

## Level 2

### Rules and tools

#### Place Value

The use of place value from earlier levels applies but is extended to all sizes of numbers. The values of columns are:

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units	.	tenths	hundredths	thousandths
1	2	3	4	5	6	7	.	8	9	1

To make reading large numbers easier digits are grouped in threes from the decimal point so the number in the table is written:

1 234 567•891

It is read aloud as:

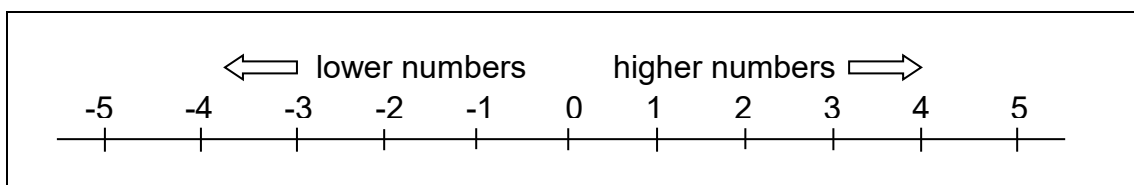
One million, two hundred and thirty-four thousand, five hundred and sixty-seven point eight, nine, one

The number

10 406•07

indicates that there are no ten-thousands, no hundreds and no tenths, but these digits may not be removed as this information is important to the place value of the other digits.

#### Negative numbers



Negative numbers are the numbers below zero.

E.g. -5 is smaller than -1.

-4 °C is colder than -2 °C

## Calculations with numbers

The methods for addition, subtraction, multiplication and division of whole numbers and decimals are the same as for Level 1.

## Multiples

The multiples of 12 are:

12, 24, 36, 48, 60, 72.....

## Factors

Factors are numbers which divide exactly into a given number.

E.g. The factors of 12 are: 1, 2, 3, 4, 6, 12.

## Prime numbers

Numbers that have no factors except the 1 and the number itself are called prime numbers.

The prime numbers to 20 are: 2, 3, 5, 7, 11, 13, 17, 19

The number 1 is a particular case and is defined **not** prime

## Ratio and Proportion

A ratio of 3 : 2 means that  
for three measures of the first ingredient, 2 measures of the second  
must be used

E.g. A concrete mix uses cement : sand in the ratio 3 : 2 means  
Use 3 buckets of cement with 2 buckets of sand or  
Use 3 lorry-loads of cement with 2 lorry-loads of sand or  
Use 9 shovelfuls of cement with 6 shovelfuls of sand

## Parts

A party drink is made from a recipe that requires  
pineapple : orange : water in the ratio 2 : 3 : 5

10 litres of drink is to be made. How much of each part will be needed?

2 parts of pineapple + 3 parts of orange + 5 parts of water  
=  $2 + 3 + 5 = 10$  parts altogether

So to make 10 litres use:

2 litres of pineapple, 3 litres of orange and 5 litres of water

To make 20 litres use:

4 litres of pineapple, 6 litres of orange and 10 litres of water

## Scale

A scale of 1 : 1000 means that every unit on the drawing represents 1000 of the same units in real life.

1 cm on a drawing represents 1000cm (which is 10m) in real life.

A scale of 1 : 100 000 means that

1 cm on a map represents 100 000cm = 1000m = 1 km on the ground

## Formulae

Letters are used to represent variable (changing) quantities in formulae.

If  $a$  represents the quantity of one item

$2a$  represents twice that quantity

The  $\times$  is understood and need not be written.

All other operating symbols (+, -,  $\div$ ) are always written.

Similarly

$ab$  represents  $a \times b$

and  $2(a + b) = 2 \times (a + b)$

## Fractions

When ordering or comparing fractions, the denominator (number on the bottom) of each fraction must be the same as it indicates the size of the fraction of the whole.

E.g. Which is the bigger  $\frac{7}{9}$  or  $\frac{5}{6}$ ?

Change both to a common denominator.

$$\frac{7}{9} = \frac{14}{18}$$

$$\frac{5}{6} = \frac{15}{18}$$

so  $\frac{5}{6}$  is larger than  $\frac{7}{9}$

E.g. Place these fractions in order of size starting with the smallest.

$$\frac{2}{5}, \frac{3}{10}, \frac{7}{15}$$

The lowest common denominator is 30.

$$\frac{2}{5} = \frac{12}{30}$$

$$\frac{3}{10} = \frac{9}{30}$$

$$\frac{7}{15} = \frac{14}{30}$$

so the order is  $\frac{9}{30}, \frac{12}{30}, \frac{14}{30}$  which is  $\frac{3}{10}, \frac{2}{5}, \frac{7}{15}$

## Fractions, Decimals and Percentages

Fractions decimals and percentages are all ways of expressing parts of a whole.

A percentage is a fraction with denominator 100.

Because it is such a common fraction it is written in a particular way:

$$17\% \text{ is } \frac{17}{100}$$

### Fractions to decimals

Change fractions to decimals by dividing the numerator by the denominator (use a calculator whenever necessary)

E.g.  $\frac{5}{8} = 5 \div 8 = 0.625$

### Decimals to percentages

Change decimals to percentages by multiplying by 100:

$$0.625 = 0.625 \times 100\% = 62.5\%$$

## Writing fractions in simplest form

Fractions are equivalent if they can be simplified to the same fraction

E.g.  $\frac{15}{18} = \frac{5}{6}$  Because both the numerator (15) and the denominator (18) have 3 as a factor.

Similarly  $\frac{20}{45} = \frac{4}{9}$  Numerator and denominator both have a factor of 5

Some fractions will not simplify:

Example:  $\frac{17}{40}$

The number 17 is a prime. Thus its only factors are 1 and itself.

Example:  $\frac{15}{22}$

Although neither 15 nor 22 are prime the fraction will not simplify as 15 or 22 do not have a common factor (other than 1).

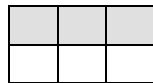
## Addition and subtraction of fractions

To add or subtract fractions the denominator must be the same

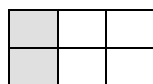
Example 1:  $\frac{1}{2} + \frac{1}{3}$

The lowest common denominator is 6.

$$\frac{1}{2} = \frac{3}{6}$$



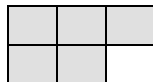
$$\frac{1}{3} = \frac{2}{6}$$



So  $\frac{1}{2} + \frac{1}{3}$

$$= \frac{3}{6} + \frac{2}{6}$$

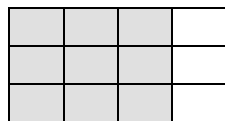
$$= \frac{5}{6}$$



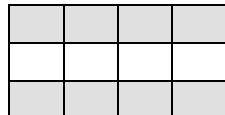
Example 2:  $\frac{3}{4} - \frac{2}{3}$

The lowest common denominator is 12

$$\frac{3}{4} = \frac{9}{12}$$

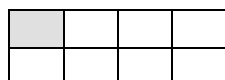


$$\frac{2}{3} = \frac{8}{12}$$



So  $\frac{3}{4} - \frac{2}{3}$

$$= \frac{9}{12} - \frac{8}{12}$$



$$= \frac{1}{12}$$

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## Rounding decimals

Numbers can be rounded to a specified degree of accuracy.

E.g. 123.456 is:

123.46	to 2 decimal places
123.5	to 1 decimal place
123	to the nearest whole number
120	to the nearest ten
100	to the nearest hundred

## Comparing Decimals

When you want to compare long decimals, rearrange the numbers in order of size. Arrange these three numbers in order of size, starting with the smallest.

284.46, 276.78, 315.54

To do this, compare the values of the digits with the same place value column

Start with the largest place value (hundreds in this example).

Pick out the smallest value (2).

If there is more than one of the same value

Compare the digits in the next place value

Column (tens in this example).

Pick out the smaller value (7).

Continue until all comparisons have been made.

2	8	4	.	8	6
2	7	6	.	7	8
3	1	5	.	5	4
Original Numbers					

2	7	6	.	7	8
2	8	4	.	8	6
3	1	5	.	5	4
Numbers in order of size					

## Calculating with decimals

There are many correct methods of carrying out calculations with decimals.

Answers should be checked by:

- Using approximate calculations
- Or by using inverse operations (E.g. use addition to check the answer to a subtraction).

## Using percentages

### Percentages of a quantity

Rules and tools for level 1 shows how to find simple percentages of a quantity.

These included:

To Find	Method
50%	Divide by 2
10%	Divide by 10
5%	Divide by 10, then divide by 2

To find 1% of a quantity, divide by 100.

This enables other percentages to be calculated

Example 1: Work out 23% of £250 (no calculator)

$$\begin{aligned}10\% \text{ of } £250 &= £25 & (£250 \div 10) \\20\% \text{ of } £250 &= 2 \times £25 = £50 \\1\% \text{ of } £250 &= £2.50 & (£250 \div 100) \\3\% \text{ of } £250 &= 3 \times £2.50 = £7.50 \\23\% \text{ of } £250 &= £50 + £7.50 \\&= £57.50\end{aligned}$$

Example 2: Work out 9% of £628.47 (use a calculator)

9% is 9 hundredths  
To find 1% of £628.47 divide by 100  
Then to calculate 9% (9 hundredths) multiply by 9

So enter  $\boxed{6}\boxed{2}\boxed{8}\boxed{.}\boxed{4}\boxed{7}\boxed{\div}\boxed{1}\boxed{0}\boxed{0}\boxed{\times}\boxed{9}\boxed{=}$  into the calculator

The calculator shows £56.56 (to the nearest penny)



## Percentage increase

VAT (Value Added Tax) is 17.5%.

It is added to the basic cost before selling many items.

17.5% is made up of 10% + 5% + 2.5%

Work out VAT like this:

Value without VAT = £150	VAT	
10%	£15	1/10 of 150
5%	£ 7 •50	Half of 10%
2.5%	£ 3 • 75	Half of 5%
17.5%	£26 •25	Add 10% +5% + 2.5%

The VAT to be added to £150 is £26.25 so the cost including VAT is £ 176.25

Although the rate of VAT is 17.5% in 2003 it can be changed at any time by the government in a budget.

## Percentage decrease

If there is 30% off (30% decrease) in a sale there is still 70% to pay .

E.g. A coat, priced at £80, is reduced in a sale by 30%.

What is the sale price?

If there is 30% off, there is 70% to pay.

10% of £80 = £8

70% of £80 = 7 X £8 = £56

So the sale price is £56

## Evaluating one number as a percentage of another

E.g.1 What is 53 as a percentage of 74?

Step 1: Make a fraction of the two numbers:  
the number that follows 'as a percentage of' is the denominator:

$$\frac{53}{74}$$

Step 2: Change the fraction to a percentage by multiplying by 100.

Use a calculator to evaluate:

$$\frac{53}{74} \times 100 \% \quad (53 \div 74 \times 100 =)$$
$$= 71.62\% \text{ correct to 2 decimal places}$$

E.g. 2 In a survey 80 people were asked what their favourite soap was.  
35 people liked Coronation Street best.

What percentage of those surveyed liked Coronation Street best?

$$\frac{35}{80} \times 100 \% = 43.75\%$$

43.75% liked Coronation Street best.

## Using a calculator

It is easy to make a mistake when keying numbers into a calculator so always check calculations done on a calculator by:-

- Doing the question twice, putting the numbers in to the calculator in a different order if this is possible e.g. for additions or multiplications.
- Using inverses to check by a different calculation e.g. doing additions to check subtractions or multiplications to check divisions.
- Round numbers to the nearest whole number or to the nearest 10 and do the calculation in your head or on paper to check the order (approximate size) of your answer.

Calculators that have functions like memory, fractions and brackets can be useful but these functions often have different ways in which they must be used. Look at the handbook that came with the calculator for instructions about how to use these functions or ask your teacher for help.

The functions that you have to be able to use are + , - , × , ÷ , = , % and  $\pi$

## Currency conversion

Banks, Post Offices and travel agencies and some building societies sell foreign currencies. They charge for this service, it is called commission. They will buy foreign money from you at a lower rate than when they sell it to you to pay for the cost of doing the business for you.

### Pounds to Euros

To change from one currency to another you need to know the exchange rate for that day.

On 24/08/02 the pound (£) to euro (€) exchange rate what £1 = €1.59 (or €1 and 59 cents).

Convert from £ to € like this.

Check your calculation by dividing by €1.59.

$$£1 = 1 \times 1.59 = €1.59$$

$$€1.59 \div €1.59 = £1$$

$$£2 = 2 \times 1.59 = €3.18$$

$$€3.18 \div €1.59 = £2$$

$$£3 = 3 \times 1.59 = €4.77$$

$$€4.77 \div €1.59 = £3$$

*So multiply the number of pounds (£) by the exchange rate.*

### Euros to Pounds

On 12/09/02 the euro (€) to pound (£) exchange rate was: €1 = £0.63

Convert from € to £ like this.

Check your calculation by dividing by £0.63.

$$€1 = 1 \times 0.63 = £0.63$$

$$£0.63 \div £0.63 = €1$$

$$€2 = 2 \times 0.63 = £1.26$$

$$£1.26 \div £0.63 = €2$$

$$€3 = 3 \times 0.63 = £1.89$$

$$£1.89 \div £0.63 = €3$$

*So multiply the numbers of Euros (€) by the exchange rate.*

The box shows the exchange rate for one pound (£1).

This indicates that:

- You have to give the bank €1.74 for every £1 they give you.
- The bank will give you only €1.59 for every £1 you give them.

Exchange Rates			
We buy		We sell	
1.74	Euros	1.59	

## Time

Earlier levels of rules and tools cover ways of recording dates and times.

The standard units of time are covered in rules and tools, Level 1.

## The Metric/Imperial system

### Length

An inch is about 2.5cm (2.54cm to 2 decimal places)

A foot is about 30cm (30.5cm to 1 decimal place)

A yard is a bit less than 1 metre

1 yard is 36 inches

1 metre is about 39 inches

### Weight

A kilogram is just over 2 pounds (about 2.2 pounds)

A pound is about 450grams.

An ounce is about 25g

### Capacity

A pint is just over half a litre

A gallon is about 4.5 litres

A litre is just under two pints

## The Metric system

The units used in the metric system and information in how to convert between different units was covered in Level 1 rules and tools.

## The Imperial system

Measure	Units	Short Form	Equivalents
Length	inch foot (feet) yard mile	in ft yd m	12 in = 1 ft 3 ft = 1 yd 1760 yds = 1m
Weight	ounces pounds stones tons	oz lb	16 oz = 1 lb 14 lb = 1 stone
Capacity	pints gallons		8 pints = 1 gallon

## Temperature

Temperature is measured in degrees using either the Celsius or Fahrenheit scales. The instrument used to measure temperatures is a thermometer.

### Celsius

Degrees Celsius are shortened to °C

Water freezes at 0 °C

Water boils at 100 °C

A warm summer's day in England would be about 20 °C

### Fahrenheit

Degrees Fahrenheit are shortened to °F

Water freezes at 32 °F

Water boils at 212 °F

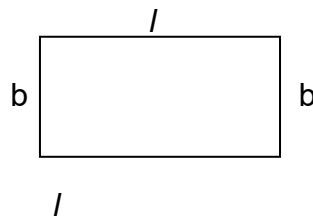
## Using formulae

A formula is a rule for working out a calculation. A formula can be written in word or in symbols.

### The rectangle

The perimeter of a shape is the distance around the edge of a shape.

To work out the perimeter of a rectangle add the length of all four sides together.

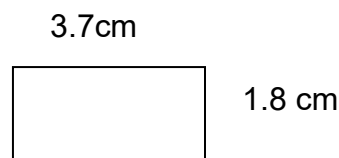


For rectangle:

$$\begin{aligned}\text{Perimeter} &= b + l + b + l \\ &= 2(b + l)\end{aligned}$$

$$\text{Area} = l \times b$$

Use the formulae to calculate the perimeter and area of this rectangle.

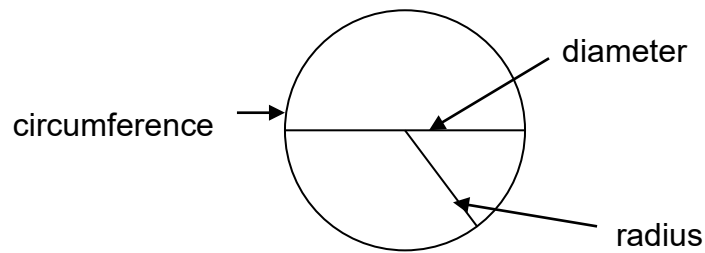


$$\begin{aligned}\text{Perimeter} &= 2(l + b) \\ &= 2(3.7 + 1.8) \text{ cm} \\ &= 2(5.5) \text{ cm} \\ &= 11 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{Area} &= l \times b \\ &= 3.7 \times 1.8 \text{ cm}^2 \\ &= 6.66 \text{ cm}^2\end{aligned}$$

## The Circle

The perimeter of a circle has a special name, 'the circumference'.



The formula for calculating the circumference (perimeter) and area of a circle are:

$$\begin{aligned}\text{Circumference} &= \pi \times \text{diameter} \\ &= \pi d\end{aligned}$$

$$\begin{aligned}\text{Area} &= \pi \times \text{radius}^2 \\ &= \pi r^2\end{aligned}$$

$\pi$  is also the number 3.142 (to 3 decimal places)

Use the  $\pi$  button on your calculator to see  $\pi$  to more decimal places.

Use the formulae to calculate the area and circumference of a circle with radius 5cm.

$$\begin{aligned}\text{Circumference} &= \pi d \\ &= 3.142 \times 10 \text{ (} r = 5\text{cm so } d = 10\text{cm)} \\ &= 31.42\text{cm}\end{aligned}$$

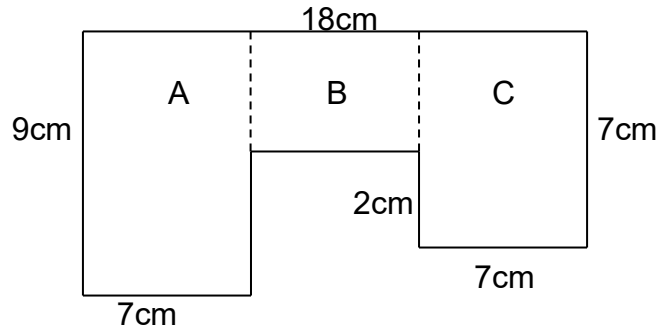
$$\begin{aligned}\text{Area} &= \pi r^2 \\ &= 3.142 \times 5 \times 5 \\ &= 85.5\text{cm}^2\end{aligned}$$

Remember measurements must be in the same unit before substituting into a formula.

## Composite shapes

The area of more difficult shapes can be calculated by breaking them up into the simple shapes that we know how to calculate the area.

E.g.



The shape shown can be divided into rectangles as shown by the dotted lines. Now the area of each rectangle (A, B and C) can be calculated and the total area calculated by adding the three areas together.

$$\text{Area A} = l \times b = 9 \times 7 = 63 \text{ cm}^2$$

$$\text{Area B} = l \times b = 5 \times 4 = 20 \text{ cm}^2$$

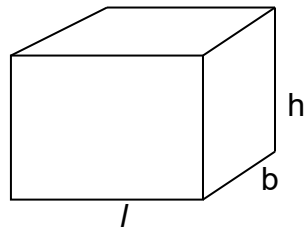
$$\text{Area C} = l \times b = 7 \times 7 = 49 \text{ cm}^2$$

$$\text{Total area} = 132 \text{ cm}^2$$



## Volumes

The volume of a cuboid is calculated using the formula:



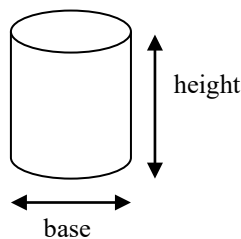
length x breadth x height

E.g.

$$\begin{aligned}\text{Volume} &= l \times b \times h \\ &= 8 \times 6 \times 5 \\ &= 240 \text{ cm}^3\end{aligned}$$

Remember:  
Ensure all lengths are  
in the same unit  
before substituting in  
the formula

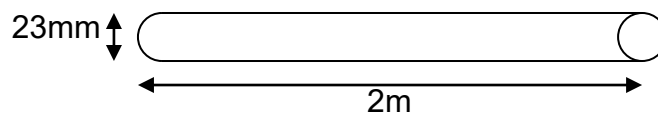
The volume of a cylinder is calculated by multiplying the area of the base by the height.



Since the area of the base is a circle the formula for the area is  $\pi r^2$

So the volume of a cylinder is  $\pi r^2 h$

Example



Calculate the volume of a drain pipe with diameter 23mm and 2m long. Give your answer in  $\text{cm}^3$

$$\begin{aligned}\text{Volume} &= \pi r^2 h \\ &= 3.142 \times 1.15^2 \times 200 \\ &= 831 \text{ cm}^3 \text{ (to the nearest cm}^3\text{)}.\end{aligned}$$

## Scale drawings

A scale of 1 : 75 means that 1 mm on the scale diagram represents 75 mm on the ground.

Scale measurement = 32 mm

Actual length is  $32 \times 75 \text{ mm} = 2400 \text{ mm} = 2.4 \text{ m}$

Scale measurement = 46 mm

Actual length is  $46 \times 75 \text{ mm} = 3450 \text{ mm} = 3.45 \text{ m}$

## Some formulae

Formula to find	Formula	Units of answer	
Perimeter of rectangle	$P = 2(l + b)$	cm or m	
Area of rectangle	$A = lb$	$\text{cm}^2$ or $\text{m}^2$	
Area of triangle	$A = \frac{1}{2}bh$	$\text{cm}^2$ or $\text{m}^2$	$h$ is the perpendicular height
Circumference of circle	$C = \pi d$	cm or m	
Area of circle	$A = \pi r^2$	$\text{cm}^2$ or $\text{m}^2$	$A = \pi \times r \times r$
Volume of cuboid	$V = lbh$	$\text{cm}^3$ or $\text{m}^3$	$b$ is breadth, also called width
Volume of cylinder	$V = \pi r^2 h$	$\text{cm}^3$ or $\text{m}^3$	$h$ is the perpendicular height

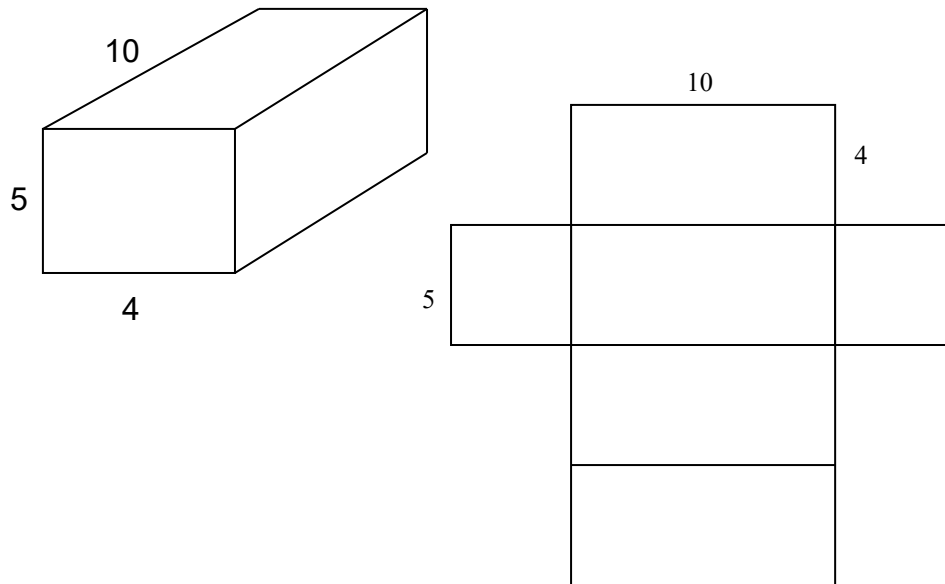
## 2D or 3D

2D shapes are shapes that are drawn on paper (or other materials)

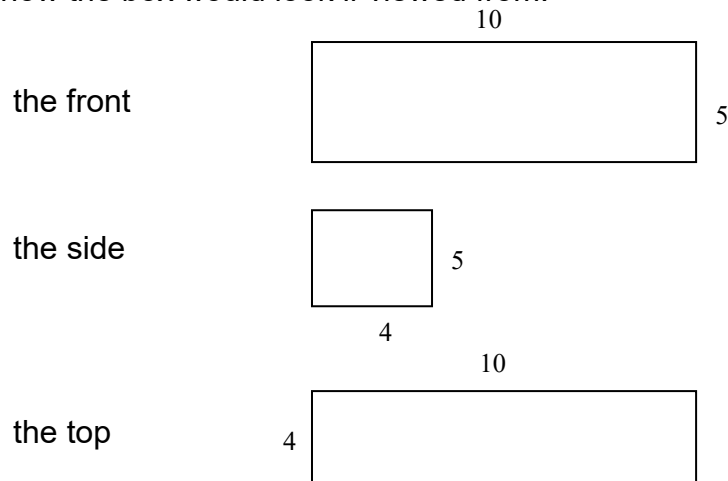
3D shapes are solid. They can be picked up

3D shapes can be represented as maps or plans in 2D on paper.

A cardboard box is a 3D-shape, Most boxes are cut from a single piece of card and folded into the required box. The shape of the box before it is folded is called a **net**.

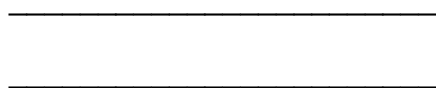


Another way of showing this box (or another 3D shape) as a 2D drawing is to show how the box would look if viewed from:



These different views are called different elevation

## Parallel



Parallel lines always stay the same distance apart.  
(Like railway lines).

## Discrete and continuous data

**Discrete data** must have particular values. These values are usually whole numbers but sometimes fractions are involved.

An example of discrete data would be shoe size: E.g. 6, 9 or  $4\frac{1}{2}$

**Continuous data** can have any value within a range of values.

An example of continuous data would be the actual length of someone's foot. This could be 24.43 cm or 27.1 cm, for example.

Continuous data can only be given to a degree of accuracy, to 2 decimal places for example, because it is measured.

## Tables, diagrams, charts and graphs

The skills to extract, collect and organise data in tables, diagrams, charts and graphs have been covered at earlier levels.

## Averages and spread

### Mean

The mean was defined at Level 1 as an average.

To calculate the mean:

add up the values of all of the data

divide this total by the number of values.

E.g. The mean of 6, 8, 7, 12 is

$$\frac{6 + 8 + 7 + 12}{4} = \frac{33}{4} = 8.25$$

This may not be the best average to represent the data if there are many items of small value and one with a very large value.

## Median

The median is another average.

The median is the middle score when the scores are put in order  
E.g. Find the median of 3,8,2,5,6,9,3,4,7

Put the scores in order:

2, 3, 3, 4, 5, 6, 7, 8, 9

Write down **all** scores, for example, there are two 3s.

Select the middle score as the median. The median is 5

E.g. 2      In this example there is no single middle score:

1, 2, 2, 3, 4, 5, 6, 6, 7, 8, 8, 9

The median is  $\frac{5 + 6}{2} = \frac{11}{2} = 5.5$

## Mode

The mode is also an average.

The mode is the score that appears most often (most fashionable).

In the data:                      3,8,2,5,6,9,3,4,7

the mode is 3.

(Because there are two 3s and only one of each of the other numbers)

Some data may have more than one mode.

In the data:                      2, 3, 2, 3, 1, 5, 3, 6, 2

The modes are 2 and 3.

(Since 2 and 3 both appear three times).

## **Advantages and disadvantages**

- Mean:** This gives a precise value, which reflects every score involved.
- It can be distorted by one or two very large or very low values.
- It is used on large populations e.g. if the average family is 2.215 children and there are 2000 families 4430 school places will be needed.
- Median:** This can give a more understandable value.
- It ignores the very high and very low values e.g. in the example above the mean of 2.215 children may be useful but is not realistic. The median gives a whole number of children!
- Mode:** This average is most useful when supplying goods for sale e.g. a shoe shop needs to order the most popular size of shoe: this is the mode of size.

## **The range**

The range is a measure of how the data spreads out.

Calculate the range by subtracting the smallest value from the largest value.

E.g. The range of 2,9,4,1,3 is  $9 - 1 = 8$

## Probability

Probability is introduced in rules and tools at level 1.

### Independence

Events are independent if one event cannot affect the outcome of the other event

E.g. Flipping a coin and rolling a die

are independent since the outcome of one does not affect the outcome of the other.

### Combined probabilities

A combined probability can be calculated for events that are independent.

E.g. The probability of scoring two 6s when two dice are rolled.

### Sample space tables and tree diagrams

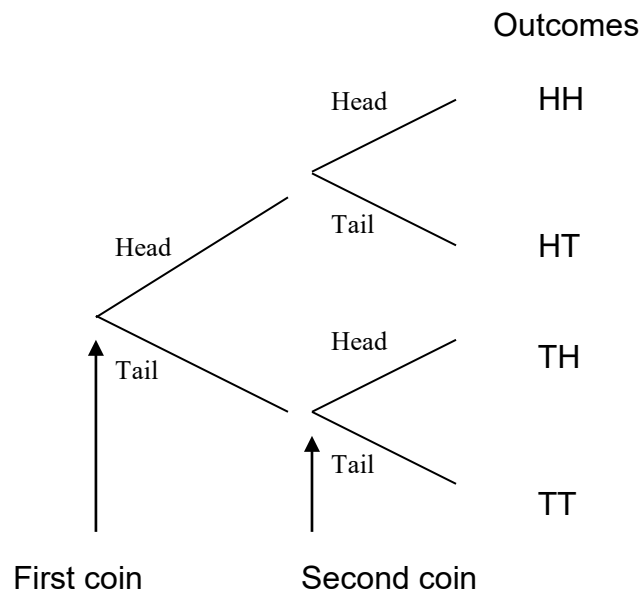
A sample space table can be useful to record all possible outcomes for two independent events.

E.g. The results obtained when the score on two dice are added:

Outcome of second die	Outcome of first die						
	+	1	2	3	4	5	6
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

## Tree diagrams

Tree diagrams can be used instead of sample space tables



This tree diagram shows all the possible outcomes of flipping two coins.

Tree diagrams can be used for more than two events but sample space tables can only show two events.

Tree diagrams for more than two events can be very large.