

Astro C10 Quiz 2 Solutions

Problem 1

- (a) We can infer the star's temperature and chemical composition from its spectrum. Circling brightness was not required for full credit, but that is also an acceptable answer since brightness appears on the vertical axis of a spectrum and we can add up (integrate) the brightness over all wavelengths to find the total brightness.
- (b) To determine the star's temperature, we can identify the peak wavelength of the star's spectrum and solve for T using Wien's law: $\lambda_{\text{peak}} T = 3 \cdot 10^6 \text{ nm K}$. To determine the star's chemical composition, we can look for absorption features in the star's spectrum. If we see absorption features characteristic of a particular element, then that element should be present in the star.
- (c) The distance d in units of parsecs can be calculated from the parallax angle p in units of arcseconds using the formula $d = \frac{1}{p}$:

$$d = \frac{1}{p} = \frac{1}{10''} = \boxed{0.1 \text{ pc}}$$

- (d) To measure the parallax of a nearby star, we need to compare its positions against the "fixed stars" (i.e., distant stars) when the Earth is on opposite sides of the Sun. It takes 12 months for the Earth to complete an orbit around the Sun, and so we'll need to measure the star's position at two times that are 6 months apart. That means it isn't possible to measure the star's parallax immediately after it is discovered.
- (e) The luminosity of a star is related to its brightness and distance by $B = \frac{L}{4\pi d^2}$, or equivalently $L = 4\pi d^2 B$. We calculated the distance from the parallax angle in part (c). Therefore, to determine the star's luminosity we'd need to measure its brightness B and then plug the brightness B and the distance d into the equation $L = 4\pi d^2 B$.

Common misconception: Many people thought of using $L = 4\pi R^2 \sigma T^4$ instead, but that will not allow us to calculate the luminosity because we don't have a way of determining the star's radius.

Problem 2

- (a) The Sun will become a white dwarf after it dies. (It will go through red giant phase and produce a planetary nebula after moving off the main-sequence, but ultimately ends up as a white dwarf.)
- (b) The main-sequence lifetime of a star is inversely proportional to the cube of the star's mass: $\ell \propto \frac{M}{L} \propto \frac{M}{M^4} = \frac{1}{M^3}$. That means if we want to increase its main-sequence lifetime by a factor of 8, we'll need to decrease its mass by a factor of 2.

Common misconception: Many people said that we should increase the star's mass by a factor of 8 because $\ell \propto \frac{M}{L}$, but we cannot change the mass while holding the luminosity fixed since $L \propto M^4$.

- (c) Any star whose mass is between roughly $0.08M_{\text{Sun}}$ and $8M_{\text{Sun}}$ will have the same fate as the Sun (i.e., will become a white dwarf). Therefore, the answer to part (a) does not change if the Sun's mass is halved.

If you did not solve part (a) and assumed the Sun's mass would need to be doubled, the answer also does not change because $2M_{\text{Sun}}$ still lies between $0.08M_{\text{Sun}}$ and $8M_{\text{Sun}}$.

- (d) The luminosity of a main-sequence star scales with its mass according to $L \propto M^4$. Therefore,

$$\frac{L_{\text{new}}}{L_{\text{old}}} = \left(\frac{M_{\text{new}}}{M_{\text{old}}} \right)^4$$

In part (b), we found that $\frac{M_{\text{new}}}{M_{\text{old}}} = \frac{1}{2}$. It follows that the ratio of Star Campbell's luminosity to the Sun's luminosity is

$$\frac{L_{\text{new}}}{L_{\text{old}}} = \left(\frac{M_{\text{new}}}{M_{\text{old}}} \right)^4 = \left(\frac{1}{2} \right)^4 = \boxed{\frac{1}{16}}$$

If you did not answer part (b) and used $\frac{M_{\text{new}}}{M_{\text{old}}} = 2$, then you should find that

$$\frac{L_{\text{new}}}{L_{\text{old}}} = \left(\frac{M_{\text{new}}}{M_{\text{old}}} \right)^4 = (2)^4 = \boxed{16}$$

- (e) The main-sequence lifetime of a star is determined by its mass. If the newly discovered star has the same mass as the Sun then it must also have the same main-sequence lifetime. Therefore, you should not believe them.