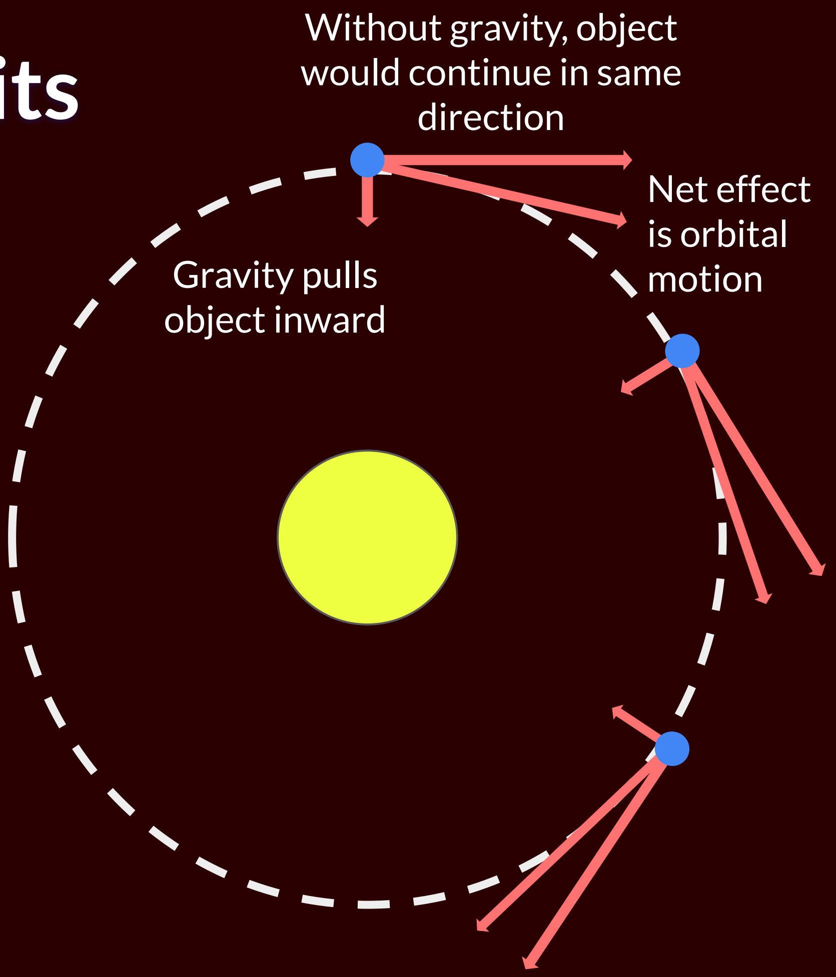
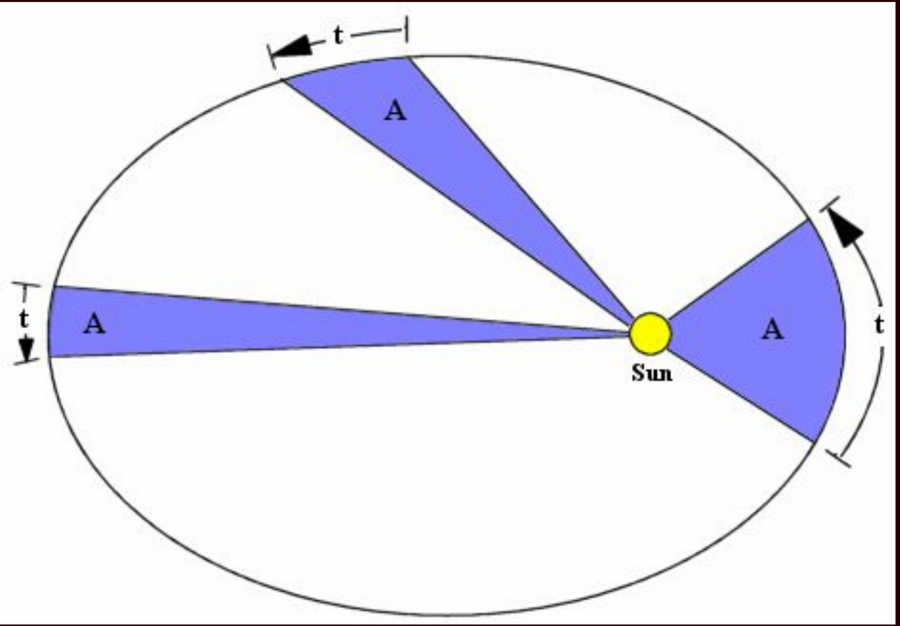


Kepler's 1st and 2nd laws

- Kepler's 1st law: planets orbit on ellipses with the Sun at one focus
- Kepler's 2nd law: planets sweep out equal areas in equal times
- Discussion question: do planets move faster at perihelion (closest to Sun) or aphelion (farthest from Sun)? Explain.



Kepler's 3rd law

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- Newton's version:

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} R^3 = k R^3 \text{ where } k = \frac{4\pi^2}{G(m_1 + m_2)}$$

Kepler's 3rd law

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- If m_1 is star's mass and m_2 is planet's mass, then to a very good approximation

$$m_1 + m_2 \approx m_1, \quad P^2 \approx \frac{4\pi^2}{Gm_1} R^3$$

Important equations for this section

$$P^2 = k\,R^3$$

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Practice Problem 2

Mercury is approximately 0.4 AU from the Sun, and Jupiter is approximately 5.2 AU from the Sun.

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c. *How would the period of Jupiter change if Jupiter's mass were halved?*

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2) $\boxed{\begin{array}{l} R_m = 0.4\ AU \\ R_j = 5.2\ AU \end{array}}$

a) $\boxed{P^2 = kR^3}$

$$P_j^2 = kR_j^3$$

$$P_m^2 = kR_m^3$$

$$\Rightarrow \frac{P_j^2}{P_m^2} = \frac{\cancel{k} R_j^3}{\cancel{k} R_m^3}$$

$$\left(\frac{P_j}{P_m}\right)^2 = \frac{R_j^3}{R_m^3}$$

$$\boxed{\frac{P_j}{P_m} = \sqrt[2]{\frac{R_j^3}{R_m^3}}}$$

b) $K = \frac{4\pi^2}{G m_{sm}}$

$$P_1^2 = K_1 R_1^3$$

$$P_1 = \sqrt{K_1 R_1^3}, \quad P_2 = \sqrt{K_2 R_1^3}$$

$$\frac{P_1}{P_2} = \frac{\sqrt{K_1 \cancel{R_1^3}}}{\sqrt{K_2 \cancel{R_1^3}}} = \sqrt{\frac{K_1}{K_2}} = \sqrt{\frac{\frac{\cancel{4\pi^2}}{\cancel{G m_{sm}}}}{\frac{\cancel{4\pi^2}}{\cancel{G}\, 4 \cancel{m_{sm}}}}} = \sqrt{\frac{1}{\frac{1}{4}}} = \sqrt{4} = \boxed{2}$$

c) Almost no change.

Since M_{sm} much > than M_j

Kepler's Laws [Demo]

https://phet.colorado.edu/sims/html/gravity-and-orbits/latest/gravity-and-orbits_en.html

Quiz Return!

Quiz

Average = Median = 32/50, Stdev = 10

These numbers were similar across other GSI sections
(slightly lower so we will curve + 3pts to everyone)

Answers in bCourses

Feedback I received:

- *Less* conceptual and *more* math
- More practice problems before quiz
- Less reading

Questions?

Attendance checkout:

See ya next time!