Level 1

Rules and tools

Numbers

The number 5 703 428 has been entered into a table. It shows the value of each column.

The 7 is in the hundred thousands column
The 0 cannot be missed out because it shows that
there are no ten thousands

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units
5	7	0	3	4	2	8

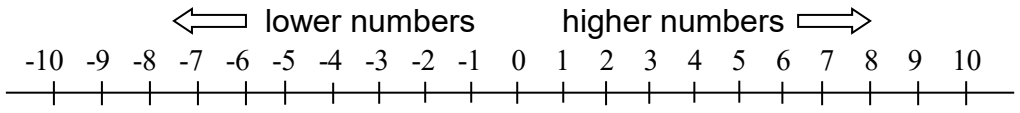
This number is read: 5million, 7hundred and 3 thousand, 4hundred and twenty-eight.

> is the symbol for 'is greater than' 5>3 is read 5 is greater than 3
< is the symbol for 'is less than' 4<9 is read 4 is less than 9

Negative numbers

Numbers that are less than 0 are **negative**.
The numbers above 0 are **positive**

A number line can help you understand negative numbers. It's a bit like a thermometer scale.



Remember

- The more you move to the right on the line, the higher the number gets
- The more you move left on the line the lower the number gets

Negative temperatures indicate cold weather.

Water freezes and becomes ice at 0 °C and below

Addition and subtraction

When adding or subtracting keep the same columns aligned: hundreds with hundreds etc.

To add 3 479 104 and 823 047 align as follows:

```
        3 479 104
add       823 047
        4 302 151
```

Multiplication

There are many ways of recording work when multiplying larger numbers (more than one digit) but they all depend on understanding what is meant by the original question

E.g. 34 × 58 means 30 × 58 add 4 × 58

```
30 × 58 = 3  ×  10 × 58 =1740
 4 × 58 =                 232
Adding gives             1972
```

This is what is being done in the standard method:

```
   58
 ×34
  232
 1740
 1972
```

Any other correct method will give the same answer.

Division

Remember that division is the inverse (opposite)) of multiplication and that, like subtraction, the order in which the numbers are entered **does** matter.

As division frequently results in a decimal answer it is usually better to use a calculator..
To work out 1972 ÷34 you must key in:

[1] [9] [7] [2] [÷] [3] [4] [=]

Multiplying by 10

$6 \times 10 = 60$
$30 \times 10 = 300$
$36 \times 10 = 360$

The effect of multiplying numbers by 10 is to move them into the next column of higher value because

10 units make one 10
and 10 tens are one hundred

Thousands	Hundreds	Tens	Units	•			×10	10 thousand	Thousands	Hundreds	Tens	Units	•		Notes
			6	•							6	**0**	•		Need a **0** to show that the 6 is in the tens column
		3	6	•						3	6	**0**	•		Need a 0 here again
		3	6	•	5					3	6	**5**	•		$10 \times .5 = 5$ so here there is a 5 in the units
	1	4	8	•	3	7			1	4	8	3	•	7	Decimals move in the same way as whole numbers
5	2	0	9	•	6	7		5	2	0	9	6	•	7	0 behaves in the same way as all other digits
	9	8	4	•	0	2			9	8	4	0	•	2	The 0 cannot be missed out - it shows that there are no units

Dividing by 10

Division is the inverse of multiplication.
So each digit will move to the next column of **lower** value.

Thousands	Hundreds	Tens	Units	•			÷10	Thousands	Hundreds	Tens	Units	•			Notes
			6	•							**0**	•	6		Need a **0** to show that the decimal point follows
		3	6	•							3	•	6		Quick check: 30 ÷10 = 3 so this is in correct columns
		3	6	•	5						3	•	6	5	
	1	4	8	•	3	4				1	4	•	8	3	
5	2	0	9	•	6	3			5	2	0	•	9	6	0 behaves in the same way as all other digits
	9	8	4	•	0	2				9	8	•	4	**0**	The 0 can be missed out here as it is at the end of the number , but if this is money in £ it must be written down
		5	0	•	4	1					5	•	**0**	4	The 0 here cannot be missed out because it is not the end of the number

Multiplying and Dividing by 100

As 100 = 10 × 10 , to multiply by 100, multiply by 10 and multiply by 10 again.

To divide by 100, divide by 10 and divide by 10 again.

There are 100 pence in pound
to change pounds to pence: multiply by 100
to change pence to pounds: divide by 100

There are 100 cm in a metre
to change m to cm: multiply by 100
to change cm to m divide by 100

There are 10 mm in a cm
to change cm to m: multiply by 10
to change mm to cm: divide by 10

Times tables

×	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Multiples

The multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20……..

The multiples of 7 are 7, 14, 21, 28, ……

Multiples of 1000 are 1000, 2000, 3000 4000,…..

Square numbers

Square numbers are made by multiplying a number by itself:
1×1, 2×2, 3×3, 4×4, 5×5, 6×6, 7×7, 8×8, 9×9, 10×10,…
or 1 4, 9, 16, 25, 36, 49, 64, 81, 100,…

Ratio and Proportion

The '**ratio** squash to water is 1 to 5' means
'one measure of squash is used with 5 measures of water'
and can be written 1:5. The measures used must be the same.

Ratios cannot be reversed - the ratio 1:3 is **not** the same as the ratio 3:1

Direct proportion means that when more of one item in a list is used more of all items are used e.g. a recipe for 2 people can be doubled to become sufficient for 4 people only if every ingredient is doubled.

Rounding large numbers

See earlier levels for the rules of rounding.

Numbers can be rounded to specific degrees of accuracy e.g. to round to the nearest hundred keep all of the digits in the hundreds place and higher, but remove others according to the rules of rounding

Number	Degree of accuracy		
123 456 789	Nearest 10	123 456 790	9 in the units column means that the ten above is nearer
123 456 789	Nearest 100	123 456 800	keep the 7 in the hundreds column but the 8 in the tens column means that the number is nearer the hundred above
123 456 789	Nearest 1000	123 457 000	
123 456 789	Nearest million (1 000 000)	123 000 000	

Rounding skills can be used to get numbers that are easy to work with to estimate answers and check calculations.

E.g. 35×754 can be checked by 40×800.
The answer will be close to 32 000

Other ways of checking include deciding if the answer is 'sensible' - knowing that multiplication of whole numbers increases the size of the numbers means that an answer less than 700 to the above question is impossible.

Equivalent fractions

Fractions are equivalent if they can be simplify to the same fraction.

E.g. $\frac{6}{20} = \frac{3}{10}$ Because both the numerator (6) and the denominator (20) can be divided by 2.

Similarly $\frac{18}{24} = \frac{3}{4}$ (Divide numerator and denominator by 6)

and $\frac{3}{6} = \frac{1}{2}$ (Divide numerator and denominator by 2).

Fractions of an amount

Finding 1/3 of something is the same as dividing it by 3.

E.g. 1/3 of 24 = 8 $(24 \div 3 = 8)$

To find 2/3, find 1/3 and then double.

E.g. 2/3 of 24 is = $2 \times 8 = 16$

Comparisons of fractions, decimals and percentages

Fractions, percentages and decimals are all ways of being able to express parts of a whole

These are the relationship between some common fractions, percentages and decimals:

Fraction	Decimal	Percentage
½	0.5	50%
¼	0.25	25%
1/5	0.2	20%
1/10	0.1	10%

Comparing decimals

To understand the value of each digit in a decimal number, put the number in a place value table.

A decimal point separates the whole numbers from the decimal fractions.

Start at the left end of a place value table - every time we move a column to the right the value of the column is 1/10 in value. If we continue this across the decimal point we find the values of the columns there

←Whole numbers Parts of number→

Hundreds	Tens	Units	•	**Tenths**	**Hundredths**	**Thousandths**
2	7	0	•	6	5	3

From the column values we can see that

270.653 is made up of:
- 2 hundreds
- 7 tens
- 0 units
- 6 tenths of a unit
- 5 hundredths of a unit
- 3 thousandths of a unit

Addition and subtraction of decimals

Remember that digits of equivalent value must be added so add in columns and the decimal point indicates where the columns should be e.g. 20.3 + 5.78 is written down:

```
  2 0 • 3
+   5 • 7 8
  2 6 • 1 8
```

Subtraction is worked in the same way as addition

Multiplication and division of decimals by single digits

When multiplying or dividing decimals by a single digit number, remember that a decimal point will occur in the answer also e.g. 23.58×2

$$\begin{array}{r} 23 \bullet 58 \\ \times \quad 2 \\ \hline 47 \bullet 16 \end{array}$$

and $23.58 \div 3$

$$3 \,\big|\, \underline{23 \bullet 58}$$
$$7 \bullet 86$$

When using a calculator to work out decimals always do a check by rounding the numbers to the nearest whole number or ten and working with the rounded numbers.

Rounding decimals

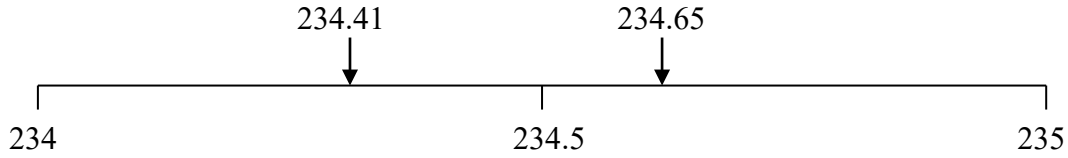
Rounding to the nearest whole number

If the first digit after the decimal point is less than 5, it is nearer to the whole number that is written.

E.g. 234.41 = 234 to the nearest whole number

If the first digit after the decimal point is 5 or more, it is nearer to the whole number above.

E.g. 234.65 = 235 to the nearest whole number.



Rounding to two decimal places

If the 3rd decimal place is 5 or more round up:

So 23.4356 rounds to 23.44 (correct to 2 decimal places)

If the 3rd decimal place is 4 or less, leave the digit alone:

So 23.4326 rounds to 23.43 (correct to 2 decimal places)

Percentages

100% means the whole. 100% correct means no wrong answers at all.

Here are some quick ways of working out common percentages

Work out ……..percent of	Ways of working
50%	halve the value
25%	halve the 50% value
75%	Add the 50% value and the 25% value
10%	Divide the value by 10
5%	Halve the 10% value
20%	1. Double the 10%value 2. Divide the original by 5

Percentage increase

To calculate a percentage increase, work out the percentage of the original and add this to the original amount.

E.g. A car costing £8000 goes up by 10%
10% of £8000 is £800
so the new price will be £8000 + £800 = £8800

Percentage decrease

To calculate a percentage decrease, work out the percentage of the original and take it away from the original amount.

E.g. In a sale the cost of a coat costing £70 is reduced by 20%
10% of £70 is £7
20% of £70 is £14
so the sale price will be £70 - £14 = £56

Converting between fractions, percentages and decimals

To change a fraction to a decimal:
divide the numerator by the denominator
E.g. ¾ = 3 ÷ 4 = 0.75

To change a decimal to a percentage:
multiply by 100
e.g. 0.75 = 0.75 × 100 = 75%

Money

When working with money we have to remember that 100 pence = £1.

In £5.23 the '5' tells you that there are 5 pounds (£5).
The number '23' after the decimal point tells you that there are 23 pence (20p).

Money, in pounds (£) is a decimal number with 2 decimal places.

It behaves in the same way as other decimals, so use the decimal part of **rules and tools** to help with addition, subtraction, multiplication and division.

Time

Dates

These are all short forms of the date twenty sixth of June 2004:

26.06.04
26.6.2004
26/06/04
26 June 2004
26 Jun 04

Twenty-four hour clock

There are 24 hours in a day. Some clocks show a 12-hour clock and repeat the times twice. Others show a 24-hour clock and show the times once.

Three o'clock in the afternoon can be written as 3 pm using the 12-hour clock, or 15:00 (15 00) using the 24-hour clock., since '15' is the number of hours and '00' the number of minutes after midnight (00 00).

When using the 24-hour clock, 4 digits must always be used. This means that 9 45 am is 09 45 on the 24 hour clock There is no need for am or pm as the times after noon are all greater than 12 00.

Timetables

Timetables are usually printed using the 24-hour clock. Read the time a train will leave a station by finding the station and looking across that line to find the train time.

	Train 1	Train2	Train3
Crewe	09 58	12 36	16 42
Sandbach	10 15	12 52	17 00
Wilmslow	10 29	13 07	17 16
Manchester	10 57	13 29	17 38

E.g. Train 1 leaves Crewe at 09 58, that is at 9 58am
Train 3 gets to Manchester at 17 38, that is 5 38 pm

Units of time

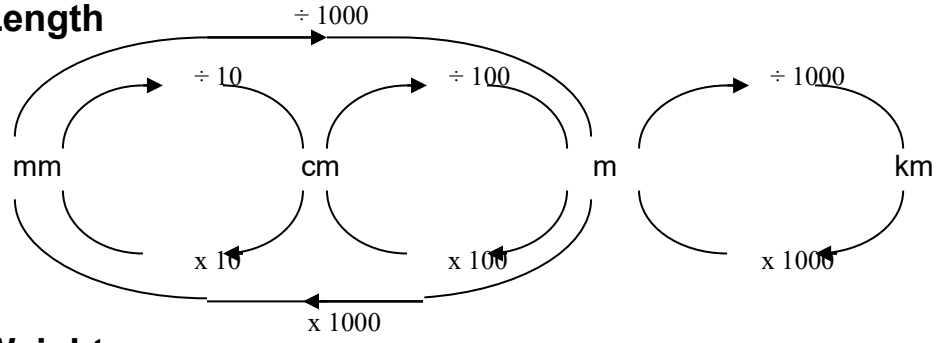
Unit	Length of time
Millennium	1000 years
Century	100 years
Year	365 days (366 in a leap year) 12 months 52 weeks
Month	January 31 February 28(29 in a leap year) March 31 April 30 May 31 June 30 July 31 August 31 September 30 October 31 November 30 December 31
Week	7 days
Day	24 hours
Hour	60 minutes
Minute	60 seconds
Second	Smallest unit of time commonly used

The metric system

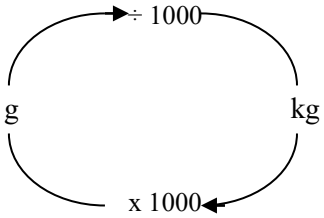
measure	unit	short form	equivalents
length	Metre Centimetre Millimetre Kilometre	m cm mm km	1m = 100 cm = 1000 mm 1000 m = 1 km
weight	Gram Kilogram	g kg	1000 g = 1 kg
capacity	Litre Millilitre	*l* ml	1000 ml = 1 *l*

The diagrams show how to convert between different units.

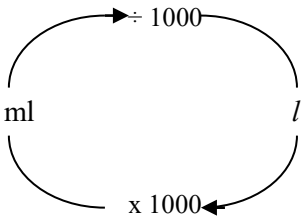
Length



Weight



Capacity



Distances

Distances between two places such as two towns can be measured either in kilometres or miles. A miles is more than a kilometre (1 mile is about 1.6 kilometres)

Distance Charts

Distance Charts record the distance between towns. Check whether the figures given are in miles or kilometres.

This chart shows that the distance between Leeds and Cardiff is 230 miles.

London					
198	Leeds				Distance in miles
79	271	Dover			
548	331	587	Aberdeen		
152	230	238	532	Cardiff	
522	335	591	156	513	Fort William

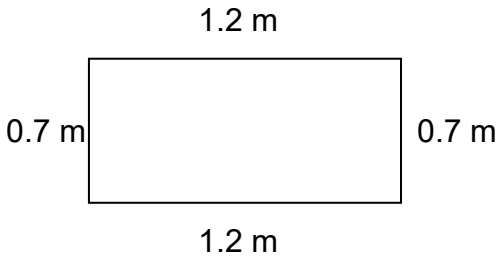
Perimeter

The perimeter of a shape is the distance round the edge of the shape.

Perimeter is a length so the units will be mm, cm, m

To work out the perimeter of a shape add the lengths of all the sides.

The perimeter of this shape is 1.2 + 0.7 + 1.2 + 0.7 = 3.8 m



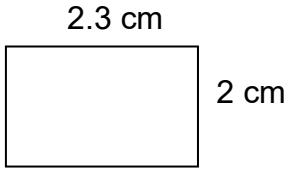
Area of rectangle

The area of a shape is the amount of surface that the shape covers.

The area of a rectangle is worked out from the length and the width.
These must be measured in the same units i.e. both measurements must be in metres or they must both be in centimetres.

The area of a rectangle = length × width.

The units of area are square units: m² or cm²
The area of this rectangle is 2.3 cm x 2 cm = 4.6 cm^2



Volume

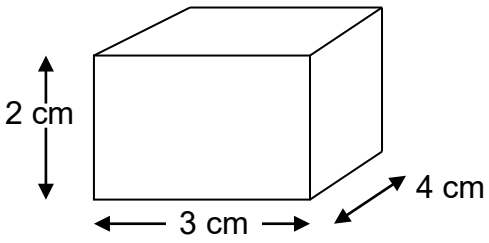
The volume of an object is the amount of space it occupies or contains.

The volume of a cuboid (box) is worked out from the length, height and width. These must be measured in the same units i.e. all measurements must be in metres or they must all be in centimetres

The volume of a cuboid = length × width × height.

The units are cubic units - m³ or cm³

The volume of this cuboid is 2 cm x 3 cm x 4cm = 24 cm3



Shape

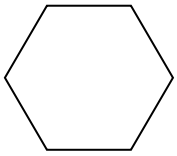
This shape is a rectangle.

It has four angles.

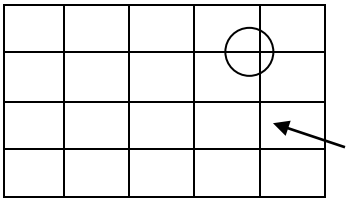
Angles are measured in degrees.

The four angles of this square are all right angles and all measure 90 degrees (90°)

This hexagon has 6 sides all the same length. All the angles are also the same size. So it is a **regular** hexagon. All the angles are bigger than a right angle so they are more than 90^{o}



Look at this tiling pattern.



Four square tiles of four right angles fit together.

Square tiles fit together perfectly without learning gaps or without overlapping so we say that they **tessellate**. For example circles do not tessellate.

Reading and Drawing Graphs

In addition to the rules and tools used at earlier levels remember to look at the scale of a graph to interpret the data shown.
Always choose a sensible scale to fit the data.

Mean

The mean is one form of average.

Find the mean by adding up all of the values and dividing the total by the number of values

E.g. Mean of 3, 6, 7, 2, 8 is $\frac{3 + 6 + 7 + 2 + 8}{5} = \frac{26}{5} = 5.2$

The mean may not be the best average to represent the data if there are many items of a small value and one with a very large value.

Range

The range is a measure of how far the data spreads out.
Work out the range by subtracting the smallest value from the largest value
E.g. the range of 3, 6, 7, 2, 8 is 8 - 2 = 6

Probability

Events can be:
impossible (seeing a blue cow)
certain (sun rises each day)
or somewhere in between

Events (or happenings) have several outcomes:
the outcomes of rolling a die are 1,2,3,4,5,6
the outcomes of tossing a coin are heads or tails.

Events can happen in various ways:
A total of 5 when throwing 2 dice can be 1 +4, 2 +3, 4 +1 or 3 +2

Probability is the likelihood or chance of something happening

The probability of an impossible event happening is 0
The probability of a certain event happening is 1

The probability of any event is $\frac{\text{Number of ways the event can happen}}{\text{Total number of outcomes}}$

E.g.
Probability of throwing a six is $\frac{1}{6}$ (there is only one six on a die)
(there are a total of six numbers on a die).

Probabilities can be written as fractions, decimals or percentages.