Homework 2

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Problem 1:

(a) $\hat{\pi} = \arg\max_{\pi} \sum_{i=1}^{n} \ln p(y_i|\pi)$ $\qquad P(y_i|\pi) = \pi^{y_i}(1-\pi)^{1-y_i}$

let $l = \sum_{i=1}^{n} \ln p(y_i|\pi) = \sum_{i=1}^{n} \ln\pi \cdot \mathbb{1}_{\{y_i=1\}} + \sum_{i=1}^{n} \ln(1-\pi)\cdot \mathbb{1}_{\{y_i=0\}}$

$= \{\#(y_i=1)\}\cdot \ln\pi + \{\#(y_i=0)\}\cdot \ln(1-\pi)$

$\frac{\partial l}{\partial \pi} = \frac{1}{\pi}\cdot\{\#(y_i=1)\} + \frac{1}{1-\pi}\{\#(y_i=0)\} \equiv 0$, and $\{\#(y_i=0)\} = n - \{\#(y_i=1)\}$

$\Rightarrow \hat{\pi} = \frac{\{\#(y_i=1)\}}{n}$, $i=1,2,\cdots n$

(b) $\hat{\lambda}_{y,d} = \arg\max_{\lambda_{y,d}} \sum_{d=1}^{D} \left(\ln p(\lambda_{y,d}) + \sum_{i=1}^{n} \ln p(x_{i,d}|\lambda_{y_i,d})\right)$

$= \arg\max_{\lambda_{y,d}} \sum_{d=1}^{D} \left(\ln(\lambda_{y,d}\cdot e^{-\lambda_{y,d}}) + \sum_{i=1}^{n} \ln\left(e^{-\lambda_{y_i,d}}\cdot \frac{\lambda_{y_i,d}^{x_{i,d}}}{x_{i,d}!}\right)\right)$

$= \arg\max_{\lambda_{y,d}} \sum_{d=1}^{D} \left(\ln(\lambda_{y,d}) - \lambda_{y,d} + \sum_{i=1}^{n}\left(-\lambda_{y_i,d} + x_{i,d}\cdot\ln\lambda_{y_i,d} - \ln(x_{i,d}!)\right)\right)$

We leave y arbitrary. Thus we make $y = y_0$.

For a specific $d = d_0$,

$\hat{\lambda}_{y_0,d_0} = \arg\max_{\lambda_{y_0,d_0}} \left(\ln(\lambda_{y_0,d_0}) - \lambda_{y_0,d_0} - \sum_{i=1}^{n}\lambda_{y_i,d_0}\cdot \mathbb{1}_{\{y_i=y_0\}} + \sum_{i=1}^{n}\left(x_{i,d_0}\cdot \ln\lambda_{y_i,d_0}\cdot \mathbb{1}_{\{y_i=y_0\}}\right)\right)$

$\equiv \arg\max_{\lambda_{y_0,d_0}} L$

$\frac{\partial L}{\partial \hat{\lambda}_{y_0,d_0}} = \frac{1}{\lambda_{y_0,d_0}} - 1 - \sum_{i=1}^{n} \mathbb{1}_{\{y_i=y_0\}} + \sum_{i=1}^{n} \frac{x_{i,d_0}\cdot \mathbb{1}_{\{y_i=y_0\}}}{\lambda_{y_0,d_0}} \equiv 0$

$\hat{\lambda}_{y_0,d} = \frac{1 + \sum_{i=1}^{n}(x_{i,d}\cdot \mathbb{1}_{\{y_i=y_0\}})}{1 + \sum_{i=1}^{n}\mathbb{1}_{\{y_i=y_0\}}} = \frac{1 + \sum_{i=1}^{n}(x_{i,d}\cdot \mathbb{1}_{\{y_i=y_0\}})}{1 + \{\#(y_i=y_0)\}}$

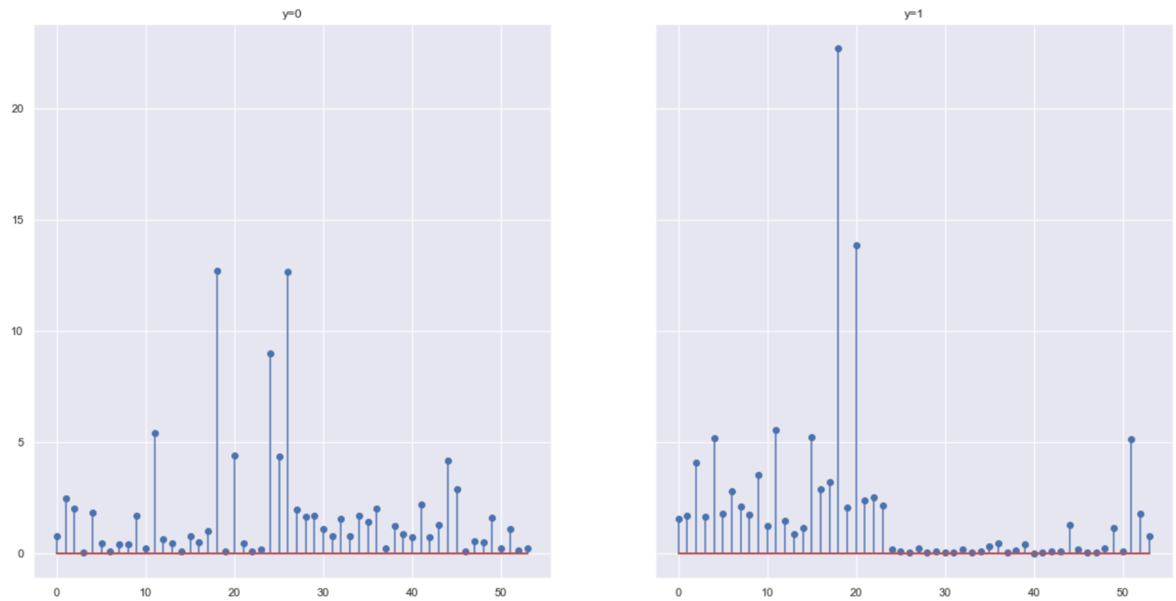
Problem 2

(a)

Prediction \ Truth	1	0
1	1703	490
0	110	2297

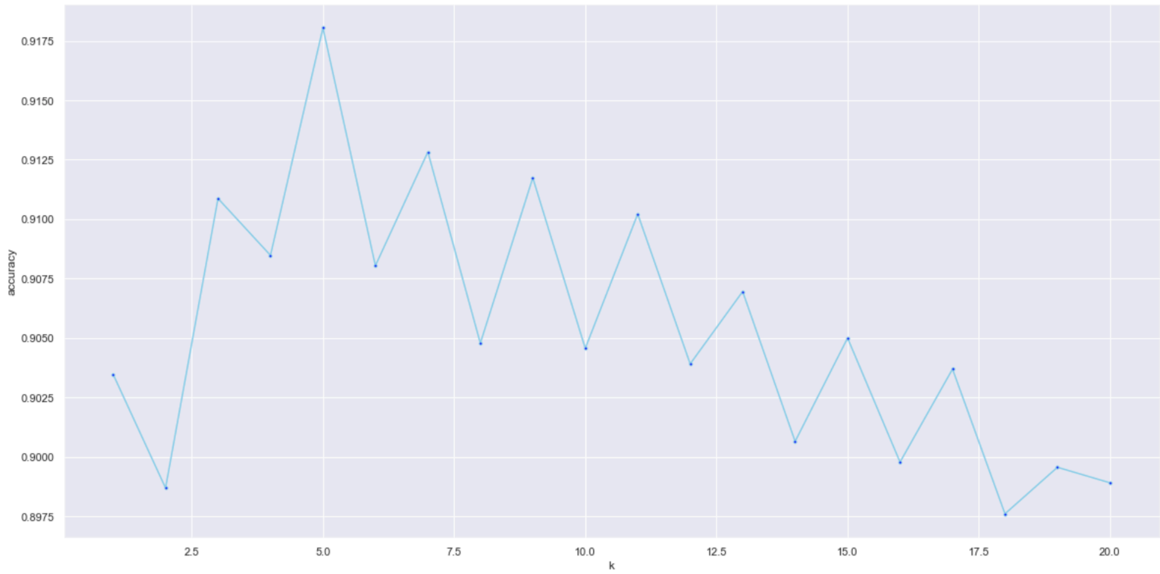
Accuracy = 0.8695652173913043

(b)

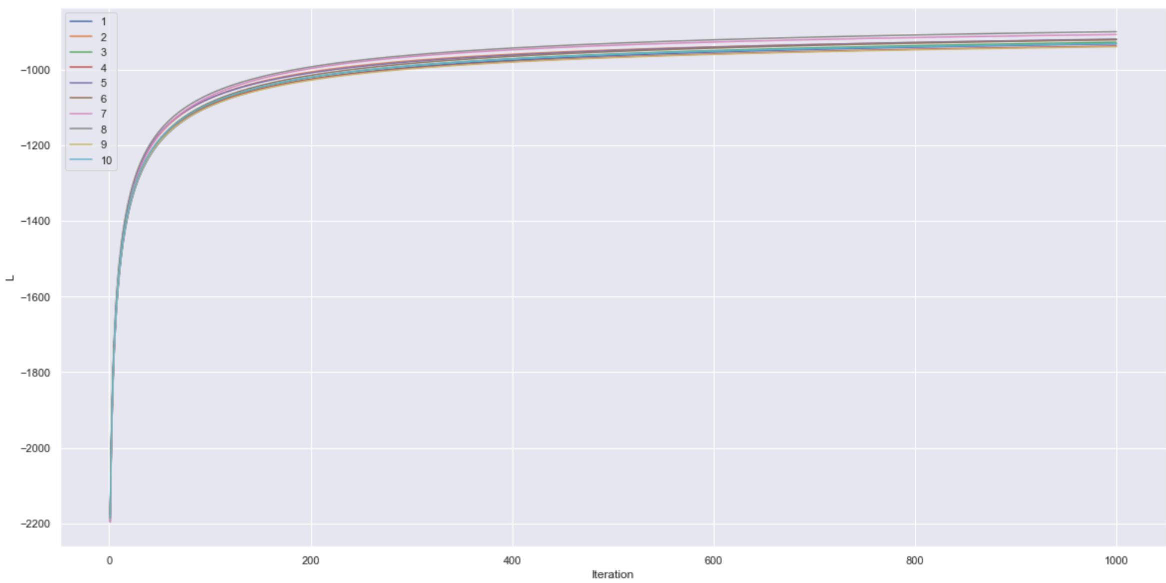


For dimension 16 and 52, the estimated value of lambda for both when y=1 are larger that the counterpart, and y=1 means spam email, which means the word 'free' and character '!' appears more frequently in spam email.

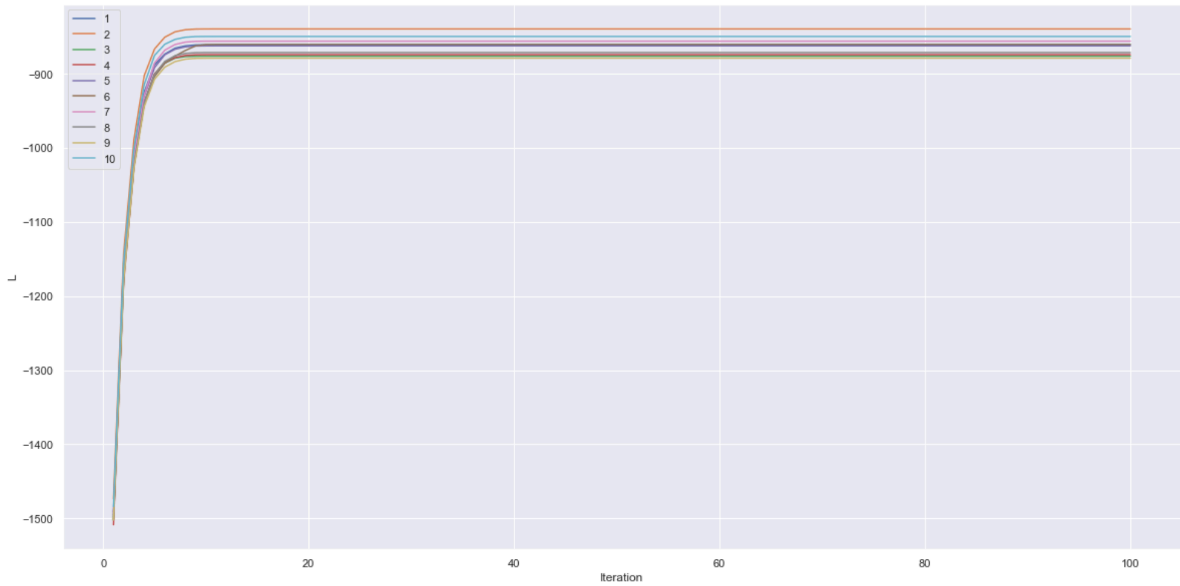
(c)


0.9175
0.9150
0.9125
0.9100
0.9075
0.9050
0.9025
0.9000
0.8975
accuracy
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
k

(d)


1
2
3
4
5
6
7
8
9
10
−1000
−1200
−1400
−1600
−1800
−2000
−2200
L
0
200
400
600
800
1000
Iteration

(e)



(f)

Truth / Prediction	1	0
1	1604	144
0	209	2643

Accuracy = 0.9232608695652174