Transit Usage in Seattle: A Spatial Investigation

Peter Silverstein

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Final Project

GIS and Spatial Analysis

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Contents

Introduction

Research Question:

1. How does transit usage percentage (percent of trips using mass transit / total trips per census tract) vary spatially in and around Seattle and Tacoma, Washington?
2. How does this variation relate to population density, median income, and median age at the census tract level?

Purpose of Study:

There are essentially two purposes to this study. The first is to better understand where there are concentrations of high and low transit usage around the region. If there is clustering and we see hotspots and/or coldspots, further policy-focused questions can be asked. For example: given clustering, what characteristics of a census tract makes in more or less likely to be in one of these hot or cold zones? How might we allocate resources across hot zones, cold zones, and those in-between to increase the adoption of transit by commuters? Is the dispersion of transit availability closely related to the demand and does the dispersion favor certain demographic groups over others?

The second research question is a very basic attempt at answering one of these follow-up questions. By understanding how each of the three independent variables (population density, median income, and median age) are related to the outcome of interest (percentage of commuter trips taken using public transit), we can begin to fill in the knowledge gaps demonstrated by the questions above.

Hypotheses

1. I believe we will see transit hotspots close to urban centers (e.g., Seattle and Tacoma, the two biggest cities in the region of interest). Further, I believe the opposite will be true for coldspots–they should exist further outside urban centers. These ideas are based on the fact that transit lines themselves tend to be clustered in high-density, urban areas, meaning opportunities for mass transit travel are more convenient and plentiful in more central urban areas.
2. I expect that transit use percentage is positively associated with population density and median income and negatively associated with age. I make this hypothesis about population density based on the reasoning above. I expect younger people to (a) be more likely to live in highly urban areas and (b) be less likely to own a personal vehicle (such as a car). Of the three variables, I am the least confident about median income, because I think the relationship could be pulled in both positive and negative directions. On one hand, urban areas tend to be more expensive and thus have a higher requirement for income to live there. On the other hand, lower income should be associated with lower rates of car ownership and thus lower income would be associated with higher transit ridership.

Data and Methodology:

Data Sources:

1. The **Puget Sound Regional Association (PSRC) Household Travel Survey (HTS) 2017-23** is a biennial survey of commuters done in the King, Kitsap, Pierce, and Snohomish counties of Washington state (the counties surrounding Seattle and Tacoma). The present analysis uses the Trips dataset from the HTS. Each observation in the dataset represents a single trip taken by a respondent and includes a variety of variables. Most important for my analysis are origin/destination tract and mode of travel, although the dataset also includes date, time, distance, speed, etc.

2. All **census tract-level ACS 2022 5-year estimates for demographic data and the associated geometries** were accessed via the R `tidycensus` package, which leverages an API connection to the US Census Bureau to provide US Census data for a specified geographic area.
3. Finally, **Stanford's Cities and Towns of the United States, 2014** dataset provided point data to allow me to add city labels to my maps for reference.

Data Preparation/Spatial Data Management:

Data Cleaning

The output from `tidycensus` is already quite clean, so the majority of data cleaning steps were conducted on the PSRC HTS dataset. After loading the dataset, I selected my columns of interest and converted their types where appropriate and useful (e.g., string to factor). The next step was to collapse the mode of travel column from around 50 unique responses (an artifact of (a) a very detailed survey and (b) some changes to response options over the years) to just 4 useful categories, outlined below:

1. *Mass Transit:* included in this category trips that used a metro bus, private bus or shuttle, urban rail/light rail, school bus, ferry, paratransit, and commuter rail. Essentially I included any multi-occupancy transit vehicle.
2. *Personal Vehicle:* trips including all single-occupancy motor vehicles. This includes personal cars, ride-shares, taxis, motorcycles, and car-share services.
3. *Active Transit:* included walking, running, biking, and skateboarding.
4. *Other:* included helicopter/plane, "other" responses.

I then implemented a number of filters to filter data that either didn't seem realistic or were outside the scope of this question. This included the following operations:

1. Filtered non-complete survey responses
2. Filtered distance to the range of 0 to 150 miles
3. Filtered trip duration to only include those greater than 0 minutes
4. Filtered speed to exclude speeds greater than 150 mph
5. Filtered to remove observations with missing value for travel mode

Spatial Joins

The next step was to count the number of trips and join these values to the geometries/ACS data from tidyverse. I used the `summarize()` function to count the number of total trips and number of trips per category for each census tracts. I then joined these counts to my geometry table via the `left_join()` function and matched on GEOID. Further, I removed any tracts from the analysis with 0 total trips. I acknowledge that this is a bit of a simplistic solution to missingness and will discuss it further in my analysis and conclusions. Finally, I calculated the percentage of trips in each tract that were made by mass transit mode (count of mass transit / count of total trips).

GIS Methodology Overview

Manipulation/Analytical Methods Used

As mentioned above, I counted the number of trips inside each census tract, then converted the total trip and mass transit trip counts to percentage. Finally, I used a simple join function to associate the counts with their respective geometries.

For the analysis in this project, I will perform a global cluster analysis (using Moran's I) and visualize any hot and cold spots using Getis-Ord Gi*. These approaches should help me understand whether transit use is clustered and visualize where it is clustered. I will then run a Spatial Lag Model (SLM) and a Spatial Error Model (SEM) regressing mass transit percentage on population density, median income, and median age. I will discuss the conceptual differences in the two regression models later in the report, but running both will provide me an opportunity to see and interpret the results from both and determine if they agree with each other.

Software Used

All data loading, cleaning, and manipulation, mapping (both choropleth and Getis-Ord Gi*), table-creation, regression modeling, and write-up were performed with R and RStudio.

Results and Analysis

Exploratory Data Analysis via Choropleth Mapping and Descriptive Tables

I will begin the analysis portion of this project with some simple choropleth maps. The purpose of this mapping is to simply visualize the spatial patterning of the dependent and independent variables. While the actual significance testing and cluster analysis will give me a scientific understanding of the problem, many of the patterns I'm looking for will be apparent with a simple eye test.

Mass Transit Trips as a % of Total Trips (classification = Jenks)

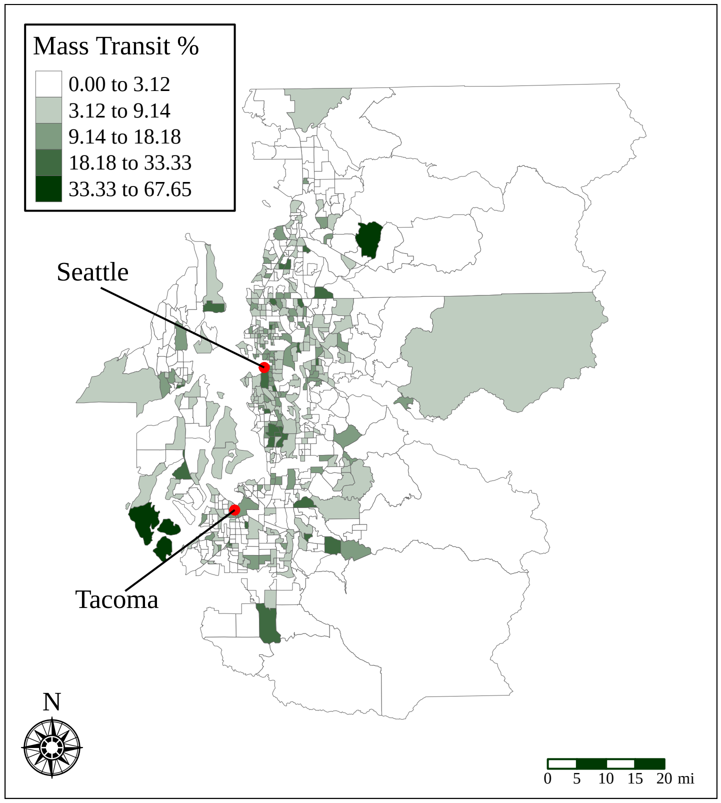


Figure 1: Sources: US Census ACS 2022 5-year estimates, Puget Sound Regional Countil, Cities and Towns of the US 2014, via Stanford University; Classification = jenks

Ignoring for a minute the top-end of the scale, it does seem that there are some clusters with higher transit usage and that these clusters tend to exist closer to the cities. Not only can we see that the apparent clustering occurs nearer to the two cities marked on the map, it also seems as though smaller census tracts tend to have higher mass transit percentages. As a general rule of thumb, smaller census tracts tend to be more urban, so this fits with my hypothesis that transit clustering will occur in more population dense areas. In the following maps, we can visually compare the pattern seen in this map to how each of the predictor variables vary spatially.

Population Density by Census Tract (classification = quantiles)

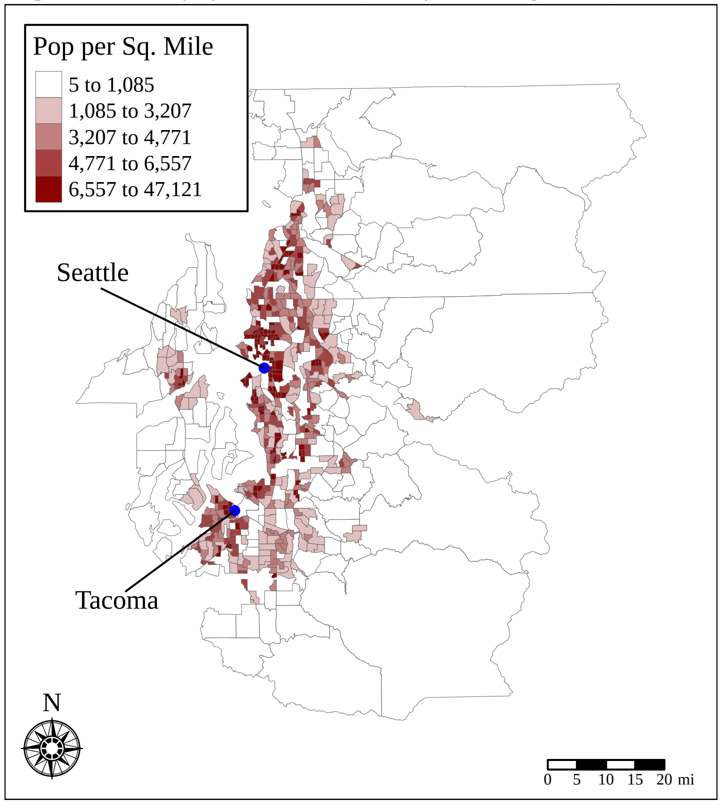


Figure 2: Sources: US Census ACS 2022 5-year estimates, Puget Sound Regional Countil, Cities and Towns of the US 2014, via Stanford University; Classification = jenks

As expected, population density (figure 2) is much higher in tracts close to the two cities (and throughout the generally-urban corridor between them). Again, visually comparing this pattern to the one seen for mass transit percentage, a lot matches up. This is not a perfect correspondence, of course.

Median Income by Census Tract (classification = quantiles)

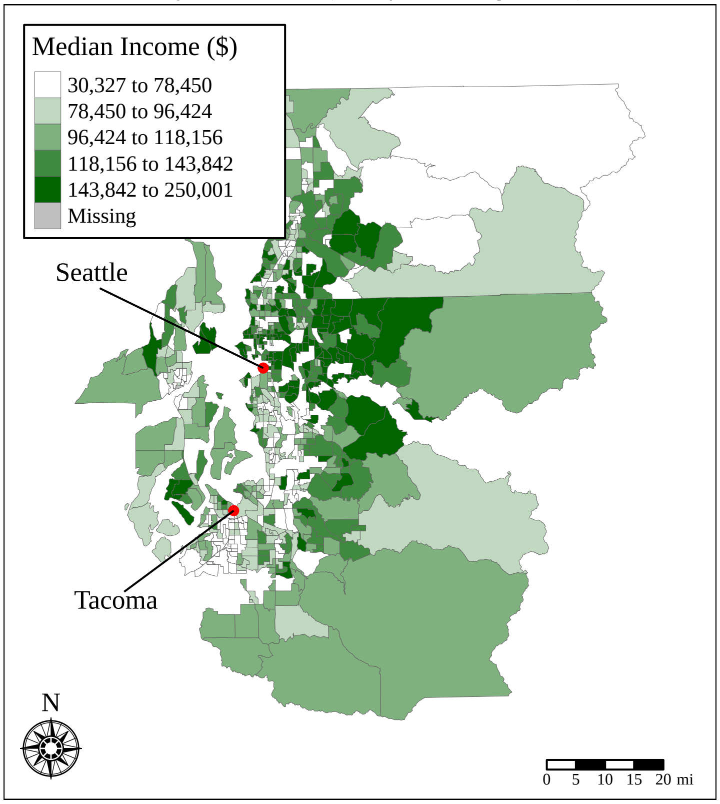


Figure 3: Sources: US Census ACS 2022 5-year estimates, Puget Sound Regional Countil, Cities and Towns of the US 2014, via Stanford University; Classification = jenks

I find the map in figure 3 to be particularly interesting. Although it is not incredibly easy to compare this spatial pattern to the mass transit percentage one, I will direct your attention to the high-income area just East of Seattle. Comparing to the transit map, we can see a pretty obvious negative association between income and transit percentage. This provides an initial piece of evidence in favor of the hypothesis that higher income tracts are likely to have lower transit usage. In the same vein, a visual inspection of the tracts directly to the south of the Seattle marker shows an area of comparatively low median income. Again, cross-referencing this with the transit map, we can see this is an area of relatively high transit usage.

Median Age by Census Tract (classification = quantiles)

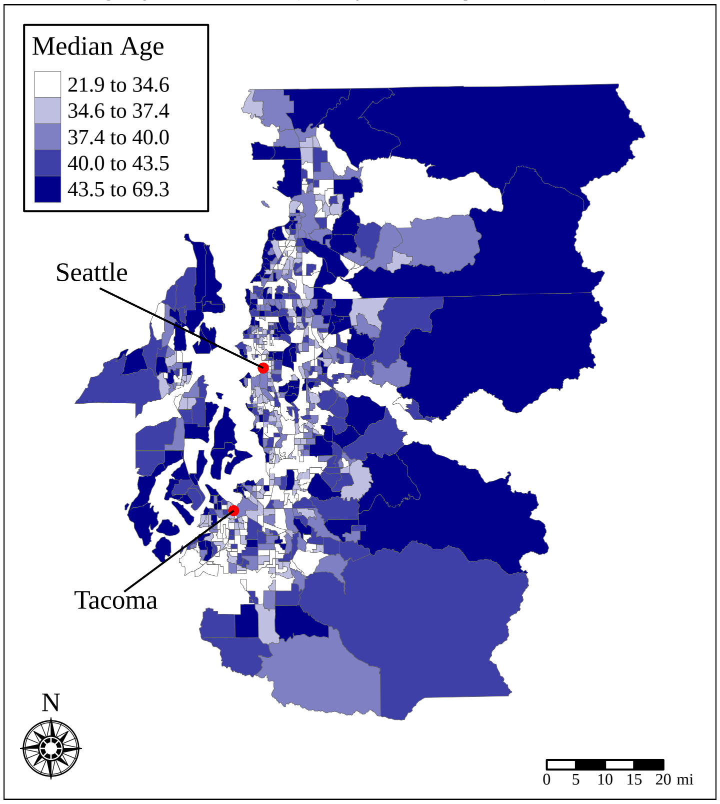


Figure 4: Sources: US Census ACS 2022 5-year estimates, Puget Sound Regional Countil, Cities and Towns of the US 2014, via Stanford University; Classification = quantiles

Finally, another urban/rural pattern can be seen for the median age variable. Generally speaking, tracts further from the centers of Seattle and Tacoma tend to be older, while tracts nearer to the cities tend to be younger. As with the other variables, this provides initial, non-statistical evidence in support of the idea that tracts with higher transit ridership should be younger.

For the final piece of exploratory data analysis, below is a table containing basic descriptive statistics for each of my variables. The only thing I will call out about this table is that it does show the high number of tracts with 0% mass transit trips. This is likely due to a low number of observations in those census tracts, since I only filtered out tracts with 0 observations. It is entirely possible there are tracts with a single observation and that the observation is for personal vehicles, active transit, or other.

Table 1: Descriptive Statistics Summary

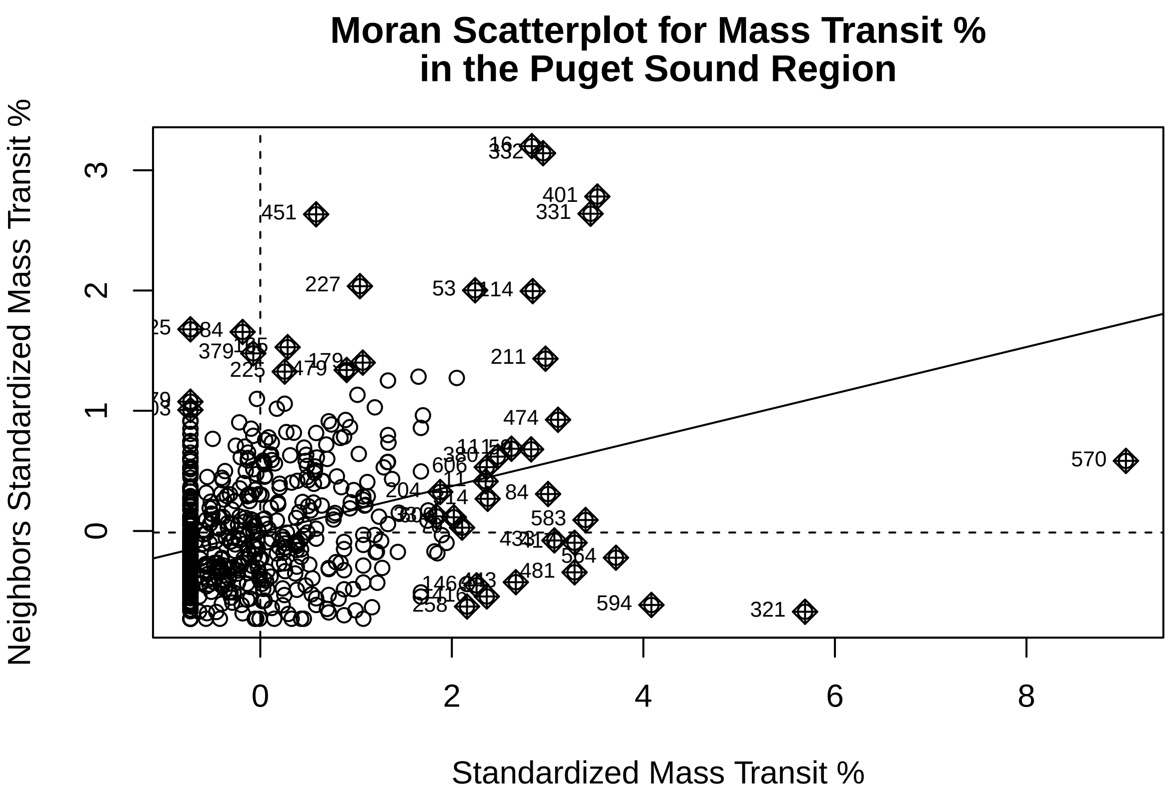
	Mass Transit %	Population Density	Median Income	Median Age
mean	5.06	4,605.37	113,140.27	39.19
sd	6.94	4,483.13	40,662.76	5.69
min	0.00	4.90	30,327.00	21.90
q25	0.00	1,779.03	84,682.75	35.60
median	2.78	3,907.93	107,472.00	38.70
q75	7.69	5,963.44	135,001.75	42.60
max	67.65	47,121.04	250,001.00	69.30

Cluster Analysis

In this section, I will apply two different methods to test and visualize the clustering of transit access in the region. First, I will run a Global Moran's I to determine if the clustering we can see visually is statistically significant. Then, I will run a hotspot analysis using the Getis-Ord Gi* statistic, primarily as a tool for creating a visualization of the clustering. I will be using the Queen's case contiguity for my weights, because it intuitively does not make sense to exclude some neighbors (as would be the case using Rook's case contiguity) for this research question.

Moran's I Test of Global Clustering

```
##
##  Moran I test under randomisation
##
## data:  psrc_table_clean$masstransit_perc
## weights: weights_clean
##
## Moran I statistic standard deviate = 7.1199, p-value = 5.4e-13
## alternative hypothesis: greater
## sample estimates:
## Moran I statistic       Expectation          Variance
##      0.1928134937     -0.0016103060      0.0007456762
```



The Global Moran's I value of 0.193 indicates a moderate, positive clustering pattern. That is, tracts are somewhat likely to be similar to their immediate neighbors in terms of their mass transit percentage. High tracts are likely to be bordered by other high tracts, and low tracts are likely to be bordered by other low tracts. Given that the Moran's I statistic ranges from -1 to 1, the 0.193 value only indicates weak-to-moderate positive clustering. Conceptually, the low bar for statistical significance (i.e., the null hypothesis is complete randomness, which is intuitively quite unlikely given the higher density of transit in more urban areas), combined with a pretty healthy sample size of n = 625, makes the fact that the test is statistically significant while being somewhat weak a reasonable result. It could also be that the choice of weights matrix could play a part in these results, but it is hard to say because only one scheme was tested. It may be that a distance-based weighting matrix would be a better fit for the analysis. All of that said, the p-value associated with this test is < 0.001, which makes me confident that this clustering pattern is not by chance - the likelihood that we would see this pattern by chance is incredibly low.

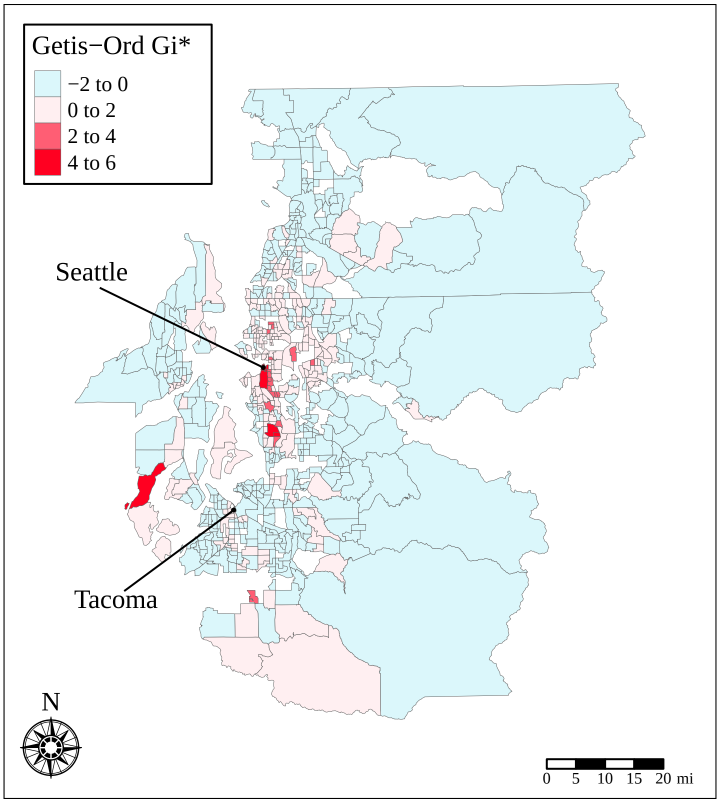
While it is useful to know that there is statistically significant spatial clustering in our variable of interest with the region, these statistics alone do a poor job of helping us to understand where this clustering is happening and whether it fits with expectations. To that end, I will employ the Getis-Ord Gi* statistic and map its values across the region to visualize mass transit hotspots.

The Moran scatterplot visualizes this moderate positive clustering with the relatively shallow, positive slope on the plot of standardized mass transit percentage vs. neighbors standardized mass transit percentage.

Hotspot Mapping with Getis-Ord Gi*

The Getis-Ord Gi* statistic evaluates each tract compared to its neighbors and finds "hotspots" (high-value tracts surrounded by other high-value tracts) and "coldspots" (low-value tracts surrounded by other low-value tracts). The output statistic, is a z-score associated with each tract. Roughly speaking, Gi* values between -2 and 2 represent areas with no significant clustering, while values outside that range (<-2 or >2), represent areas with significant clustering. A negative Gi* statistic indicates a coldspot, while a positive Gi* statistic indicates a hotspot. I am using the same spatial weights matrix as I used for the Global Moran's I.

Puget Sound Mass Transit Hotspot Tracts (Getis−Ord Gi)*



As can be seen in the map above, I chose to give the tracts with non-significant Gi* values directional coloration. Tracts with non-significant negative values are light blue and tracts with non-significant positive values are light pink. Despite the non-significance, I do believe this coloration paints a picture generally consistent with my expectation and thus deserves to be seen, especially given that sample size is a concern for many of these tracts.

Speaking of significant values, however, it is clear to see that the predominant occurrences of clustering are nearby Seattle, with most happening in South Seattle (noted earlier for having high population density and relatively lower median incomes). There are a couple of other instances of significant clustering towards the southwest corner of the map, but I do not have an intuitive explanation for why those are occurring there. More research on locations of transit lines and tract characteristics would have to be done in order to get a better understanding of what is happening.

In the next section, I will take my analysis further with spatial models that regress mass transit percentage onto the independent variables: population density, median income, and median age.

Regression using Spatial Error Model and Spatial Lag Model

Spatial Lag Model

First, I will run a spatial lag model, which essentially includes the value of mass transit percentage in surrounding tracts as defined by the spatial weights matrix. This allows the model to take into account the spatial clustering we saw in the Moran's I test and produce coefficients for the other predictor variables that are more efficient and accurate. For this testing, I will use the `lagsarlm()` function from the `spatialreg` package. I will log both population density and median income, which is a common practice for variables that are always above 0 and can theoretically increase without limit.

```
##
## Call:
## lagsarlm(formula = masstransit_perc ~ log(pop_per_sqmile) + log(medincomeE) +
##     medageE, data = psrc_table_clean, listw = weights_clean)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.7888 -4.1372 -2.0656  2.3349 61.9643
##
## Type: lag
## Coefficients: (asymptotic standard errors)
##                      Estimate Std. Error z value Pr(>|z|)
## (Intercept)          0.906643   8.856855  0.1024  0.91847
## log(pop_per_sqmile)  0.491216   0.229086  2.1442  0.03201
## log(medincomeE)     -0.250715   0.777386 -0.3225  0.74707
## medageE              0.038179   0.053273  0.7167  0.47358
##
## Rho: 0.32971, LR test value: 37.7, p-value: 8.2494e-10
## Asymptotic standard error: 0.049143
##     z-value: 6.7092, p-value: 1.9574e-11
## Wald statistic: 45.013, p-value: 1.9574e-11
##
## Log likelihood: -2055.995 for lag model
## ML residual variance (sigma squared): 42.829, (sigma: 6.5444)
## Number of observations: 621
## Number of parameters estimated: 6
## AIC: 4124, (AIC for lm: 4159.7)
## LM test for residual autocorrelation
## test value: 2.519, p-value: 0.11248
```

As can be seen in this output, there are two predictors for which the coefficient is statistically significant. These are the lag term (Rho), as expected given the clustering already seen above, and the log(pop_per_sqmile) term. The positive coefficient on log(pop_per_sqmile) indicates that greater population density is associated with greater mass transit percentage. This was hypothesized. I will not directly try to interpret the values of the coefficients because the spatial lag term makes direct interpretations inaccurate. The other predictors are non-significant.

I will also calculate a pseudo R^2 value for this regression model using the formula $1 - \frac{SSE}{TSS}$.

```
## Pseudo R-squared: 0.1069
```

This indicates our model explains approximately 10% of the variation seen in the dependent variable, mass transit percentage.

Spatial Error Model

Rather than modeling lag terms, a spatial error model assumes that the residuals (error) of the model are spatially autocorrelated. It seeks to address the same issues as the SLM: ensuring the the coefficients and standard errors for our other predictors are efficient and accurate. I will use the `errorsarlm()` function, also via the `spatialreg` package.

```
##
## Call:errorsarlm(formula = masstransit_perc ~ log(pop_per_sqmile) +
##     log(medincomeE) + medageE, data = psrc_table_clean, listw = weights_clean)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.1359 -4.1456 -2.1598  2.4106 61.7270
##
## Type: error
## Coefficients: (asymptotic standard errors)
##                      Estimate Std. Error z value Pr(>|z|)
## (Intercept)          4.179806  10.937819  0.3821   0.7024
## log(pop_per_sqmile)  0.463804   0.289750  1.6007   0.1094
## log(medincomeE)     -0.352675   0.938844 -0.3756   0.7072
## medageE              0.032458   0.057055  0.5689   0.5694
##
## Lambda: 0.32786, LR test value: 35.675, p-value: 2.3318e-09
## Asymptotic standard error: 0.049346
##     z-value: 6.6442, p-value: 3.0484e-11
## Wald statistic: 44.146, p-value: 3.0484e-11
##
## Log likelihood: -2057.008 for error model
## ML residual variance (sigma squared): 42.982, (sigma: 6.5561)
## Number of observations: 621
## Number of parameters estimated: 6
## AIC: 4126, (AIC for lm: 4159.7)
```

Interestingly, none of the predictors in this model are statistically significant, although the significance in the error term lambda indicates, once again, that it is important to account for spatial autocorrelation in our modeling. I will again calculate and print the pseudo r-squared.

```
## Pseudo R-squared: 0.1037
```

This value is nearly identical to the one for the spatial lag model (as are the values of log likelihood and AIC), which indicates similar predictive power between the models.

From the relatively low r-squared values and the lack of statistical significance in most of my predictors, it is clear to me that this analysis would benefit from a new selection of predictor variables. A simple first step in this direction would be to do a more thorough analysis with the ACS demographic variables, but I think there are richer extensions beyond this. There are a plethora of built-environment and transportation-relevant variables that could be summarized per tract, such as land use percentage (industrial, residential, commercial, etc.), parking availability/price, and traffic congestion (particularly relevant in Seattle, where buses and at-grade light rail are common), among others. Each of these variables are accessible, but would require a more involved validation and joining procedure than the ACS census tract variables. Nonetheless, I believe the more model complexity is the answer here, rather than simplicity. This is particularly true as the goal of the analysis is to better understand what impacts transit usage, rather than whether a single variable does or does not impact transit usage.

Discussion and Interpretation

Key Findings

This investigation put into statistical terms a pattern that was (a) probably easy to intuit and (b) visually apparent from the initial mapping: high values of transit usage clusters close to city centers (particularly for Seattle) in the Puget Sound region. A further corroborating relationship was revealed with the SLM: that population density is positively associated with mass transit usage.

The other predictor variables (median income and median age) were not statistically significantly related to my outcome variable. The analysis did not present me with any surprising or counterintuitive results, merely a lack of significance for some of my variables.

As an aside, I was interested in how some of the weird variation seen in areas further outside the city might be affecting my results. I cloned the R script I used for the analysis and ran all the same analysis, but filtering to just King country (where Seattle is located), removing Kitsap, Pierce, and Snohomish counties. Although my regression outputs remained non-significant (possible reasons why are discussed in the limitations section below), my clustering statistic became stronger (still significant) and my regression diagnostics for predictive power (pseudo R^2, AIC, log likelihood) improved.

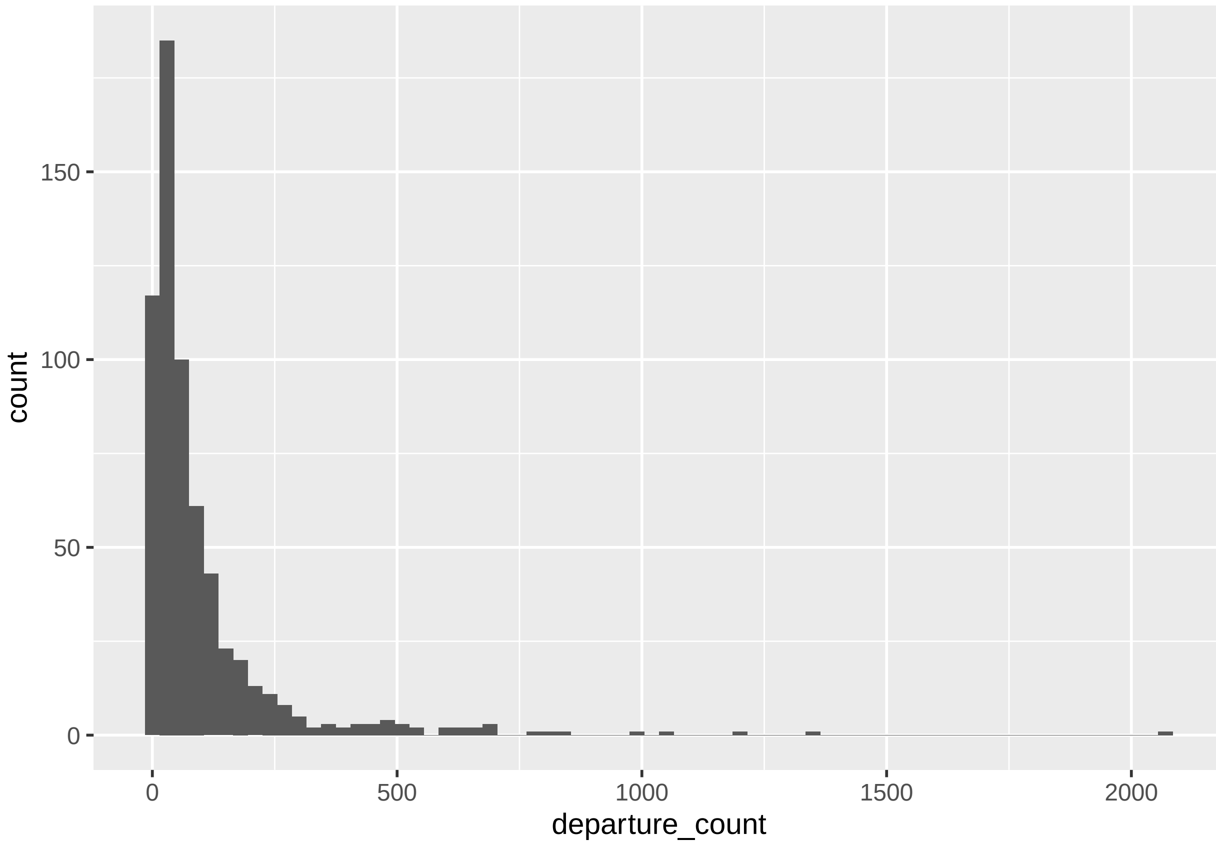
Implications/Areas for Future Inquiry

With statistical evidence in favor of urban clustering of transit usage, I can now confidently ask the question: "what is it about urban areas that encourages transit ridership?" I believe this can be decomposed into two pieces. First, there are likely certain pro-transit characteristics of urban areas. A high density of stops and transfer options mean transit is more convenient for riders, as does better walkability and a lower distance between likely origin and destination. On the other hand, some characteristics of urban areas have a more anti-car flavor. Limited and expensive parking options are a good example. If increasing transit adoption and usage is a priority (and it should be, for sustainability and equitability reasons), further research should investigate these characteristics to better understand how to design desirable transit in areas with low usage rates.

This sort of investigation could remain a spatial one. I imagine a survey of people in the region, each linked to a census tract, and would imagine that their public transit usage considerations might vary with geography. People in urban areas might consider transfer reliability, walkability, or anti-car factors more than suburban or rural respondents, who might be more interested in things like overall speed. Seattle, in particular, has a big suburban population that commutes either to the city or nearby tech campuses and, as we can see from the initial choropleth mapping, tends to do so by car. A better understanding of their reasons for this would allow for better policy and infrastructure design to increase transit usage and adoption.

Limitations

There are two major limitations to this analysis. First of all, I have serious concerns about sample size for this project. As can be seen in the simple histogram below (binwidth = 30), a large percentage of tracts have below 30 observations. This gives me concerns about noise. The structure of my analysis was such that observation count was lost in translation - my response variable was aggregated and in the form of a rate. This means that a tract with 5 departures (very likely to be noisy) was treated the same as one with 1000 departures (unlikely to be noisy). Missingness is a further issue with sample size - there are a number of tracts completely omitted from my analysis because they had no observations, period. The solution to the problem is not entirely clear to me at this stage, although it may be that some sort of multilevel modeling or Bayesian approach could at least help me propagate uncertainty from the initial calculations through to my final models.



The second limitation to this analysis (and probably the one more easily solved) is the lack of predictor variables in my modeling. Given more time, I would have (a) included more demographic variables and (b) looked for some built-environment and tract-characteristic variables (e.g., parking cost, walkability, congestion) to associate with each tract. Although this would be a time-consuming process, it would add a lot of color to my model and, at the very least, be likely to improve the predictive power of my models.

Conclusion

This report can be thought of as a jumping-off point for my personal research interests. Having this sort of statistical and spatial understanding of how transit usage is distributed in the region gives me a concrete foundation off of which to build further analyses. Although I did not find much in the way of interesting regression results, the analysis has given me pause to consider what other types of variables (especially outside of basic ACS demographics) might be useful and/or interesting to consider adding to future work. Finally, I think the number of questions that this analysis provokes within me will be helpful for thinking about future research directions and how the relate to policy decisions.

References

Data Sources

1. National Atlas of the United States. (2013). Cities and Towns of the United States, 2014. National Atlas of the United States. Available at: http://purl.stanford.edu/bx729wr3020.

2. Puget Sound Regional Council. (2023). Household travel survey program. Puget Sound Regional Council. https://www.psrc.org/our-work/household-travel-survey-program
3. U.S. Census Bureau. (n.d.). *American Community Survey 5-year estimates: 2022*. U.S. Department of Commerce. Retrieved December 11, 2024, from https://data.census.gov/

Programming Languages/Software

1. R Core Team (2024). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria. https://www.R-project.org/.
2. RStudio Team. (2020). RStudio: Integrated Development for R. RStudio, PBC, Boston, MA. http://www.rstudio.com/.

R Packages

1. Baddeley A, Rubak E, Turner R (2015). *Spatial Point Patterns: Methodology and Applications with R*. Chapman and Hall/CRC Press, London. ISBN 9781482210200, https://www.routledge.com/Spatial-Point-Patterns-Methodology-and-Applications-with-R/Baddeley-Rubak-Turner/p/book/9781482210200/.
2. Bivand R (2022). "R Packages for Analyzing Spatial Data: A Comparative Case Study with Areal Data." *Geographical Analysis*, *54*(3), 488-518. https://doi.org/10.1111/gean.12319.
3. Bivand R, Millo G, Piras G (2021). "A Review of Software for Spatial Econometrics in R." *Mathematics*, *9*(11). https://doi.org/10.3390/math9111276, https://www.mdpi.com/2227-7390/9/11/1276.
4. Bivand R, Pebesma E, Gómez-Rubio V (2013). *Applied spatial data analysis with R, Second edition*. Springer, NY. https://asdar-book.org/.
5. Bivand R, Wong D (2018). "Comparing implementations of global and local indicators of spatial association." *TEST*, *27*(3), 716-748. https://doi.org/10.1007/s11749-018-0599-x.
6. Müller K (2020). *here: A Simpler Way to Find Your Files*. R package version 1.0.1, https://CRAN.R-project.org/package=here.
7. Pebesma, E., & Bivand, R. (2023). Spatial Data Science: With Applications in R. Chapman and Hall/CRC. https://doi.org/10.1201/9780429459016
8. Pebesma E, Bivand R (2023). *Spatial Data Science With Applications in R*. Chapman & Hall. https://r-spatial.org/book/.
9. Pebesma, E., 2018. Simple Features for R: Standardized Support for Spatial Vector Data. The R Journal 10 (1), 439-446, https://doi.org/10.32614/RJ-2018-009
10. Tennekes M (2018). "tmap: Thematic Maps in R." *Journal of Statistical Software*, *84*(6), 1-39. https://doi.org/10.18637/jss.v084.i06.
11. Walker K, Herman M (2024). *tidycensus: Load US Census Boundary and Attribute Data as 'tidyverse' and 'sf'-Ready Data Frames*. R package version 1.6.7, https://CRAN.R-project.org/package=tidycensus.
12. Wickham H (2023). *forcats: Tools for Working with Categorical Variables (Factors)*. R package version 1.0.0, https://CRAN.R-project.org/package=forcats.
13. Wickham H, Averick M, Bryan J, Chang W, McGowan LD, François R, Grolemund G, Hayes A, Henry L, Hester J, Kuhn M, Pedersen TL, Miller E, Bache SM, Müller K, Ooms J, Robinson D, Seidel DP, Spinu V, Takahashi K, Vaughan D, Wilke C, Woo K, Yutani H (2019). "Welcome to the tidyverse." *Journal of Open Source Software*, *4*(43), 1686. https://doi.org/10.21105/joss.01686.
14. Xie Y (2024). *knitr: A General-Purpose Package for Dynamic Report Generation in R*. R package version 1.48, https://yihui.org/knitr/.
15. Yihui Xie (2015) Dynamic Documents with R and knitr. 2nd edition. Chapman and Hall/CRC. ISBN 978-1498716963
16. Yihui Xie (2014) knitr: A Comprehensive Tool for Reproducible Research in R. In Victoria Stodden, Friedrich Leisch and Roger D. Peng, editors, Implementing Reproducible Computational Research. Chapman and Hall/CRC. ISBN 978-1466561595