Ex: Subjects in a sleep deprivation study were asked to solve a set of addition problems after having been deprived of sleep for a specified number of hours.

Hours w/o sleep, x	8	8	12	12	16	16	20	20	24	24
Errors, y	8	6	6	10	14	8	14	12	12	16

(a) Plot the data (use R if available)

(b) Fit a linear regression model to the data.

(c) Get confidence intervals for all 3 parameters in the model $(\beta_0, \beta_1, \sigma^2)$

(d) Do hypothesis tests for the slope and intercept parameters using a zero-valued null.

(e) Do a hypothesis test for $\sigma > 2$.

(f) Compare your computations with the summary output for a linear model in R.

Inference for $E(Y\,|x)$: Let's say we want to estimate the average number of errors committed by someone deprived of sleep for $x^* = 18$ hours. We have been using the estimator $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 x^*$ to estimate

$$E(Y\,|x^*) = \beta_0 + \beta_1 x^*.$$

Note that since $\hat{\beta}_0$ and $\hat{\beta}_1$ are normal, then $\hat{Y}$ is normally distributed as well.

(a) Find $E(\hat{Y})$. Is the estimator biased?

(b) Show that $V(\hat{Y}) = \sigma^2 \left(\dfrac{1}{n} + \dfrac{(x^* - \bar{x})^2}{S_{xx}} \right)$ [if time]

(c) Use the previous result to construct a 95% CI for the average number of errors committed by someone deprived of sleep for 18 hours.