

To what extent does the efficiency of Dijkstra's Algorithm and Shortest Path Algorithm differentiate on randomly generated unweighted bi-directional graphs?

Compare and contrast the efficiency of the Dijkstra shortest path algorithm and the Shortest Path Faster Algorithm (SPFA) on randomly generated bi-directional weighted graphs at various input sizes.

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A computer Science Extended Essay

3578 Words

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1: Introduction

Graph theory is the study of mathematical structures that model pairwise relationships between variables, known as graphs. Each graph is composed of a set of vertices which are connected by another set of edges (Weisstein, 2021). The concept of graphs was first proposed by famous mathematician Leonhard Euler in his "Seven Bridges of Königsberg" paper (Paoletti, 2011). Euler observed that the city of Königsberg, in Prussia had seven bridges which connected the two large islands of Kneiphof and Lomse to the each other and the mainland. In his publication, Euler proposed that there was no way to walk through the city by crossing each bridge exactly one time (Paoletti, 2011). His work marked the beginning of the branch of mathematics known as graph theory.

Graphs come in shapes and forms, such as Tree Graphs, where each pair of vertices is connected by exactly one edge, or weighted graphs, where each edge has specific value that has to be expended to travel across. However, the two most simple and common type of graphs are directed and undirected graphs. The edges within a directed graph denote that the path can only be crossed in a specified direction and an undirected graph contains edges that are bidirectional.

In the modern day, the study of graph theory also has many real world applications. An example of which would be studying the spread of infectious diseases such as the Coronavirus in the field of epidemiology or criminal networks in criminology (Levenkova,

2014). The Dijkstra's Shortest Path Algorithm, which will be investigated in this paper has also been implemented as the primary algorithm for finding the shortest path in Google Maps (Crovari, 2019). New research is constantly being conducted on the development of faster and more efficient algorithms that can traverse graphs.

Even though there are many different problems involving graph theory, one of the most common and important issue to this day would be the "single source shortest path problem". The problem requires an algorithm to find the shortest path between a given source node, and every other node within the graph (Chapuis & Andonov, 2017). In order for an algorithm to accomplish this, it must be provided a list of all the edges between nodes as well as the source node where the query must be conducted. There are currently two contenders for the most efficient single source shortest path algorithm, namely the Dijkstra algorithm (Richards, 1972), first proposed by computer scientist Edsger W. Dijkstra in 1956 and the Shortest Path Faster Algorithm (SPFA), an improvement to the Bellman-Ford Algorithm made by Chinese researcher FanDing Duan (玉人, 2010).

This investigation aims to compare and contrast the efficiency of both graph theory algorithms by comparing their time complexity at various input data sizes and record the performance of each algorithm over a series of a hundred trials.

2: Background Information

2.1: Algorithmic Efficiency

To measure the algorithmic efficiency of both methods, the time and space complexities must be taken into consideration. However, as most modern day computers have at least 4 Gigabytes of internal Randomly Accessed Memory (RAM), the issue of memory allocation will not be a factor considered in this investigation. Therefore, the primary dependent variable of this paper would be the time complexity of the two algorithms. The time complexity of most algorithms is divided into three parts, the best case, average case and worst case time complexity. This indicates the theoretical upper and lower bound of the algorithm's run time.

The time complexity of an algorithm can be denoted by the "Big O Notation" in the form of $O(N)$. The number within the brackets following the capital 'O' denotes the theoretical time complexity of the algorithm based on the size of the input, which is generally represented by the capital letter 'N'. Most of the time, the time complexity of an algorithm can be determined by analyzing the number of operations that the computer needs to conduct. Take the example of a linear searching algorithm designed to find an element in an array. If the array contains 'N' elements, then the time complexity of the algorithm would be $O(N)$, because in the worst case scenario, the algorithm will have to search through all N elements before it can find the desired result. The average time of an algorithm is much harder to determine, as different input could result in different average time complexities.

2.2: Dijkstra's Algorithm for Single Source Shortest Path

2.2.1: Algorithm Explanation

Dijkstra's shortest path algorithm employs the use of Breadth First Search (BFS) to determine the distance between a source node and every other node within a graph. Dijkstra's algorithm works by creating a first in first out data structure, known as a Queue, which keeps track of all the nodes that is being processed. While the queue is not empty, the top node in the queue is popped out and denoted as the current node. After this has been completed, the algorithm checks for all of the nodes the the current node is connected to and adds them all to the queue. This process is repeated until every node in the graph has been updated and the shortest distance between the source node and every other nodes in the graph can be determined (Gregg, 2018).

2.2.2: Time Complexity

The theoretical time complexity of the algorithm is $O(V^2)$ where V is the number of vertices within the graph. However, by implementing a priority queue, the time complexity can be optimized to $O(E\log(V) + V)$ where E is the number of edges. This algorithm can be further optimized to $O(E + V\log(V))$ through the use of a combination of Priority Queue and the Fibonacci heap data structure. For the purpose of this investigation, the the Priority Queue Dijkstra will be tested.

2.3: Shortest Path Faster Algorithm (SPFA)

2.3.1: Algorithm Explanation

Shortest Path Faster Algorithm is an improvement to the Bellman-Ford shortest path algorithm and involves the use of lazy propagation to "relax" vertices. Similar to Dijkstra, SPFA also finds the shortest distance between a single source node and every other nodes in the graph. Furthermore, SPFA also maintains a queue to keep track of all the nodes that are going to be visited. However, it also has an array to check if an element is already in the queue. If the element is already in the queue, it will not be added (玉人, 2010). This reduces the number of nodes that has to be checked and is able to improve the time complexity of the algorithm.

2.3.2: Time Complexity

The theoretical worst case time complexity would be $O(V \times E)$ (玉人, 2010). Even though not enough research has been done to confirm this, experimental data suggests that the average time complexity of the algorithm on randomly generated graphs is near $O(E)$ (玉人, 2010). Furthermore, SPFA is always preferred over the original Bellman-Ford algorithm because it is significantly faster on randomly generated graphs and has the same worst case time complexity.

2.4: Data Set Constraints

The processing power of computers are measured in Million Instructions per Second (MIPS), which denotes the number of operations the computer can compute each second. As this value is different for every processor, a simple program can be written to calculate the MIPS.

The following simple Java program determines the MIPS of a given processor by calculating the starting time and stopping until one second has completed. The program calls the built in Java function, "System.nanoTime()" which returns the current time in nanoseconds (ns). Therefore, while the difference between the current time and starting time is less than 10^9 ns, or exactly one second, the counter will increment by one. After the time period has concluded, the number of instructions done by the processor will be printed out.

```
long startTime = System.nanoTime(), count = 0;
while(System.nanoTime() - startTime < 1e9) {
    count++;
}
System.out.println(count);
```

After conducting exactly one hundred trials on the average instructions per second, the following results were recorded. Only the first ten trials are shown.

Trials	Million Instructions per Second
1	0.044981391
2	0.046862592
3	0.046891600
4	0.046870524
5	0.046744232
6	0.046890223
7	0.046893581
8	0.046886984
9	0.046890159
10	0.046808171
Average	0.0466719457

Table 1: Million Instructions per Second of a I5-6600K 3.50GHz CPU

As shown in the primary data above, the processor used for this experiment can execute around 0.0467 million instructions each second, which is around 4.6672×10^7 individual operations. Therefore, to determine a visible difference in the run time of the two algorithms, each will be tested with rigorous amounts of data. Based on the theoretical time complexity mentioned in the previous section, both algorithms will be tested with data set of at most 10^6

vertices and edges to ensure that there will be visible differences in the run time. However, as many real world applications of this algorithm does not require such large data sets, both algorithms will also be tested at 10^5 and 10^4 vertices and edges.

3: Experimental Methodology

The main source of data used for this investigation comes from primary sources of data. Both Dijkstra and SPFA were programmed in Java and were fed random locally generated data at the specified constraints. Primary sources of data were chosen for this experiment because SPFA has limited secondary and tertiary data and no comprehensive research has been conducted to verify its average time complexity. Furthermore, primary data collection allows for the manipulation of the independent variables involved in the algorithm. However, this experimental methodology also comes with its limitations. The scope of this investigation was severely limited as all the code used for data collection was executed on a home desktop with mediocre computational power, hence larger data sets could not be tested.

3.1: Experimental Procedure

The experimental procedure for both the Dijkstra's algorithm and SPFA are relatively similar. Both programs start off by generating the test data and printing it off to a ".txt" file. Then the data is scanned in my the program and the timer begins. The program will either execute Dijkstra's algorithm or SPFA and when the program has stopped running, the run time will be printed out. This procedure is clearly demonstrated in the flow chart below.

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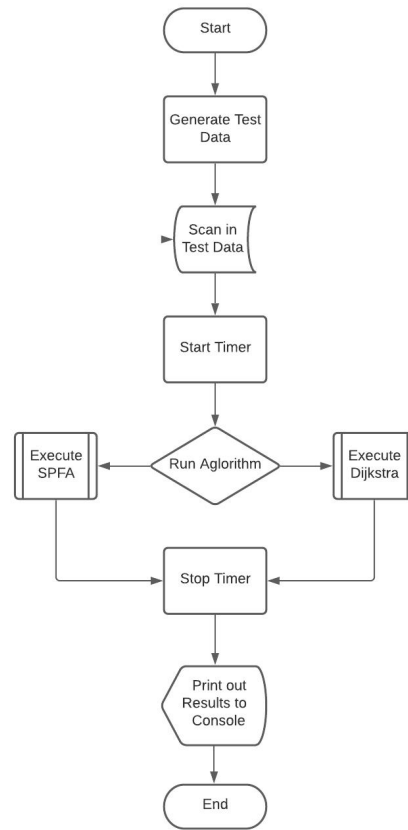


Figure 1: Visualization of Experimental Procedure

The procedure can also be demonstrated through the use of Pseudo code as shown below.

```
int t = readInteger();
for(int i = 0; i < t; i++) {
    generateTestData();
    int v = readInteger(), e = readInteger();
    for(int i = 0; i < e; i++) {
        int start = readInteger(), end = readInteger(), dist = readInteger();
        makeGraph();
    }
    long timeStart = System.nanoTime();
    callDijkstra();
    /* or */
    callSPFA ();
    long timeEnd = System.nanoTime();
```

```
    System.out.println(timeEnd - timeStart);  
}
```

3.2: Measurement of Data

To measure the time complexity of both algorithms at varying sizes, Java's `System.nanoTime()` can be implemented. The total run time can be determined by finding the difference in the initial value recorded before the program was run and the final value.

```
long start = System.nanoTime();
```

// Execute Code

```
long end = System.nanoTime();  
System.out.println("Total Run Time in Nanoseconds is: " + (end-start))
```

3.3: Experimental Accuracy

The theoretical experimental accuracy of the `System.nanoTime` function should be $\pm 1\text{ns}$. However, this appears to not be the case as repeated trials of the functions indicated that the rightmost three digits of the returned value is always zero. This results in the experimental accuracy of the function to be $\pm 1\mu\text{s}$. As this is around 0.001% of the experimental results, the uncertainty values can be neglected.

3.4: Variables

3.4.1: Independent Variable

The independent variable of this investigation is the data constraints of the datasets that is being inputted into the algorithms. The amount of input size will directly correlate to the time complexity.

3.4.2: Dependent Variable

The dependent variable of this investigation is the time complexity of the algorithm measured in seconds. The run time will directly affected by changes in the input size, making it the dependent variable.

3.4.3: Controlled Variable

1. **Processor:** The same processor must be used to compute both the programs as different processing units can compute different number of instructions per second.
2. **Programming Language:** The programming language used to code and execute the algorithms must also be kept the same as different programming languages take different amount of time to compile and execute code.
3. **Measurement of Time:** To ensure that the time taken is measured correctly, the time must be measured after the data has been generated and inputted to ensure that the collected data has high accuracy.

4: Experimental Results

4.1: Tabular Representation of Experimental Data

The following table displays the performance of both the Dijkstra's Algorithm and Shortest Path Faster Algorithm at various data sizes from 10^6 to 10^4 data values. Furthermore, the final two graphs shows the correlation between input data size and time taken of both programs.

For each data set, only the first ten out of one fifty trials were shown and the average time for the trials were shown on the final line of each table. Furthermore, a large number of decimal places were used to maintain high accuracy in the recorded results, as the difference between a selective number of trails were fairly minuscule.

Trials	Shortest Path Faster Algorithm (s)	Dijkstra's Algorithm (s)
1	2.49675	2.20985
2	2.08050	3.35393
3	1.69652	3.07306
4	1.93442	2.46844
5	2.00193	2.50254
6	1.94521	2.36200
7	1.55685	2.39478
8	1.52814	2.49976

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9	1.96018	2.15172
10	1.87326	2.13480
Average	1.90738	2.51509

Table 1: Time Complexity of Dijkstra and SPFA at 1, 000, 000 Data Values

Trials	Shortest Path Faster Algorithm (s)	Dijkstra's Algorithm (s)
1	0.26428	0.25972
2	0.20451	0.19840
3	0.14483	0.17919
4	0.17780	0.18377
5	0.12722	0.24069
6	0.18332	0.15402
7	0.16731	0.20284
8	0.16175	0.24611
9	0.15707	0.15519
10	0.11558	0.17455
Avg	0.17037	0.19945

Table 2: Time Complexity of Dijkstra and SPFA at 100, 000 Data Values

Trials	Shortest Path Faster Algorithm (s)	Dijkstra's Algorithm (s)
---------------	---	---------------------------------

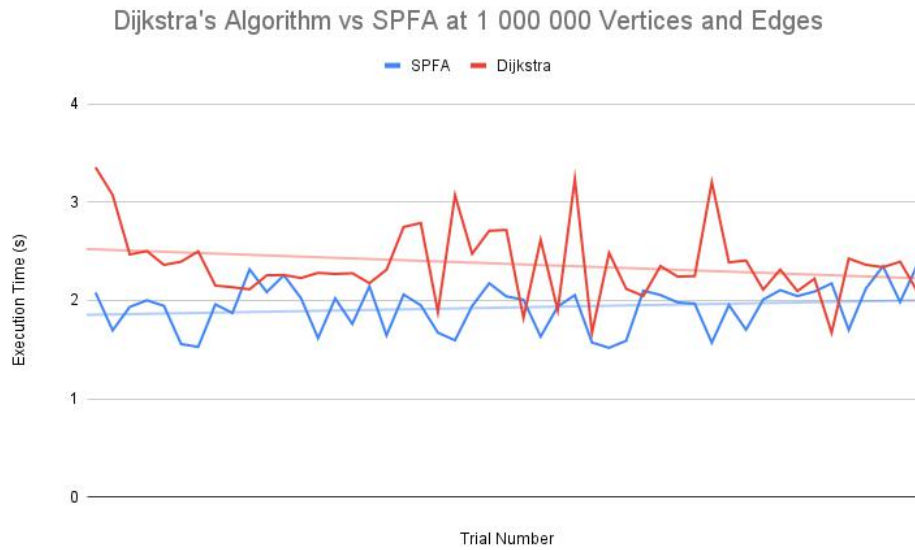
1	0.05457	0.05463
2	0.03146	0.03753
3	0.01957	0.01984
4	0.02277	0.01818
5	0.01906	0.01786
6	0.02073	0.02173
7	0.02041	0.02026
8	0.01885	0.0197
9	0.01494	0.01512
10	0.01541	0.01494
Avg	0.02378	0.02398

Table 3: Time Complexity of Dijkstra and SPFA at 10, 000 Data Values

4.2: Graphical Representation of Data

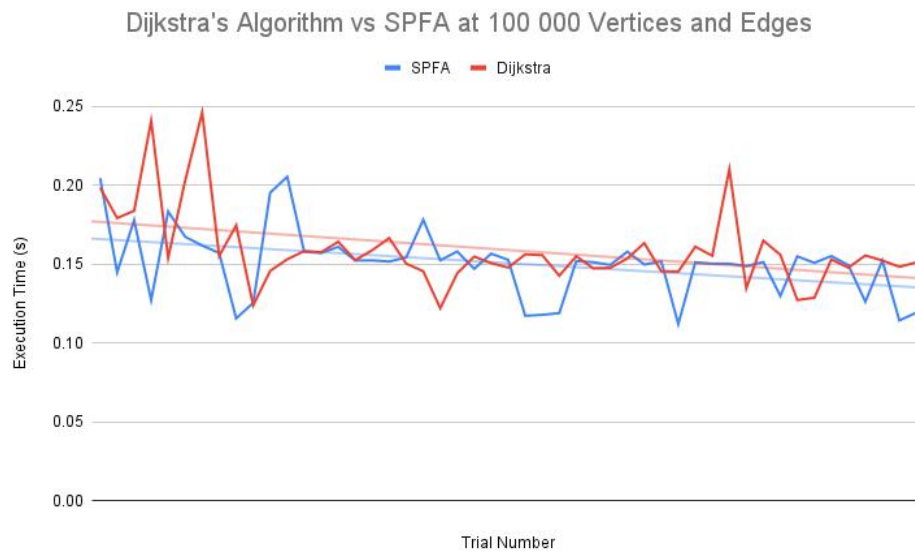
In order to better understand the trends in performance, the collected experimental data has been converted into a double line graph. The red line denotes the performance of the Dijkstra' Algorithm and the blue line represents SPFA. The x-axis of the graph is the trial number and has no units, while the y-axis is the amount of time taken to complete execution of the program and is measured in seconds.

The correlation between input values and run time can be shown in the final two graphs. The x-axis of the graph is the number of data values inputted into the program and the y-axis is the run time measured in seconds.



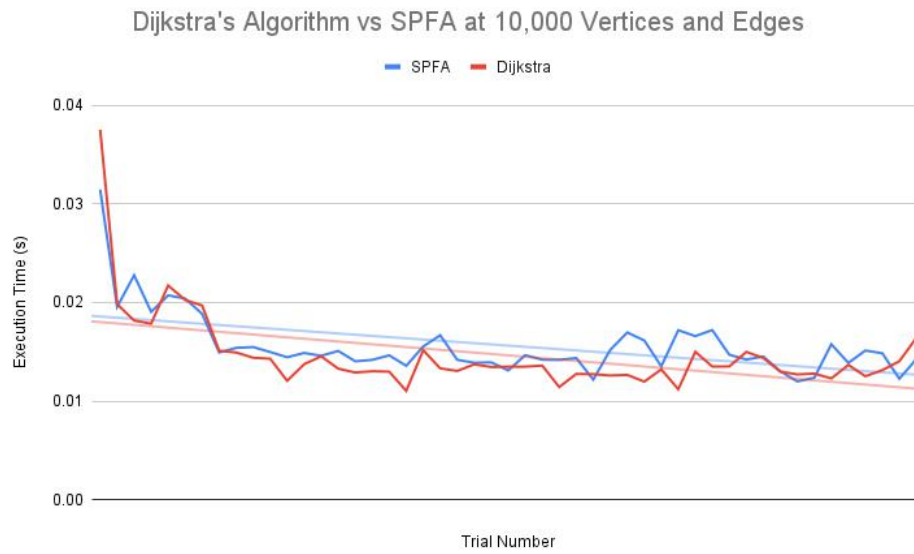
Graph 1: Dijkstra and SPFA Performance at 1, 000, 000 Data Values

To what extent does the efficiency of Dijkstra's Algorithm and Shortest Path Algorithm differentiate on randomly generated unweighted bi-directional graphs?

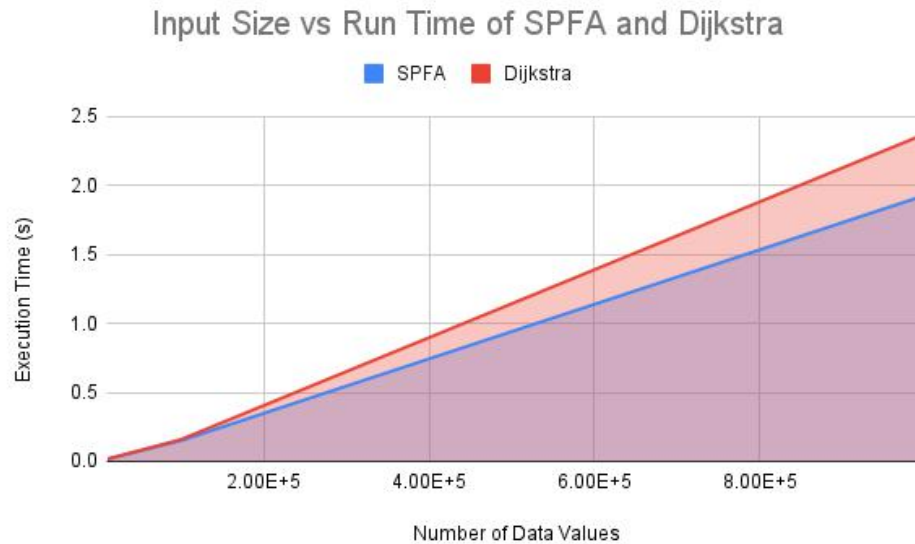


Graph 2: Dijkstra and SPFA Performance at 100, 000 Data Values

To what extent does the efficiency of Dijkstra's Algorithm and Shortest Path Algorithm differentiate on randomly generated unweighted bi-directional graphs?



Graph 3: Dijkstra and SPFA Performance at 10, 000 Data Values



Graph 4: Correlation between Input Size and Run Time of Dijkstra and SPFA

4.3: Data Analysis

The collected data is very conclusive with regards to the performance of both algorithms at various data sizes. In both the tabular and graphical representation of the collected data, it is evident that SPFA and Dijkstra have extremely similar run times at smaller data sizes. However, as the number of data values increases, so did the difference in run time. When the graph tested has 10^6 vertices and edges, the average run time of SPFA is nearly half a second faster.

4.3.1: Determining Experimental Average Time Complexity

In an earlier section of this paper, the processing speed of the computer used was determined to be around 4.6672×10^7 instructions per second. Therefore, given the number of instructions and time taken, the average time complexity of the algorithms can be calculated by the following algorithm:

$$\text{Number of Operations} = \text{Time (s)} \times \text{Instructions per Second}$$

A sample calculation for the number of operations of SPFA at 10^6 data values is shown below:

$$\begin{aligned}\text{Number of Operations} &= 1.9074 \times 4.6672 \times 10^7 \\ &= 8.9022 \times 10^7 \\ &= 89.022 N\end{aligned}$$

Given the input size of the test was 10^6 data values, it can be shown that the number of operations done by the computer is around 89 times the input values. Therefore, the experimental average time complexity of SPFA at 10^6 data values would be $O(89N)$. The following tables indicates the average time complexity of both algorithms at all three recorded data ranges.

Number of Data Values	SPFA Time Complexity	Dijkstra Time Complexity
1, 000, 000	$O(89N)$	$O(110N)$
100, 000	$O(79N)$	$O(93N)$
10, 000	$O(111N)$	$O(112N)$
Average	$O(93N)$	$O(105N)$

As seen from the table above, the average experimental time complexity of both algorithm is around 10^2 times the number of input values. Generally a relatively small constant multiplier can neglected. Therefore the average time experimentally determined time complexity of both algorithms could be simplified to $O(N)$.

4.3.2: Run Time Disparity

As seen in the final graph which shows the correlation between the run time of the two algorithms at different data sizes, as the number of input value increase, so does the run time disparity between the two algorithm. It is conclusive from the collected data that the Shortest Path Faster Algorithm outperforms Dijkstra's Shortest Path Algorithm at all three tested data values and is far more efficient at higher data sizes.

4.3.3: Inconsistency

Another interesting observation that can be made from the results of the comparison between the two algorithms would be that both algorithms are rather inconsistent. Take the example of the performance of SPFA at 10^6 data values. The error range of the experimental data is large as shown in the calculations below:

$$\begin{aligned}\text{Error Range} &= \frac{\text{Max} - \text{Min}}{2} \\ &= \frac{2.49675 - 1.52814}{2} \\ &= 0.48431\end{aligned}$$

The error range of 0.48431 accounts for a relative error range of over 25%. This clearly indicates that both algorithms are rather inconsistent and have large disparity in time complexity in randomized graphs between best and worst cases.

A possible explanation to the inconsistency noticed in the two algorithms would be edge cases. Take the example of SPFA, the algorithm adds the current nodes in a queue into a visited array to avoid adding repeated vertices into the queue. This is particularly efficient when a node has many connected edges to it. However, if many of the nodes only have a single connected edge, then the optimization would prove to be less than impactful.

5: Further Research Questions

5.1: Graphs with Negative Weight Edges

This investigation mostly focused on determining the efficiency of Dijkstra's shortest path algorithm and Shortest Path Faster Algorithm (SPFA) in bidirectional non-negative weighted graphs at various data set constraints. However, further research can be conducted on the performance of these algorithms in graphs with negative weight edges. Having negative weight as edges can significantly impact the efficiency of the algorithm or even render some useless as it affects the calculation of minimum distance.

5.2: Directional Graphs

The graphs investigated in this experiment were all bidirectional, meaning each edge can be travelled in both directions. However, this may affect the efficiency of algorithms if edges can only be travelled in a single direction, as it significantly limits the number of possible paths.

5.3: Cyclic Graphs

Another possible expansion to this research topic would be to investigate whether graphs with cycles will affect the time complexity of the algorithm. There is also the possibility that some of the algorithms will become stuck in an infinite loop within the cycle and eventually crash the program

6. Conclusion

In this paper, two single source shortest path algorithm, namely the Shortest Path Faster Algorithm (SPFA) and Dijkstra's Shortest Path Algorithm were tested with randomly generated graphs at three different data sizes to determine the average run time over fifty trials. The experimental time complexity of both algorithms were also calculate and mathematical explanations for the observations made were also provided.

From the experimental results, the average experimental time complexity were determined to be $O(93N)$ for SPFA and $O(105N)$ for Dijkstra. The time complexity can be simplified to be $O(N)$ as the constant multiplier is rather small. Furthermore, the data conclusively indicate that SPFA is more efficient than Dijkstra at all three tested data sizes and the trend indicates that the more data values inputted, the greater the run time disparity between the two algorithms.

However, both algorithms are noticed to be rather inconsistent with large disparities between their best case and worst case run times. This could be problematic at larger data values as the best and worst case scenarios can results in large differences in run times.

7: Works Cited

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8: Appendix

The following code was used for my experiments. I used them to generate the random test data, which were then fed into the Dijkstra and SPFA algorithms. Each program has a single aspect that can be changed, which is the number of edges and vertices. The only input required for the program would be to enter the number of trials to be conducted and the program will output the time complexity for each trial as well as the average time taken over all the trials at the end.

The following code were compiled in IntelliJ IDEA 2021.2.1 Community Edition on a i5500k home desktop with 8GB of Randomly-Accessed Memory (RAM).

8.1 Dijkstra

```
import java.util.*;
import java.io.*;

class Graph {
    class Edge implements Comparable<Edge>{
        int node;
        int weight;
        Edge(int node, int weight) {
            this.node = node;
            this.weight = weight;
        }

        public int compareTo(Edge o) {
            return this.weight - o.weight;
        }
    }
    int N;
    ArrayList<LinkedList<Edge>> adj = new ArrayList<>();
    boolean[] visited;
    int[] distance;
    Graph(int N) {
        this.N = N;
        visited = new boolean[N];
        distance = new int[N];
    }
}
```

```

        for (int i = 0; i < N; i++) {
            adj.add(new LinkedList<>());
        }
    }

    void add(int source, int dest, int weight) {
        adj.get(source).add(new Edge(dest,weight));
        adj.get(dest).add(new Edge(source,weight));
    }

    void dijkstra(int root) {
        PriorityQueue<Edge> q = new PriorityQueue<>();

        Arrays.fill(distance,Integer.MAX_VALUE);
        distance[root] = 0;
        q.add(new Edge(root,0));

        while (!q.isEmpty()) {
            int a = q.remove().node;
            if (visited[a]) continue;
            visited[a] = true;

            for (Edge e: adj.get(a)) {
                int b = e.node;
                int w = e.weight;
                if (distance[a] +w < distance[b]) {
                    distance[b] = distance[a] + w;
                    q.add(new Edge(b,distance[b]));
                }
            }
        }
    }
}

public class finalDijkstra {
    public static final int mx = (int) Math.pow(10, 3) - 1, d = (int) Math.pow(10, 3) -
1;
    public static double sum = 0.0;
    public static void main(String[] args) throws IOException {
        Scanner in = new Scanner(System.in);
        double trials = in.nextDouble();
        for(int f = 0; f < (int) trials; f++) {
            PrintWriter pw = new PrintWriter("test-data.txt");
            pw.println(mx + " " + mx);

            for(int i = 0; i < mx; i++) {
                pw.println(GenerateRandInt(mx) + " " + GenerateRandInt(mx) + " " + Gene
rateRandInt(d));
            }
            pw.close();

            long start = System.nanoTime(); // track start time
            Scanner sc = new Scanner(new File("test-data.txt"));
            int N = sc.nextInt(), M = sc.nextInt();
            Graph g = new Graph(N+1);
            for (int i = 0; i < M; i++) {
                int u = sc.nextInt(), v = sc.nextInt(), w = sc.nextInt();
                g.add(u,v,w);
            }

            g.dijkstra(1);
            long end = System.nanoTime(); // track the final time

```

```

        System.out.println("Trial: " + f);
        System.out.println(roundNum((end-start)/Math.pow(10, 9)));
        sum += (end-start)/Math.pow(10, 9);
        System.out.println();
    }
    System.out.println("Average: " + roundNum(sum/trials));
}
}
public static double roundNum (double x) {
    return Math.round(x * 100000.0) / 100000.0;
}
public static int GenerateRandInt(int bound) {
    Random rand = new Random();
    return rand.nextInt(bound);
}
}
}

```

8.2 SPFA

```

import java.util.*;
import java.io.*;

public class SPFA {
    public static final int mx = (int) Math.pow(10, 3) - 1, d = (int) Math.pow(10, 3) - 1;

    public static void main(String[] args) throws IOException {
        Scanner in = new Scanner(System.in);
        double trials = in.nextDouble(), sum = 0.00;
        for(int f = 0; f < (int) trials; f++) {
            PrintWriter pw = new PrintWriter("test-data.txt");
            pw.println(mx + " " + mx);

            for(int i = 0; i < mx; i++) {
                pw.println(GenerateRandInt(mx) + " " + GenerateRandInt(mx) + " " + GenerateRandInt(d));
            }
            pw.close();

            long start = System.nanoTime();

            Scanner sc = new Scanner(new File("test-data.txt"));

            int V = sc.nextInt(), E = sc.nextInt();
            ArrayList<ArrayList<Edge>> adj = new ArrayList<ArrayList<Edge>>();
            for (int i = 0; i <= V; i++) {
                adj.add(new ArrayList<Edge>());
            }
            int dis[] = new int[V + 1];
            Arrays.fill(dis, 0x3f3f3f3f);
            boolean inQ[] = new boolean[V + 1];
            Arrays.fill(inQ, false);

            for (int i = 1; i <= E; i++) {
                int u = sc.nextInt(), v = sc.nextInt(), w = sc.nextInt();
                adj.get(u).add(new Edge(v, w));
                adj.get(v).add(new Edge(u, w));
            }
        }
    }
}

```

```

LinkedList<Integer> q = new LinkedList<Integer>();
dis[1] = 0;
inQ[1] = true;
q.add(1);

while (!q.isEmpty()) {
    int u = q.poll();
    inQ[u] = false;
    for (Edge e : adj.get(u)) {
        if (dis[u] + e.w < dis[e.v]) {
            dis[e.v] = dis[u] + e.w;
            if (inQ[e.v] == false) {
                q.add(e.v);
                inQ[e.v] = true;
            }
        }
    }
}

long end = System.nanoTime();
System.out.println("Trial: " + f);
System.out.println(roundNum((end-start)/Math.pow(10, 9)));
sum += (end-start)/Math.pow(10, 9);
System.out.println();
}
System.out.println("Average: " + roundNum(sum/trials));
}

public static double roundNum (double x) {
    return Math.round(x * 100000.0) / 100000.0;
}

public static int GenerateRandInt(int bound) {
    Random rand = new Random();
    return rand.nextInt(bound);
}

public static class Edge {
    int v, w;
    public Edge(int v, int w) {
        this.v = v;
        this.w = w;
    }
}
}

```