

# A Comparison Study of PSO Neighborhoods

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**Abstract.** A comparative study is performed to reveal the convergence characteristics and the robustness of three local neighborhoods in the particle swarm optimization algorithm (PSO): ring, Von Neumann and singly-linked ring. In the PSO algorithm, a neighborhood enables different communication paths among its members, and therefore, the way the swarm searches the landscape. Since the neighborhood structure changes the flying pattern of the swarm, convergence and diversity differ from structure to structure. A set of controlled experiments is developed to observe the transmission behavior (convergency) of every structure. The comparison results illustrate similarities and differences in the three topologies. A brief discussion is provided to further reveal the reasons which may account for the difference of the three neighborhoods.

## 1 Introduction

The particle swarm optimization (PSO) algorithm is a population-based optimization technique inspired by the motion of a bird flock [7]. Such groups are social organizations whose overall behavior relies on some sort of communication between members. Any member of the flock is called a “particle”. Most models for flock’s motion are based on the interaction between the particles, and the motion of the particle as an independent entity. In PSO, the particles fly over a real valued  $n$ -dimensional search space, where each particle has three attributes: position  $x$ , velocity  $v$ , and best position visited after the first fly  $P_{Best}$ . The best of all  $P_{Best}$  values is called global best  $G_{Best}$ . Its position is communicated to all flock members such that, at the time of the next fly, all the particles are aware of the best position visited. By “flying” a particle we mean to apply the effect of the local and global attractors

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to the current motion vector. As a result, every particle gets a new position. The flock must keep flying and looking for better positions even when the current one seems good. Thus, finding the next best position is the main task of the flock for which exploration and therefore population diversity is crucial.

In PSO, the source of diversity, called *variation*, comes from two sources. One is the difference between the particle's current position and the global best, and the other is the difference between the particle's current position and its best historical value.

$$\begin{aligned} v_{t+1} &= w * v_t + c_1 * U(0, 1) * (x_t - P_{Best}) \\ &\quad + c_2 * U(0, 1) * (x_t - L_{Best}) \\ x_{t+1} &= x_t + v_{t+1} \end{aligned} \tag{1}$$

The equation above reflects the three main features of the PSO paradigm: distributed control, collective behavior, and local interaction with the environment [3]. The first term is the previous velocity (inertia path), the second term is called the cognitive component (particle's memory), and the last term is called the social component (neighborhood's influence).  $w$  is the inertia weight, and  $c_1$  and  $c_2$  are called acceleration coefficients.

The whole flock moves following the leader, but the leadership can be passed from member to member. At every PSO iteration the flock is inspected to find the best member. Whenever a member is found to improve the function value of the current leader, that member takes the leadership.

A leader can be global to all the flock, or local to a flock's neighborhood. In the latter case there are as many local leaders as neighborhoods. Having more than one leader in the flock translates into more attractors, possibly scattered over the search space. Therefore, the use of neighborhoods is a natural approach to fight premature convergence in the PSO algorithm [13].

For updating the particle position, the equation for local best PSO is similar to that of the global best PSO. One would simply substitute  $G_{Best}$  by  $L_{Best}$  in Equation 2.

$$\begin{aligned} v_{t+1} &= w * v_t + c_1 * U(0, 1) * (x_t - P_{Best}) \\ &\quad + c_2 * U(0, 1) * (x_t - L_{Best}) \end{aligned} \tag{3}$$

In the PSO algorithm, a neighborhood structure enables different communication paths among its members, and therefore, the way the swarm searches the landscape. Since the neighborhood topology changes the flying pattern of the swarm, convergence and diversity differ from structure to structure.

This work studies three effective PSO neighborhoods: ring [8], Von Neumann [9] and singly-linked ring [16]. In Section 2 a brief analysis of these structures is

presented. The developed experiments are described in Section 3. The results of the performed comparison is discussed in Section 4. Finally, in Section 5 the conclusions are presented.

## 2 Neighborhood Topologies

There are several neighborhood structures for PSO. Their importance have been addressed by several researchers [7], [12], [14], [15]. Flock neighborhoods have a structure which define the way the information flows among members. Virtually, the information of the best particle, or leader, is concentrated and then distributed among its members.

The organization of the flock affects search and convergence capacity. The first particle swarm version used a kind of fully connected structure that became known as global PSO (Gbest) [7]. The fully connected structure has reported the fastest convergence speed [3, 8]. In a fully connected structure all the particles are neighbors of each other. The communication between particles is expeditious; thereby, the flock moves quickly toward the best solution found. Nevertheless, on non-smooth functions, the population will fail to explore outside of locally optimal regions. Namely, if the global optimum is not close to the best particle, it may be impossible to the swarm to explore other areas; this means that the swarm can be trapped in local optima.

The local PSO (Lbest) was proposed as a way to deal with more difficult problems. It offered the advantage that subpopulations could search diverse regions of the problem space [10]. In the local PSO, only a specific number of particles  $N_k$  (neighborhood) can influence the motion of a given particle  $k$ . Every particle is initialized with a permanent label which is independent of its geographic location in space. The swarm will converge slower but can locate the global optimum with a greater chance.

Various types of local PSO topologies are investigated and presented in literature [6]. Kennedy and Mendes compared traditional gbest topology to some lbest topologies like von Neumann, star, ring and pyramid [9]. They also have suggested a new methodology for involving neighborhood in PSO, called Fully-Informed Particle Swarm, which uses some portion of each neighbor's findings instead of the best neighbor and the best experience of the particle [10]. They have indicated that the individual's experience tends to be overwhelmed by social influence.

In [19], Safavieh et al. apply Voronoi diagram, which supports geometric dynamic neighborhood, to choose neighbors in PSO algorithm. The running time of the algorithm depends on the dimension of search space. By increasing the dimension the running time of the computing the Voronoi neighbors will increase.

Researchers who have suggested methods that use neighborhood, discussed that by this kind of neighborhood, a society is constructed between particles [19].

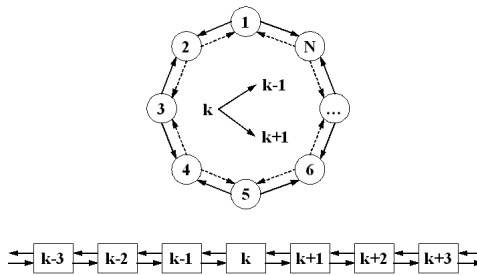
Once a particle finds a good result, it reports its location to its friends, and by some iterations all of the particles know something about good locations and try to move to them. The neighborhoods topologies study in this paper are: ring, Von Neumann and singly-linked ring.

## 2.1 Ring Topology

Flock members organized in a ring structure communicate with  $N_k$  immediate neighbors,  $N_k/2$  on each side (usually  $N_k = 2$ ). Finding the local best  $L_{Best}$  neighbor of particle  $k$  is done by inspecting the particles in the neighborhood:  $k+1, k+2, \dots, k+n/2$  and  $k-1, k-2, \dots, k-n/2$ .

In a ring structure, the information is slowly distributed among the flock members. This behavior does not contribute necessarily to improve the performance because it may be very inefficient during the refining phase of the solutions. However, using the ring topology will slow down the convergence rate because the best solution found has to propagate through several neighborhoods before affecting all particles in the swarm [4]. This slow propagation will enable the particles to explore more areas in the search space and thus decreases the chance of premature convergence [8].

The ring topology is used by most PSO implementations. In its simplest version, every particle  $k$  has two neighbors, particles  $k-1$  and  $k+1$ . Likewise, particles  $k-1$  and  $k+1$  have particle  $k$  as a neighbor. Therefore, there is a mutual attraction between consecutive particles because they are shared by two neighborhoods. This can be represented as a doubly-linked list [16], as shown in Figure 1.



**Fig. 1** Ring topology of neighborhood  $N_k = 2$

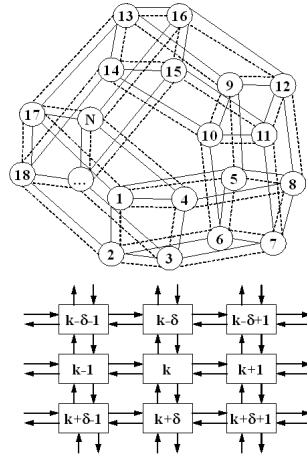
## 2.2 Von Neumann Topology

The Von Neumann topology, so-named after its use in cellular automata pioneered by John Von Neumann, was proposed for the PSO algorithm by Kennedy

and Mendes [9]. In a Von Neumann neighborhood, particles are connected using a rectangular matrix and every particle is connected to the individuals above, below, and to each side of it, wrapping the edges [10].

Kennedy and Mendes analyzed the effects of various neighborhoods structures on the PSO algorithm [9]. They recommended the Von Neumann structure because performed more consistently in their experiments than other neighborhoods tested [9].

In a Von Neumann topology every particle  $k$  has four neighbors, particles  $k - 1$  and  $k + 1$  at the sides, and particles  $k - \delta$  and  $k + \delta$  at the top and bottom, where  $\delta$  is a distance determined by the flock size. Likewise, particles  $k - 1$ ,  $k + 1$ ,  $k - \delta$  and  $k + \delta$  have particle  $k$  as a neighbor. Therefore, the Von Neumann topology possesses a doubly-linked list, like ring topology [16]. Figure 2.2 illustrates the Von Neumann neighborhood topology.



**Fig. 2** Von Neumann topology

### 2.3 Singly-Linked Ring Topology

The singly-linked ring topology, introduced by Muñoz-Zavala et al. [16], organizes the flock in a modified ring fashion. This topology avoids the double list, that there are in the ring and Von Neumann topologies, applying an asymmetric connection. Algorithm 1 shows the procedure to find the neighbors for particle  $k$  in a singly-linked ring topology; where  $N_k$  is the neighborhood, and  $P_{(k+m)}$  is the particle located  $m$  positions beyond particle  $k$ .

As it is shown in Figure 3, in the singly-linked ring topology, every particle  $k$  has particles  $k - 2$  and  $k + 1$  as neighbors (not  $k - 1$  and  $k + 1$  as in the ring topology). In turn, particle  $k + 1$  has particles  $k - 1$  and  $k + 2$  as neighbors, and  $k - 1$  has

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**Algorithm 1** Singly-linked ring neighborhood
 

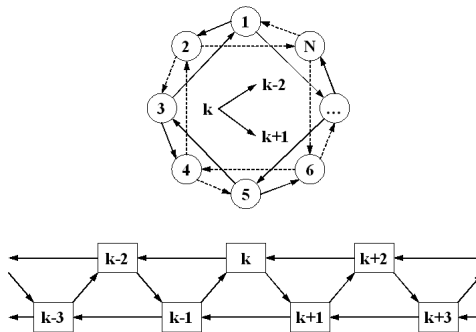
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1:  $N_k = \emptyset$ 
2:  $Step = 1$ 
3:  $Switch = 1$ 
4: repeat
5:    $N_k = N_k \cup P_{(k+Switch*Step)}$ 
6:    $Step = Step + 1$ 
7:    $Switch = -1 * Switch$ 
8: until  $N_k = N$ 
  
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particles  $k-3$  and  $k$  as neighbors. Then,  $k$  attracts  $k-1$ , but  $k-1$  only attracts  $k$  through particle  $k+1$ . Therefore, the particle in between cancels the mutual attraction. Additionally, the information of the leader is transmitted to the particles at a lower speed [16].



**Fig. 3** Singly-linked ring topology of neighborhood  $N_k = 2$

The finding reported by Muñoz-Zavala et al. [16] is that by reducing the transfer information speed, the singly-linked ring keeps the exploration of the search space. The singly-linked ring structure allows neighborhoods of size  $N_k = flock/2 - 1$  without mutual attraction.

### 3 Experiments

A controlled test set was developed to show the convergence of each algorithm. The objective is to perform a fair comparison between the 3 structures using the same PSO model with the same parameters; thereby, a swarm size of 20 particles with a neighborhood size of  $N_k = 4$  particles is used for every topology. The neighbors of each particle  $k = \{1, \dots, 20\}$  are listed in Table 1 for every topology: ring (R), Von Neumann (VN) and singly-linked ring (SLR).

**Table 1** Neighbors of size  $N_k = 4$  for every topology

Particle	R	VN	SLR
1	2,3,19,20	2,4,5,17	2,4,17,19
2	1,3,4,20	1,3,6,18	3,5,18,20
3	1,2,4,5	2,4,7,19	1,4,6,19
4	2,3,5,6	1,3,8,20	2,5,7,20
5	3,4,6,7	1,6,8,9	1,3,6,8
6	4,5,7,8	2,5,7,10	2,4,7,9
7	5,6,8,9	3,6,8,11	3,5,8,10
8	6,7,9,10	4,5,7,12	4,6,9,11
9	7,8,10,11	5,10,12,13	5,7,10,12
10	8,9,11,12	6,9,11,14	6,8,11,13
11	9,10,12,13	7,10,12,15	7,9,12,14
12	10,11,13,14	8,9,11,16	8,10,13,15
13	11,12,14,15	9,14,16,17	9,11,14,16
14	12,13,15,16	10,13,15,18	10,12,15,17
15	13,14,16,17	11,14,16,19	11,13,16,18
16	14,15,17,18	12,13,15,20	12,14,17,19
17	15,16,18,19	1,13,18,20	13,15,18,20
18	16,17,19,20	2,14,17,19	1,14,16,19
19	1,17,18,20	3,15,18,20	2,15,17,20
20	1,2,18,19	4,16,17,19	1,3,16,18

In the field of evolutionary computation, it is common to compare different algorithms using a large test set. However, the effectiveness of an algorithm against another algorithm cannot be measured by the number of problems that it solves better [17]. The “no free lunch” theorem [22] shows that, if we compare two search algorithms with all possible functions, the performance of any two algorithms will be, on average, the same.

That is the reason why, when an algorithm is evaluated, we must look for the kind of problems where its performance is good, in order to characterize the type of problems for which the algorithm is suitable. In the experiments, we used three test functions well-known in the state-of-the-art and also used in [11]: Sphere, Rastrigin and Schwefel.

- Sphere function, proposed by De Jong [5] is a unimodal function and so very easy to find the minimum. It is a continuous and strongly convex function.

$$f(x) = \sum_{j=1}^2 x_j^2$$

$$x_j \in [-100, 100], \quad f^*(0, 0) = 0$$

- The Rastrigin function was constructed from Sphere adding a modulator term  $\alpha \cdot \cos(2\pi x_i)$ . It was proposed by Rastrigin [18] and is considered as difficult

for most optimization methods [17]. Rastrigin function has the property to have many local minima whose value increases with the distance to the global minimum.

$$f(x) = \sum_{j=1}^2 (x_j^2 - 10\cos(2\pi x_j) + 10)$$

$$x_j \in [-5.12, 5.12], \quad f^*(0, 0) = 0$$

- The Schwefel function is the hardest among three functions [20]. The surface of Schwefel function is composed of a great number of peaks and valleys. The function has a second best minimum far from the global minimum where many search algorithms are trapped [17]. Besides, the global minimum is near the bounds of the domain.

$$f(x) = \sum_{j=1}^2 (x_j \sin(\sqrt{|x_j|}) + 418.9829)$$

$$x_j \in [-500, 500], \quad f^*(420.96874, 420.96874) = 0$$

### 3.1 Test I

In the first test, we use the Sphere function with the initial particle positions presented in Table 2.

**Table 2** Initial positions for test I

Particle	$x_1$	$x_2$	Particle	$x_1$	$x_2$
1	0	0	11	50	50
2	5	5	12	55	55
3	10	10	13	60	60
4	15	15	14	65	65
5	20	20	15	70	70
6	25	25	16	75	75
7	30	30	17	80	80
8	35	35	18	85	85
9	40	40	19	90	90
10	45	45	20	95	95

In test I, particles are linearly arranged at different contour levels. The global optimum is assigned to particle  $k = 1$ . The aim is to illustrate the topology effect in the swarm motion.



### 3.2 Test II

The second test applies the initial particle positions given in Table 3 in the Rastrigin function.

**Table 3** Initial positions for test II

Particle	$x_1$	$x_2$	Particle	$x_1$	$x_2$
1	1	1	11	5	-5
2	-1	-1	12	-5	5
3	2	2	13	4	-4
4	-2	-2	14	-4	4
5	3	3	15	3	-3
6	-3	-3	16	-3	3
7	4	4	17	2	-2
8	-4	-4	18	-2	2
9	5	5	19	1	-1
10	-5	-5	20	-1	1

In test II, particles are grouped by contours levels (4 particles in each contour level), whose function value  $f(x_1, x_2)$  are approximately in local minima. The test try to show the ability of each topology to handle symmetric multimodal functions.

### 3.3 Test III

This test uses the Schwefel function with the initial particle positions shown in Table 4.

In test III, particles are grouped by neighborhood according Table 1. The test try to show the ability of each topology to handle asymmetric multimodal functions. The aim is to known the robustness of the studied neighborhood structures to find a global optimum away from the center.

## 4 Comparative Study

The aim is to perform an unbiased study. Thereby, we use the same PSO parameter in the next three local PSO approaches: PSO-R (ring topology), PSO-VN (Von Neumann topology) and PSO-SLR (singly-linked ring topology).

Van den Bergh analyzed PSO's convergence [21] and provided a model to obtain convergent trajectories  $0.5(c_1 + c_2) < w$ . In his work, Van den Bergh used the standard parameter values proposed by Clerc and Kennedy [2], and also applied by Bratton and Kennedy [1]:  $w = 0.7298$ ,  $c_1 = 1.49618$ ,  $c_2 = 1.49618$ .

**Table 4** Initial positions for test III

	PSO-R	PSO-VN	PSO-SLR
1	( 420 , -305 )	( 420 , -305 )	( 420 , -305 )
2	( 420 , -305 )	( 420 , -305 )	( 420 , -305 )
3	( 420 , -305 )	( -305 , -305 )	( -305 , -305 )
4	( -305 , -305 )	( 420 , -305 )	( 420 , -305 )
5	( -305 , -305 )	( 420 , -305 )	( -305 , -305 )
6	( -305 , -305 )	( -305 , 420 )	( -305 , 420 )
7	( -305 , -305 )	( -305 , -305 )	( -305 , -305 )
8	( -305 , 420 )	( -305 , -305 )	( -305 , 420 )
9	( -305 , 420 )	( -305 , 420 )	( -305 , -305 )
10	( -305 , 420 )	( -305 , 420 )	( -305 , 420 )
11	( -305 , 420 )	( -305 , 420 )	( -305 , 420 )
12	( -305 , 420 )	( -305 , -305 )	( -305 , -305 )
13	( -305 , -305 )	( -305 , -305 )	( -305 , 420 )
14	( -305 , -305 )	( -305 , 420 )	( -305 , -305 )
15	( -305 , -305 )	( -305 , -305 )	( -305 , -305 )
16	( -305 , -305 )	( -305 , -305 )	( -305 , -305 )
17	( -305 , -305 )	( 420 , -305 )	( 420 , -305 )
18	( -305 , -305 )	( -305 , -305 )	( -305 , -305 )
19	( 420 , -305 )	( -305 , -305 )	( 420 , -305 )
20	( 420 , -305 )	( -305 , -305 )	( -305 , -305 )

### 4.1 Comparative Test I

For each PSO algorithm (PSO-SLR, PSO-Ring and PSO-VN) 1000 runs were developed. Each algorithm performs the function evaluations required to reach a  $P_{Best} \leq 1 \times 10^{-16}$  for every particle  $k$ . The results are shown in Table 5.

**Table 5** Convergence results for test I

	PSO-R	PSO-VN	PSO-SLR
Mean	7027.86	7008.68	6997.38
Median	6980	6940	6940
Min	5500	5540	5340
Max	9240	10360	10020
Std. Devs.	601.55	588.34	604.11

The fitness evaluations performed by the 3 PSO algorithms to reach a  $P_{Best} \leq 1E - 16$  in the whole swarm are similar. In fact, there are insignificant differences for the mean, median and standard deviation values, between the 3 topologies. Table 6 presents the average function evaluations required for every particle to reach a  $P_{Best} \leq 1E - 16$ .

Table 6 shows that there are significant difference in the convergence results. For instance, particles  $k = 2$  and  $k = 3$  presents a difference in the average convergence results between the singly-linked ring topology and the other two topologies. In the

**Table 6** Average convergence results of each particle for test I

Particle	PSO-R	PSO-VN	PSO-SLR
1	0	0	0
2	203.17	201.387	223.676
3	211.784	217.691	212.698
4	217.236	215.054	226.659
5	223.457	218.267	220.337
6	226.445	227.273	234.319
7	231.381	233.364	233.243
8	236.835	237.91	242.043
9	241.988	249.668	245.614
10	246.318	254.491	252.521
11	252.622	256.428	257.582
12	260.623	262.283	264.312
13	264.678	274.137	268.656
14	273.87	278.755	273.141
15	282.573	281.023	280.896
16	289.207	283.961	285.631
17	293.286	273.323	288.304
18	294.25	275.572	273.079
19	277.517	283.239	283.639
20	280.056	282.14	279.244

PSO-SLR, particle  $k = 3$  converges before particle  $k = 2$ , despite that particle  $k = 2$  is closer to the optimum than particle  $k = 3$  (see Table 2), because particle  $k = 1$  (global optimum) belongs to the neighborhood of particle  $k = 3$  (see Table 1). In the PSO-Ring, particle  $k = 1$  is neighbor of both particles,  $k = 2$  and  $k = 3$ ; thereby, there is not an advantage for any particle. Finally, particle  $k = 1$  belongs only to the neighborhood of particle  $k = 2$  in the PSO-VN algorithm.

**4.2 Comparative Test II**

The function evaluations required to reach a  $P_{Best} \leq 1E - 16$ , for the whole swarm, were performed in each PSO algorithm described above. The results are shown in Table 7.

**Table 7** Convergence results for test II

	PSO-R	PSO-VN	PSO-SLR
Mean	8097.22	7842.82	8127.3
Median	8040	7750	8070
Min	6240	6540	6780
Max	10900	11460	10600
Std. Desv.	676.59	623.70	617.86

The fitness evaluations performed by the PSO-SLR and PSO-R algorithms to reach a  $P_{Best} \leq 1E - 16$  in the whole swarm are similar. There is a small difference between the Von Neumann and the other topologies in the mean and median convergence results. Table 8 presents the average function evaluations required for every particle to reach a  $P_{Best} \leq 1E - 16$ .

**Table 8** Average convergence results of each particle for test II

Particle	PSO-R	PSO-VN	PSO-SLR
1	331.887	288.467	332.236
2	328.927	290.827	334.689
3	315.006	313.608	334.403
4	305.924	316.664	332.756
5	302.951	291.887	322.194
6	303.158	292.875	325.509
7	311.764	311.444	325.935
8	315.415	311.573	328.919
9	325.772	317.6	326.452
10	329.055	316.867	330.452
11	328.102	316.095	332.605
12	323.983	316.573	331.993
13	313.553	312.848	331.085
14	309.071	310.601	331.13
15	300.749	293.242	333.313
16	304.937	292.516	331.503
17	304.742	315.487	331.984
18	313.174	316.23	335.084
19	328.718	288.913	335.95
20	330.49	289.531	331.294

Table 8 shows that there are several differences in the convergence results between the 3 topologies. For instance, in the PSO-R the particles allocated in the contour level  $f(x_1, x_2) = 18$  (particles  $k = \{5, 6, 15, 16\}$ ) reach the global optimum faster than the particles allocated in contour levels nearby to the global optimum (see Table 3. Similarly, in the PSO-VN, these particles reach the global optimum straightway the particles in the contour level  $f(x_1, x_2) = 2$  (particles  $k = \{1, 2, 19, 20\}$ ), outperform the average convergence results of particles  $k = \{3, 4, 17, 18\}$  allocated in the contour level  $f(x_1, x_2) = 8$ . On the other hand, the PSO-SLR convergence results are very similar for all the particles, we may say that the swarm “flies together”. Besides, the singly-linked ring shows a slower motion than the other two topologies.

### 4.3 Comparative Test III

Each PSO algorithm performs the function evaluations required to reach a  $P_{Best} \leq 1E - 04$  for every particle  $k$ . If the algorithm is trapped in local minima, the run is finished at 20,000 function evaluations and reported as unsuccessful run. The results are shown in Table 9.

**Table 9** Convergence results for test III

	PSO-R	PSO-VN	PSO-SLR
Mean	3966.46	3962.40	3922.04
Median	3800	3740	3700
Min	2580	2700	2760
Max	18740	19580	14920
Std. Devs.	1192.91	1266.26	996.14
Unsuccesful Run	3.1%	3.4%	0.9%

The fitness evaluations performed by the PSO-R and PSO-VN algorithms to reach a  $P_{Best} \leq 1E - 04$  in the whole swarm are similar. In fact, there are insignificant differences for the mean, median and standard desviation values, between the

**Table 10** Average convergence results of each particle for test III

Particle	PSO-R	PSO-VN	PSO-SLR
1	141.1987891	139.8146998	139.623323
2	139.9566095	139.4658385	139.3065015
3	141.2926337	139.373706	138.7997936
4	140.8193744	138.8022774	139.2693498
5	140.5005045	141.245614	140.7997936
6	139.7719475	138.7225673	139.8167702
7	141.2431887	139.8799172	139.1671827
8	141.1210898	138.6335404	139.4427245
9	140.7830474	141.5093168	140.2786378
10	141.6801211	138.4633643	139.5614035
11	139.2179617	140.0786749	139.0890269
12	139.6528759	140.7401656	139.380805
13	140.938446	141.1362229	139.7089783
14	140.1029263	138.1697723	140.0310559
15	138.7860747	138.073499	140.2641899
16	138.5095863	139.9285714	139.5675955
17	137.7134208	140.4761905	140.4499484
18	137.864783	139.2142857	140.2373581
19	138.7093845	138.8395445	139.6955624
20	139.6801211	139.1242236	139.1795666

2 topologies. Nevertheless, the PSO-SLR results outperform significantly the standard deviation and succesful runs values obtained by the other two topologies. Table 10 presents the average function evaluations required for every particle to reach a  $P_{Best} \leq 1E - 04$ .

Table 10 shows that there is not a significant difference in the convergence results for all the swarm between the topologies tested. These results are consistent with the mean and median convergence results show in Table 9. Nevertheless, we can not find a relation between these results and the unsuccessful run rates show in Table 9. There is a marked difference in the PSO-SLR succesful rate 99.1% to reach the global optimum against the succesful rates 96.9% and 96.4%, obtained by the PSO-R and PSO-VN, respectively.

The results obtained in test I and test II can help us to understand the effect of the topology structure in the communication of the swarm, which is illustrated in the unsuccessful rate results obtained with a singly-linked ring topology. Muñoz-Zavala et al. [16] conclude that the singly-linked ring topology outperforms the ring and Von Neumann topologies due to the double list that exist between the particles in these topologies; which are eliminated in their approach.

## 5 Conclusions

In the PSO algorithm the communication between particles is essential. The information is transmitted according to an interaction structure (neighborhood), which can be global or local. The neighborhood affects the transmission speed and influences the PSO convergence.

This papers proposed a study comparison between three topologies (neighborhood structures) which are the *state-of-the-art*: ring, Von Neumann and singly-linked ring. Three controled tests were applied to illustrate the global and individual convergence of the PSO algorithm applying the 3 topologies. The convergence results show that the global convergence is similar for the 3 topologies. On the other hand, the individual (particle) convergences are different in the 3 neighborhood approaches.

Although the singly-linked ring outperforms the ring and Von Neumann topologies in test III, the tests are not conclusive and more experiments are being performed. A future work is to apply the topologies studied in this paper in another kind of functions. An analysis of indirect neighbors may provide additional information about swarm motion mechanism.

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