# EC_ExpectationMaximizationDemofor2ComponentGMM

April 25, 2019

## 1 Extra Credit: Expectation Maximization Demo for 2 Component GMM

This is an OPTIONAL extra credit assignment worth 10 points. Any points earned on this assignment will be applied to one of your exam grades, excluding the final exam (Exam 1 and Exam 2 only).

Assignment

Recreate the demo video showed in class on 4/17, or show iterated frames from the simulation. The video is also in the slides from class as well as posted under the respective module on the Modules page.

Generate synthetic data drawn from 2 independent Gaussian distributions with non-circular covariance matrices and means at least 5 units apart (for easy convergence). Apply the EM algorithm to fit a 2-component GMM to the data. Record the iterations either in a video or in successive plots showing the data and the contour plots of the individual Gaussian models at each iteration. Also show the "ground-truth" means and covariances you defined in step 1, as well as the estimates the EM algorithm converged to. A table might be the best way to show this. Your deliverables are:

Either a video or a series of plots showing successive iterations of the algorithm. A comparison of the ground-truth means/covariances and the estimated means/covariances. Your code. You may not use built in GMM or EM functions for this assignment.

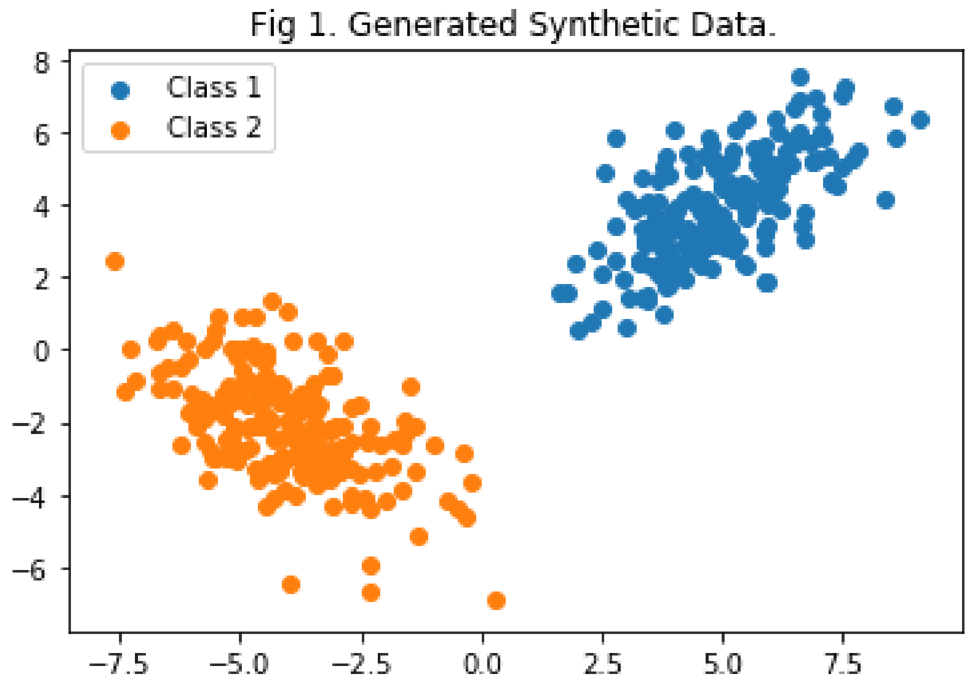
```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        import random
        import math
        import scipy.stats as ss

        #### Given parameters
        mean1 = np.array([5,4])
        mean2 = np.array([-4,-2])
        cov1  = np.array([[2,1.2],[1.2,2]])
        cov2  = np.array([[2,-1],[-1,2]])

        # Generating Training and Testing Data
        # Creating a class for the dataset
        class Data_set:
            def __init__(self, mean, cov):
```

```
            self.mean = mean
            self.cov  = cov

        def split_data(self):
            np.random.shuffle(self.data)
            self.train = self.data[:len(self.data)//2]
            self.test  = self.data[len(self.data)//2:]

            return self

        def multivariate_normal(self, num):
            # asself.data.shape = num *  2
            self.data = np.random.multivariate_normal(self.mean,self.cov,size=num)

            return self
```

In [2]:
```
# Generating Data sets
c1 = Data_set(mean1, cov1)
c2 = Data_set(mean2, cov2)

c1.multivariate_normal(200)
c2.multivariate_normal(200)
```

Out[2]: `<__main__.Data_set at 0x7f10b81402e8>`

In [3]:
```
# Ploting synthetic data
f1 = plt.figure(1)
p1 = f1.add_subplot(111)
p1.scatter(c1.data[:,0],c1.data[:,1], label='Class 1')
p1.scatter(c2.data[:,0],c2.data[:,1], label='Class 2')
p1.legend()
p1.set_title("Fig 1. Generated Synthetic Data.")
plt.show()
```




```
In [4]: all_data = np.vstack((c1.data, c2.data))
        np.random.shuffle(all_data)
        # plt.scatter(all_data[:,0], all_data[:,1])

In [9]: # def my_multiply(a,b):
        #     # a is an array
        #     # b is a element
        #     result = []
        #     for item in a:
        #         result.append(item*b)

        #     return result

        def EW_multiply(a,b):
            # they are both arrays
            # this function multply them one by one
            result = []
            for i in range(len(a)):
                result.append(a[i]*b[i])

            return result

        def Expectation_Max(data):
            # Step 1: Initial Guess
            # - choose random values for means
```

```
    # - set covariance to overall covariance
    _mean1 = random.choice(data)
    _mean2 = random.choice(data)
    _cov1  = np.cov(data.T)
    _cov2  = np.cov(data.T)
    _pi    = 0.5

    print("Iteration | Pi")

    iterations = 0
    while(1):
        iterations = iterations+1
        # Expectation step, calculating responsibility
        _gamma = []
        _prev_pi = _pi


        # Calculating means
        for sample in data:
            _rv1  = ss.multivariate_normal(_mean1, _cov1)
            _rv2  = ss.multivariate_normal(_mean2, _cov2)
            num   = _pi * _rv2.pdf(sample)
            denom = (1 - _pi)*_rv1.pdf(sample) + _pi*_rv2.pdf(sample)
            _gamma.append(num/denom)
#         plt.clf()
#         plt.plot(_gamma)
        _gamma = np.array(_gamma)

        # Maximization step, calculating
        temp1 = EW_multiply((1-_gamma), data)
        temp2 = EW_multiply(_gamma, data)
        _mean1 = sum(temp1)/sum(1-_gamma)
        _mean2 = sum(temp2)/sum(_gamma)


        # Calculating _cov1
        num = []
        denom = []
        for i in range(len(all_data)):
            num.append((1-_gamma[i])*((data[i][None]-_mean1).T.dot(data[i][None]-_mean
            denom.append(1-_gamma[i])
        _cov1 = sum(num)/sum(denom)


        # Calculating _cov2
        num = []
        denom = []
        for i in range(len(all_data)):
```

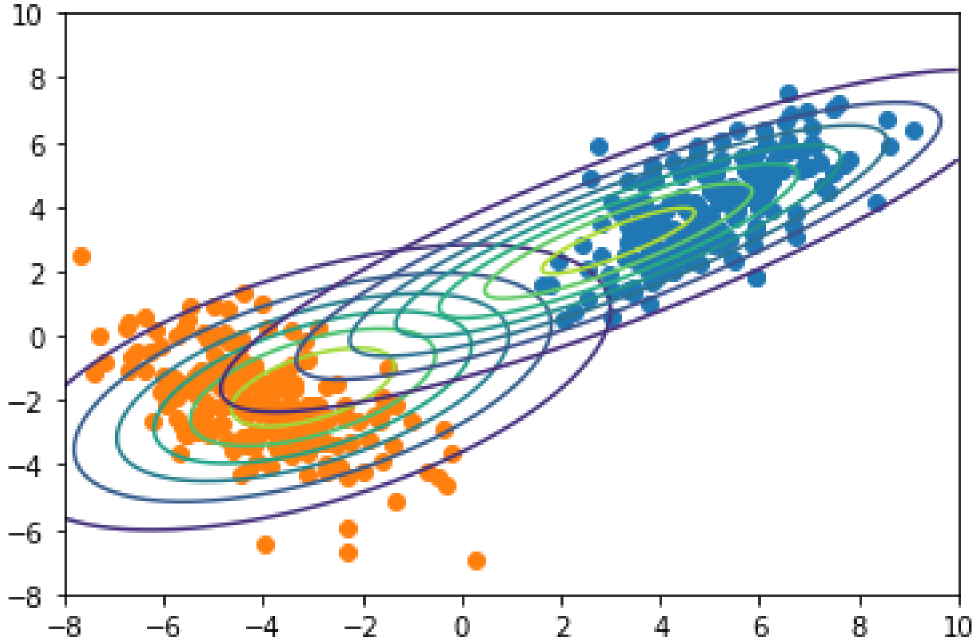
```
                num.append(_gamma[i]*((data[i][None]-_mean2).T.dot(data[i][None]-_mean2)))
                denom.append(_gamma[i])
            _cov2 = sum(num)/sum(denom)

            _pi    = sum(_gamma)/len(_gamma)
            print(iterations,_pi)

            # Plotting
            plt.clf()

            rv1 = ss.multivariate_normal(_mean1, _cov1)
            rv2 = ss.multivariate_normal(_mean2, _cov2)

            x, y = np.mgrid[-8:10:.01, -8:10:.01]
            pos = np.dstack((x, y))
            f2 = plt.figure(2)
            p2 = f2.add_subplot(111)
            p2.scatter(c1.data[:,0],c1.data[:,1], label='Class 1')
            p2.scatter(c2.data[:,0],c2.data[:,1], label='Class 2')
            p2.contour(x,y,rv1.pdf(pos))
            p2.contour(x,y,rv2.pdf(pos))
            plt.pause(0.05)


            # converge condition
            if abs(_prev_pi-_pi)<0.00001:
                print("Converged!")
                return _mean1, _mean2, _cov1, _cov2, _pi
```

In [10]:
```
mean1, mean2, cov1, cov2
```

Out[10]:
```
(array([5, 4]), array([-4, -2]), array([[2. , 1.2],
        [1.2, 2. ]]), array([[ 2, -1],
        [-1,  2]]))
```

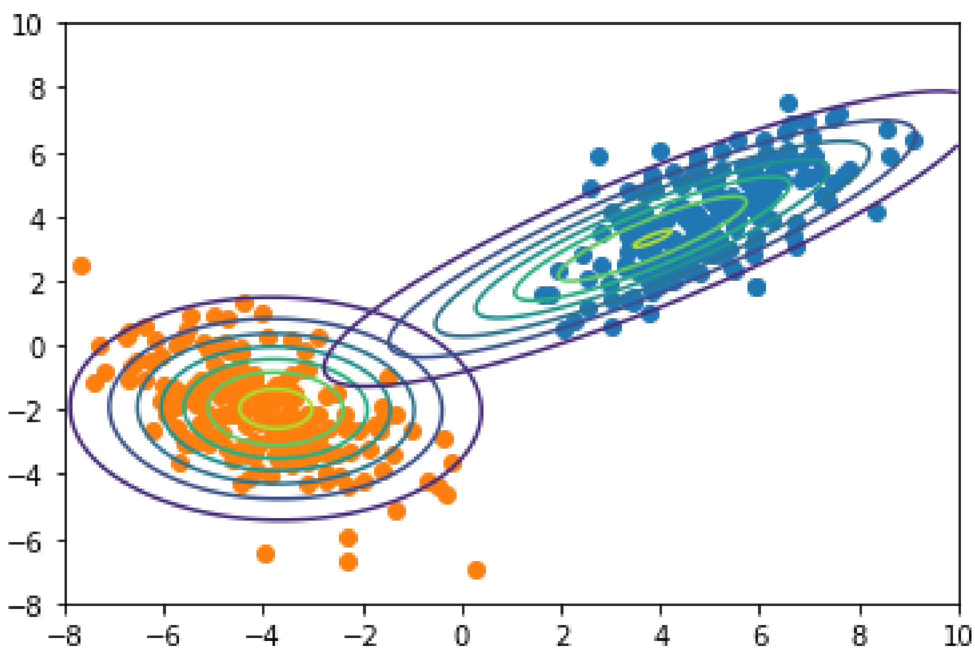
In [11]:
```
[_mean1, _mean2, _cov1, _cov2, _pi] = Expectation_Max(all_data)
```

```
Iteration | Pi
1 0.5627038136305361


<Figure size 432x288 with 0 Axes>
```



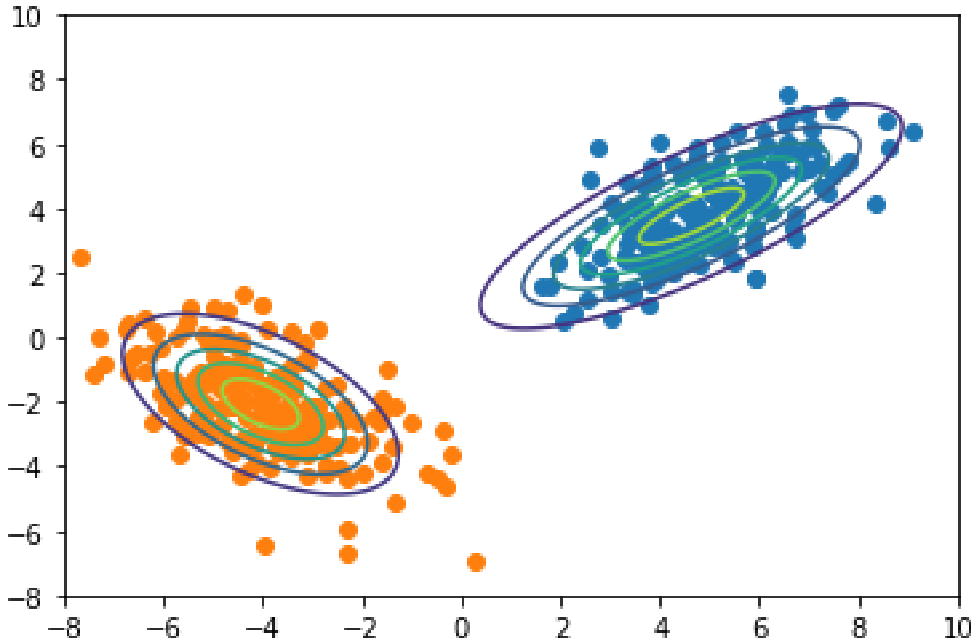

```
2 0.5550164048404276
```

```
<Figure size 432x288 with 0 Axes>
```



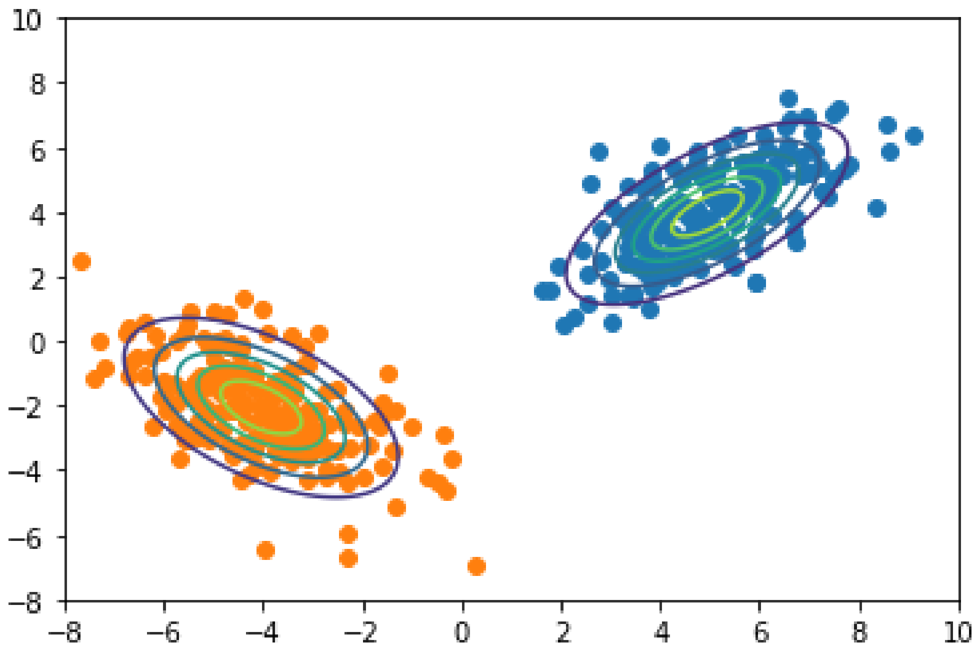
```
3 0.5199314195792062
```

```
<Figure size 432x288 with 0 Axes>
```



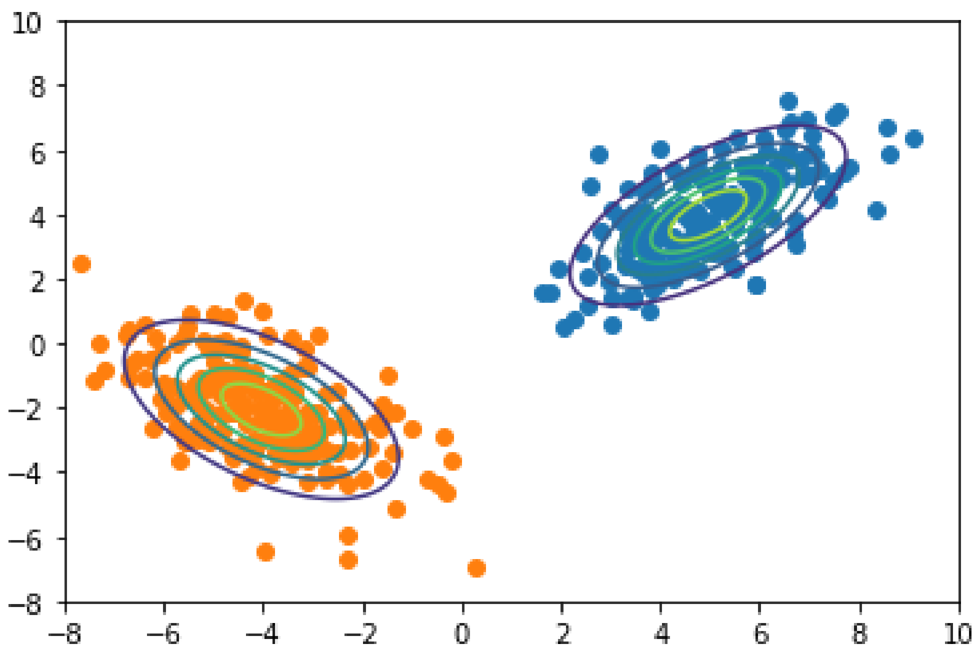

```
4 0.5013712822168638
```

```
<Figure size 432x288 with 0 Axes>
```



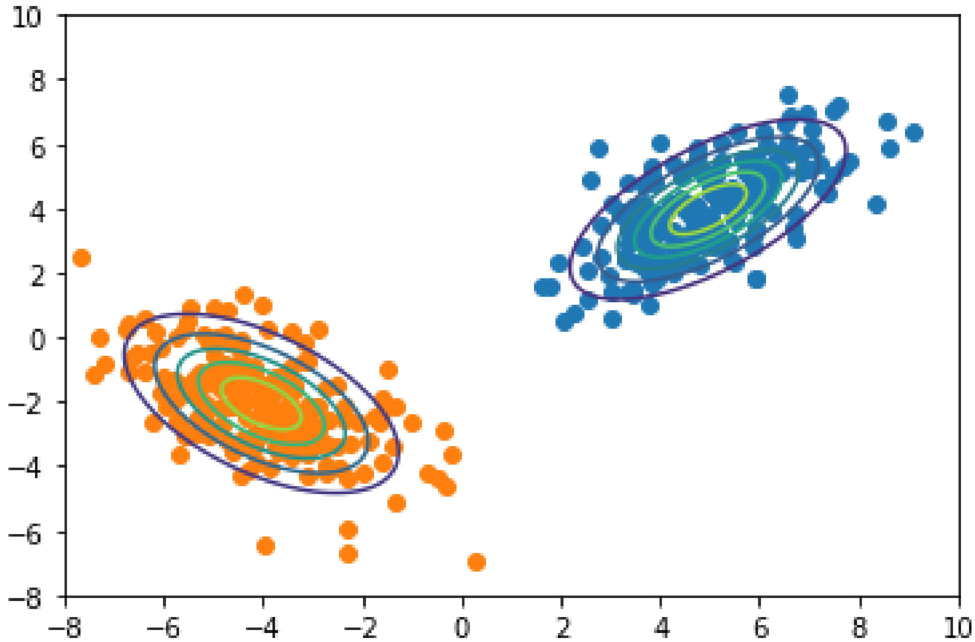

```
5 0.5000070368062124
```

```
<Figure size 432x288 with 0 Axes>
```



```
6 0.5000035041187288
```

```
<Figure size 432x288 with 0 Axes>
```




```
Converged!
```

```
In [12]: print("True Means are:", mean1, mean2)
         print("Estimate Means are:", _mean1, _mean2)
         print("\n")
         print("True Covariances are:\n", cov1, "\n\n", cov2)
         print("Estimate Covariances are:\n", _cov1, "\n\n", _cov2)
```

```
True Means are: [5 4] [-4 -2]
Estimate Means are: [-4.03031968 -2.0588895 ] [4.96437693 3.9651535 ]


True Covariances are:
 [[2.  1.2]
 [1.2 2. ]]
```

```
 [[ 2 -1]
 [-1  2]]
Estimate Covariances are:
 [[ 2.16838263 -1.18480835]
 [-1.18480835  2.19269594]]

 [[1.98819759 1.29722499]
 [1.29722499 2.01187409]]
```