Astro C10 Quiz 1 Solutions

1 Star Athena

(a) Athena's spectrum peaks at a wavelength of $\lambda_{\text{peak}} = 300$ nm. By Wien's Law, the star's temperature must be

$$T = \frac{3.0 \times 10^6 \text{ nm K}}{300 \text{ nm}} = \boxed{10^4 \text{ K}}$$

(b) The absorption line occurs at a wavelength of $\lambda = 600$ nm. To find the energy of a photon with wavelength λ, we will need to use the equation $E = \frac{hc}{\lambda}$. Also, to ensure proper unit cancellation we must plug in the wavelength *in units of meters*.

$$E = \frac{hc}{\lambda} \Longrightarrow E = \frac{hc}{\lambda} = \frac{hc}{600 \text{ nm}} = \frac{hc}{600 \times 10^{-9} \text{ m}} = \boxed{\frac{hc}{6 \times 10^{-7} \text{ m}}}$$

(c) An electron gains energy when it absorbs a photon, so absorption causes electron to move to *higher* energy levels. Therefore, the absorption line occurs because of electrons jumping from E_1 to E_2.

(d) An absorption spectrum can be used to measure velocities using the Doppler effect. Suppose we know the wavelength λ_0 at which a particular feature in the absorption spectrum occurs when the source is at rest. If we instead observe the feature at a wavelength λ, then we can use the Doppler effect equation to solve for the velocity of the source:

$$\frac{\lambda - \lambda_0}{\lambda_0} = \frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}$$

In this equation, v is the quantity we are trying to solve for (the velocity), c is the speed of light, λ_0 is the rest wavelength, and λ is the observed wavelength.

2 Stars Apollo and Hera

(a) The problem states that the temperature of Apollo is twice the temperature of Hera. This can be expressed as $T_A = 2T_H$, where T_A denotes the temperature of Apollo and T_H denotes the temperature of Hera. To relate temperature to peak wavelength, we'll need to use Wien's Law: $\lambda_{\text{peak}} T = 3.0 \times 10^6$ nm K. Let's start by solving Wien's Law for the quantity we are interested in, λ_{peak}.

$$\lambda_{\text{peak}} T = 3.0 \times 10^6 \text{ nm K} \Longrightarrow \lambda_{\text{peak}} = \frac{3.0 \times 10^6 \text{ nm K}}{T}$$

Since we know λ_{peak} for Star Hera and are trying to find λ_{peak} for Star Apollo, we should look at the *ratio* between the two peak wavelengths:

$$\frac{\lambda_{\text{peak, A}}}{\lambda_{\text{peak, H}}} = \frac{\left(3.0 \times 10^6 \text{ nm K}\right)/T_A}{\left(3.0 \times 10^6 \text{ nm K}\right)/T_H} = \frac{1/T_A}{1/T_H} = \frac{T_H}{T_A}$$

Lastly, we can plug in $T_A = 2T_H$ and $\lambda_{\text{peak,H}} = 1000$ nm to solve for the peak wavelength of Apollo:

$$\lambda_{\text{peak,A}} = \frac{T_H}{T_A} \cdot \lambda_{\text{peak,H}} = \frac{T_H}{2T_H} \cdot 1000 \text{ nm} = \frac{1}{2} \cdot 1000 \text{ nm} = \boxed{500 \text{ nm}}$$

(b) Yes. 500 nm, or 600 nm if you did not solve part (a), falls in the visible part of the electromagnetic spectrum.

(c) To determine Star Hera's speed from the shift in peak wavelength, we will need to use the Doppler effect equation:

$$\frac{\lambda - \lambda_0}{\lambda_0} = \frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}$$

In this case, the rest wavelength is $\lambda_0 = 1000$ nm and the observed wavelength is $\lambda = 1200$ nm. Therefore, the star's velocity expressed in terms of the speed of light c is

$$\frac{v}{c} = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{1200 \text{ nm} - 1000 \text{ nm}}{1000 \text{ nm}} = \frac{200 \text{ nm}}{1000 \text{ nm}} = \frac{1}{5} \Longrightarrow \boxed{v = \frac{1}{5}c}$$

(d) Star Hera is moving away from us because the observed peak wavelength is longer than the rest wavelength. In general, a source moving away from the observer will be redshifted because the distance between successive wave crests (i.e., the wavelength) is longer than it would have been if the source were stationary.

(e) The speed of light is $c = 3 \times 10^5$ km/s. Therefore, Star Hera's speed in units of km/s is

$$v = \frac{1}{5}c = \frac{1}{5} \times \left(3 \times 10^5 \text{ km/s}\right) = \boxed{6 \times 10^4 \text{ km/s}}$$

If you used $v = \frac{1}{4}c$, then the speed in km/s should be

$$v = \frac{1}{4}c = \frac{1}{4} \times \left(3 \times 10^5 \text{ km/s}\right) = \boxed{7.5 \times 10^4 \text{ km/s}}$$

3 Element Celestium

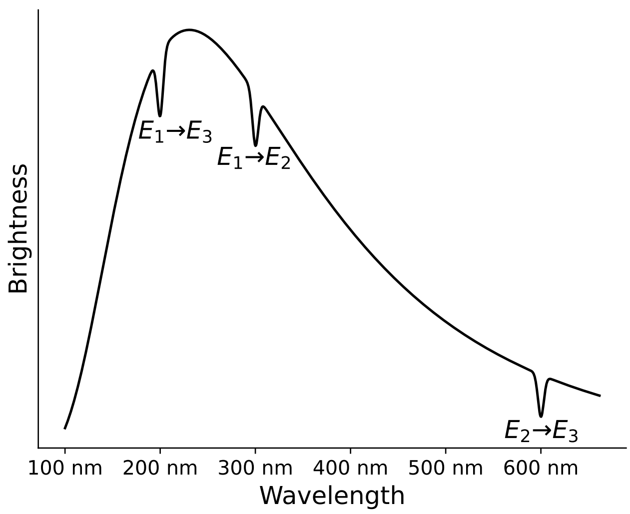
(a) There are three possible electronic transitions: $E_1 \rightarrow E_2$, $E_2 \rightarrow E_3$, and $E_1 \rightarrow E_3$. The energies of these transitions (in arbitrary units) are

$$E_{1\to2} = E_2 - E_1 = 2 - 0 = 2$$
$$E_{2\to3} = E_3 - E_2 = 3 - 2 = 1$$
$$E_{1\to3} = E_3 - E_1 = 3 - 0 = 3$$

We can use the equation $E = \frac{hc}{\lambda}$ to determine the ratios between the wavelengths of the photons needed for the three transitions. In particular,

$$\frac{\lambda_{1\to2}}{\lambda_{1\to3}} = \frac{hc/E_{1\to2}}{hc/E_{1\to3}} = \frac{E_{1\to3}}{E_{1\to2}} = \frac{3}{2} \Longrightarrow 2\lambda_{1\to2} = 3\lambda_{1\to3}$$

$$\frac{\lambda_{2\to3}}{\lambda_{1\to3}} = \frac{hc/E_{2\to3}}{hc/E_{1\to3}} = \frac{E_{1\to3}}{E_{2\to3}} = \frac{3}{1} = 3 \Longrightarrow \lambda_{2\to3} = 3\lambda_{1\to3}$$



The absorption lines in the spectrum occur at wavelengths of 200 nm, 300 nm, and 600 nm. We can ensure $2\lambda_{1\to2} = 3\lambda_{1\to3}$ and $\lambda_{2\to3} = 3\lambda_{1\to3}$ are both satisfied by choosing $\lambda_{1\to3} = 200$ nm, $\lambda_{1\to2} = 300$ nm, and $\lambda_{2\to3} = 600$ nm. The correct labeling is shown in the diagram above.

(b) The energy levels are the same as in part (a), but this time instead of looking at the ratios of wavelengths we will instead look at the ratios of frequencies. One approach is to invert the ratios from part (a) since wavelength and frequency are inversely related (e.g., $\frac{f_{1\to2}}{f_{1\to3}} = \frac{\lambda_{1\to3}}{\lambda_{1\to2}}$). Alternatively, we can use the equation $E = hf$ to solve for the ratios of frequencies directly from the energy differences:

$$\frac{f_{1\to2}}{f_{1\to3}} = \frac{E_{1\to2}/h}{E_{1\to3}/h} = \frac{E_{1\to2}}{E_{1\to3}} = \frac{2}{3} \Longrightarrow 3f_{1\to2} = 2f_{1\to3}$$

$$\frac{f_{2\to3}}{f_{1\to3}} = \frac{E_{2\to3}/h}{E_{1\to3}/h} = \frac{E_{2\to3}}{E_{1\to3}} = \frac{1}{3} \Longrightarrow 3f_{2\to3} = f_{1\to3}$$

The problem statement indicates that the frequency of the photon that must be absorbed for an electron to jump from the second energy level to the third energy level is $f_{2\to3} = f$. Using the two equations above, we can show the following:

$$f_{1\to2} = \frac{2}{3} \cdot f_{1\to3} = \frac{2}{3} \cdot 3f_{2\to3} = 2f_{2\to3} = 2f$$

This means that the frequency of the photon that must be absorbed for an electron to jump from the first energy level to the second energy level is equal to $2f$. Hence a photon with frequency $2f$ *could* be absorbed by an electron in the first energy level, and after absorbing the photon that electron would end up in the second energy level.