SIR Investigation

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Introduction

The aim of this project is to examine the different factors that affect the spread of infectious diseases. Specifically, I will examine transmission rate, recovery time, population size, and incubation period to determine how they affect the spread of a disease. Additionally, I examine potential responses and how they could potentially slow down the spread of the disease. This investigation mainly focuses on the percentage of the infected that quarantine, how many attendants come into contact with quarantined individuals, and masking to determine if they would be effective options to combat different infectious diseases. Because much of the numerical investigations rely on Euler's method for simulations, I utilized the programming language python and specifically the library Matplotlib to help generate and visualize data. All of the code can be found in this GitHub repository. There are three general models: a basic model, a quarantine model, and an incubation period model in increasing complexity. This document details my numerical and analytical investigations with these models.

Basic SIR Modeling

In this project, a compartment model is used to simulate the spread of a disease in a population. We will use the three general groups of general groups of susceptible, infected, and recovered. The assumption is that people in the recovered group are either dead or immune, so they cannot be infected twice. In the basic model, the only parameters we will tweak are the population size, the transmission rate of the disease, and the recovery rate of those who have been infected.

Definition of Variables

N = Total Population

S = Susceptible Population

I = Infected Population

R = Recovered Population

α - transmission rate

β - recover rate

Differential Equation Setup

$$\frac{dS}{dt} = -\alpha \bullet S \bullet I$$

$$\frac{dI}{dt} = \alpha \bullet S \bullet I - \beta \bullet I$$

$$\frac{dR}{dt} = \beta \bullet I$$

Numerical Investigation

Using Euler's Method to Simulate the Disease Spread

Because we only have differential equations, we must use Euler's method to simulate the spread of the disease. Using python, we can utilize Matplotlib to graph the results and visualize this process. Starting with a base simulation with parameters as follows, we will run Euler's method.

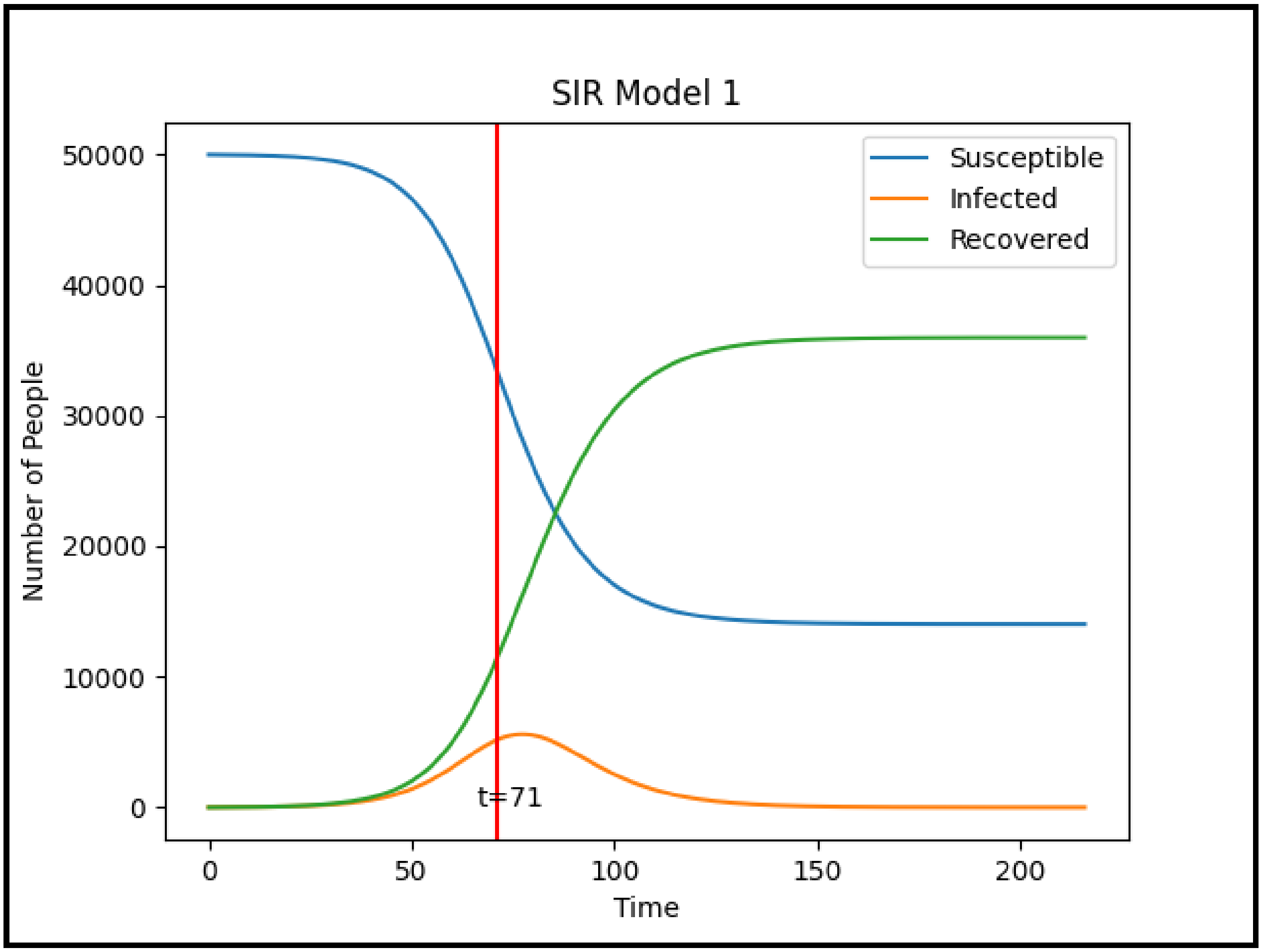


Figure 1: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_0 = 10, $\alpha = 5*10^{-6}$, = 1/7, Δt = 1)

Base Simulation:

- Total infected: 35975
- Maximum ill at once: 5596 (at t=77)
- Time of fastest spread: 71 (862.71 p/t)
- Total lifespan of disease: 216

Altering the Initial Population Size

The first parameter we will examine is the initial population size. Euler's Method Simulations with a smaller initial susceptible population yield a slower spreading disease which peeters out quickly. This is because there aren't enough people who continue to get infected, which limits the spread. This results in a disease which is spreading the fastest at the very beginning (t=0), before the infected population begins to recover which even further decreases transmission. In terms of the differential equations, there is always a lower S value which decreases the value of $\frac{dI}{dt}$. The simulations with a starting populations of 20,000, 10,000, and 1,000 show this pattern.

Simulation A: N = 20,000

- Total infected: 32
- Maximum ill at once: 10
- Time of fastest spread: 0
- Total lifespan of disease: 69

Simulation B: N = 10,000

- Total infected: 15
- Maximum ill at once: 10
- Time of fastest spread: 0
- Total lifespan of disease: 31

Simulation C: N = 1,000

- Total infected: 10
- Maximum ill at once: 10
- Time of fastest spread: 0
- Total lifespan of disease: 21

Altering Recovery Time

The second parameter we will examine is the recovery time. Shorter recovery times means there will consistently be a lower percentage of the population infected, so there will be less people spreading the disease. In the differential equations, this represents a larger $\frac{dR}{dt}$ value, and thus a lower $\frac{dI}{dt}$ value. So, for example, a treatment which halves recovery time would be effective to slow the spread of the illness because the infected people recover faster than susceptible people become infected. This results in a disease which is spreading the fastest and has the largest infected population at the very beginning (t=0), before the infected population begins to recover. It also means overall, less people will become infected with the disease. The simulation, returning to N = 50,000, backs this up.

Simulation A: = 2/7

- Total Infected: 76
- Maximum ill at once: 10
- Time of fastest spread: 0
- Total lifespan of disease: 82

Altering Transmission Rate

The last parameter of the basic model we will alter is the transmission rate. The transmission rate is important because it determines how many of the susceptible population will become infected. In terms of the differential equations, a higher α value would decrease $\frac{dS}{dt}$ and increase $\frac{dI}{dt}$. Interestingly, it produces the same number of total infected as part 4, just in twice the total time. This is likely because a lower transmission rate means it takes longer for the disease to spread, whereas increasing the rate of recovery means that there are less people to spread the disease without affecting the time. This can be seen in simulations.

Simulation A: $\alpha = 2.5*10^{-6}$

- Total Infected: 76
- Maximum ill at once: 10
- Time of fastest spread: 0
- Total lifespan of disease: 165

Simulation B: $\alpha = 1.0*10^{-5}$

- Total Infected: 48,688
- Maximum ill at once: 19013 (at time t=30)
- Time of fastest spread: 26 (3926.62 p/t)
- Total lifespan of disease: 111

(graph on next page)

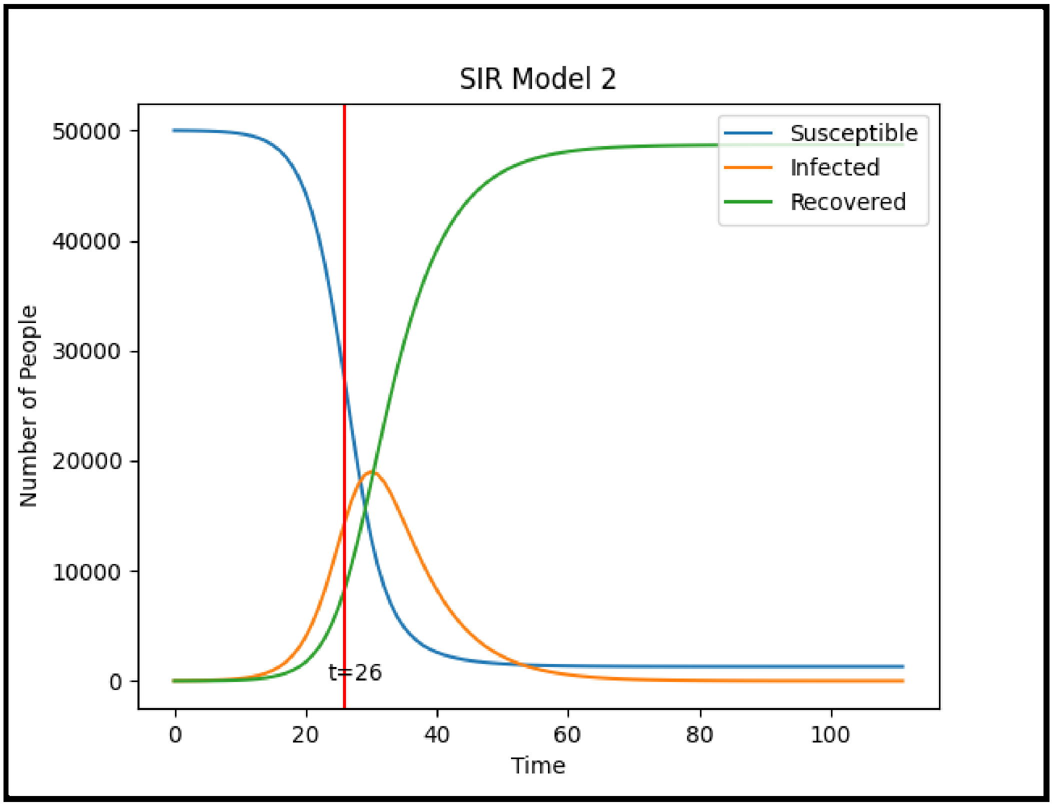


Figure 2: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_0 = 10, $\alpha = 1*10^{-5}$, $= 1/7$, $\Delta t = 1$)

One method that can be used to decrease the transmission rate of diseases is masks. They can block airborne particles which otherwise would transfer the disease from one person to another. Simulation C utilizes standard masks which decrease the transmission of airborne particles by 13%. Assuming this is the primary method of transmission, we will simulate using a 13% decrease in transmission. As can be seen below, these were not effective in significantly altering the transmission as the vast majority of the population was still infected.

Simulation C: $\alpha = 8.7*10^{-6}$

- Total Infected: 47,617
- Maximum ill at once: 16189 (at time t=35)
- Time of fastest spread: 31 (3102.24 p/t)
- Total lifespan of disease: 121

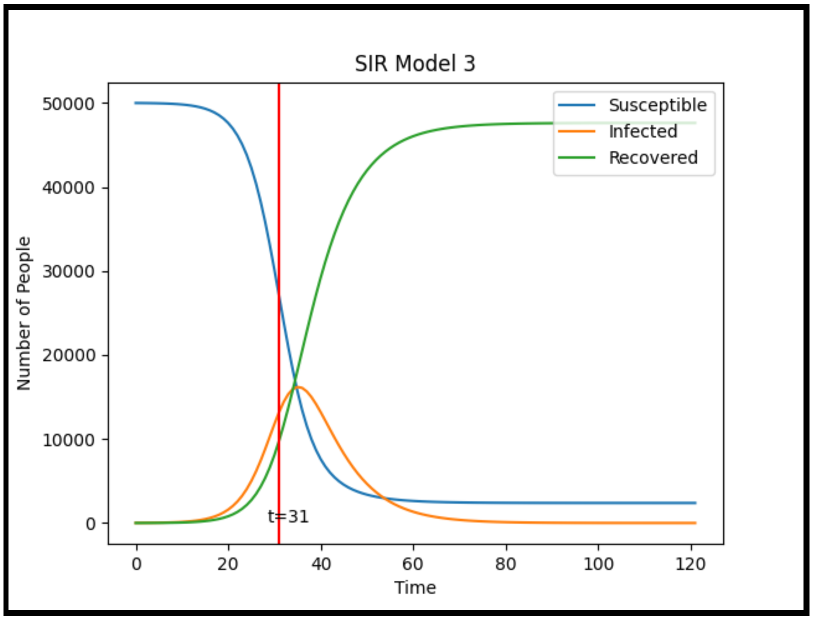


Figure 3: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_0 = 10, ɑ = $8.7*10^{-6}$, = 1/7, Δt = 1)

Simulation D utilizes N-95 masks which decrease the transmission of airborne particles by 98%. Assuming this is the primary method of transmission, we will simulate using a 98% decrease in transmission. These were very effective in stopping the spread quickly because they greatly decreased the transmission rate. However, a caveat with both mask simulations is that they assume every person wears a mask at all times, and they assume that airborne particles are the only method of transmission.

Simulation D: ɑ = $2.0*10^{-7}$

- Total Infected: 11
- Maximum ill at once: 10
- Time of fastest spread: 0
- Total lifespan of disease: 22

Now we can examine the required transmission rate needed to ensure half of the population never gets infected. Because the simulations happen relatively quickly, we can just input experimental values for ɑ to determine the value. After trial and error, we found the closest value to the nearest thousandth where less than half the population is infected. The result, to the nearest thousandth, was ɑ = $3.948*10^{-6}$. Details are shown below.

Simulation E: $\alpha = 3.948*10^{-6}$

- Total Infected: 24992
- Maximum ill at once: 2154 (at time t=129)
- Time of fastest spread: 122 (315.46 p/t)
- Total lifespan of disease: 343

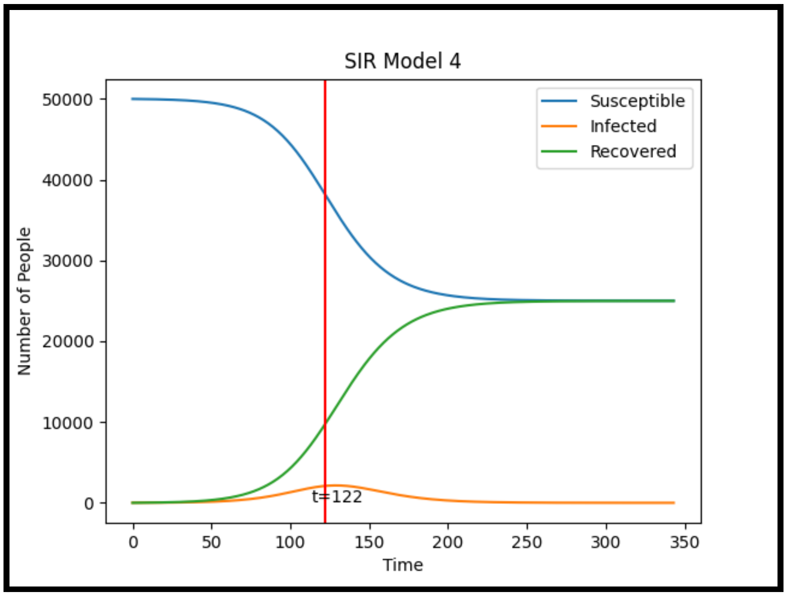


Figure 4: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_0 = 10, $\alpha = 3.948*10^{-6}$, = 1/7, $\Delta t = 1$)

Analytical Investigation

Logical Analysis

Based on the differential equation $\frac{dI}{dt}$, $I(t)$ will be increasing when the transmission rate multiplied by the number of susceptible people is greater than the recovery rate. This gives us insight about how previous diseases were able to spread so quickly. For example, we can deduce that the spread of the H1N1 flu happened quicker than people were recovering, which is why the number of infected continued to rise until the transmission rates decreased because of things like vaccines and pandemic control measures. Because of this, quarantining would likely be effective against the flu because it is one of the best ways to limit the transmission rate. If the infected don't come into contact with any of the susceptible population, then the virus would spread slower than people recovered which decreases the number of infected.

Deducing other important functions

Finding $\frac{dI}{dS}$, $\frac{d^2 I}{dS^2}$, and $I(S)$

$\frac{dS}{dt} = \; - \alpha \bullet S \bullet I$

$\frac{dI}{dt} = \alpha \bullet S \bullet I \; - \; \beta \bullet I$

$\frac{dI}{dS} = \frac{\alpha \bullet S - \beta}{-\alpha \bullet S}$

$\frac{dI}{dS} = \frac{\beta}{\alpha \bullet S} - 1$

$\frac{dI}{dS} = \frac{\beta}{\alpha} \bullet \frac{1}{S} - 1$

$dI = (\frac{\beta}{\alpha} \bullet \frac{1}{S} - 1)dS$

$I(S) \; = \frac{\beta}{\alpha} ln(S) \; - \; S \; + \; C$

$I(S) \; = \frac{\beta}{\alpha} ln(S) \; - \; S \; + \; C$

$I_0 = \frac{\beta}{\alpha} \ln(S_0) - S_0 + C$

$C = I_0 + S_0 - \frac{\beta}{\alpha} \ln(S_0)$

$I(S) \; = \frac{\beta}{\alpha} ln(S) \; - \; S \; + I_0 + S_0 - \frac{\beta}{\alpha} \ln(S_0)$

$I(S) \; = \frac{\beta}{\alpha} ln(\frac{S}{S_0}) \; - \; S \; + \; N$

Important equations:

$\frac{dI}{dS} = \frac{\beta}{\alpha} \bullet \frac{1}{S} - 1$

$\frac{d^2 I}{dS^2} = -\frac{\beta}{\alpha} \bullet \frac{1}{S^2}$

$I(S) \; = \frac{\beta}{\alpha} ln(\frac{S}{S_0}) \; - \; S \; + \; N$

The $I(S)$ function is especially important because it gives us a way to determine the exact values of the infected population based on the susceptible population, so we don’t have to rely on Euler’s method which will be slightly inaccurate.

Finding when the infected population starts to decrease

$\frac{dI}{dS} = \frac{\beta}{\alpha} \bullet \frac{1}{S} - 1$

$0 \; = \frac{\beta}{\alpha} \bullet \frac{1}{S} - 1$

$1 \; = \frac{\beta}{\alpha} \bullet \frac{1}{S}$

$S \; = \frac{\beta}{\alpha}$

This is consistent with the answer based on the differential equation $\frac{dI}{dt}$ described in the logical analysis. While the susceptible population is greater than the recovery rate divided by the transmission rate, the infected population is increasing. When the two are equal, the infected population stays constant. When the susceptible population is less than the recovery rate divided by the transmission rate, the infected population decreases.

Finding the maximum number of infected people at any given time

$$S = \frac{\beta}{\alpha}$$

$$I_{max} = \frac{\beta}{\alpha} ln(\frac{\beta}{\alpha \bullet S_0}) - \frac{\beta}{\alpha} + \text{N}$$

Because we know the infected population starts to decrease when S is less than $\frac{\beta}{\alpha}$, we can plug this value into our I(S) function to find the maximum number of infected. This general result is consistent with the experimental data from Euler's method during numerical analysis. In the base simulation in part 1, the expected maximum illness value was 5445, while the experimental value was 5596. This yields a proportion of 5445 infected to 50,000 total people, meaning 10.89% of the population was infected. In simulation B in part 4 the expected value was 17821 while the experimental value was 19013. In simulation C in part 4 the expected value was 15299 while the experimental value was 16189. In simulation E in part 4 the expected value was 2121 while the experimental value was 2154. All of these values are close to each other, and the difference between the expected and experimental values are on account of the error in Euler's method. If Δt were to be decreased to a smaller value, then the expected and experimental values would be closer to each other. As Δt approaches 0, the two are equal.

How many people never contracted the flu when the pandemic ends?

When the pandemic is over, we know that the number of infected people is equal to 0 ($I(S) = 0$). Thus, we can write a function in python to loop through each possible value, plug it into our I(S) function, and find when it is equal to 0. The specific function is shown below.

```python
def calc_total_not_infected() -> int:
    """
    Loops through all possible values of the susceptible
    population and finds when I(S) is equal to 0.
    :return: the value of S for which I(S) equals 0.
    """
    for sus in range(N).__reversed__():
        if round(calc_infected(sus)) == 0:
            return sus
    return 0
```

Implementing Quarantines

A common tactic used to slow the spread of diseases is quarantining those who become infected. This lets them have less contact with the susceptible population, which means the less new people become infected. Instead of having to factor in the infected populations' contact with the entire susceptible population, we only have to take into account their contact with a specific number of attendants who they have contact with. Furthermore, we should factor in that not all of the infected population will quarantine because that is unrealistic. Instead, it will likely only be a given percentage. So, we will need a few more constants: average attendants per infected and percentage of population that quarantines. In our differential equations, this means $\frac{dS}{dt}$ and $\frac{dI}{dt}$ must be changed. To do this, we can calculate the total number of attendants to the infected, and assuming that whether someone is an attendant is independent of whether they are susceptible or infected, the proportion of those that are susceptible. Then we factor in the number of infected who aren't quarantined and how much they spread the disease to have our completed equations. We will also implement deaths from disease into this model with an additional constant of a death rate.

Definition of Variables

N = Total Population
S = Susceptible Population
I = Infected Population
R = Recovered Population
T = Dead Population
α - transmission rate
β - recover rate
f - death rate
a - average attendants in contact with each quarantined infected person
q - percentage of population that quarantines

Differential Equation Setup

$$\frac{dS}{dt} = -\alpha \bullet S \bullet (1 - (1 - (\frac{a}{S+R}))^{q \bullet I}) \bullet I - a \bullet S \bullet I \bullet (1 - q)$$
$$\frac{dI}{dt} = \frac{dS}{dt} - \beta \bullet I$$
$$\frac{dR}{dt} = (1 - f) \bullet \beta \bullet I$$
$$\frac{dT}{dt} = f \bullet \beta \bullet I$$

Implementing Incubation Periods

We will also factor a potential incubation period of the disease as well as quarantines. This means that there is a set period of time that people remain asymptomatic before they develop symptoms. However, they can still spread the disease during this time. So, in this model, we need to differentiate between the symptomatic and asymptomatic population. We also need an additional constant, for the average rate at which people become symptomatic (1/incubation period). Lastly, we need to tweak our differential equations because asymptomatic people are unlikely to be quarantined, as they have not been identified as infected.

Definition of Variables

N = Total Population
S = Susceptible Population
I_a = Asymptomatic Infected Population
I_s = Symptomatic Infected Population
R = Recovered Population
T = Dead Population
α - transmission rate
β - recover rate
f - death rate
a - average attendants in contact with each quarantined infected person
q - percentage of population that quarantines
c - symptom development rate

Differential Equation Setup

$$\frac{dS}{dt} = -\alpha \bullet S \bullet (1 - (1 - (\frac{a}{S+R}))^{q \bullet I_s}) \bullet I_s - a \bullet S \bullet (I_s \bullet (1 - q) + I_a)$$

$$\frac{dI_a}{dt} = \frac{dS}{dt} - c \bullet I_a$$

$$\frac{dI_s}{dt} = c \bullet I_a - \beta \bullet I_s$$

$$\frac{dR}{dt} = (1 - f) \bullet \beta \bullet I_s$$

$$\frac{dT}{dt} = f \bullet \beta \bullet I_s$$

Numerical Investigations

Altering fraction of people quarantined (q):

Using a base transmission rate of $5 \bullet 10^{-6}$, recovery rate of 1/7, symptom development rate of 1/3, and attendants of 5, we can tweak the values of q to determine the effectiveness of quarantining different percentages of the population. As can be seen in the graph below, as well as the simulations in Appendix A, quarantining can be a very effective way to slow the spread of the disease. However, it will only have a limited impact until a larger percentage of the population quarantines. Until at least half of the population is quarantining, there is no significant change in how many become infected. However, once around 85% of the population quarantines, the spread will be largely contained to a small group of people.

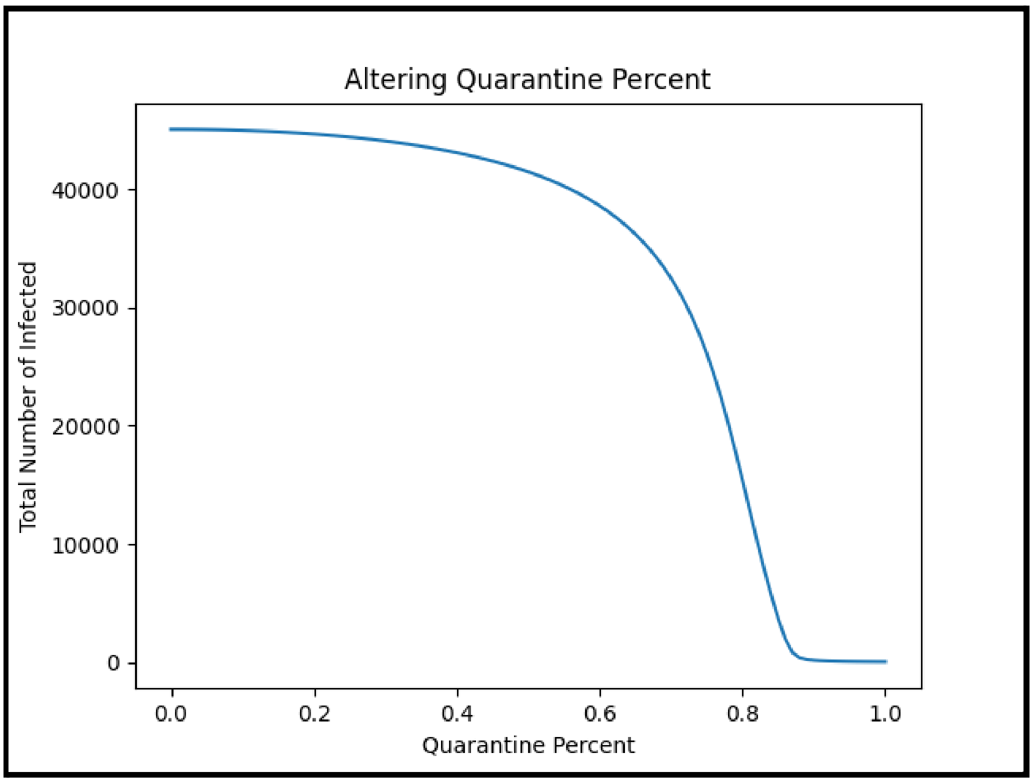


Figure 22: The effect of altering quarantine percent on total number of infected

Altering Number of Attendants (a)

Another important factor in determining the effectiveness of a quarantine is how many attendants come into contact with an infected person, on average. The more contact a quarantined individual has with others, the more likely they are to spread the disease. For these simulations, we will return to a baseline of 75% of the population quarantining.

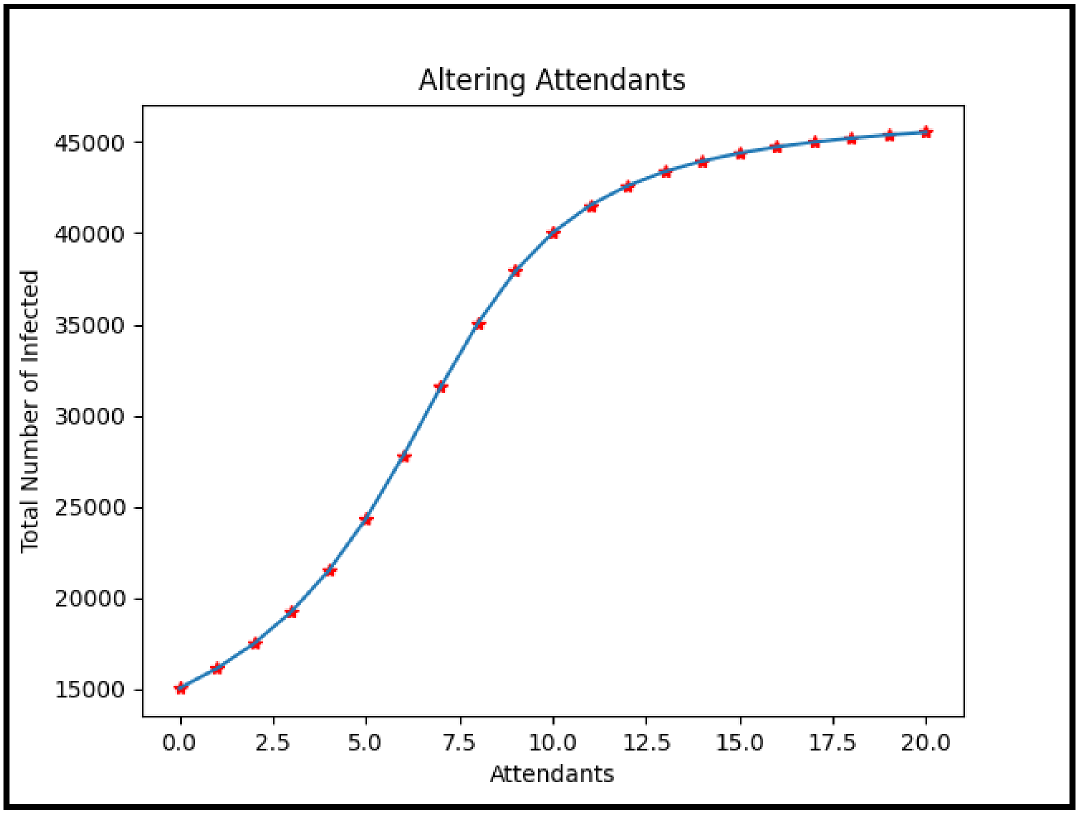


Figure 15: Visualization of the effect of attendants on the population that becomes infected

The figure above as well as other simulations shown in Appendix B reveal several interesting findings. First, even if quarantined individuals have no outside contact, a significant portion of the population still becomes infected. This is both because the incubation period means the infected can spread disease before quarantining, and because 25% of the infected never quarantine. We also see that quarantining is most effective when the number of attendants is kept low, ideally under 2 or 3. After that any additional attendants means the disease will spread as a whole much more quickly, until quarantining has almost no effect when there are more than 10 attendants on average.

Altering Incubation period

The last parameter to be examined is the incubation period of the disease. Logically, a longer incubation period should mean the disease spreads faster and to more people. This is because there will be more time for asymptomatic infected people to have contact with the broader population before they realize they need to quarantine. For these simulations, we will return to 5 attendants and move to 70% of the population quarantining.

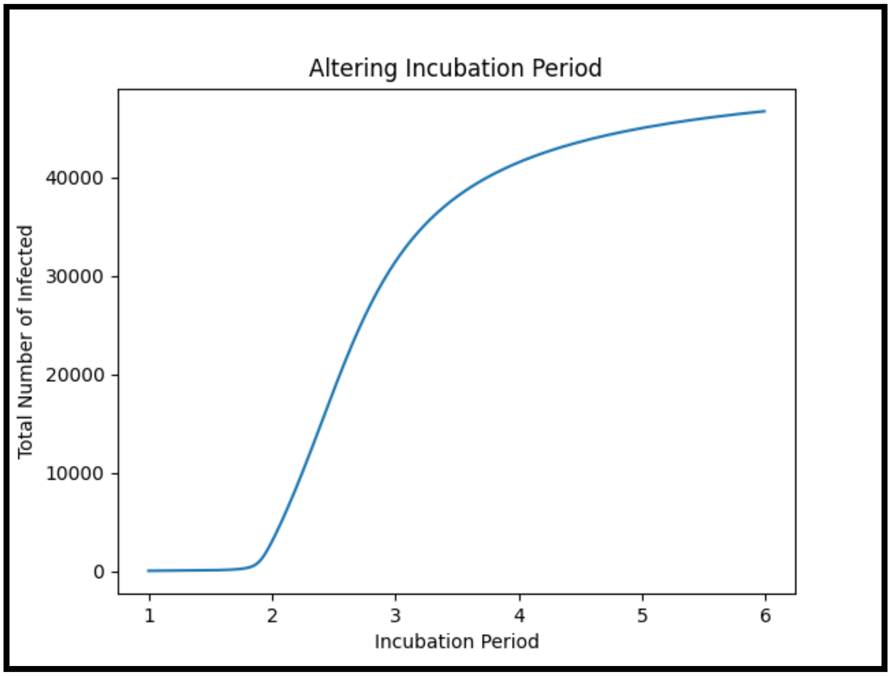


Figure 21: The effect of altering the incubation period on the total infected population

Figure 21 above as well as simulations in Appendix C show that incubation period has a drastically large effect on the spread of the disease. Even small increases in the incubation period can result in a much larger portion of the population becoming infected. For example, at an incubation period of 2 (c = ½), 3,012 become infected whereas at 3 (c = ⅓), the total infected population is 31,458. This has a few implications for disease prevention as a whole. First, it means that the incubation period is likely one of the deadliest aspects of a disease, so strains with higher incubation periods should be a focus for organizations. Second, it means that widespread preventive measures which target the entire population will often be necessary when the incubation period is high, because it might be too late to quarantine. This would lend itself to options such as frequent handwashing and hygiene practices, widespread masking, or even national lockdowns in serious cases. Overall, it seems that altering the incubation period had one of the largest effects of all the different parameters, save perhaps the transmission rate.

Further Explorations

Implementing Masking

Especially during the COVID pandemic over the past few years, masking has arisen as one of the primary ways to combat airborne infections diseases. Specifically, they aim to block the transmission of particles from the mouth and nose, effectively lowering the transmission rates. In this section, we examine the efficacy of utilizing different quality masks, as well as the demographic of the population that is masking. Simulation A is a baseline with no masking, Simulation B has only the quarantined population wearing highly effective masks which reduce transmission by 97% for their attendants, and Simulation C is the same as B except the rest of the general population also wears cheaper, less effective masks which lower transmission by 50%.

Simulation A (No Masking):

- Total Infected: 31458
- Maximum ill at once: 5129 (at time t=140)
- Time of fastest spread: 133 (535.99 p/t)
- Total lifespan of disease: 260

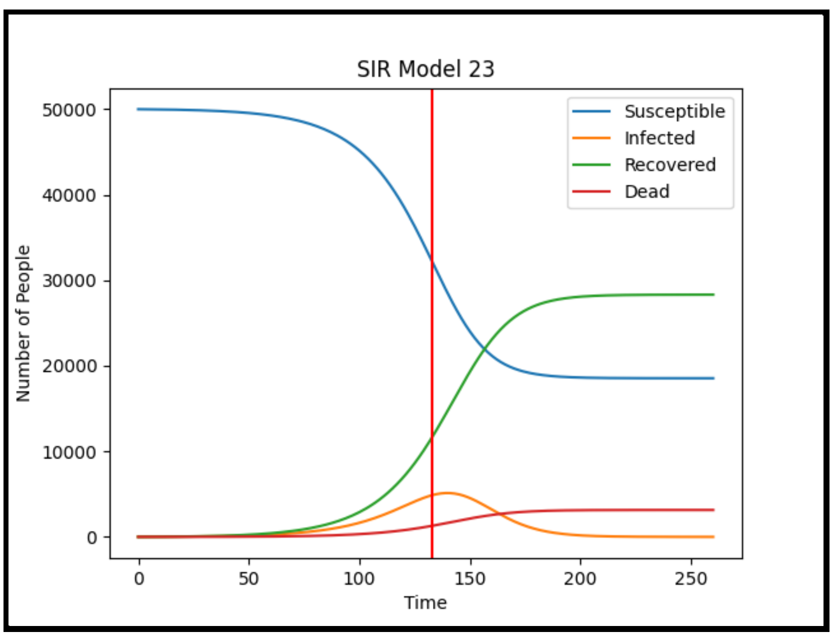


Figure 23: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_{a0} = 10, $\alpha = 5*10^{-6}$, = 1/7, *f=0.1*, Δt = 1, a = 5, q = 0.7, c = 1/3)

Simulation B (Quarantined only wear effective masks that reduce ɑ by 97%):

- Total Infected: 20239
- Maximum ill at once: 2217 (at time t=136)
- Time of fastest spread: 129 (226.46 p/t)
- Total lifespan of disease: 357

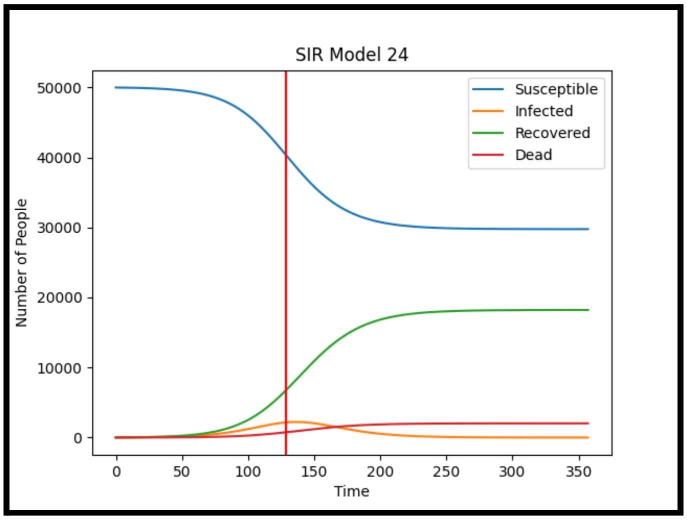


Figure 24: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_{a0} = 10, ɑ = $5*10^{-6}$, = 1/7, *f=0.1*, Δt = 1, a = 5, q = 0.7, c = 1/3)

Simulation C (Quarantined wear highly effective masks that reduce ɑ by 97%, general public wears cheap masks that reduce ɑ by 50%):

- Total Infected: 27
- Maximum ill at once: 12 (at time t=3)
- Time of fastest spread: 0 (1.25 p/t)
- Total lifespan of disease: 48

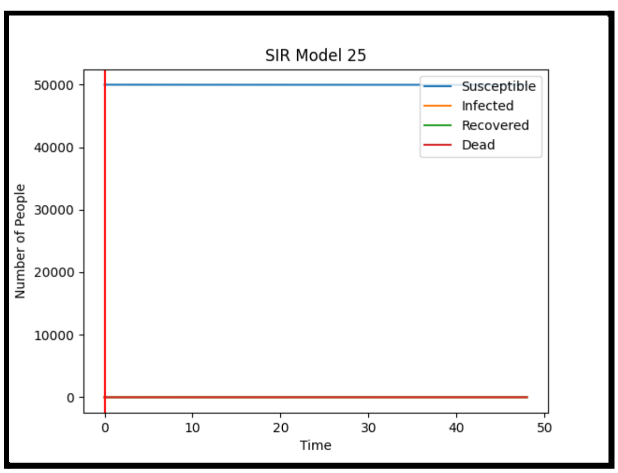


Figure 25: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_{a0} = 10, ɑ = $5*10^{-6}$, = 1/7, *f=0.1*, Δt = 1, a = 5, q = 0.7, c = 1/3)

As can be seen in both simulations B and C, masks can definitely be an effective option to decrease transmission. When just those who are quarantined and their attendants wear masks, transmission is already greatly decreased. However, because of the incubation period as well as the portion of the infected who don't quarantine, the disease still spreads to a decent amount of the population. Thus, the most effective option is to have the general population also wear masks when in public, to reduce any possible transmission amongst the public. Indeed, in simulation C, we see the disease stops almost before it begins, only spreading to 48 total people from the 10 who were originally infected. However, to avoid being too idealistic, there are of course barriers to widespread masking. First, the cost can add up to having an entire population mask so it isn't necessarily always feasible, depending on the resources of the individuals and their government. Second, this model assumes complete compliance with masking, which is not realistic, especially as we've seen in the past few years with COVID. However, if possible, widespread masking early in the disease's lifespan can greatly limit the number who become infected.

Quarantining people as groups

The last potential solution to examine is quarantining in groups. For example, the infected could be quarantined together in a hospital wing or infirmary. The idea of this solution is to limit the total number of attendants needed to care for the infected, which would limit the overall transmission. However, we must also take into account that these new attendants will have a greater exposure to the infected population, because each attendant will be caring for a greater number of infected people. Ultimately, this balance should determine how effective group quarantining as a strategy is. For these simulations, we return to assuming no masking as well 70% quarantine compliance, and an incubation period of 3.

Simulation A (5 attendants, no increased exposure for attendants):

- Total Infected: 31458
- Maximum ill at once: 5129 (at time t=140)
- Time of fastest spread: 133 (535.99 p/t)
- Total lifespan of disease: 260

(graph below)

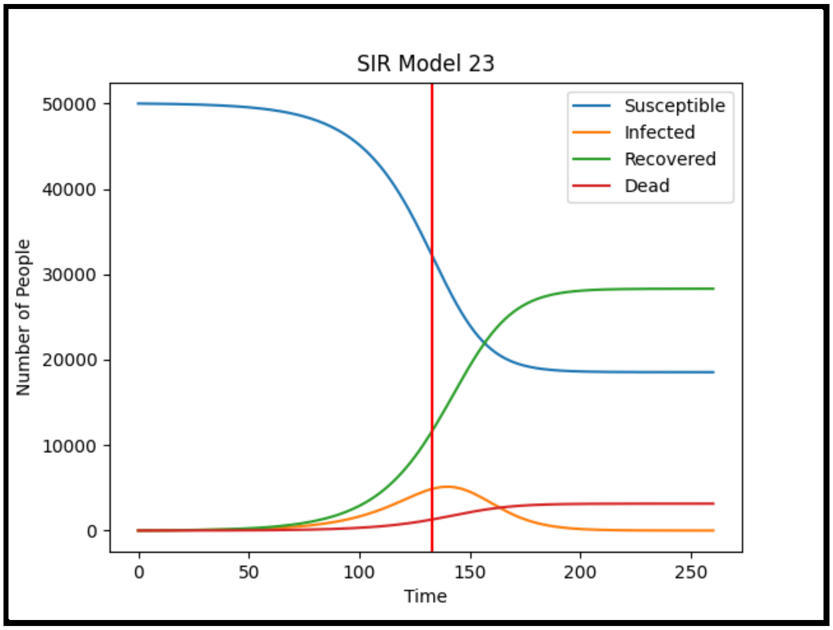


Figure 23: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_{a0} = 10, $\alpha = 5*10^{-6}$, = 1/7, *f=0.1*, Δt = 1, a = 5, q = 0.7, c = 1/3)

Simulation B (1 attendant, 5x Increased Exposure):

- Total Infected: 33036
- Maximum ill at once: 5593 (at time t=141)
- Time of fastest spread: 134 (586.07 p/t)
- Total lifespan of disease: 254

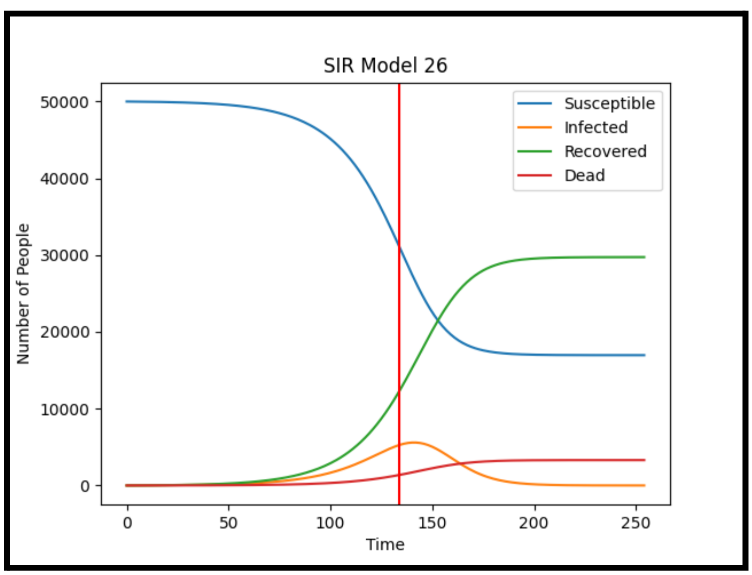


Figure 26: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_{a0} = 10, $\alpha = 5*10^{-6}$, = 1/7, *f=0.1*, Δt = 1, a = 5, q = 0.7, c = 1/3)

Simulation C (1 attendant, 2.5x Increased Exposure):

- Total Infected: 24662
- Maximum ill at once: 3223 (at time t=139)
- Time of fastest spread: 132 (331.63 p/t)
- Total lifespan of disease: 303

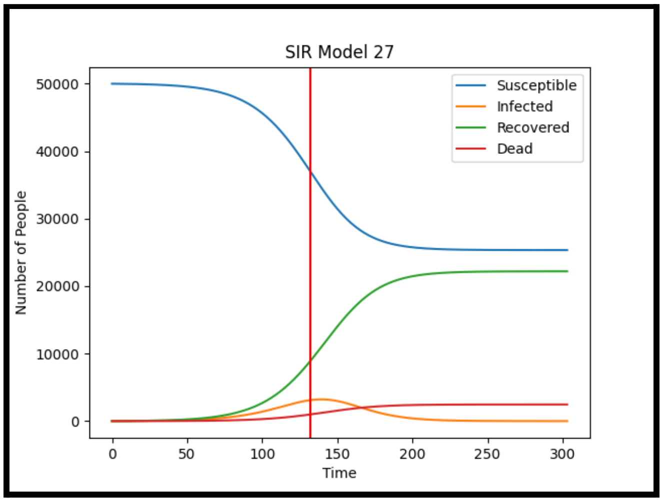


Figure 27: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_{a0} = 10, $\alpha = 5*10^{-6}$, = 1/7, *f=0.1*, Δt = 1, a = 5, q = 0.7, c = 1/3)

Simulation D (1 attendant, 10x Increased Exposure):

- Total Infected: 48743
- Maximum ill at once: 17044 (at time t=129)
- Time of fastest spread: 125 (2081.81 p/t)
- Total lifespan of disease: 202

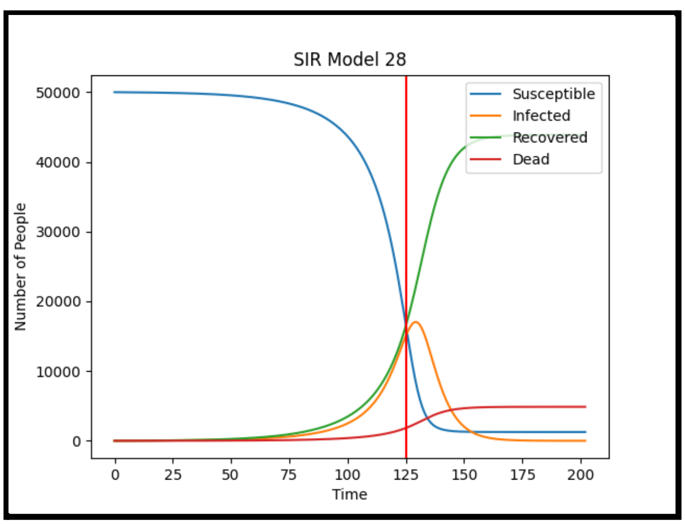


Figure 28: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_{a0} = 10, $\alpha = 5*10^{-6}$, = 1/7, *f=0.1*, Δt = 1, a = 5, q = 0.7, c = 1/3)

Simulation E (0.5 attendants, 10x Increased Exposure):

- Total Infected: 33273
- Maximum ill at once: 5664 (at time t=141)
- Time of fastest spread: 135 (593.73 p/t)
- Total lifespan of disease: 253

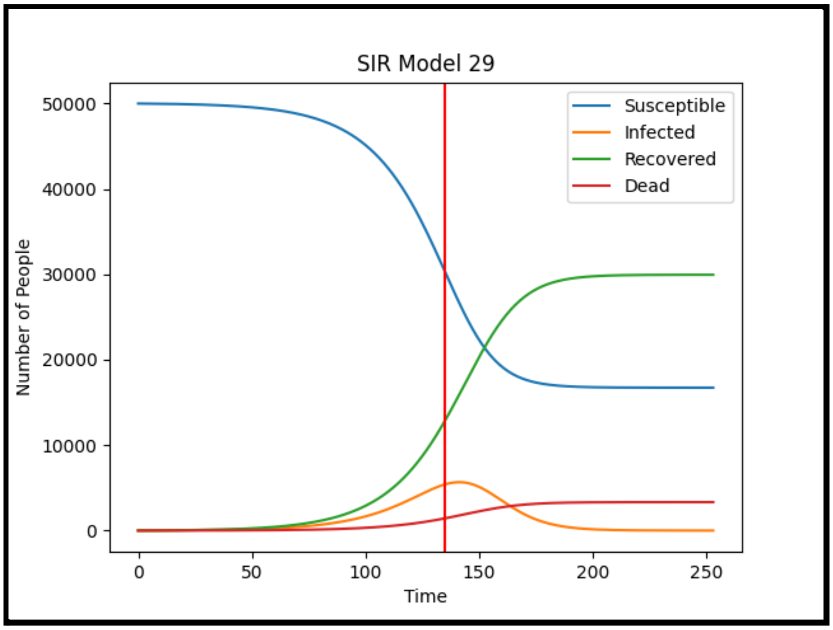


Figure 29: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_{a0} = 10, $\alpha = 5*10^{-6}$, = 1/7, *f=0.1*, Δt = 1, a = 5, q = 0.7, c = 1/3)

From the simulations, group quarantining seems to have mixed results. Simulation B and E show little change from the baseline, C shows a slower spread, and D shows a much faster spread. Overall, the success of group quarantining seems to be dependent on whether the additional exposure is more impactful than the fewer number of attendants. When the decrease is proportional, there is no effect. When the number of attendants decreases by a greater factor than exposure increases, quarantining is more successful. If not, it is less successful. So, this would lead to the conclusion that the attendants should take great precautions to decrease their exposure and lower the transmission rate. One option for this would be using high quality masks or other protection which can significantly decrease their own risk. So, the simulations would suggest that group quarantining is only effective under specific conditions. Additionally, they don't take into account the fact that there are limited hospital staff, which could present problems if they become infected.

Conclusion

Overall, the investigations, aided by computer simulations, indicate that a few factors are especially important in determining the lifespan and devastation caused by a disease. The most important is likely transmission rate. This makes sense, as the faster a disease spreads, the more people it will be able to infect. As such, efforts that target the transmission rate and attempt to decrease it are important in flattening the curve and stopping the spread. In this investigation, high quality masks and widespread quarantine among the infected population are some of the best ways to do this. Another important factor is the recovery rate. When people are sick for longer periods of the time, there is a greater chance they will spread it to more people. While it is hard to decrease this, it could be helpful to explore treatments that help people recover quicker so the disease spreads less quickly. Lastly, the incubation period of the disease is another determining factor in how quickly it spreads. When people are asymptomatic, they are unlikely to quarantine so they will come into contact with more people. One of the only ways to target this is to utilize widespread measures like universal masking and cleanliness in an attempt to stop the spread as a whole. Overall, the simulations and models developed provide great insight into the spread of diseases.

Appendix A: Quarantine Percent Simulations

Simulation A (q=0)

- Total Infected: 45,045
- Maximum ill at once: 15228 (at time t=54)
- Time of fastest spread: 48 (1821.56 p/t)
- Total lifespan of disease: 153

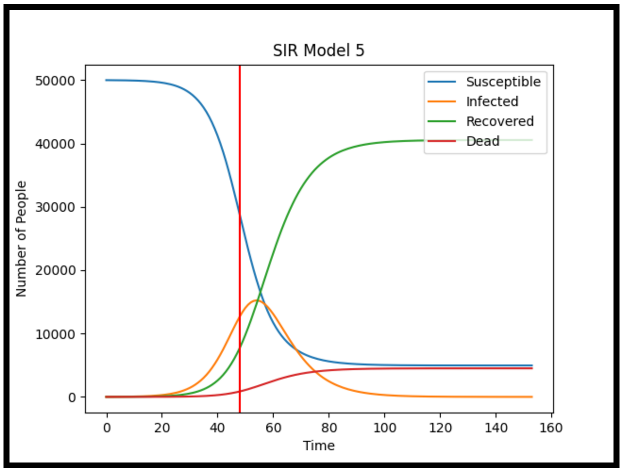


Figure 5: Euler's Method Simulation ($N = 50{,}000$, $S_0 = 49{,}990$, $I_{a0} = 10$, $\alpha = 5*10^{-6}$, = 1/7, *f = 0.1* $\Delta t = 1$, $a = 5$, $q = 0$, $c = 1/3$)

Simulation B (q=0.5)

- Total Infected: 41,242
- Maximum ill at once: 10679 (at time t=87)
- Time of fastest spread: 80 (1184.90 p/t)
- Total lifespan of disease: 185

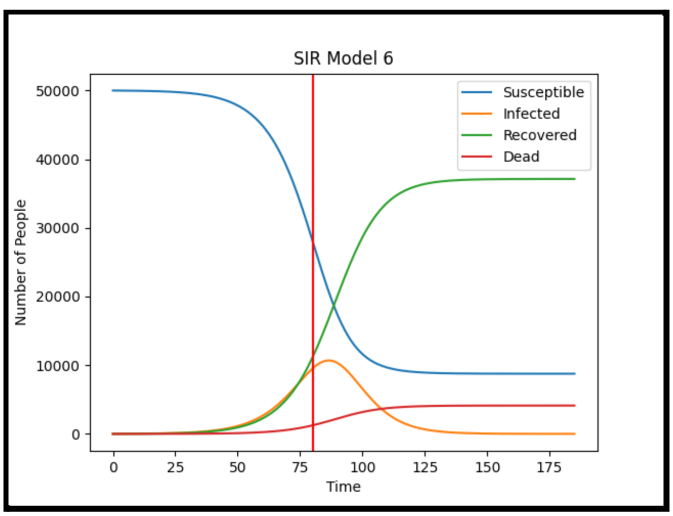


Figure 6: Euler's Method Simulation ($N = 50{,}000$, $S_0 = 49{,}990$, $I_{a0} = 10$, $\alpha = 5*10^{-6}$, = 1/7, *f = 0.1* $\Delta t = 1$, $a = 5$, $q = 0.5$, $c = 1/3$)

Simulation C (q=0.7)

- Total Infected: 31,457
- Maximum ill at once: 5129 (at time t=140)
- Time of fastest spread: 133 (535.99 p/t)
- Total lifespan of disease: 260

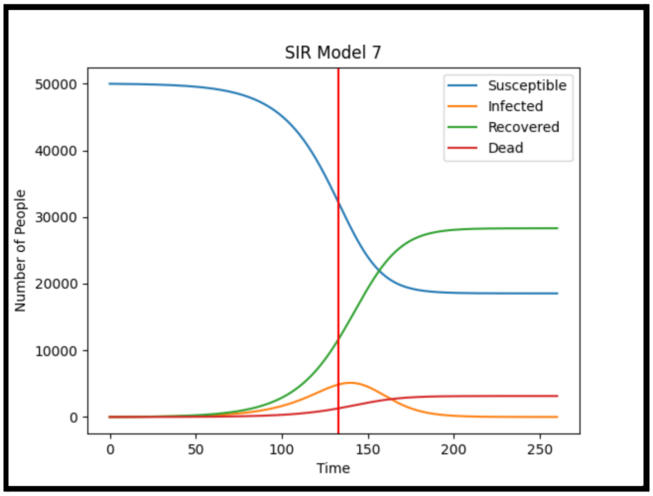


Figure 7: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_{a0} = 10, $\alpha = 5*10^{-6}$, = 1/7, *f = 0.1* Δt = 1, a = 5, q = 0.7, c = 1/3)

Simulation D (q=0.8)

- Total Infected: 12,886
- Maximum ill at once: 831 (at time t=244)
- Time of fastest spread: 237 (83.73 p/t)
- Total lifespan of disease: 493

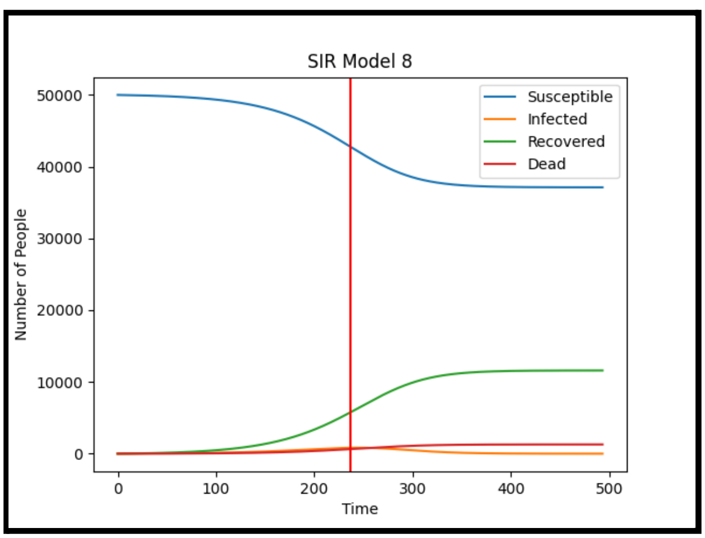
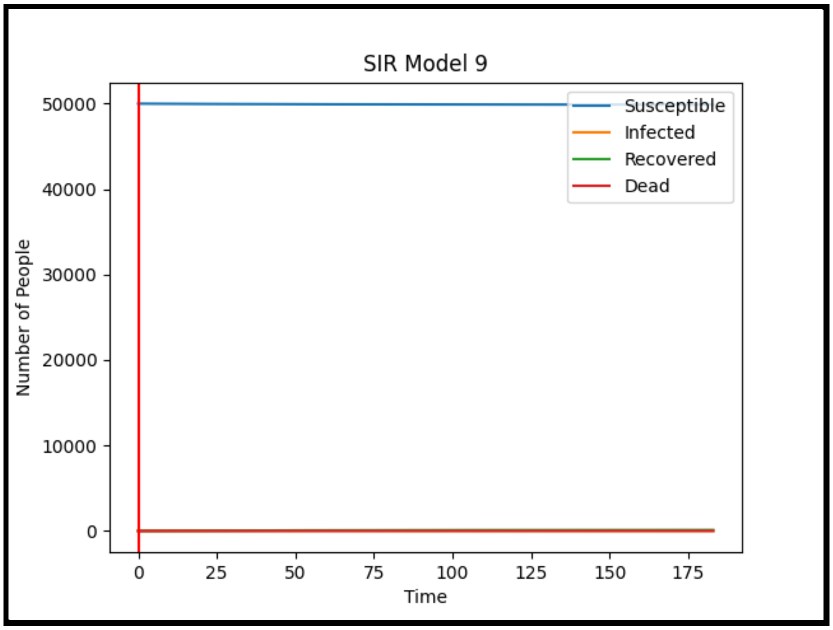


Figure 8: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_{a0} = 10, $\alpha = 5*10^{-6}$, = 1/7, *f = 0.1* Δt = 1, a = 5, q = 0.8, c = 1/3)

Simulation E (q=0.9)

- Total Infected: 128
- Maximum ill at once: 19 (at time t=10)
- Time of fastest spread: 0 (2.5 p/t)
- Total lifespan of disease: 183



Appendix B: Attendant Simulations

Simulation A (a = 0)

- Total Infected: 15022
- Maximum ill at once: 1206 (at time t=168)
- Time of fastest spread: 160 (122.04 p/t)
- Total lifespan of disease: 447

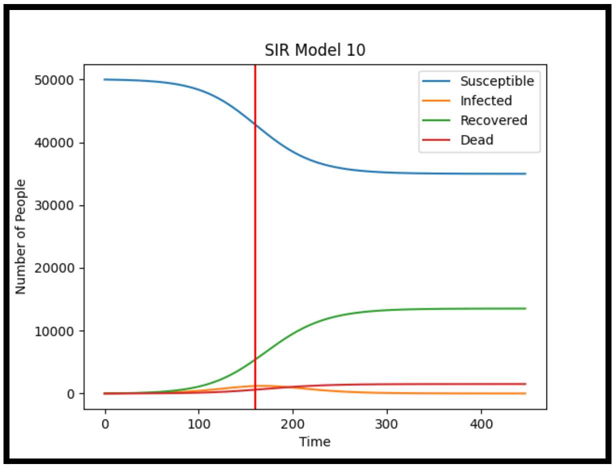


Figure 10: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_{a0} = 10, $\alpha = 5*10^{-6}$, = 1/7, *f=0.1*, Δt = 1, a = 0, q = 0.75, c = 1/3)

Simulation B (a = 3)

- Total Infected: 19222
- Maximum ill at once: 1907 (at time t=173)
- Time of fastest spread: 165 (193.92 p/t)
- Total lifespan of disease: 368

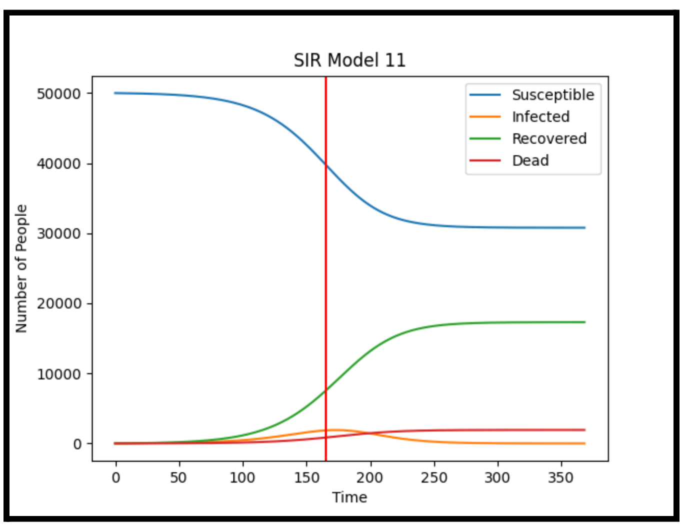


Figure 11: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_{a0} = 10, $\alpha = 5*10^{-6}$, = 1/7, *f=0.1*, Δt = 1, a = 3, q = 0.75, c = ⅓)

Simulation C (a = 5)

- Total Infected: 24336
- Maximum ill at once: 2912 (at time t=175)
- Time of fastest spread: 168 (298.43 p/t)
- Total lifespan of disease: 323

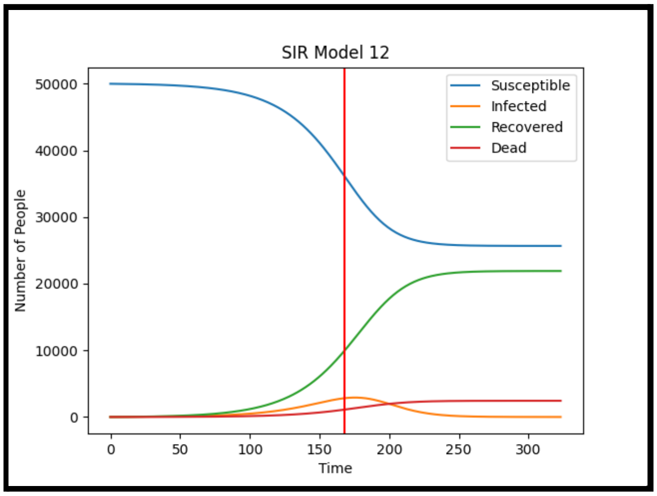


Figure 12: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_{a0} = 10, ɑ = $5*10^{-6}$, = 1/7, *f=0.1*, Δt = 1, a = 5, q = 0.75, c = 1/3)

Simulation D (a = 7)

- Total Infected: 31561
- Maximum ill at once: 4793 (at time t=174)
- Time of fastest spread: 167 (498.98 p/t)
- Total lifespan of disease: 287

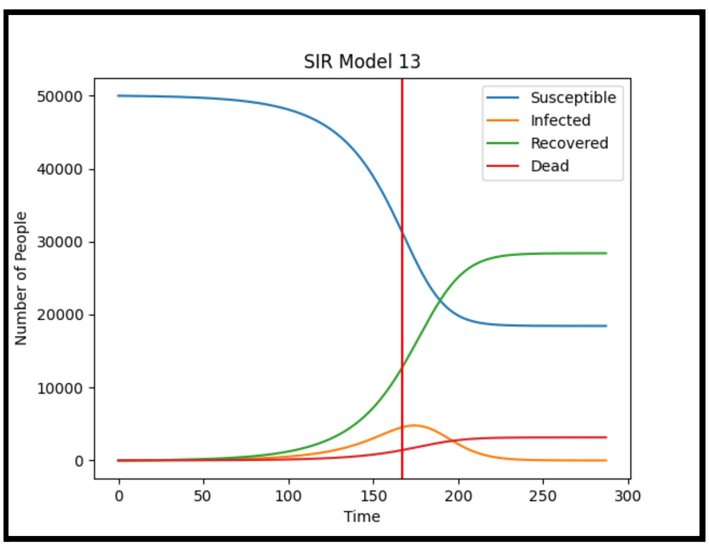


Figure 13: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_{a0} = 10, ɑ = $5*10^{-6}$, = 1/7, *f=0.1*, Δt = 1, a = 7, q = 0.75, c = ⅓)

Simulation E (a = 10)

- Total Infected: 40033
- Maximum ill at once: 8516 (at time t=164)
- Time of fastest spread: 158 (924.98 p/t)
- Total lifespan of disease: 255

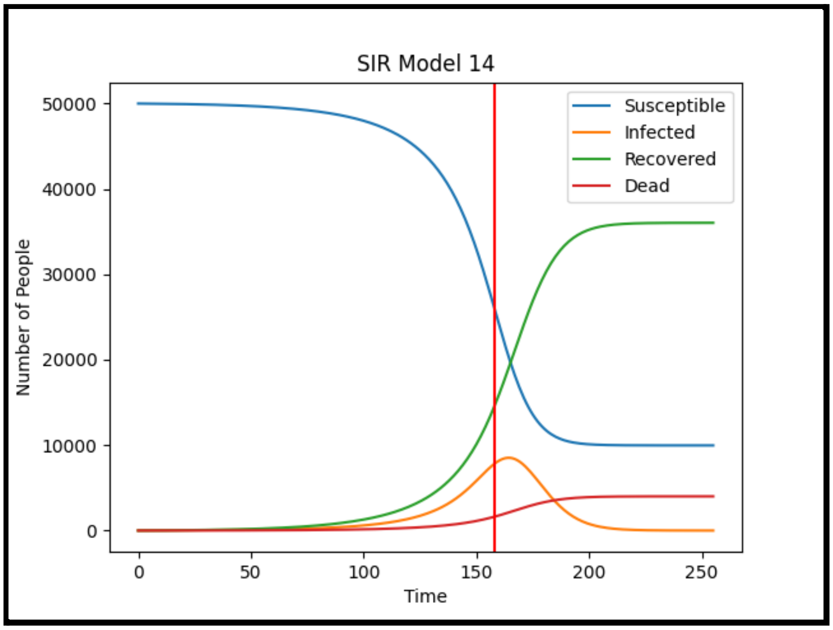


Figure 14: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_{a0} = 10, $\alpha = 5*10^{-6}$, = 1/7, *f=0.1*, Δt = 1, a = 10, q = 0.75, c = 1/3)

Appendix C: Incubation Period Simulations

Simulation A (c = 1)

- Total Infected: 42
- Maximum ill at once: 12 (at time t=1)
- Time of fastest spread: 0 (2.5 p/t)
- Total lifespan of disease: 68

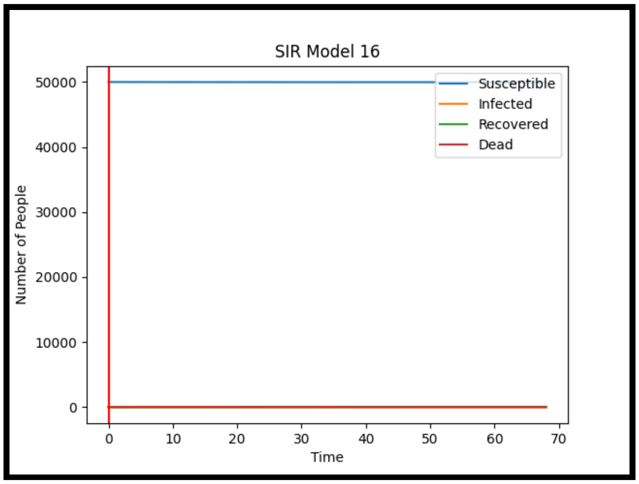


Figure 16: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_{a0} = 10, $\alpha = 5*10^{-6}$, = 1/7, *f=0.1*, Δt = 1, a = 5, q = 0.7, c = 1)

Simulation B (c = 1/2)

- Total Infected: 3012
- Maximum ill at once: 47 (at time t=378)
- Time of fastest spread: 370 (5.2 p/t)
- Total lifespan of disease: 1181

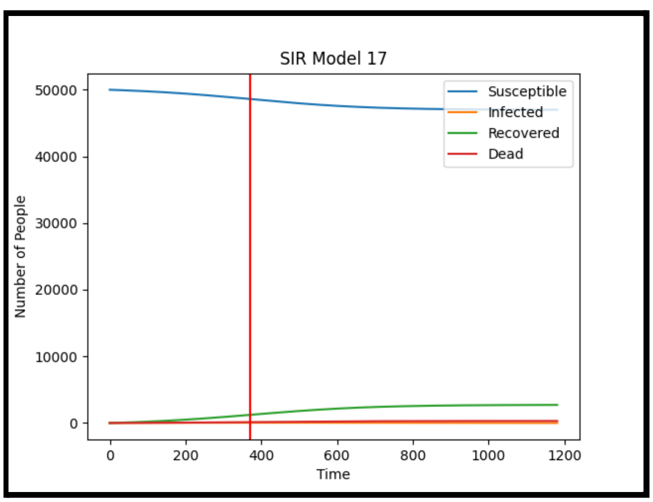


Figure 17: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_{a0} = 10, $\alpha = 5*10^{-6}$, = 1/7, *f=0.1*, Δt = 1, a = 5, q = 0.7, c = 1/2)

Simulation C (c = 1/3)

- Total Infected: 31458
- Maximum ill at once: 5129 (at time t=140)
- Time of fastest spread: 133 (535.99 p/t)
- Total lifespan of disease: 260

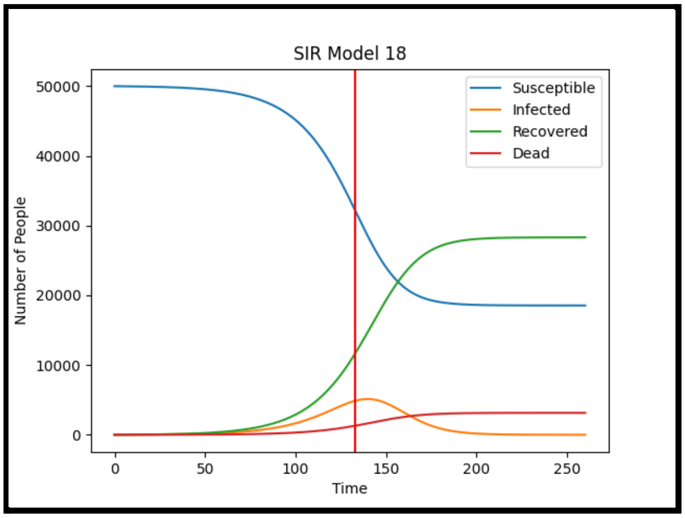


Figure 18: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_{a0} = 10, $\alpha = 5*10^{-6}$, = 1/7, *f=0.1*, Δt = 1, a = 5, q = 0.7, c = 1/3)

Simulation D (c = 1/4)

- Total Infected: 41558
- Maximum ill at once: 11360 (at time t=94)
- Time of fastest spread: 87 (1148.74 p/t)
- Total lifespan of disease: 190

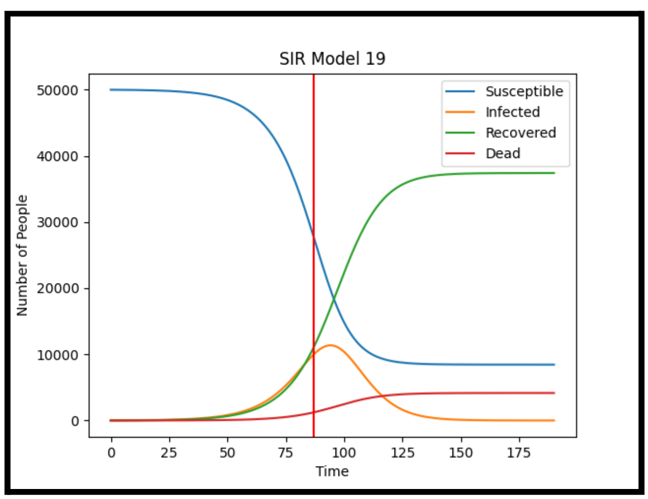


Figure 19: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_{a0} = 10, $\alpha = 5*10^{-6}$, = 1/7, *f=0.1*, Δt = 1, a = 5, q = 0.7, c = 1/4)

Simulation E (c = 1/5)

- Total Infected: 45044
- Maximum ill at once: 15550 (at time t=78)
- Time of fastest spread: 71 (1520.42 p/t)
- Total lifespan of disease: 170

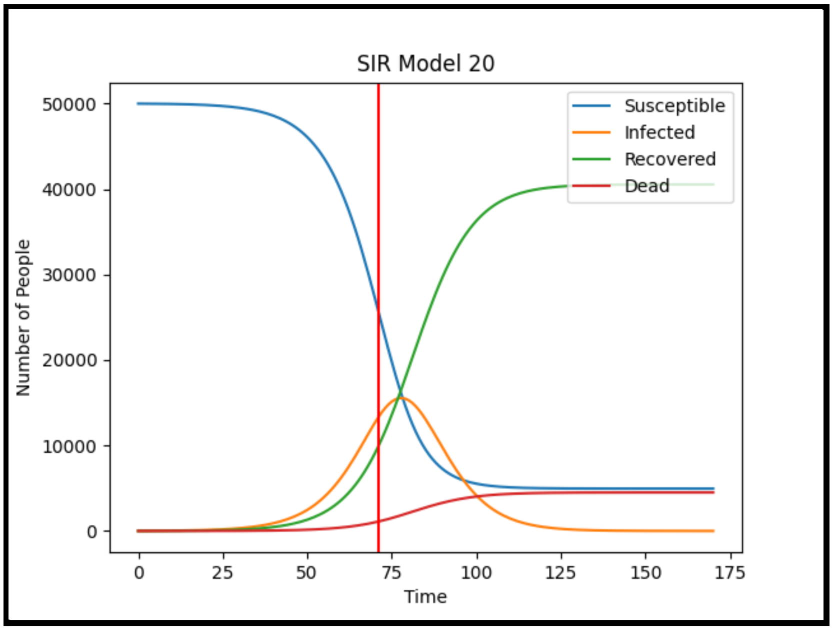


Figure 20: Euler's Method Simulation (N = 50,000, S_0 = 49,990, I_{a0} = 10, $\alpha = 5*10^{-6}$, = 1/7, *f=0.1*, Δt = 1, a = 5, q = 0.7, c = 1/5)