Groups 12 Members:
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Group Activity: Cryptarithms
Due Oct 9 by 11:59pm

Group process overall description:

The process that our team took towards solving the 5 problems was that we first broke our team down into sub teams with two team members each for the first 3 problems. By the team not all focusing on a single problem, we were able to make progress on multiple problems at the same time and develop techniques for solving the problems in tandem. As the team members begin to solve the problems, the member that ultimately solved the problem wrote up their approach to allow the other team members to review and then apply those techniques to the new problems.

This approach allowed for rapid iteration of the techniques being used as well the ability for our team to move through all the problems. It also allowed members to have some independence, and free up time towards finding solutions and processes in smaller groups for faster turnaround. While members used different variants of processes to solve the problems, they all had a underlying core process at work that is listed below.

Generalized process steps:

1. Diagram out the values to map to variables that can match and cannot match for testing.
2. Deconstruct the problem down into smaller, more manageable pieces. This also allows to more easily identify rules and relationships.
3. Test the standard arithmetic rules as they related to the specific type of arithmetic operation (i.e. Multiplication (example, - 0 x variable, 1 x variable, 5 x variable, 6 x variable.), Division, Addition, Subtraction).
4. Closely examine the problem(s) to identify relationship between variables after identifying rules.
5. Begin extensive testing with these identified rules and relationships.

Listed below are problems 1-5 with the generalized process used and a more refined process explained and how it was used specifically for solving the problem.

-The following 5 problems for our group 12 team, have the team member who ultimately solved the problem credited to the problem.

Problem 1:
Peter Moldenhauer

```
   N  C  I  S  A  A    0123456789 the key has 2  words
 √EM' AN' CI' PA' TI' ON
  PI
   S AN
   N OC
     NM CI
     PN TP
     TC MN PA
     TC MN AT
           CS TI ON
```

Answer to Problem 1: ATOMIC PENS

Steps taken to solve Problem 1:

To start, I first made a chart that lists all of the possible letters vertically in one column and then next to each letter I listed all of the possible numbers that each letter could hold (0 - 9). Example below:
A - 0 1 2 3 4 5 6 7 8 9
C - 0 1 2 3 4 5 6 7 8 9
E - 0 1 2 3 4 5 6 7 8 9
I - 0 1 2 3 4 5 6 7 8 9
M - 0 1 2 3 4 5 6 7 8 9
N - 0 1 2 3 4 5 6 7 8 9
O - 0 1 2 3 4 5 6 7 8 9
P - 0 1 2 3 4 5 6 7 8 9
S - 0 1 2 3 4 5 6 7 8 9
T - 0 1 2 3 4 5 6 7 8 9

Then, as I work through the problem, I gradually cross off specific numbers that I know that a particular letter can not be.

Now to solve the problem:

- N, E, P, S, N P, T and C cannot be 0 because per the instructions, the first digit cannot be a 0 (cross of 0's in the rows to these letters)
- We know that N^2 = PI, so therefore N cannot be 0 1 2 or 3
- I is the last digit of N^2
- C is the last digit of C^2 (so C has to be 1,5 or 6)
- P is the last digit of I^2
- I is the last digit of S^2
- S + PI = EM

With all of these above clues, it is just a matter of trial and error of "if-then" statements to slowly decipher what letter goes to what number. For example: "If N = 4, then P = 1 and then I = 6, but I cannot = 6 so we know that P does not = 1 and that N does not = 4….so now let's see what happens when N = 7...". I started with the Top left corner and then slowly worked down throughout the whole problem. I think you have to start from the top left because that is where most of the letters with corresponding clues begins. As you work through the problem and crossing off what numbers can't be associated with what letters, then more clues will come up for the letters at the bottom right.

Once you finally decipher all of the letter/number combinations, you end up with:
A = 0, C = 5, E = 7, I = 4, M = 3, N = 8, O = 2, P = 6, S = 9 and T = 1

Then you substitute all of these letters into the string of numbers at the top of the problem: 0123456789 and you get ATOMICPENS. Since per the key, this contains 2 words, the only logical 2 words in this string is ATOMIC PENS. Got your answer!

Problem 2:
Peter Moldenhauer

```
   F   O   R         1234567890 the key has 2 words
 √TH' RU' SH
  AF
   L  RU
   U  UL
      AL  SH
      RF  UT
      WR  RW
```

Answer to Problem 2: WRATH FOULS

Steps taken to solve Problem 2:

To start, I begin by writing out a chart that lists all of the potential letter/number combinations (see above how I solved problem 1 for an example of this). Then, I write out all of the clues that are known right off the bat:

- All of the first digits in each number cannot equal 0
- F^2 = AF
- F is the last digit of F^2 (so F has to be a 5 or 6)
- L is the last digit of O^2
- T is the last digit of R^2
- AF + L = TH

Working off all of these clues and starting in the top left corner of the problem, proceed to solve the problem by testing various “if-then” statements of number/letter combinations.

Once you decipher all of the numbers/letters, you get: F = 6, O = 7, R = 2, T = 4, H = 5, A = 3, L = 9, U = 8, W = 1 and S = 0.

Then substitute the corresponding letters to the string of numbers at the top of the problem: 1234567890. You will then get: WRATHFOULS. Since it is known that this string contains 2 words, the only logical words are: WRATH FOULS. Got your answer!

<u>Problem 3:</u>

Jason Silber

```
   LA          IZ    9876543210 the key has no words
 * RM        * RM
  OLD         RNO
 NEA         ORM
 OOND        OLAO
```

Answer to Problem 3: <u>key</u>= E Z I L A M R O N D
9 8 7 6 5 4 3 2 1 0

To begin the multiplication problem, we diagram out the digits so that we have a visual aid to place the respective variable against the number. By placing a letter on top we indicate the letter satisfies some of the conditions for the number and is in active testing. If the letter is below, then the number cannot be one of those letters.

<u>Example:</u>

```
           A
 9 8 7 6 5 4 3 2 1 0
                   L
                   O
                   N
                   I
                   R
```

Now we begin to simplify the multiplication problem by deconstructing it down into more manageable parts. By splitting the top values into their respective groupings of multiplication, we can more easily examining the problems and look for consistencies.

Partitioned out problem:

1.	2.	3.	4.
LA	LA	IZ	IZ
* M	* R	* M	* R
OLD	NEA	RNO	ORM

Now the problem becomes more accessible. At this stage we need to use standard arithmetic rules in multiplication for identifying variables that pose these characteristics.

The first check that we do is look for 0 and it's common characteristics. (i.e. 0 x any number = 0). Looking closely at the problem in column 2, A x R holds this characteristic. However, when applied to column 1, this characteristic does not hold for A because when A x M does not equal M and vice versa. Therefore we rule out 0 for A, M and R. Also, no multi digits may begin with 0 in cryptarithms, so we rule out L, O, N, I as well. This leads us to an important discovery that only E and D are remaining and could satisfy the requirements of 0.

Next we apply the rule for the number 1 (i.e. 1 x any number = the same number). Examining the problems, again column 2, A x R holds this characteristic. However, when applied to column 1, this characteristic does not hold for A because when A x M does not equal A. We can rule out 1 for M and R because if M = 1 then LA x M would equal LA and it does not and would the same logic would apply to R. We can also apply this logic to L, I, Z.

Next we apply rule for the number 5 (i.e. 5 x 3 = 15, 5 x 7 = 35, 5 x 9 = 45). It is also worth noting the 5 also provides the unique property of producing trailing 0's (i.e. 5 x 4 = 20, 5 x 6 = 30, 5 x 8 = 40) which can be used in testing A x M. We assign A the value 5 and then we assign R as the value of 3.

Now, before we get heavy into testing these values choices, we need to step back and examine any other characteristics of the problems are obvious rules that we can apply. After examination, no others stand out.

Next we look closely at all 4 problems together to identify any important relationships that will help guide us on assigning any other values when testing A as 5 and R as 3. We identify that R is the furthermost left digit in column 3, which tells us the R is likely a low value digit as it is both one of the multipliers and holds this furthermost left digit. Also, we know that D is one of the two candidates for 0 and we see that D is the product of A x M. We remember that one of the traits of 5 is that it can have a trailing digit of 0 when multiplied by 2,4,6,8. This helps guide us on further testing. We then assign D = 0 and M = 2.

Now we begin our extensive testing with these identified rules and relationships in place. The values of A = 5, R = 3, D = 0, M = 2, do not yield results. Given the probabilities of the relationships identified (D has a 50% chance of being 0, R has a 33% chance of being 3,7,9 and M has a weighted chance of 2,4,6,8 based off D = 0) M is the most logical choice to assign a new test value. We assign M = 4, this begins to yield results in column 1 as we identify the M x L = O and we assign L = 6 with a carry of 2 to equal O = 2 and L = 6. Assigning these further values further unlocking the problems. Then testing column 3 to satisfy the relationship of R holding the furthermost left value and necessitating a low value by assigning I = 7 further unlocks the problem to finally allow simple test of the remaining values to solve.

<u>Summary of the process for problem 3:</u>

1. Diagram out the values to map to variables that can match and cannot match for testing.
2. Deconstruct the problem down into smaller, more manageable pieces. This also allows to more easily identify rules and relationships.
3. Test the standard arithmetic rules, for multiplication - 0 x variable, 1 x variable, 5 x variable, 6 x variable.
4. Closely examine the problem(s) to identify relationship between variables after identifying rules.
5. Begin extensive testing with these identified rules and relationships.

Problem 4:
Brent Irwin

```
         AL     1234567890 the key has 2 words
      ______
GREY |  LARGE
        GREY
        AOLCE
        AEGRK
         RRSE
```

The first two letters were easiest to find, as they were immediately available.

1. **Since A * GREY = GREY, we know that A = 1.**
2. **Since E – K = E in the digits column, we know that K = 0.**

```
        1L
     ______
GREY / L1RGE
       GREY
       10LCE
       1EGR0
        RRSE
```

Next, notice that L – G appears to be equal to 1. But in the next column over, 1 – R (which is not 0) equals something positive, which is not possible in this situation. So the 1 must borrow a 10 from L, meaning that actually L – G = 2.

In the bottom subtraction set, L – G appears again. There are 4 possible ways this could be solved, so I tried each one. R could equal 2, but there are other possibilities. Assuming R ≠ 2, since R ≠ 1, L must either lose a digit to the column to the right, gain a 10 from the column to the left, or both. In each case, substituting (L – G) for 2 when needed:

Gain a 10: (10 + L) – G = R

10 + 2 = R

12 is not a possible digit

Lose 1: (L – 1) – G = R

2 – 1 = R

1 is already taken by A

Both: (10 + L – 1) – G = R

9 – 2 = R

11 is not a possible digit either.

3. R = 2, because none of the alternatives produced a viable result.

```
           1L
G2EY / L12GE
       G2EY
       10LCE
       1EG20
        22SE
```

We discussed before that the 1 in the dividend borrows a 10 from L. We know that 2 must also borrow a 10 from 1, since all the digits less than 2 are taken already. This makes the "A" position in LARGE now equal to 10. Thus:

4. O = 8, since (10 + 1 – 1) – 2 = R.
5. E = 6, since 8 – E = 2.

```
           1L
G26Y / L12G6
       G26Y
       18LC6
       16G20
        22S6
```

Now, we know the R(2) in LARGE takes 10. But 12 – 6 = L. L couldn't equal 6 because E already does. So we know that G must borrow a 10. This means it's actually 11 – 6.

6. L = 5, because (10 + 2 – 1) – 6 = L.
7. G = 3, because we stated before that L – G = 2, and L = 5.

```
      15
326Y / 51236
       326Y
       185C6
       16320
        22S6
```

At this point, we know the following values:
1 = A
2 = R
3 = G
4 =
5 = L
6 = E
7 =
8 = O
9 =
0 = K

Knowing that the leftover letters are Y, S, and C, it may be easy enough to piece together that the key is ARGYLE SOCK, but we want to prove it to the end.

All of these letters appear in one column, making it straightforward to find their values.

13 – Y = C and C – 2 = S.

Only 2 of these numbers are 2 apart (7 and 9), so they must be C and S. These are the only available numbers left.

8. S = 7 and C = 9 because C – 2 = S.

Y then must be the last digit, and we can know it's true based on the equations above.

9. Y = 4 because 13 – Y = 9.

The answer is: ARGYLE SOCK

Problem 5:

Zachary Williams/David Rider

```
  HEAD     LOW    0123456789 the key has no words
    TO   - FAT
+ TOES     SAE
 TWOFF
```

1) First, **we try T as 1**, because it is a left hand single digit on the addition line "TWOFF"

2) With T being 1 in "TWOFF", **W must be 0**. The most that will carry over to W's column is 1, since there

3) With T in "FAT" being 1 and W in "LOW" being 0. we know that **E in "SAE" must be 9**. This can be seen through the equation E + 1 = 10. This leads to a carry over of 1 to the column to the left.

4) **H in "HEAD" now cannot be 9, so therefore it must be 8.** The carry over from the column to the right confirms this. This can be seen by the equation 1 + H + 1 = 0 + 10.

   It's a little less straightforward from here.

5) There is a carry over of 1 to the column containing O in "LOW". We know that O must be odd because of the carry over and the fact that there are 2 A's above. This leads to the equation O = 2A +1. We also know that O cannot equal 1, because 1 or 9 because they are already taken by T and E, respectively. We also know that O cannot equal 3, because through the equation O = 2A + 1, that would lead to A equaling 1, which again, cannot happen because 1 is taken by 1. This leaves O only able to equal 5 or 7.

   Trying O = 5, A must equal 2 due to the equation O = 2A + 1. This leads to only 3,4,6,7 being the only numbers left. Column S in "SAE" yields the equation S + F = L. The only combination of numbers that works here is 3 + 4 = 7 or 4 + 3 = 7. In that column, let’s try setting S to 4 and F to 3. This leaves only 6 left which is taken by D of “HEAD”. In D’s column, setting S to 4 and F to 3 **WILL NOT WORK**, because equation 6 + 5 + 4 = 3 + 10 is not true. Next, in column S of “SAE”, we can try setting S to 3 and F to 4…

6) In column D of “HEAD”, with O equaling 5 (this is what we’re trying), 6 is only left over for D and S equals 3 with F equaling 4. *This combination works* (6 + 5 + 3 = 4 + 10). This leads to a carry over of 1 to the next left column (A of “HEAD”). In this column you are left with A equaling 2 (because we tried O = 5), which leads to the equation 1 + A + T + E = F + 10 *or* 1 + 2 + 1 + 9 = 4 + 10, which is a false statement. **THIS COMBINATION WILL NOT WORK.**

7) Therefore the only odd left is 7. **O must equal 7.** Because O equals 7, **A must equal 3** due to the equation O = 2A + 1 that was found earlier. This leaves 2,4,5,6 as the only numbers that are left. Once again, column S in “SAE” yields the equation S + F = L. The only combination of numbers that works here is 2 + 4 = 6 or 4 + 2 = 6. In that column, let’s try setting S to the lower number (2) and F to the higher number (4) like last time.

   In column D of “HEAD”, with O equaling 7 (which it must be as seen earlier), 5 is only left over for D and S equals 2 with F equaling 4. *This combination works* (5 + 7 + 2 = 4 + 10). This leads to a carry over of 1 to the next left column (A of “HEAD”). In this column you are left with A equaling 3 (as seen above), which leads to the equation 1 + A + T + E = F + 10 *or* 1 + 3 + 1 + 9 = 4 + 10, which is a true statement. **THIS COMBINATION WILL WORK. Therefore S = 2, F = 4, L = 6 and D = 5.**

8)

0	1	2	3	4	5	6	7	8	9
W	T	S	A	F	D	L	O	H	E