

CS 325 - Homework 7

1. (7 pts) Let X and Y be two decision problems. Suppose we know that X reduces to Y in polynomial time. Which of the following can we infer? Explain

- a. If Y is NP-complete then so is X .
- b. If X is NP-complete then so is Y .
- c. If Y is NP-complete and X is in NP then X is NP-complete.
- d. If X is NP-complete and Y is in NP then Y is NP-complete.
- e. X and Y can't both be NP-complete.
- f. If X is in P, then Y is in P.
- g. If Y is in P, then X is in P.

2. (4 pts) Consider the problem COMPOSITE: given an integer y , does y have any factors other than one and itself? For this exercise, you may assume that COMPOSITE is in NP, and you will be comparing it to the well-known NP-complete problem SUBSET-SUM: given a set S of n integers and an integer target t , is there a subset of S whose sum is exactly t ? Clearly explain whether or not each of the following statements follows from that fact that COMPOSITE is in NP and SUBSET-SUM is NP-complete:

- a. $\text{SUBSET-SUM} \leq_p \text{COMPOSITE}$.
- b. If there is an $O(n^3)$ algorithm for SUBSET-SUM, then there is a polynomial time algorithm for COMPOSITE.
- c. If there is a polynomial algorithm for COMPOSITE, then $P = NP$.
- d. If $P \neq NP$, then **no** problem in NP can be solved in polynomial time.

3. (3 pts) Two well-known NP-complete problems are 3-SAT and TSP, the traveling salesman problem. The 2-SAT problem is a SAT variant in which each clause contains at most two literals. 2-SAT is known to have a polynomial-time algorithm. Is each of the following statements true or false? Justify your answer.

- a. $3\text{-SAT} \leq_p \text{TSP}$.
- b. If $P \neq NP$, then $3\text{-SAT} \leq_p 2\text{-SAT}$.
- c. If $P \neq NP$, then no NP-complete problem can be solved in polynomial time.

4. (6 pts) A Hamiltonian path in a graph is a simple path that visits every vertex exactly once. Show that $\text{HAM-PATH} = \{ (G, u, v) : \text{there is a Hamiltonian path from } u \text{ to } v \text{ in } G \}$ is NP-complete. You may use the fact that HAM-CYCLE is NP-complete

5. (5 pts) LONG-PATH is the problem of, given (G, u, v, k) where G is a graph, u and v vertices and k an integer, determining if there is a simple path in G from u to v of length at least k . Prove that LONG-PATH is NP-complete.