# Homework 2

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Problem 1:

(a) $\hat{\pi} = \arg\max_{\pi} \sum_{i=1}^{n} \ln p(y_i|\pi)$ $\quad P(y_i|\pi) = \pi^{y_i}(1-\pi)^{1-y_i}$

let $l = \sum_{i=1}^{n} \ln p(y_i|\pi) = \sum_{i=1}^{n} \ln\pi \cdot \mathbb{1}_{\{y_i=1\}} + \sum_{i=1}^{n} \ln(1-\pi)\cdot \mathbb{1}_{\{y_i=0\}}$

$= \{\#(y_i=1)\}\cdot \ln\pi + \{\#(y_i=0)\}\cdot \ln(1-\pi)$

$\frac{\partial l}{\partial \pi} = \frac{1}{\pi}\cdot\{\#(y_i=1)\} + \frac{1}{1-\pi}\{\#(y_i=0)\} \equiv 0$, and $\{\#(y_i=0)\} = n - \{\#(y_i=1)\}$

$\Rightarrow \hat{\pi} = \frac{\{\#(y_i=1)\}}{n}$, $i=1,2,\cdots n$

(b) $\hat{\lambda}_{y,d} = \arg\max_{\lambda_{y,d}} \sum_{d=1}^{D} \left(\ln p(\lambda_{y,d}) + \sum_{i=1}^{n} \ln p(x_{i,d}|\lambda_{y_i,d})\right)$

$= \arg\max_{\lambda_{y,d}} \sum_{d=1}^{D} \left(\ln(\lambda_{y,d}\cdot e^{-\lambda_{y,d}}) + \sum_{i=1}^{n} \ln\left(e^{-\lambda_{y_i,d}}\cdot \frac{\lambda_{y_i,d}^{x_{i,d}}}{x_{i,d}!}\right)\right)$

$= \arg\max_{\lambda_{y,d}} \sum_{d=1}^{D} \left(\ln(\lambda_{y,d}) - \lambda_{y,d} + \sum_{i=1}^{n}\left(-\lambda_{y_i,d} + x_{i,d}\cdot\ln\lambda_{y_i,d} - \ln(x_{i,d}!)\right)\right)$

We leave $y$ arbitrary. Thus we make $y=y_0$.

For a specific $d=d_0$,

$\hat{\lambda}_{y_0,d_0} = \arg\max_{\lambda_{y_0,d_0}} \left(\ln(\lambda_{y_0,d_0}) - \lambda_{y_0,d_0} - \sum_{i=1}^{n}\lambda_{y_i,d_0}\cdot\mathbb{1}_{\{y_i=y_0\}} + \sum_{i=1}^{n}\left(x_{i,d_0}\cdot\ln\lambda_{y_i,d_0}\cdot\mathbb{1}_{\{y_i=y_0\}}\right)\right)$

$\equiv \arg\max_{\lambda_{y_0,d_0}} L$

$\frac{\partial L}{\partial \hat{\lambda}_{y_0,d_0}} = \frac{1}{\lambda_{y_0,d_0}} - 1 - \sum_{i=1}^{n}\mathbb{1}_{\{y_i=y_0\}} + \sum_{i=1}^{n}\frac{x_{i,d_0}\cdot\mathbb{1}_{\{y_i=y_0\}}}{\lambda_{y_0,d_0}} \equiv 0$

$\hat{\lambda}_{y_0,d} = \frac{1+\sum_{i=1}^{n}(x_{i,d}\cdot\mathbb{1}_{\{y_i=y_0\}})}{1+\sum_{i=1}^{n}\mathbb{1}_{\{y_i=y_0\}}} = \frac{1+\sum_{i=1}^{n}(x_{i,d}\cdot\mathbb{1}_{\{y_i=y_0\}})}{1+\{\#(y_i=y_0)\}}$