

Read the cryptarithms as the following:

$$\begin{array}{r} \text{HEAD} \\ \text{TO} \\ + \text{TOES} \\ \hline \text{TWOFF} \end{array} \qquad \begin{array}{r} \text{SAE} \\ + \text{FAT} \\ \hline \text{LOW} \end{array}$$

- 1) First, **we try T as 1**, because it is a left hand single digit on the addition line "TWOFF"
- 2) With T being 1 in "TWOFF", **W must be 0**. The most that will carry over to W's column is 1, since there are only 2 variables in the column to the right. W in "TWOFF" cannot be 1.
- 3) With T in "FAT" being 1 and W in "LOW" being 0. we know that **E in "SAE" must be 9**. This can be seen through the equation  $E + 1 = 10$ . This leads to a carry over of 1 to the column to the left.
- 4) **H in "HEAD" now cannot be 9, so therefore it must be 8**. The carry over from the column to the right confirms this. This can be seen by the equation  $1 + H + 1 = 0 + 10$ .

Its a little less straight forward from here.

- 5) There is a carry over of 1 to the column containing O in "LOW". We know that O must be odd because of the carry over and the fact that there are 2 A's above. This leads to the equation  $O = 2A + 1$ . We also know that O cannot equal 1, because 1 or 9 because they are already taken by T and E, respectively. We also know that O cannot equal 3, because through the equation  $O = 2A + 1$ , that would lead to A equaling 1, which again, cannot happen because 1 is taken by 1. This leaves O only able to equal 5 or 7.

Trying  $O = 5$ , A must equal 2 due to the equation  $O = 2A + 1$ . This leads to only 3,4,6,7 being the only numbers left. Column S in "SAE" yields the equation  $S + F = L$ . The only combination of numbers that works here is  $3 + 4 = 7$  or  $4 + 3 = 7$ . In that column, lets try setting S to 4 and F to 3. This leaves only 6 left which is taken by D of "HEAD". In D's column, setting S to 4 and F to 3 **WILL NOT WORK**, because equation  $6 + 5 + 4 = 3 + 10$  is not true. Next, in column S of "SAE", we can try setting S to 3 and F to 4...

- 6) In column D of "HEAD", with O equaling 5 (this is what were trying), 6 is only left over for D and S equals 3 with F equaling 4. *This combination works* ( $6 + 5 + 3 = 4 + 10$ ). This leads to a carry over of 1 to the next left column (A of "HEAD"). In this column you are left with A equaling 2 (because we tried  $O = 5$ ), which leads to the equation  $1 + A + T + E = F + 10$  or  $1 + 2 + 1 + 9 = 4 + 10$ , which is a false statement.

**THIS COMBINATION WILL NOT WORK.**

- 7) Therefore the only odd left is 7. **O must equal 7**. Because O equals 7, **A must equal 3** due to the equation  $O = 2A + 1$  that was found earlier. This leaves 2,4,5,6 as the only numbers that are left. Once again, column S in "SAE" yields the equation  $S + F = L$ . The only combination of numbers that works here is  $2 + 4 = 6$  or  $4 + 2 = 6$ . In that column, lets try setting S to the lower number (2) and F to the higher number (4) like last time.

In column D of "HEAD", with O equaling 7 (which it must be as seen earlier), 5 is only left over for D and S equals 2 with F equaling 4. *This combination works* ( $5 + 7 + 2 = 4 + 10$ ). This leads to a carry over of 1 to the next left column (A of "HEAD"). In this column you are left with A equaling 3 (as seen above), which leads to the equation  $1 + A + T + E = F + 10$  or  $1 + 3 + 1 + 9 = 4 + 10$ , which is a true statement. **THIS COMBINATION WILL WORK. Therefore S = 2, F = 4, L = 6 and D = 5.**

- 8) 

0	1	2	3	4	5	6	7	8	9
W	T	S	A	F	D	L	O	H	E