Quiz 1 Solutions

"Fueling up!"

Explorer Lyra and her crew are traveling through the Andromeda sector aboard the science vessel **Aurora**. They're on the hunt for a rare fuel source: **tritium**, a heavier isotope of hydrogen that powers their fusion drives. The crew uses spectrographs to scan nebulae for tritium's signature emission lines, which are known to occur at wavelengths of **500 nm** and **600 nm**. Spotting these characteristic lines tells them whether a cloud of gas is rich in tritium and worth harvesting.

i) (4) Which emission line is more energetic? Which one has a higher frequency? No need for calculation, just explain in words why that is the case through mathematical relationships (like proportional or inversely proportionally etc.).
Ans: 500nm is more energetic because of shorter wavelengths as wavelength is inversely prop. To energy. 500nm would have higher frequency, frequency is inversely proportional to wavelength.

ii) (8) The science vessel *Aurora* is scanning a gas cloud for tritium, whose known emission lines are at **500 nm** and **600 nm (at rest)**. The ship is moving away from the cloud at a velocity 1/100th speed of light. The spectrograph detects two emission lines at **550 nm** and **660 nm (while moving)**.

Answer:
Step one: lambda_obs = lambda_emit (1+v/c)
Step two: lambda_obs/lambda_emit = (1+v/c)
Step three: lambda_obs/lambda_emit = (1+1/100)
Step four: lambda_obs/lambda_emit = (1.01)
Step five: check…
550/500 = 1.10
660/600 = 1.10
Conclusion, the ratios are very different from each other and is NOT the right cloud move on

iii) (2) If we are now moving **Towards** from the cloud in search of more tritium, what will happen to these lines? Why is this? No need for math, just explain in words.

Answer: The lines will **Blueshift** (move to shorter wavelengths). Physically this happens because relative motion toward compresses the electromagnetic waves between emission and detection, increasing their frequency and decreasing wavelength (Doppler effect). Higher frequency → higher photon energy.

iv) (2) If *Aurora* is moving at velocity v, how fast is the light from the emission lines moving as they reach the starship *Aurora*?

Answer: The light always arrives at speed c (the speed of light in vacuum) relative to Aurora, regardless of Janeway's velocity or the source's velocity. This is a postulate/result of special relativity: c is invariant.

Vacation time!

You've booked a vacation trip to the exoplanet **Kepler-16b**, which orbits a striking pair of suns. The two stars are named **Aurelia** and **Borealis**. While both stars emit the same total luminosity, **Borealis has a radius four times smaller than Aurelia**.

i) (4) Which star is hotter, **Aurelia** or **Borealis**? Without doing the calculations, why is that?

Answer: Borealis is hotter.

Both stars have the **same luminosity** L.
Luminosity of a star: L=4πR2σT^4L = 4\pi R^{2}\sigma T^{4}L=4πR2σT4.
For the same L, if R is smaller, then T must be **larger** to compensate.
Since Borealis's radius is 4× smaller, it must have a much higher temperature.

ii) (8) By how much is one star hotter than the other? Make sure to show your work.

Answer:

Step 1: Luminosity formula:

$$L = 4\pi R^2 \sigma T^4$$

Step 2: Equating luminosities for both stars:

$$R_1^2 T_1^4 = R_2^2 T_2^4$$

Step 3: Solve for temperature ratio:

$$\left(\frac{T_2}{T_1}\right)^4 = \left(\frac{R_1}{R_2}\right)^2$$

Step 4: Given $R_2 = \frac{1}{4}R_1$,

$$(\frac{R_2}{R_1})^2 = (\frac{R_2}{\frac{1}{4}R_1})^2 = (4)^2 = 16$$

So,

$$(\frac{T_2}{T_1})^4 = 16 \Rightarrow T_1 = 2$$

Final Answer: Borealis is **twice as hot** as Aurelia.

iii) (8) If the thermal radiation of Borealis peaks at a wavelength of 400 nm, what wavelength does Aurelia peak at? Which star would then appear redder, and which would appear to be bluer?

Answer

Step 1: Wien's Law:

$$\lambda_{\text{peak}} T = b$$

Step 2: Ratio form:

$$\frac{\lambda_1}{\lambda_2} = \frac{T_2}{T_1}$$

Step 3: From part (ii), $\frac{T_2}{T_1} = 2$

So,

$$\frac{\lambda_1}{400 \text{ nm}} = 2 \quad \Rightarrow \quad \lambda_1 = 800 \text{ nm}$$

Step 4: Compare colors:

- Aurelia peaks at **800 nm** (infrared, redder).
- Borealis peaks at **400 nm** (violet-blue).

Final Answer: Aurelia would appear **redder**, Borealis **bluer**.

vi) (4) Draw a rough estimate of the blackbody diagram of these two stars below using your previous answers. Assume Tatoo I has a brightness of 4 units. Make sure to label the x-axis position of your peaks, as well as which star is which!

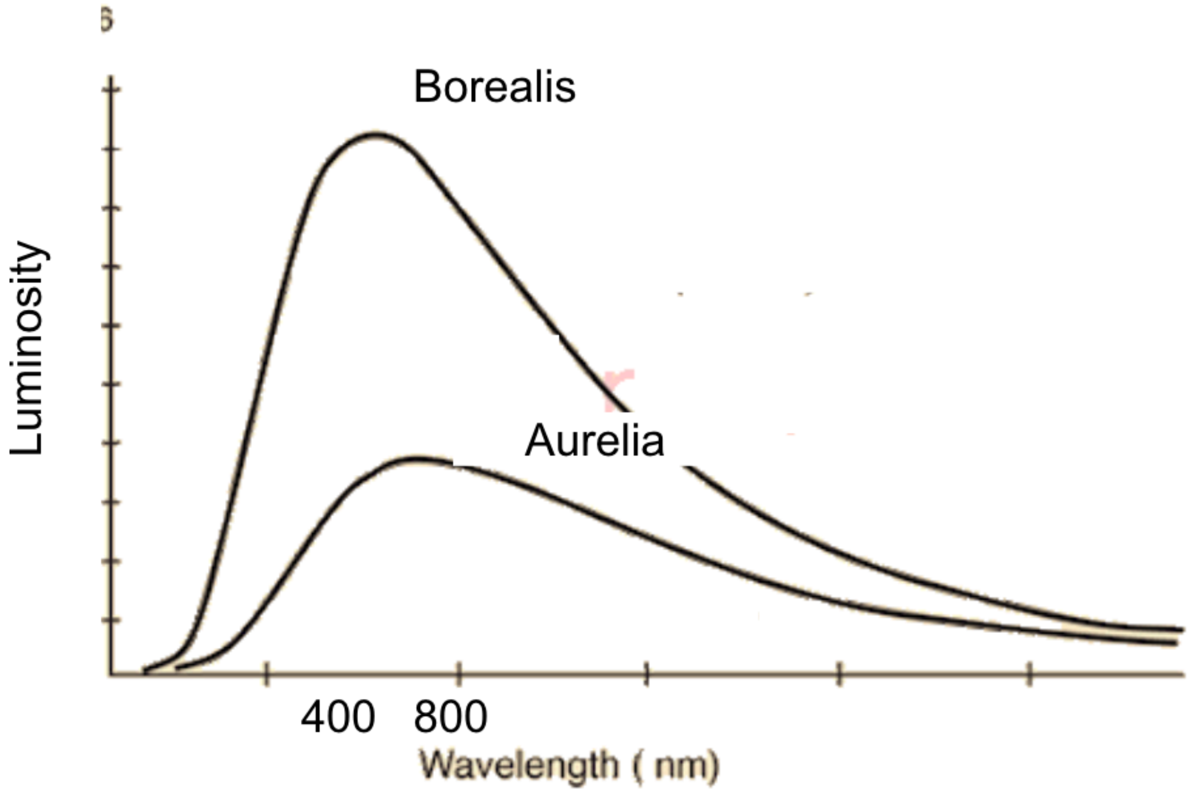
Answer

What we know from earlier parts:

- **Aurelia**: T1T_1T1, cooler, peak wavelength at **800 nm**, brightness normalized to **4 units** (given).
- **Borealis**: T2=2T1T_2 = 2T_1T2=2T1, hotter, peak wavelength at **400 nm**, same total luminosity, but since it's hotter the curve is **taller and narrower** than **Aurelia**s (to give same integrated area).

Sketch description:

- X-axis: Wavelength (nm), labeled from ~200 to 1000 nm.
- Y-axis: Intensity (arbitrary units).
- Curve for **Aurelia** peaks at **800 nm**, height about **4 units**.
- Curve for Borealis peaks at **400 nm**, taller than 4 units (to keep total luminosity equal but shifted blueward).
- Label each peak with star name and wavelength:
    - Aurelia → redder, peak at 800 nm.
    - Borealis → bluer, peak at 400 nm.



Observing time!

You bring a friend to an observatory that houses three telescopes, each tuned to a different part of the spectrum: one designed for X-rays, one for radio waves, and one for visible light.

i) (2) While peering through the optical telescope, your friend notices a star positioned near Saturn in the eyepiece. They observe that the star appears to twinkle, whereas Saturn does not, and ask you to explain the reason.

Answer:
Stars appear as **point sources** of light. When their light passes through Earth's turbulent atmosphere, the refraction shifts quickly, so the light seems to flicker — this is twinkling.
Jupiter, on the other hand, has an **apparent disk** (finite angular size), so the turbulence averages out across its larger surface → no visible twinkling.

ii) (6) Assume that all three telescopes have the same angular resolution. Rank the telescopes in increasing order of how big they would have to be, and explain your reasoning.

Key idea: Angular resolution depends on wavelength and aperture size, roughly

$$\theta \sim \frac{\lambda}{D}$$

For the **same resolution** (θ fixed), larger λ→ larger required aperture D.

Step 1 — compare wavelengths:

- Radio: longest wavelength (cm–m).
- Infrared: intermediate (μm).
- Optical: shortest (~hundreds of nm).

Step 2 — increasing order of required telescope diameter:

[smallest] Xray < Optical < Radio [largest]

Answer: Optical telescope would be smallest, infrared bigger, radio telescope largest.

(c) (1 point) What is unrealistic about this scenario in this question?

Key idea: X-rays can't pass through Earth's atmosphere!