Having completed the project of forecasting potential future crop yields in Minnesota, you get a follow-up assignment from bank management. They realize that existing clients could be engaged in further business by helping them hedge their risks related to climate change. Therefore, they want you to develop a method that provides an optimal partial hedging strategy for clients growing corn under each of three high-level scenarios (Scenario 1, 2 and 3 in the attachment).

In the partial hedging strategy, the client uses agricultural derivatives (specifically, call and put options on the market price of corn) to reduce the likelihood of having a low income after harvest. Derivatives are purchased in advance, when the harvested yield and the market price at which the corn will be sold are still uncertain. Rather, yield and market price are only known probabilistically, represented by 1000 equally likely (yield,price) pairs in each scenario. Your task is to suggest a set of derivatives that would result in the highest expected (i.e., average) income for the client, with the constraint that both the lowest 10% and 25% percentile points of the income distribution are at or above the specific values in Table 1 below. Make sure to also include the cost of derivatives in the income and the income distribution.

First, develop a general method/program that solves this problem. Then, apply that to find the most optimal set of derivatives in the 3 scenarios. For each scenario, you are expected to return a vector that gives how many of each kind of derivative the client should purchase. Each vector must maximize the expected income whilst meeting the constraints shown in Table 1. (For example, with the hedge, in Scenario 1 the maximum of the lowest 10% of incomes should be at least $1,320,000 and the maximum of the lowest 25% of incomes should be at least $1,400,000.)

Scenario	25th percentile lower limit [thousands of USD]	10th percentile lower limit [thousands of USD]
Scenario 1	1400	1320
Scenario 2	1300	1200
Scenario 3	1370	1260

Table 1.

There are 2x7 = 14 derivatives available for hedging: put and call options at strikes 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, and 8.0 USD/bushel. These may be abbreviated as P5.0, P5.5, P6.0, P6.5, P7.0, P7.5, P8.0, C5.0, C5.5, C6.0, C6.5, C7.0, C7.5, and C8.0. Clients can only buy options (not sell), and no fractional options can be bought. I.e., your solution vectors should only contain nonnegative integers. Prices for all 14 options are provided in each scenario.

All options are cash-settled, i.e., pay out in dollars (there is no need to purchase/sell actual crops when an option is exercised). When the market price is *S,* put options of strike *K* have the payoff

$$if\ S < K:\ \ Payoff_{Put}\,(S,\,K)\ = A(K-S)$$

$$if\ S \geq K:\ \ Payoff_{Put}\,(S,\,K)\ = 0\,,$$

while call options of strike *K* have the payoff

$$if\ S > K:\ \ Payoff_{Call}\,(S,\,K)\ = A(S-K)$$

$$if\ S \leq K: \quad Payoff_{Call}\,(S, K) = 0\ .$$

Here *A* = 100 bushels. Thus, if the price is *S* = \$5/bushel, then each put option of strike *K* = 2*S* pays \$500.

You need to submit

1) the 3 vectors that lead to the highest expected gain while meeting the constraints, and
2) the codes that you wrote to get to these vectors.

Finally,

3) you will have 15 minutes to present your methodology and results to the senior leadership of the bank (10-minute presentation, followed by a 5-minute Q&A session).