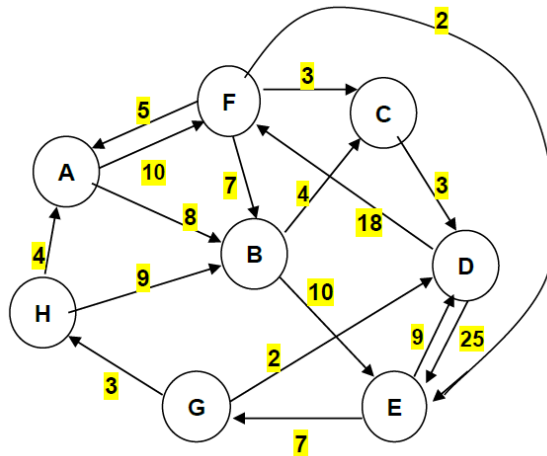


CS 325 Homework Assignment 6  
Peter Moldenhauer  
11/6/17

- 1 Shortest paths can be cast as an LP using distances  $d_v$  from the source  $s$  to a particular vertex  $v$  as variables.  
We can compute the shortest path from  $s$  to  $t$  in a weighted directed graph by solving:  
max  $d_t$   
subject to  
 $d_s = 0$   
 $d_v - d_u \leq w(u,v)$  for all  $(u,v)$  in  $E$   
We can compute the single source by changing the objective function to:  
max  $\sum_{v \in V} d_v$   
Use linear programming to answer the questions below. Submit a copy of the LP code and output.



- a) Find the distance of the shortest path from G to C in the graph above

The distance of the shortest path from G to C is 16.

LP code and output is on the following page...

```

LINDO
File Edit Solve Reports Window
<untitled>
max dc
ST
dg = 0
dh - dg <= 3
dd - dg <= 2
da - dh <= 4
db - dh <= 9
df - dd <= 18
de - dd <= 25
db - da <= 8
df - da <= 10
dc - db <= 4
de - db <= 10
dg - de <= 7
dd - de <= 9
dc - df <= 3
db - df <= 7
da - df <= 5
dd - dc <= 3
de - df <= 2

```

LP OPTIMUM FOUND AT STEP 6

OBJECTIVE FUNCTION VALUE

1) 16.00000

VARIABLE	VALUE	REDUCED COST
DC	16.000000	0.000000
DG	0.000000	0.000000
DH	3.000000	0.000000
DD	0.000000	0.000000
DA	4.000000	0.000000
DB	12.000000	0.000000
DF	13.000000	0.000000
DE	0.000000	0.000000

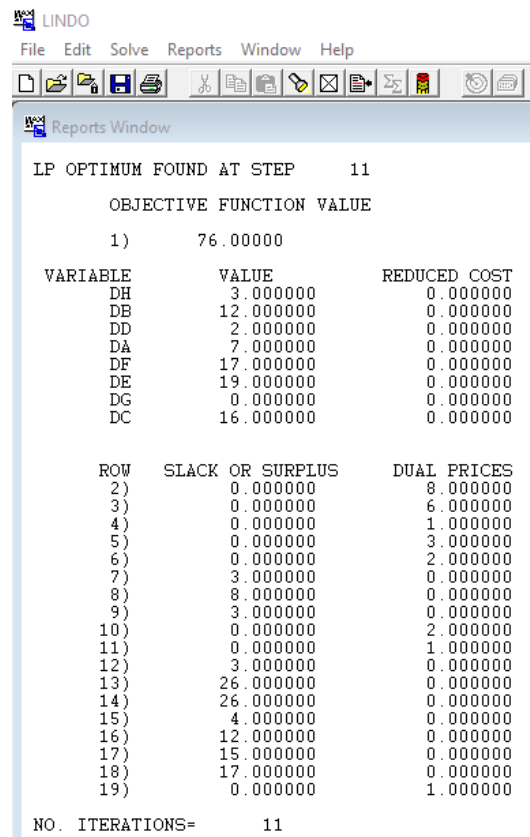
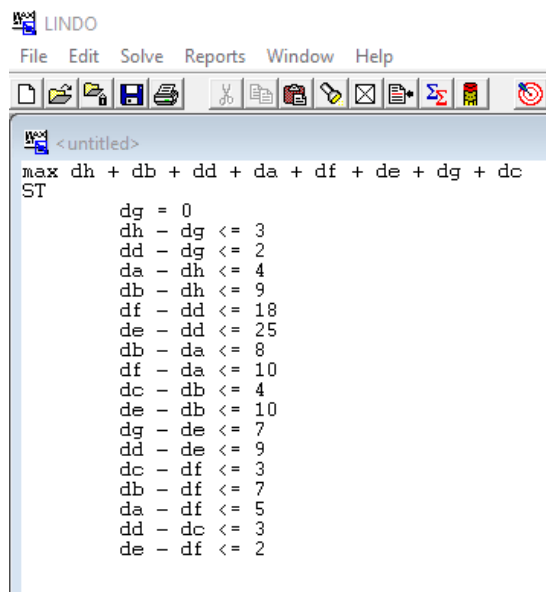
ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000
3)	0.000000	1.000000
4)	2.000000	0.000000
5)	3.000000	0.000000
6)	0.000000	1.000000
7)	5.000000	0.000000
8)	25.000000	0.000000
9)	0.000000	0.000000
10)	1.000000	0.000000
11)	0.000000	1.000000
12)	22.000000	0.000000
13)	7.000000	0.000000
14)	9.000000	0.000000
15)	0.000000	0.000000
16)	8.000000	0.000000
17)	14.000000	0.000000
18)	19.000000	0.000000
19)	15.000000	0.000000

NO. ITERATIONS= 6

b) Find the distances of the shortest paths from G to all other vertices

- The shortest path distance from G to A is 7.
- The shortest path distance from G to B is 12.
- The shortest path distance from G to C is 16.
- The shortest path distance from G to D is 2.
- The shortest path distance from G to E is 19.
- The shortest path distance from G to F is 17.
- The shortest path distance from G to G is 0.
- The shortest path distance from G to H is 3.

LP code and output on the following page...



- 2 Acme Industries produces four types of men's ties using three types of material. Your job is to determine how many of each type of tie to make each month. The goal is to maximize profit, profit per tie = selling price – labor cost – material cost. Labor cost is \$0.75 per tie for all four types of ties. The material requirements and costs are given below.

Material	Cost per yard	Yards available per month
Silk	\$20	1,000
Polyester	\$6	2,000
Cotton	\$9	1,250

Product Information	Type of Tie			
	Silk = s	Poly = p	Blend1 = b	Blend2 = c
Selling Price per tie	\$6.70	\$3.55	\$4.31	\$4.81
Monthly Minimum units	6,000	10,000	13,000	6,000
Monthly Maximum units	7,000	14,000	16,000	8,500

Material Information in yards	Type of Tie			
	Silk	Polyester	Blend 1 (50/50)	Blend 2 (30/70)
Silk	0.125	0	0	0
Polyester	0	0.08	0.05	0.03
Cotton	0	0	0.05	0.07

type	selling price	labor	material	profit per tie
silk s	6.7	0.75	2.5	3.45
poly p	3.55	0.75	0.48	2.32
blend1 b	4.31	0.75	0.75	2.81
blend2 c	4.81	0.75	0.81	3.25

Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. What are the optimal numbers of ties of each type to maximize profit? Include a copy of the code and output.

Goal: maximize profit (profit for each of the four types of ties)

Profit per tie = selling price – labor cost – material cost

Profit for 1 silk tie = 3.45

Profit for 1 Polyester tie = 2.32

Profit for 1 Blend 1 tie = 2.81

Profit for 1 Blend 2 tie = 3.25

$x_1$  = silk,  $x_2$  = Polyester,  $x_3$  = Blend 1,  $x_4$  = Blend 2

Using the above information, this problem can be formulated as a linear program with an objective function and all of the constraints by the following:

Maximize Profit =  $3.45x_1 + 2.32x_2 + 2.81x_3 + 3.25x_4$

Subject To

$0.125x_1 \leq 1000$  (constraint on yards avail. for silk)

$.08x_2 + .05x_3 + .03x_4 \leq 2000$  (constraint on yards avail. for polyester)

$.05x_3 + .07x_4 \leq 1250$  (constraint on yards avail. for cotton)

$x_1 \geq 6000$  (constraint on min. silk ties)

$x_1 \leq 7000$  (constraint on max. silk ties)

$x_2 \geq 10000$  (constraint on min. polyester ties)

$x_2 \leq 14000$  (constraint on max. polyester ties)

$x_3 \geq 13000$  (constraint on min. blend 1 ties)

$x_3 \leq 16000$  (constraint on max. blend 1 ties)

$x_4 \geq 6000$  (constraint on min. blend 2 ties)

$x_4 \leq 8500$  (constraint on max. blend 2 ties)

$x_1, x_2, x_3, x_4 \geq 0$  (constraint on no non-negative tie amounts)

The optimal solution (max. profit) for this linear program is: \$120196.00

The optimal numbers of ties of each type to maximize profit is the following:

# of silk ties → 7000  
 # of polyester ties → 13625  
 # of blend 1 ties → 13100  
 # of blend 2 ties → 8500

LP code and output below...

```

LINDO
File Edit Solve Reports Window Help
<untitled>
max 3.45x1 + 2.32x2 + 2.81x3 + 3.25x4
ST
.125x1 <= 1000
.08x2 + .05x3 + .03x4 <= 2000
.05x3 + .07x4 <= 1250
x1 >= 6000
x1 <= 7000
x2 >= 10000
x2 <= 14000
x3 >= 13000
x3 <= 16000
x4 >= 6000
x4 <= 8500
x1 >= 0
x2 >= 0
x3 >= 0
x4 >= 0

```

LINDO

File Edit Solve Reports Window Help

Reports Window

OBJECTIVE FUNCTION VALUE

1) 120196.0

VARIABLE	VALUE	REDUCED COST
X1	7000.000000	0.000000
X2	13625.000000	0.000000
X3	13100.000000	0.000000
X4	8500.000000	0.000000

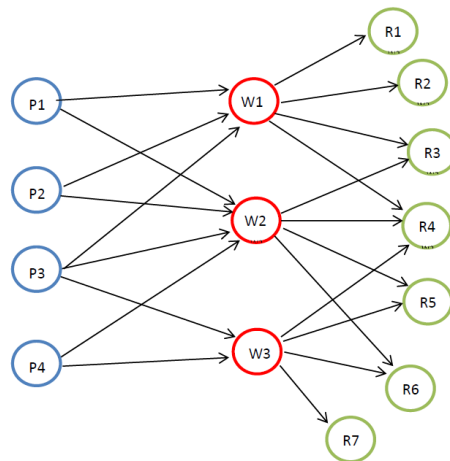
  

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	125.000000	0.000000
3)	0.000000	29.000000
4)	0.000000	27.200001
5)	1000.000000	0.000000
6)	0.000000	3.450000
7)	3625.000000	0.000000
8)	375.000000	0.000000
9)	100.000000	0.000000
10)	2900.000000	0.000000
11)	2500.000000	0.000000
12)	0.000000	0.476000
13)	7000.000000	0.000000
14)	13625.000000	0.000000
15)	13100.000000	0.000000
16)	8500.000000	0.000000

NO. ITERATIONS= 4

- 3 This is an extension of the transportation model. There are now intermediate transshipment points added between the sources (plants) and destinations (retailers). Items being shipped from a Plant ( $p_i$ ) must be shipped to a Warehouse ( $w_j$ ) before being shipped to the Retailer ( $r_k$ ). Each plant will have an associated supply ( $s_i$ ) and each retailer will have a demand ( $d_k$ ). The number of plants is  $n$ , number of warehouses is  $q$  and the number of retailers is  $m$ . The edges ( $i,j$ ) from plant ( $p_i$ ) to warehouse ( $w_j$ ) have costs associated denoted  $cp(i,j)$ . The edges ( $j,k$ ) from a warehouse ( $w_j$ ) to a retailer ( $r_k$ ) have costs associated denoted  $cw(j,k)$ .

The graph below shows the transshipment map for a manufacturer of refrigerators. Refrigerators are produced at four plants and then shipped to a warehouse (weekly) before going to the retailer.



Below are the costs of shipping from a plant to a warehouse and then a warehouse to a retailer. If it is impossible to ship between the two locations an X is placed in the table.

cost	W1	W2	W3
P1	\$10	\$15	X
P2	\$11	\$8	X
P3	\$13	\$8	\$9
P4	X	\$14	\$8

cost	R1	R2	R3	R4	R5	R6	R7
W1	\$5	\$6	\$7	\$10	X	X	X
W2	X	X	\$12	\$8	\$10	\$14	X
W3	X	X	X	\$14	\$12	\$12	\$6

The tables below give the capacity of each plant (supply) and the demand for each retailer (per week).

	P1	P2	P3	P4
Supply	150	450	250	150

	R1	R2	R3	R4	R5	R6	R7
Demand	100	150	100	200	200	150	100

Your goal is to determine the number of refrigerators to be shipped plants to warehouses and then warehouses to retailers to minimize the cost. Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. What are the optimal shipping routes and minimum cost. Include a copy of the code and output.

Goal: Minimize the cost of shipping refrigerators from plants to warehouses to retailers. To do this we need to add up all of the costs per each possible route (plants to warehouses and warehouses to retailers). We then take the min. of

this total amount with using the constraints of the supply and demand for the plants and retail locations.

This problem can be formulated as a linear program with an objective function and all constraints in the following manner:

$$\begin{aligned} \text{Minimize Cost} = & 10(p_1w_1) + 15(p_1w_2) + 11(p_2w_1) + 8(p_2w_2) + 13(p_3w_1) + \\ & 8(p_3w_2) + 9(p_3w_3) + 14(p_4w_2) + 8(p_4w_3) + 5(w_1r_1) + 6(w_1r_2) + 7(w_1r_3) + \\ & 12(w_2r_3) + 10(w_1r_4) + 8(w_2r_4) + 14(w_3r_4) + 10(w_2r_5) + 12(w_3r_5) + \\ & 14(w_2r_6) + 12(w_3r_6) + 6(w_3r_7) \end{aligned}$$

Subject to

$$\begin{aligned} p_1w_1 + p_1w_2 & \leq 150 \\ p_2w_1 + p_2w_2 & \leq 450 \\ p_3w_1 + p_3w_2 + p_3w_3 & \leq 250 \\ p_4w_2 + p_4w_3 & \leq 150 \end{aligned}$$

$$\begin{aligned} w_1r_1 & \geq 100 \\ w_1r_2 & \geq 150 \\ w_1r_3 + w_2r_3 & \geq 100 \\ w_1r_4 + w_2r_4 + w_3r_4 & \geq 200 \\ w_2r_5 + w_3r_5 & \geq 200 \\ w_2r_6 + w_3r_6 & \geq 150 \\ w_3r_7 & \geq 100 \end{aligned}$$

$$\begin{aligned} w_1r_1 + w_1r_2 + w_1r_3 + w_1r_4 - p_1w_1 - p_2w_1 - p_3w_1 & = 0 \\ w_2r_3 + w_2r_4 + w_2r_5 + w_2r_6 - p_1w_2 - p_2w_2 - p_3w_2 - p_4w_2 & = 0 \\ w_3r_4 + w_3r_5 + w_3r_6 + w_3r_7 - p_3w_3 - p_4w_3 & = 0 \end{aligned}$$

$$p_1w_1, p_1w_2, p_2w_1, p_2w_2, p_3w_1, p_3w_2, p_3w_3, p_4w_2, p_4w_3, w_1r_1, w_1r_2, w_1r_3, w_2r_3, w_1r_4, w_2r_4, w_3r_4, w_2r_5, w_3r_5, w_2r_6, w_3r_6, w_3r_7 \geq 0$$

The optimal solution (minimal cost) for this linear program is: \$17100.00

The optimal shipping routes are as follows:

p1 to w1 (150 units)  
p2 to w1 (200 units)  
p2 to w2 (250 units)  
p3 to w2 (150 units)  
p3 to w3 (100 units)  
p4 to w3 (150 units)  
w1 to r1 (100 units)  
w1 to r2 (150 units)  
w1 to r3 (100 units)  
w2 to r4 (200 units)  
w2 to r5 (200 units)

w3 to r6 (150 units)

w3 to r7 (100 units)

Note: p = plant, w = warehouse, r = retailer and refrigerators are the units

LP code and output below...

```
LINDO
File Edit Solve Reports Window Help
<untitled>
min 10 p1w1 + 15 p1w2 + 11 p2w1 + 8 p2w2 + 13 p3w1 +
8 p3w2 + 9 p3w3 + 14 p4w2 + 8 p4w3 + 5 w1r1 + 6 w1r2 +
7 w1r3 + 12 w2r3 + 10 w1r4 + 8 w2r4 + 14 w3r4 + 10 w2r5 +
12 w3r5 + 14 w2r6 + 12 w3r6 + 6 w3r7
ST
    p1w1 + p1w2 <= 150
    p2w1 + p2w2 <= 450
    p3w1 + p3w2 + p3w3 <= 250
    p4w2 + p4w3 <= 150

    w1r1 >= 100
    w1r2 >= 150
    w1r3 + w2r3 >= 100
    w1r4 + w2r4 + w3r4 >= 200
    w2r5 + w3r5 >= 200
    w2r6 + w3r6 >= 150
    w3r7 >= 100

    w1r1 + w1r2 + w1r3 + w1r4 - p1w1 - p2w1 - p3w1 = 0
    w2r3 + w2r4 + w2r5 + w2r6 - p1w2 - p2w2 - p3w2 - p4w2 = 0
    w3r4 + w3r5 + w3r6 + w3r7 - p3w3 - p4w3 = 0

    p1w1 >= 0
    p1w2 >= 0
    p2w1 >= 0
    p2w2 >= 0
    p3w1 >= 0
    p3w2 >= 0
    p3w3 >= 0
    p4w2 >= 0
    p4w3 >= 0
    w1r1 >= 0
    w1r2 >= 0
    w1r3 >= 0
    w2r3 >= 0
    w1r4 >= 0
    w2r4 >= 0
    w3r4 >= 0
    w2r5 >= 0
    w3r5 >= 0
    w2r6 >= 0
    w3r6 >= 0
    w3r7 >= 0
```

OBJECTIVE FUNCTION VALUE		
1)	17100.00	
VARIABLE	VALUE	REDUCED COST
P1W1	150.000000	0.000000
P1W2	0.000000	8.000000
P2W1	200.000000	0.000000
P2W2	250.000000	0.000000
P3W1	0.000000	2.000000
P3W2	150.000000	0.000000
P3W3	100.000000	0.000000
P4W2	0.000000	7.000000
P4W3	150.000000	0.000000
W1R1	100.000000	0.000000
W1R2	150.000000	0.000000
W1R3	100.000000	0.000000
W2R3	0.000000	2.000000
W1R4	0.000000	5.000000
W2R4	200.000000	0.000000
W3R4	0.000000	7.000000
W2R5	200.000000	0.000000
W3R5	0.000000	3.000000
W2R6	0.000000	1.000000
W3R6	150.000000	0.000000
W3R7	100.000000	0.000000
ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000
3)	0.000000	0.000000
4)	0.000000	0.000000
5)	0.000000	1.000000
6)	0.000000	-16.000000
7)	0.000000	-17.000000
8)	0.000000	-18.000000
9)	0.000000	-16.000000
10)	0.000000	-18.000000
11)	0.000000	-21.000000
12)	0.000000	-15.000000
13)	0.000000	11.000000
14)	0.000000	8.000000
15)	0.000000	9.000000
16)	150.000000	0.000000
17)	0.000000	0.000000
18)	200.000000	0.000000
19)	250.000000	0.000000
20)	0.000000	0.000000
21)	150.000000	0.000000
22)	100.000000	0.000000
23)	0.000000	0.000000
24)	150.000000	0.000000
25)	100.000000	0.000000
26)	150.000000	0.000000
27)	100.000000	0.000000
28)	0.000000	0.000000
29)	0.000000	0.000000
30)	200.000000	0.000000
31)	0.000000	0.000000
32)	200.000000	0.000000
33)	0.000000	0.000000
34)	0.000000	0.000000
35)	150.000000	0.000000
36)	100.000000	0.000000
NO. ITERATIONS=		4

Minimum cost is \$17100.00



- 4 Veronica the owner of Very Veggie Vegeria is creating a new healthy salad that is low in calories but meets certain nutritional requirements. A salad is any combination of the following ingredients: tomato, lettuce, spinach, carrot, smoked tofu, sunflower seeds, chickpeas, oil

Each salad must contain:

- At least 15 grams of protein
- At least 2 and at most 8 grams of fat
- At least 4 grams of carbohydrates
- At most 200 milligrams of sodium
- At least 40% leafy greens by mass

The nutritional contents of these ingredients (per 100 grams) and cost are:

Ingredient	Energy (Cal)	Protein (grams)	Fat (grams)	Carbohydrate (grams)	Sodium (mg)	Cost (100g)
Tomato	21	0.85	0.33	4.64	9.00	\$1.00
Lettuce	16	1.62	0.20	2.37	28.00	\$0.75
Spinach	40	2.86	0.39	3.63	65.00	\$0.50
Carrot	41	0.93	0.24	9.58	69.00	\$0.50
Sunflower Seeds	585	23.4	48.7	15.00	3.80	\$0.45
Smoked Tofu	120	16.00	5.00	3.00	120.00	\$2.15
Chickpeas	164	9.00	2.6	27.0	78.00	\$0.95
Oil	884	0	100.00	0	0	\$2.00

**Part A:** Determine the combination of ingredients that minimizes calories but meets all nutritional requirements. Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. What is the cost of the low calorie salad? Include a copy of the code and output.

Goal: Minimize calories but still meet the nutritional requirements. To do this we need to add up all of the total calories per ingredient. We then take the min. of that total amount with using the constraints of the protein, fat, carbohydrates, sodium and leafy greens percentage.

This problem can be formulated as a linear program with an objective function and all constraints in the following manner:

Minimize Calories =  $21T + 16L + 40S + 41C + 585SS + 120ST + 164CP + 884O$

Subject to

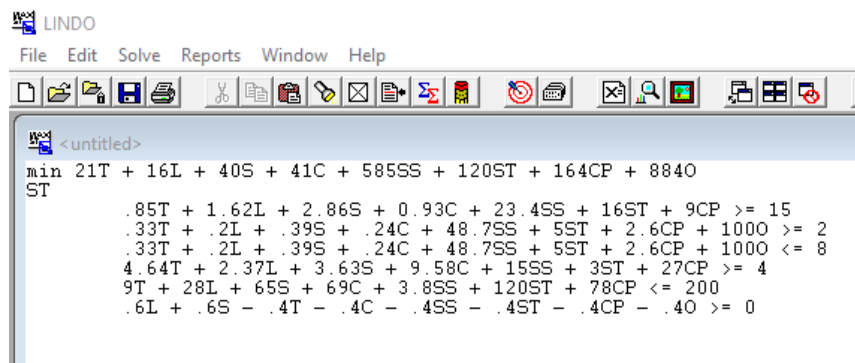
- $.85T + 1.62L + 2.86S + 0.93C + 23.4SS + 16ST + 9CP \geq 15$  (protein)
- $.33T + .2L + .39S + .24C + 48.7SS + 5ST + 2.6CP + 100O \geq 2$  (fat min)
- $.33T + .2L + .39S + .24C + 48.7SS + 5ST + 2.6CP + 100O \leq 8$  (fat max)

$$\begin{aligned}
4.64T + 2.37L + 3.63S + 9.58C + 15SS + 3ST + 27CP &\geq 4 \text{ (carbs)} \\
9T + 28L + 65S + 69C + 3.8SS + 120ST + 78CP &\leq 200 \text{ (sodium)} \\
.6L + .6S - .4T - .4C - .4SS - .4ST - .4CP - .4O &\geq 0 \text{ (leafy greens \%)}
\end{aligned}$$

The optimal solution (min. calorie salad) is a 114.75 calorie salad. The combination of ingredients that makes up this salad while still meeting the nutritional requirements is: Lettuce (0.585) and Smoked Tofu (0.878). The cost of this low calorie salad is \$2.33.

The cost was derived by:  $(0.75 \cdot 0.585) + (2.15 \cdot 0.878) = 2.33$

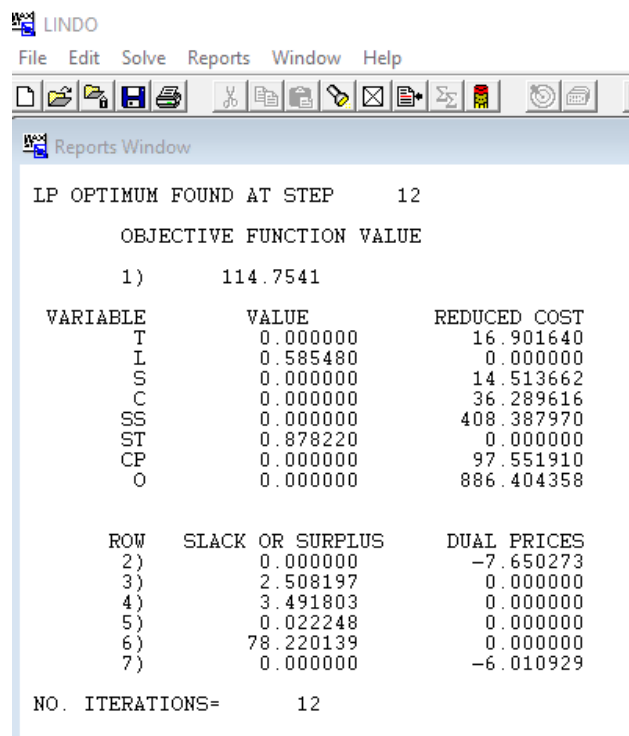
The LP code and output to this problem is below...



```

LINDO
File Edit Solve Reports Window Help
min 21T + 16L + 40S + 41C + 585SS + 120ST + 164CP + 8840
ST
.85T + 1.62L + 2.86S + 0.93C + 23.4SS + 16ST + 9CP >= 15
.33T + .2L + .39S + .24C + 48.7SS + 5ST + 2.6CP + 1000 >= 2
.33T + .2L + .39S + .24C + 48.7SS + 5ST + 2.6CP + 1000 <= 8
4.64T + 2.37L + 3.63S + 9.58C + 15SS + 3ST + 27CP >= 4
9T + 28L + 65S + 69C + 3.8SS + 120ST + 78CP <= 200
.6L + .6S - .4T - .4C - .4SS - .4ST - .4CP - .4O >= 0

```



```

LINDO
File Edit Solve Reports Window Help
Reports Window
LP OPTIMUM FOUND AT STEP      12
      OBJECTIVE FUNCTION VALUE
    1)      114.7541
      VARIABLE           VALUE           REDUCED COST
        T              0.000000             16.901640
        L              0.585480              0.000000
        S              0.000000             14.513662
        C              0.000000             36.289616
        SS             0.000000            408.387970
        ST             0.878220              0.000000
        CP             0.000000             97.551910
        O              0.000000            886.404358
      ROW    SLACK OR SURPLUS    DUAL PRICES
    2)              0.000000          -7.650273
    3)              2.508197           0.000000
    4)              3.491803           0.000000
    5)              0.022248           0.000000
    6)              78.220139           0.000000
    7)              0.000000          -6.010929
      NO. ITERATIONS=          12

```

**Part B:** Veronica realizes that it is also important to minimize the cost associated with the new salad. Unfortunately some of the ingredients can be expensive. Determine the combination of ingredients that minimizes cost. Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. How many calories are in the low cost salad? Include a copy of the code and output.

Goal: Minimize cost of ingredients for salad that still meets the requirements. To do this we need to add up the total cost of each ingredient. We then take the min. of that total amount with using the constraints of the protein, fat, carbohydrates, sodium and leafy greens percentage.

This problem can be formulated as a linear program with an objective function and all constraints in the following manner:

$$\text{Minimize Cost} = 1T + .75L + .5S + .5C + .45SS + 2.15ST + .95CP + 2O$$

Subject to

$$\begin{aligned} .85T + 1.62L + 2.86S + 0.93C + 23.4SS + 16ST + 9CP &\geq 15 \text{ (protein)} \\ .33T + .2L + .39S + .24C + 48.7SS + 5ST + 2.6CP + 100O &\geq 2 \text{ (fat min)} \\ .33T + .2L + .39S + .24C + 48.7SS + 5ST + 2.6CP + 100O &\leq 8 \text{ (fat max)} \\ 4.64T + 2.37L + 3.63S + 9.58C + 15SS + 3ST + 27CP &\geq 4 \text{ (carbs)} \\ 9T + 28L + 65S + 69C + 3.8SS + 120ST + 78CP &\leq 200 \text{ (sodium)} \\ .6L + .6S - .4T - .4C - .4SS - .4ST - .4CP - .4O &\geq 0 \text{ (leafy greens \%)} \end{aligned}$$

The optimal solution (min. salad cost) is \$1.55. The combination of ingredients that makes up this salad while still meeting the nutritional requirements is: Spinach (0.832), Sunflower Seeds (0.096) and Chickpeas (1.152). In this low cost salad, there are 278.49 calories.

The LP code and output to this problem is on the following page...

LINDO

File Edit Solve Reports Window Help

min 1T + .75L + .5S + .5C + .45SS + 2.15ST + .95CP + 20  
ST  
.85T + 1.62L + 2.86S + 0.93C + 23.4SS + 16ST + 9CP >= 15  
.33T + .2L + .39S + .24C + 48.7SS + 5ST + 2.6CP + 1000 >= 2  
.33T + .2L + .39S + .24C + 48.7SS + 5ST + 2.6CP + 1000 <= 8  
4.64T + 2.37L + 3.63S + 9.58C + 15SS + 3ST + 27CP >= 4  
9T + 28L + 65S + 69C + 3.8SS + 120ST + 78CP <= 200  
.6L + .6S - .4T - .4C - .4SS - .4ST - .4CP - .40 >= 0

LINDO

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Reports Window

LP OPTIMUM FOUND AT STEP 3

OBJECTIVE FUNCTION VALUE

1) 1.554133

VARIABLE	VALUE	REDUCED COST
T	0.000000	1.002081
L	0.000000	0.402912
S	0.832298	0.000000
C	0.000000	0.486914
SS	0.096083	0.000000
ST	0.000000	0.405609
CP	1.152364	0.000000
O	0.000000	7.281258

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	-0.131261
3)	6.000000	0.000000
4)	0.000000	0.051847
5)	31.576324	0.000000
6)	55.651089	0.000000
7)	0.000000	-0.241358

NO. ITERATIONS= 3