UC Berkeley
Astronomy Department

Astro C10 – QUIZ #2

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Name: ______________________________

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This quiz contains 3 questions and will be graded out of 50 points. The questions focus primarily on material from after the midterm, with an emphasis on content we have discussed in section. I suggest that you start by looking over all the questions to see which ones you feel most comfortable answering first, then circling back to the challenging questions at the end. **To receive full credit, you must show your work and include units.** Good luck!

Distribution of Marks

Question	Points	Score
1	20	
2	22	
3	8	
Total:	50	

Useful Constants and Equations

Constants (approximate)

$$c = 3 \times 10^8 \text{ m/s} = 3 \times 10^5 \text{ km/s}$$

$$h = 6 \times 10^{-34} \text{ J s}$$

$$1 \text{ m} = 10^9 \text{ nm} = 10^{-3} \text{ km}$$

$$1 \text{ pc} = 3 \times 10^{16} \text{ m} = 3 \text{ light-years}$$

$$1 L_\odot = 4 \times 10^{26} \text{ W}$$

$$1 R_\odot = 7 \times 10^8 \text{ m}$$

$$1 M_\odot = 2 \times 10^{30} \text{ kg}$$

$$1 T_\odot = 5 \times 10^3 \text{ K}$$

$$\sigma = 6 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{ K}^4}$$

Equations

$$c = \lambda f$$

$$E = hf = \frac{hc}{\lambda}$$

$$\lambda_{\text{peak}} \cdot T \approx 3.0 \times 10^6 \text{ nm K}$$

$$L = A\sigma T^4$$

$$\frac{v}{c} = \frac{\Delta\lambda}{\lambda_0} = \frac{\lambda - \lambda_0}{\lambda_0}$$

$$d = \frac{\text{arcsec}}{\theta} \text{pc}$$

$$B = \frac{L}{4\pi d^2}$$

$$L \propto M^4$$

$$t_{\text{life}} \propto M/L \propto 1/M^3$$

$$A = 4\pi R^2$$

1. **You want to measure the distance to a pair of stars (binary) but you can't use parallax :(However, you (a motivated Berkeley student) want to find how far it is away anyways.**

   (a) (8 points) You see that Star 1 is much bigger than Star 2 ($M_1 >> M_2$), and you notice that the stars orbit each other with a period of **2 years**. You also know that their separation is **10 AU**. What is the mass (M_1) of Star 1?

$$P^2 = KR^3$$

$$P^2 = \frac{4\pi^2}{Gm_1}R^3$$

$$m_1 = \frac{4\pi^2}{GP^2}R^3$$

$$\frac{m_1}{M_\odot} = \frac{1}{(P/P_{Earth})^2}(R/R_{earth})^3$$

$$\frac{m_1}{M_\odot} = \frac{1}{(2)^2}(10)^3$$

$$\frac{m_1}{M_\odot} = 250$$

   (b) (2 points) Knowing the mass of Star 1 (M_1) (if you didn't get part a) use $M_1 = 25M_\odot$) and knowing it is a main sequence star, how much more luminous is star 1 L_1 compared to our own sun $L_\odot$ (you can leave it as an exponent)?

$$L/L_\odot = [\frac{m_1}{M_\odot}]^4 = 250^4$$

$$L/L_\odot = [\frac{m_1}{M_\odot}]^4 = 25^4$$

   (c) (8 points) Now having calculated their luminosity, we go out and measure the of the brightest star which has a **brightness as** $\frac{B}{B_\odot} = 1/250$ (it is 250 times **LESS** bright than the sun). What is their distance to us? (you can leave it as an exponent)

$$L = 4\pi d^2 B$$

$$L/L_\odot = (d/d_\odot)^2 B/B_\odot$$

$$d/d_\odot = \sqrt{L/L_\odot \cdot B_\odot/B}$$

$$d/d_\odot = \sqrt{250^4/(1/250} = 250^{5/2}$$

$$d/d_\odot = \sqrt{25^4/(1/25)} = 25^{5/2}$$

(d) (2 points) Which star of the two stars is the brightest? And why?
The most massive star because of $L \propto M^4$

2. **In a distant world, a star has a small planet that is observed to have transited it.**

(a) (6 points) You measure a planet's transit by a star. This star has a **radius** of $R = 2R_\odot$. The brightness **dips by 1%** $= \frac{1}{100}$. What is the **radius** of this planet relative to the host star?

$$\text{dip fraction} = R^2/R_\odot^2$$

$$R/R_\odot = \sqrt{\frac{1}{100}}$$

$$R/R_\odot = \frac{1}{10}$$

(b) (8 points) The planet is made of rock. And the rock is **2× denser** than the sun (on average), what is the **mass** of the planet relative to our sun? (Remember density $= \frac{\text{mass}}{\text{volume}}$)

$$\rho = \frac{M}{V} \Rightarrow M = \rho V$$

$$\frac{M}{M_\odot} = \frac{\rho}{\rho_\odot}\frac{V}{V_\odot}$$

$$\frac{M}{M_\odot} = 2(\frac{R}{R_\odot})^3$$

$$\frac{M}{M_\odot} = 2(\frac{1}{10})^3$$

$$\frac{M}{M_\odot} = \frac{1}{500}$$

(c) (2 points) Repeat the similar process, what would be the mass of the star relative to our sun? Assume it has the same density as our sun.

$$\frac{M}{M_\odot} = (\frac{R}{R_\odot})^3$$

$$\frac{M}{M_\odot} = (2)^3 = 8$$

(d) (2 points) What is the ratio of its lifetime compared to our sun?

$$\tau \propto \frac{1}{M^3}$$

$$\tau = \frac{1}{8^3}\tau_\odot$$

(e) (4 points) What would be the ratio of their distances to their center of mass?

$$M_1 r_1 = M_2 r_2$$

$$\frac{M_s}{M_p} = \frac{r_p}{r_s}$$

$$\frac{r_p}{r_s} = \frac{M_s}{M_p}$$

$$\frac{r_p}{r_s} = \frac{8}{1/500}$$

$$\frac{r_p}{r_s} = 40,000$$

3. **Conceptual Questions**

(a) (4 points) Imagine the Earth's orbit around the sun was smaller. What would happen to the parallax angles? More specifically, **would the parallax angle of a star at a fixed distance become larger or smaller (can draw a picture)?**

For a fixed distance, a smaller orbital radius means smaller parallax angles — you'd need to measure smaller shifts in position. That makes distance measurements harder, since smaller angles are more difficult to detect precisely.

(b) (4 points) Why do more massive main-sequence stars have shorter lifetimes than less massive ones? Specifically explain the proportionality $t_{life} \propto M/L$ (hint: candle analogy from section) A star's main-sequence lifetime depends on: How much fuel it has (M), and How fast it uses up that fuel (its luminosity). Faster it burns (large L) the less it lives so it is inverse prop to L and more fuel it lives longer, directly prop to M.