modelnotebook

April 19, 2024

1 DEDS_Portfolio_Week-10

Van Pjotr en Sennen

De opdracht van deze week is:

1.1 Opdracht:

Bouw een neuraal netwerk met generatieve AI

1.2 Doel:

Het doel van deze opdracht is om een basisbegrip van neurale netwerken te ontwikkelen door een eenvoudig neuraal netwerk te implementeren, zonder gebruik te maken van backpropagation en gradient descent. Er dient voor het trainen van het model een simpel algoritme gebruikt te worden. Je moet de code zelf goed kan uitleggen.

1.2.1 Requirements:

1. NN heeft 4 input nodes, 1 hidden layer met een door de student gekozen aantal nodes en 1 output node.
2. Gebruik 1 tot 5 input datapunten met bijbehorende output (antwoorden)
3. Maak gebruik van arrays
4. Het mag geen backpropagation gebruiken en ook niet Gradient Descent algoritme.

1.2.2 Stappen:

De volgende algemene stappen zou je terug moeten kunnen vinden of herkennen in de gegenereerde code van je NN. Dit is een hulpmiddel voor je om de code te begrijpen. Het is niet erg als jou gegenereerde code hier iets van afwijkt. 1. Definieer de structuur van het neurale netwerk, inclusief het aantal input nodes, het aantal nodes in de hidden layer en het aantal output nodes. 2. Initialiseer de gewichten van het netwerk willekeurig. 3. Implementeer de feedforward-methode om de input door het netwerk te sturen en de output te berekenen. 4. Bereken de error of fout tussen de voorspelde output en de werkelijke output. 5. Pas de gewichten niet aan met behulp van backpropagation en gradient descent. In plaats daarvan kunnen de studenten ervoor kiezen om de gewichten met een eenvoudige regel aan te passen. 6. Train het netwerk met behulp van de gegeven training samples en evalueer de prestaties ervan.

1.2.3 Training:

Het kan zijn dat de LLM alsnog, direct of indirect, de backpropagation en gradient descent trainingsalgoritme gebruikt. Andere termen die hier direct te maken mee hebben zijn de 'afgeleide' (in het engels de derivative). Dat zie je als je bijvoorbeeld het volgende ziet in de code die de LLM genereerd: - inputToHiddenWeights[i, j] += error * input[i] * hiddenOutput[j] * (1 - hiddenOutput[j]); Metname als het "… (1 - …)" gedeelte. Je wil het liefst code zien dat er als volgt uit ziet: - inputToHiddenWeights[i, j] += error * input[i] * hiddenOutput[j];

1.3 Maak het model from scratch:

Dit is een model die we hebben gemaakt met behulp van ChatGPT. Het model is een simpel neuraal netwerk met 4 input nodes, 1 hidden layer met 3 nodes en 1 output node. Het model maakt gebruik van 1 input datapunt met bijbehorende output. Het model maakt gebruik van arrays en maakt geen gebruik van backpropagation en gradient descent algoritme.

[ ]:
```python
import numpy as np
import matplotlib.pyplot as plt

class NeuralNetwork:
    def __init__(self, input_size, hidden_size, output_size):
        self.input_size = input_size
        self.hidden_size = hidden_size
        self.output_size = output_size

        # Initialize weights randomly
        self.input_to_hidden_weights = np.random.randn(input_size, hidden_size)
        self.hidden_to_output_weights = np.random.randn(hidden_size,␣
↪output_size)

    def sigmoid(self, x):
        return 1 / (1 + np.exp(-x))

    def feedforward(self, inputs):
        # Input to hidden layer
        hidden_sum = np.dot(inputs, self.input_to_hidden_weights)
        hidden_output = self.sigmoid(hidden_sum)

        # Hidden to output layer
        output_sum = np.dot(hidden_output, self.hidden_to_output_weights)
        output = self.sigmoid(output_sum)

        return output

    def train(self, inputs, targets, epochs):
        errors = []
        for epoch in range(epochs):
            epoch_error = 0
```

```
            for i in range(len(inputs)):
                # Feedforward
                input_data = inputs[i]
                target = targets[i]

                hidden_sum = np.dot(input_data, self.input_to_hidden_weights)
                hidden_output = self.sigmoid(hidden_sum)

                output_sum = np.dot(hidden_output, self.
 ↪hidden_to_output_weights)
                output = self.sigmoid(output_sum)

                # Calculate error
                error = target - output
                epoch_error += np.mean(np.abs(error))

                # Update weights (no backpropagation)
                self.input_to_hidden_weights += np.outer(input_data,␣
 ↪hidden_output) * error
                self.hidden_to_output_weights += np.outer(hidden_output,␣
 ↪output) * error

            # Append average error for this epoch
            errors.append(epoch_error / len(inputs))

            # Print average error for this epoch
            print(f'Epoch {epoch + 1}, Average Error: {errors[-1]}')

        # Plot the training error over epochs
        plt.plot(range(1, epochs + 1), errors)
        plt.xlabel('Epoch')
        plt.ylabel('Average Error')
        plt.title('Training Error Over Epochs')
        plt.show()

# Example usage
inputs = np.array([[0, 0, 1, 1],
                   [0, 1, 1, 0],
                   [1, 0, 1, 1],
                   [1, 1, 1, 0]])
targets = np.array([[0], [1], [1], [0]])

nn = NeuralNetwork(input_size=4, hidden_size=3, output_size=1)
nn.train(inputs, targets, epochs=100)
nn.feedforward([0, 0, 1, 1])
```

```
Epoch 1, Average Error: 0.5105571902842014
Epoch 2, Average Error: 0.5041654016133469
Epoch 3, Average Error: 0.502018387057321
Epoch 4, Average Error: 0.5038056409321545
Epoch 5, Average Error: 0.5032324541562966
Epoch 6, Average Error: 0.5000931161826749
Epoch 7, Average Error: 0.49606796091092453
Epoch 8, Average Error: 0.4919766465647807
Epoch 9, Average Error: 0.48808307227265035
Epoch 10, Average Error: 0.484461383443363
Epoch 11, Average Error: 0.481091783248895
Epoch 12, Average Error: 0.47787336850110873
Epoch 13, Average Error: 0.47465423819328834
Epoch 14, Average Error: 0.47129311236241955
Epoch 15, Average Error: 0.4677138223822269
Epoch 16, Average Error: 0.4639169444833012
Epoch 17, Average Error: 0.4599530702797703
Epoch 18, Average Error: 0.45588753925491016
Epoch 19, Average Error: 0.451776989683938
Epoch 20, Average Error: 0.44766029026146303
Epoch 21, Average Error: 0.4435584054172921
Epoch 22, Average Error: 0.4394776288194856
Epoch 23, Average Error: 0.4354131315191134
Epoch 24, Average Error: 0.4313521671717597
Epoch 25, Average Error: 0.4272776939523346
Epoch 26, Average Error: 0.4231734872507178
Epoch 27, Average Error: 0.4190310616140589
Epoch 28, Average Error: 0.4148571185093611
Epoch 29, Average Error: 0.4106786016840833
Epoch 30, Average Error: 0.4065422004081922
Epoch 31, Average Error: 0.40250724430214085
Epoch 32, Average Error: 0.3986343814429553
Epoch 33, Average Error: 0.39497461295783964
Epoch 34, Average Error: 0.3915625086017043
Epoch 35, Average Error: 0.3884147160293335
Epoch 36, Average Error: 0.3855324548609125
Epoch 37, Average Error: 0.3829058174247415
Epoch 38, Average Error: 0.3805181154686392
Epoch 39, Average Error: 0.37834936286941856
Epoch 40, Average Error: 0.3763786750511824
Epoch 41, Average Error: 0.37458573116894256
Epoch 42, Average Error: 0.3729515578849124
Epoch 43, Average Error: 0.37145887542747386
Epoch 44, Average Error: 0.37009218521495013
Epoch 45, Average Error: 0.3688377172193877
Epoch 46, Average Error: 0.36768330893554224
Epoch 47, Average Error: 0.36661825708018
Epoch 48, Average Error: 0.3656331642933235
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Epoch 49, Average Error: 0.36471979213758404
Epoch 50, Average Error: 0.3638709255442175
Epoch 51, Average Error: 0.36308025053144233
Epoch 52, Average Error: 0.36234224530311093
Epoch 53, Average Error: 0.3616520840125794
Epoch 54, Average Error: 0.36100555214053487
Epoch 55, Average Error: 0.36039897235576335
Epoch 56, Average Error: 0.3598291397742468
Epoch 57, Average Error: 0.35929326563224584
Epoch 58, Average Error: 0.35878892850428823
Epoch 59, Average Error: 0.35831403230787684
Epoch 60, Average Error: 0.35786677043513304
Epoch 61, Average Error: 0.3574455954357218
Epoch 62, Average Error: 0.3570491937467352
Epoch 63, Average Error: 0.35667646502665007
Epoch 64, Average Error: 0.3563265057052036
Epoch 65, Average Error: 0.3559985964119447
Epoch 66, Average Error: 0.35569219299559374
Epoch 67, Average Error: 0.3554069208957493
Epoch 68, Average Error: 0.35514257267886573
Epoch 69, Average Error: 0.3548991086020913
Epoch 70, Average Error: 0.3546766601213953
Epoch 71, Average Error: 0.35447553631380035
Epoch 72, Average Error: 0.3542962332364657
Epoch 73, Average Error: 0.3541394462962456
Epoch 74, Average Error: 0.3540060857498524
Epoch 75, Average Error: 0.35389729549350113
Epoch 76, Average Error: 0.35381447532695
Epoch 77, Average Error: 0.3537593068830872
Epoch 78, Average Error: 0.35373378339059475
Epoch 79, Average Error: 0.35374024336984905
Epoch 80, Average Error: 0.3537814082324123
Epoch 81, Average Error: 0.35386042353791414
Epoch 82, Average Error: 0.35398090332858767
Epoch 83, Average Error: 0.354146976475654
Epoch 84, Average Error: 0.3543633332948271
Epoch 85, Average Error: 0.35463526978488724
Epoch 86, Average Error: 0.3549687256907313
Epoch 87, Average Error: 0.35537031119636686
Epoch 88, Average Error: 0.35584731547102855
Epoch 89, Average Error: 0.35640768865957373
Epoch 90, Average Error: 0.3570599874726786
Epoch 91, Average Error: 0.3578132736703042
Epoch 92, Average Error: 0.35867695494945206
Epoch 93, Average Error: 0.35966055963116633
Epoch 94, Average Error: 0.36077344064603073
Epoch 95, Average Error: 0.36202441097563554
Epoch 96, Average Error: 0.36342132179386116
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Epoch 97, Average Error: 0.36497060525567543
Epoch 98, Average Error: 0.3666768146024494
Epoch 99, Average Error: 0.36854220273635896
Epoch 100, Average Error: 0.37056638419234983
```

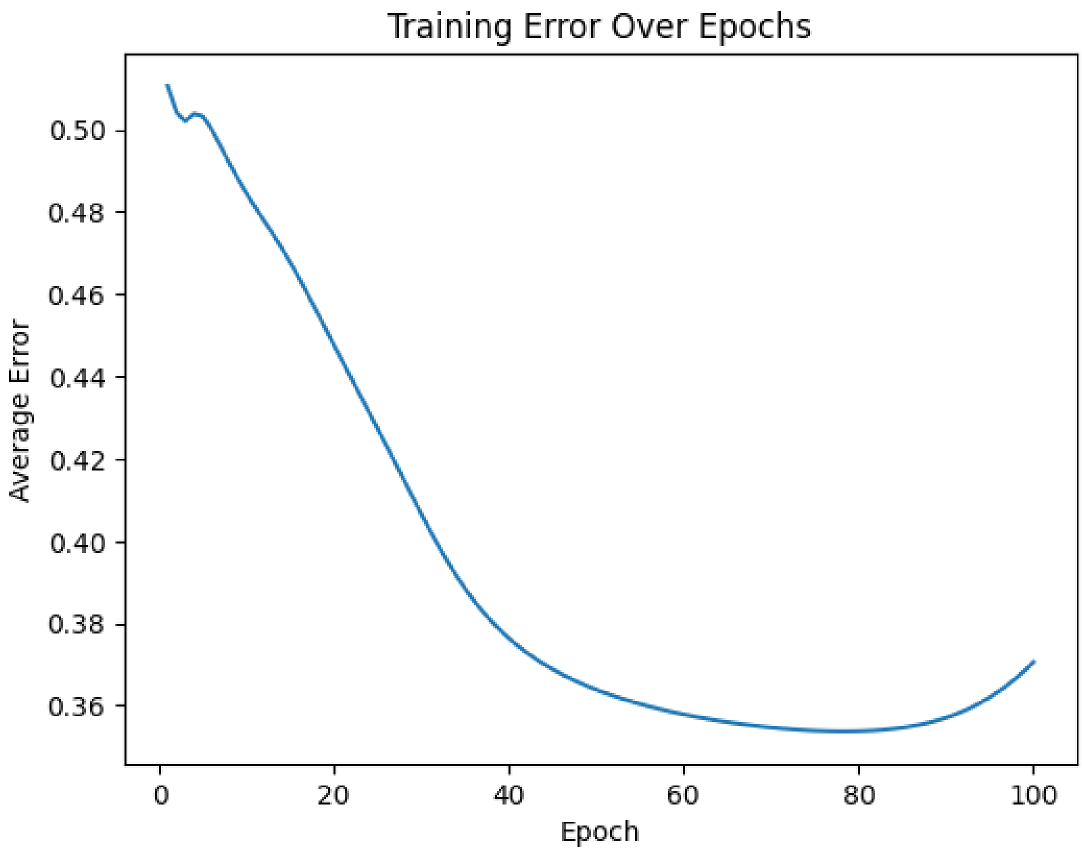

[ ]:
```
array([0.27233558])
```

1.4 Gebaseerd op het C# Script:

In het begin van deze opdracht moesten we ook een c# script maken. Waar we ook ons eigen neural network moesten bouwen. Hieronder is een python script gebasseerd op het c# script.

Hier voegen we ook backpropagation toe. Dat zorgt ervoor dat het model beter kan leren, door fouten te corrigeren en veranderingen te maken in de gewichten van het model.

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
import sklearn.metrics as metrics

class NeuralNetwork:
    def __init__(self, input_size, hidden_layer_size, learning_rate):
```

```
        self.input_size = input_size
        self.hidden_layer_size = hidden_layer_size
        self.learning_rate = learning_rate

        self.weights_input_to_hidden = np.random.uniform(-1, 1, (input_size,␣
↪hidden_layer_size))
        self.weights_hidden_to_output = np.random.uniform(-1, 1,␣
↪hidden_layer_size)

    def sigmoid(self, x):
        return 1 / (1 + np.exp(-x))

    def feed_forward(self, inputs):
        hidden_layer_output = self.sigmoid(np.dot(inputs, self.
↪weights_input_to_hidden))
        output = self.sigmoid(np.dot(hidden_layer_output, self.
↪weights_hidden_to_output))
        return output

    def train(self, inputs, target, epochs):
        errors = []
        for epoch in range(epochs):
            hidden_layer_output = self.sigmoid(np.dot(inputs, self.
↪weights_input_to_hidden))
            output = self.sigmoid(np.dot(hidden_layer_output, self.
↪weights_hidden_to_output))

            error = target - output

            self.weights_hidden_to_output += error * self.learning_rate *␣
↪hidden_layer_output
            self.weights_input_to_hidden += np.outer(inputs, error * self.
↪learning_rate * self.weights_hidden_to_output * hidden_layer_output * (1 -␣
↪hidden_layer_output))

            errors.append(np.mean(np.abs(error)))
            print(f'Epoch {epoch + 1}, Average Error: {errors[-1]}')

        # Plot the training error over epochs
        plt.plot(range(1, epochs + 1), errors)
        plt.xlabel('Epoch')
        plt.ylabel('Average Error')
        plt.title('Training Error Over Epochs')
        plt.show()

# Example usage
```

```python
np.random.seed(0)  # for reproducibility

neural_network = NeuralNetwork(4, 3, 0.1)

inputs = np.array([0.1, 0.2, 0.3, 0.4])
target = 0.5

neural_network.train(inputs, target, epochs=1000)

test_inputs = np.array([0.5, 0.6, 0.7, 0.8])
output = neural_network.feed_forward(test_inputs)

print("Output:", output)

print("Mean Absolute Error:", metrics.mean_absolute_error([target], [output]))
print("Mean Squared Error:", metrics.mean_squared_error([target], [output]))
```

```
Epoch 1, Average Error: 0.020241418340183115
Epoch 2, Average Error: 0.01974383285036374
Epoch 3, Average Error: 0.019258445687551418
Epoch 4, Average Error: 0.018784960203589662
Epoch 5, Average Error: 0.018323086793048504
Epoch 6, Average Error: 0.017872542738384123
Epoch 7, Average Error: 0.017433052057591825
Epoch 8, Average Error: 0.01700434535438522
Epoch 9, Average Error: 0.016586159670923695
Epoch 10, Average Error: 0.016178238343110718
Epoch 11, Average Error: 0.015780330858468886
Epoch 12, Average Error: 0.01539219271660297
Epoch 13, Average Error: 0.015013585292249276
Epoch 14, Average Error: 0.01464427570090876
Epoch 15, Average Error: 0.014284036667056466
Epoch 16, Average Error: 0.013932646394915538
Epoch 17, Average Error: 0.01358988844178044
Epoch 18, Average Error: 0.013255551593873705
Epoch 19, Average Error: 0.01292942974471245
Epoch 20, Average Error: 0.012611321775964357
Epoch 21, Average Error: 0.012301031440766885
Epoch 22, Average Error: 0.011998367249481445
Epoch 23, Average Error: 0.011703142357856189
Epoch 24, Average Error: 0.01141517445756457
Epoch 25, Average Error: 0.011134285669089472
Epoch 26, Average Error: 0.010860302436919267
Epoch 27, Average Error: 0.010593055427022935
Epoch 28, Average Error: 0.010332379426568838
Epoch 29, Average Error: 0.010078113245852505
Epoch 30, Average Error: 0.009830099622396449
Epoch 31, Average Error: 0.009588185127187843
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Epoch 32, Average Error: 0.009352220073015616
Epoch 33, Average Error: 0.009122058424870572
Epoch 34, Average Error: 0.008897557712371662
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Epoch 36, Average Error: 0.00846498652437877
Epoch 37, Average Error: 0.00825664817073024
Epoch 38, Average Error: 0.008053434834863316
Epoch 39, Average Error: 0.007855220624263026
Epoch 40, Average Error: 0.00766188272608026
Epoch 41, Average Error: 0.007473301332708959
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Epoch 43, Average Error: 0.007109943421743425
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Epoch 49, Average Error: 0.006122443145988421
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Epoch 52, Average Error: 0.005681368577107215
Epoch 53, Average Error: 0.00554151985742557
Epoch 54, Average Error: 0.005405112759791608
Epoch 55, Average Error: 0.0052720626430079465
Epoch 56, Average Error: 0.005142286943562002
Epoch 57, Average Error: 0.005015705124905012
Epoch 58, Average Error: 0.0048922386279474095
Epoch 59, Average Error: 0.0047718108227464695
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Epoch 61, Average Error: 0.004539774131813279
Epoch 62, Average Error: 0.0044280212132356
Epoch 63, Average Error: 0.0043190188320013645
Epoch 64, Average Error: 0.004212699318990931
Epoch 65, Average Error: 0.004108996667861353
Epoch 66, Average Error: 0.004007846494331613
Epoch 67, Average Error: 0.003909185996455622
Epoch 68, Average Error: 0.003812953915857009
Epoch 69, Average Error: 0.0037190904999062635
Epoch 70, Average Error: 0.003627537464815367
Epoch 71, Average Error: 0.003538237959630597
Epoch 72, Average Error: 0.003451136531099408
Epoch 73, Average Error: 0.0033661790893941834
Epoch 74, Average Error: 0.0032833128746684315
Epoch 75, Average Error: 0.0032024864244288853
Epoch 76, Average Error: 0.0031236495417028554
Epoch 77, Average Error: 0.0030467532639811834
Epoch 78, Average Error: 0.002971749832919146
Epoch 79, Average Error: 0.002898592664776989
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Epoch 80, Average Error: 0.002827236321581661
Epoch 81, Average Error: 0.002757636482993764
Epoch 82, Average Error: 0.002689749918860951
Epoch 83, Average Error: 0.0026235344624430113
Epoch 84, Average Error: 0.0025589489842913204
Epoch 85, Average Error: 0.002495953366766779
Epoch 86, Average Error: 0.0024345084791824734
Epoch 87, Average Error: 0.0023745761535530763
Epoch 88, Average Error: 0.0023161191609391008
Epoch 89, Average Error: 0.0022591011883701384
Epoch 90, Average Error: 0.002203486816333089
Epoch 91, Average Error: 0.00214924149681206
Epoch 92, Average Error: 0.002096331531866835
Epoch 93, Average Error: 0.0020447240527354804
Epoch 94, Average Error: 0.001994386999450093
Epoch 95, Average Error: 0.0019452891009520412
Epoch 96, Average Error: 0.0018973998556949256
Epoch 97, Average Error: 0.0018506895127234912
Epoch 98, Average Error: 0.0018051290532162811
Epoch 99, Average Error: 0.0017606901724818158
Epoch 100, Average Error: 0.001717345262396086
Epoch 101, Average Error: 0.0016750673942714789
Epoch 102, Average Error: 0.0016338303021461442
Epoch 103, Average Error: 0.0015936083664835898
Epoch 104, Average Error: 0.001554376598272289
Epoch 105, Average Error: 0.0015161106235167532
Epoch 106, Average Error: 0.0014787866681078565
Epoch 107, Average Error: 0.0014423815430663067
Epoch 108, Average Error: 0.0014068726301480483
Epoch 109, Average Error: 0.001372237867803272
Epoch 110, Average Error: 0.0013384557374800377
Epoch 111, Average Error: 0.0013055052502658482
Epoch 112, Average Error: 0.0012733659338557413
Epoch 113, Average Error: 0.0012420178198415677
Epoch 114, Average Error: 0.0012114414313141308
Epoch 115, Average Error: 0.0011816177707689723
Epoch 116, Average Error: 0.0011525283083103632
Epoch 117, Average Error: 0.0011241549701448417
Epoch 118, Average Error: 0.0010964801273577462
Epoch 119, Average Error: 0.001069486584966084
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Epoch 121, Average Error: 0.0010174767272855822
Epoch 122, Average Error: 0.000992428096889153
Epoch 123, Average Error: 0.0009679961166034756
Epoch 124, Average Error: 0.000944165606083458
Epoch 125, Average Error: 0.0009209217586583218
Epoch 126, Average Error: 0.0008982501321341818
Epoch 127, Average Error: 0.000876136639824554
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Epoch 128, Average Error: 0.0008545675418007992
Epoch 129, Average Error: 0.0008335294363573942
Epoch 130, Average Error: 0.000813009251688146
Epoch 131, Average Error: 0.0007929942377659094
Epoch 132, Average Error: 0.0007734719584235883
Epoch 133, Average Error: 0.0007544302836285377
Epoch 134, Average Error: 0.0007358573819481462
Epoch 135, Average Error: 0.0007177417131996044
Epoch 136, Average Error: 0.0007000720212820832
Epoch 137, Average Error: 0.0006828373271836607
Epoch 138, Average Error: 0.0006660269221614445
Epoch 139, Average Error: 0.0006496303610894483
Epoch 140, Average Error: 0.000633637455968783
Epoch 141, Average Error: 0.0006180382695997189
Epoch 142, Average Error: 0.0006028231094079572
Epoch 143, Average Error: 0.0005879825214228918
Epoch 144, Average Error: 0.0005735072844046396
Epoch 145, Average Error: 0.0005593884041154018
Epoch 146, Average Error: 0.0005456171077319327
Epoch 147, Average Error: 0.0005321848383946781
Epoch 148, Average Error: 0.0005190832498921383
Epoch 149, Average Error: 0.0005063042014756824
Epoch 150, Average Error: 0.0004938397528018168
Epoch 151, Average Error: 0.00048168215899890843
Epoch 152, Average Error: 0.00046982386585558977
Epoch 153, Average Error: 0.00045825750512751284
Epoch 154, Average Error: 0.0004469758899591225
Epoch 155, Average Error: 0.0004359720104191167
Epoch 156, Average Error: 0.0004252390291445973
Epoch 157, Average Error: 0.0004147702770939121
Epoch 158, Average Error: 0.0004045592494019701
Epoch 159, Average Error: 0.00039459960133969574
Epoch 160, Average Error: 0.0003848851443718493
Epoch 161, Average Error: 0.0003754098423116581
Epoch 162, Average Error: 0.0003661678075705943
Epoch 163, Average Error: 0.0003571532975008562
Epoch 164, Average Error: 0.0003483607108262232
Epoch 165, Average Error: 0.00033978458416372703
Epoch 166, Average Error: 0.0003314195886268134
Epoch 167, Average Error: 0.0003232605265163224
Epoch 168, Average Error: 0.0003153023280900724
Epoch 169, Average Error: 0.00030754004841326843
Epoch 170, Average Error: 0.0002999688642859599
Epoch 171, Average Error: 0.0002925840712458827
Epoch 172, Average Error: 0.0002853810806461299
Epoch 173, Average Error: 0.0002783554168037661
Epoch 174, Average Error: 0.00027150271421871874
Epoch 175, Average Error: 0.0002648187148613923
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Epoch 176, Average Error: 0.00025829926552745075
Epoch 177, Average Error: 0.0002519403152567712
Epoch 178, Average Error: 0.0002457379128164572
Epoch 179, Average Error: 0.00023968820424646875
Epoch 180, Average Error: 0.00023378743046365003
Epoch 181, Average Error: 0.00022803192492759639
Epoch 182, Average Error: 0.00022241811136125555
Epoch 183, Average Error: 0.00021694250152959338
Epoch 184, Average Error: 0.00021160169307177235
Epoch 185, Average Error: 0.00020639236738728695
Epoch 186, Average Error: 0.00020131128757383543
Epoch 187, Average Error: 0.00019635529641626182
Epoch 188, Average Error: 0.00019152131442456977
Epoch 189, Average Error: 0.00018680633792078627
Epoch 190, Average Error: 0.00018220743717267673
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Epoch 192, Average Error: 0.00017334650286304765
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Epoch 965, Average Error: 7.427392034742297e-13
Epoch 966, Average Error: 7.244205235679146e-13
Epoch 967, Average Error: 7.065459328714496e-13
Epoch 968, Average Error: 6.892264536872972e-13
Epoch 969, Average Error: 6.722400414105323e-13
Epoch 970, Average Error: 6.556977183436175e-13
Epoch 971, Average Error: 6.395994844865527e-13
Epoch 972, Average Error: 6.238343175368755e-13
Epoch 973, Average Error: 6.084022174945858e-13
Epoch 974, Average Error: 5.934142066621462e-13
Epoch 975, Average Error: 5.788702850395566e-13
Epoch 976, Average Error: 5.645484080218921e-13
Epoch 977, Average Error: 5.506706202140776e-13
Epoch 978, Average Error: 5.371258993136507e-13
Epoch 979, Average Error: 5.239142453206114e-13
Epoch 980, Average Error: 5.110356582349596e-13
Epoch 981, Average Error: 4.984901380566953e-13
Epoch 982, Average Error: 4.86166662483356e-13
Epoch 983, Average Error: 4.741762538174044e-13
Epoch 984, Average Error: 4.625189120588402e-13
Epoch 985, Average Error: 4.510836149052011e-13
Epoch 986, Average Error: 4.4009240696141205e-13
Epoch 987, Average Error: 4.292122213200855e-13
Epoch 988, Average Error: 4.1866510258614653e-13
Epoch 989, Average Error: 4.083400284571326e-13
Epoch 990, Average Error: 3.9823699893304365e-13
Epoch 991, Average Error: 3.8846703631634227e-13
```
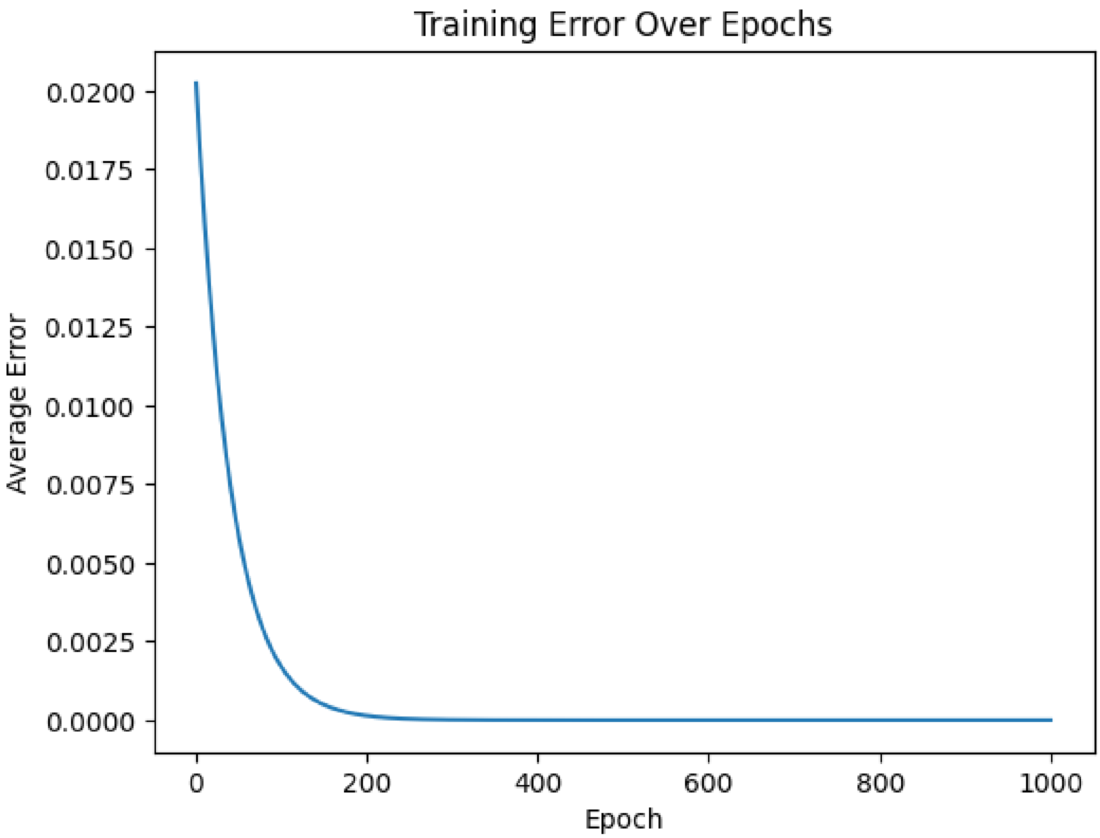

```
Epoch 992, Average Error: 3.7891911830456593e-13
Epoch 993, Average Error: 3.695932448977146e-13
Epoch 994, Average Error: 3.6048941609578833e-13
Epoch 995, Average Error: 3.516076318987871e-13
Epoch 996, Average Error: 3.4294789230671086e-13
Epoch 997, Average Error: 3.3451019731955967e-13
Epoch 998, Average Error: 3.262945469373335e-13
Epoch 999, Average Error: 3.183009411600324e-13
Epoch 1000, Average Error: 3.1041835768519377e-13
```




```
Output: 0.4971570953933695
Mean Absolute Error: 0.002842904606630503
Mean Squared Error: 8.082106602400935e-06
```

1.5 AO die nummers kan voorspellen.

We wilden een Neural Netwerk maken die makkelijk geschreven nummers kan begrijpen en voorspellen welke nummer opgeschreven is. We maken hierbij een model gemaakt die gebruikt maakt van scipy minimize functie, die ervoor zorgt dat

[ ]:
```
import numpy as np
from scipy.optimize import minimize
from scipy.io import loadmat
from sklearn.metrics import mean_absolute_error, mean_squared_error


def predict(Model, Dummies, X):
        m = X.shape[0]
        one_matrix = np.ones((m, 1))
        X = np.append(one_matrix, X, axis=1) # Adding bias unit to first layer
        z2 = np.dot(X, Model.transpose())
        a2 = 1 / (1 + np.exp(-z2)) # Activation for second layer
        one_matrix = np.ones((m, 1))
        a2 = np.append(one_matrix, a2, axis=1) # Adding bias unit to hidden␣
↪layer
        z3 = np.dot(a2, Dummies.transpose())
        a3 = 1 / (1 + np.exp(-z3)) # Activation for third layer
        p = (np.argmax(a3, axis=1)) # Predicting the class on the basis of max␣
↪value of hypothesis
        return p


def initialise(a, b):
    epsilon = 0.15
    c = np.random.rand(a, b + 1) * (
      # Randomly initialises values of data between [-epsilon, +epsilon]
      2 * epsilon) - epsilon
    return c


def neural_network(nn_params, input_layer_size, hidden_layer_size, num_labels,␣
↪X, y, lamb):
    # Weights are split back to Model, Dummies
    Model = np.reshape(nn_params[:hidden_layer_size * (input_layer_size + 1)],
                        (hidden_layer_size, input_layer_size + 1))
    Dummies = np.reshape(nn_params[hidden_layer_size * (input_layer_size + 1):],
                        (num_labels, hidden_layer_size + 1))

    # Forward propagation
    m = X.shape[0]
    one_matrix = np.ones((m, 1))
    X = np.append(one_matrix, X, axis=1)  # Adding bias unit to first layer
    a1 = X
    z2 = np.dot(X, Model.transpose())
    a2 = 1 / (1 + np.exp(-z2))  # Activation for second layer
    one_matrix = np.ones((m, 1))
    a2 = np.append(one_matrix, a2, axis=1)  # Adding bias unit to hidden layer
```

```
    z3 = np.dot(a2, Dummies.transpose())
    a3 = 1 / (1 + np.exp(-z3))  # Activation for third layer

    # Changing the y labels into vectors of boolean values.
    # For each label between 0 and 9, there will be a vector of length 10
    # where the ith element will be 1 if the label equals i
    y_vect = np.zeros((m, 10))
    for i in range(m):
        y_vect[i, int(y[i])] = 1

    # Calculating cost function
    J = (1 / m) * (np.sum(np.sum(-y_vect * np.log(a3) - (1 - y_vect) * np.log(1␣
↪- a3)))) + (lamb / (2 * m)) * (
                sum(sum(pow(Model[:, 1:], 2))) + sum(sum(pow(Dummies[:, 1:],␣
↪2))))

    # backprop
    Delta3 = a3 - y_vect
    Delta2 = np.dot(Delta3, Dummies) * a2 * (1 - a2)
    Delta2 = Delta2[:, 1:]

    # gradient
    Model[:, 0] = 0
    Model_grad = (1 / m) * np.dot(Delta2.transpose(), a1) + (lamb / m) * Model
    Dummies[:, 0] = 0
    Dummies_grad = (1 / m) * np.dot(Delta3.transpose(), a2) + (lamb / m) *␣
↪Dummies
    grad = np.concatenate((Model_grad.flatten(), Dummies_grad.flatten()))

    return J, grad


# Loading mat file
data = loadmat('mnist-original.mat')

# Extracting features from mat file
X = data['data']
X = X.transpose()

# Normalizing the data
X = X / 255

# Extracting labels from mat file
y = data['label']
y = y.flatten()

# Splitting data into training set with 60,000 examples
```

```
X_train = X[:60000, :]
y_train = y[:60000]

# Splitting data into testing set with 10,000 examples
X_test = X[60000:, :]
y_test = y[60000:]

m = X.shape[0]
input_layer_size = 784  # Images are of (28 X 28) px so there will be 784␣
↪features
hidden_layer_size = 100
num_labels = 10  # There are 10 classes [0, 9]

# Randomly initialising The Model itself and the Dummy variables
initial_Model = initialise(hidden_layer_size, input_layer_size)
initial_Dummies = initialise(num_labels, hidden_layer_size)

# Unrolling parameters into a single column vector
initial_nn_params = np.concatenate((initial_Model.flatten(), initial_Dummies.
↪flatten()))
maxiter = 100
lambda_reg = 0.1  # To avoid overfitting
myargs = (input_layer_size, hidden_layer_size, num_labels, X_train, y_train,␣
↪lambda_reg)

# Calling minimize function to minimize cost function and to train weights
results = minimize(neural_network, x0=initial_nn_params, args=myargs,
          options={'disp': True, 'maxiter': maxiter}, method="L-BFGS-B",␣
↪jac=True)

nn_params = results["x"]  # Trained Data is extracted

# Weights are split back to Model, Dummies
Model = np.reshape(nn_params[:hidden_layer_size * (input_layer_size + 1)], (
                              hidden_layer_size, input_layer_size + 1))  #␣
↪shape = (100, 785)
Dummies = np.reshape(nn_params[hidden_layer_size * (input_layer_size + 1):],
                      (num_labels, hidden_layer_size + 1))  # shape = (10, 101)

# Checking test set accuracy of our model
pred = predict(Model, Dummies, X_test)
print('Test Set Accuracy: {:f}'.format((np.mean(pred == y_test) * 100)))

# Checking train set accuracy of our model
pred = predict(Model, Dummies, X_train)
print('Training Set Accuracy: {:f}'.format((np.mean(pred == y_train) * 100)))
```

```python
# Evaluating precision of our model
true_positive = 0
for i in range(len(pred)):
    if pred[i] == y_train[i]:
        true_positive += 1
false_positive = len(y_train) - true_positive
print('Precision =', true_positive/(true_positive + false_positive))

# Saving the data in .txt file
np.savetxt('Model.txt', Model, delimiter=' ')
np.savetxt('Dummies.txt', Dummies, delimiter=' ')
```

```
Test Set Accuracy: 97.470000
Training Set Accuracy: 99.441667
Precision = 0.9944166666666666
```

[ ]:
```python
from tkinter import *
import numpy as np
from PIL import ImageGrab

window = Tk()
window.title("Handwritten digit recognition")
l1 = Label()

def predict(Model, Dummies, X):
        m = X.shape[0]
        one_matrix = np.ones((m, 1))
        X = np.append(one_matrix, X, axis=1) # Adding bias unit to first layer
        z2 = np.dot(X, Model.transpose())
        a2 = 1 / (1 + np.exp(-z2)) # Activation for second layer
        one_matrix = np.ones((m, 1))
        a2 = np.append(one_matrix, a2, axis=1) # Adding bias unit to hidden␣
 ↪layer
        z3 = np.dot(a2, Dummies.transpose())
        a3 = 1 / (1 + np.exp(-z3)) # Activation for third layer
        p = (np.argmax(a3, axis=1)) # Predicting the class on the basis of max␣
 ↪value of hypothesis
        return p

def Prediction():
    global l1

    widget = cv
    # Setting co-ordinates of canvas
    x = window.winfo_rootx() + widget.winfo_x()
    y = window.winfo_rooty() + widget.winfo_y()
    x1 = x + widget.winfo_width()
```

```python
    y1 = y + widget.winfo_height()

    # Image is captured from canvas and is resized to (28 X 28) px
    img = ImageGrab.grab().crop((x, y, x1, y1)).resize((28, 28))

    # Converting rgb to grayscale image
    img = img.convert('L')

    # Extracting pixel matrix of image and converting it to a vector of (1, 784)
    x = np.asarray(img)
    vec = np.zeros((1, 784))
    k = 0
    for i in range(28):
        for j in range(28):
            vec[0][k] = x[i][j]
            k += 1

    # Loading the Text.
    Model = np.loadtxt('Model.txt')
    Dummies = np.loadtxt('Dummies.txt')

    # Calling function for prediction
    pred = predict(Model, Dummies, vec / 255)

    # Displaying the result
    l1 = Label(window, text="Digit = " + str(pred[0]), font=('Calibri', 20))
    l1.place(x=260, y=420)


lastx, lasty = None, None


# Clears the canvas
def clear_widget():
    global cv, l1
    cv.delete("all")
    l1.destroy()


# Activate canvas
def event_activation(event):
    global lastx, lasty
    cv.bind('<B1-Motion>', draw_lines)
    lastx, lasty = event.x, event.y


# To draw on canvas
```

```python
def draw_lines(event):
    global lastx, lasty
    x, y = event.x, event.y
    cv.create_line((lastx, lasty, x, y), width=20, fill='white',␣
 ↪capstyle=ROUND, smooth=TRUE, splinesteps=12)
    lastx, lasty = x, y


# Label
L1 = Label(window, text="Handwritten Digit Recoginition", font=('Calibri', 25),␣
 ↪fg="blue")
L1.place(x=100, y=10)

# Button to clear canvas
b1 = Button(window, text="1. Clear Canvas", font=('Calibri', 15), bg="orange",␣
 ↪fg="black", command=clear_widget)
b1.place(x=120, y=370)

# Button to predict digit drawn on canvas
b2 = Button(window, text="2. Prediction", font=('Calibri', 15), bg="white",␣
 ↪fg="red", command=Prediction)
b2.place(x=355, y=370)

# Setting properties of canvas
cv = Canvas(window, width=350, height=290, bg='black')
cv.place(x=120, y=70)

cv.bind('<Button-1>', event_activation)
window.geometry("600x500")
window.mainloop()
```