Molecular Mechanisms for Human Diseases: Lab 1

Bacteria’s Brain – Part A

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## Abstract

## Experiment

### Method

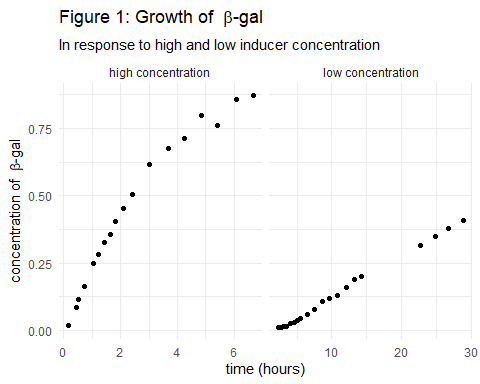
### Setup

## Results:

The complete resulting data can be seen in tables 1 and 2 in the appendix, and as shown in table 1a and 1b:

| t | concentration |
| --- | --- |
| 2.38 | 0.01090 |
| 2.72 | 0.00934 |
| 3.04 | 0.01460 |
| 3.52 | 0.01450 |
| 4.00 | 0.02420 |
| 4.57 | 0.02860 |
| 5.05 | 0.03760 |
| 5.54 | 0.04350 |
| 6.45 | 0.05920 |
| 7.53 | 0.07640 |
| 8.58 | 0.10700 |
| 9.63 | 0.11800 |
| 10.80 | 0.12800 |
| 12.00 | 0.16000 |
| 13.20 | 0.18800 |
| 14.30 | 0.19900 |
| 22.70 | 0.31700 |
| 24.90 | 0.34900 |
| 26.80 | 0.37800 |
| 28.90 | 0.40800 |

### Visualization of Data set A and B



## Analysis:

### Derivation of Functions used for fitting both datasets:

I will first define the different terms that will be used.

is the number of individual E.coli bacteria in the chemostat, which is maintained at a constant value.

is the flow rate both in- and outward from the chemostat, which is kept at a constant value so that can be kept constant.

is the volume of the chemostat

we define as the average rate of production by any individual bacterium at a given time. Note this is not the rate of production by individual bacterium but the mean of the population. Hence this value would change based on how many bacterium are “activated” within the population.

is defined as the concentration of -gal within the system at any moment. With these terms defined, we observe that at any moment, the growth of -gal is , which equates two terms: the rate of production by the bacteria and the rate of loss due to flow rate. Hence we get the equation:

#### A: High Concentration of Inducers

For data set A, when the inducers are introduced at high concentration, we assume that is constant as all bacteria rapidly “switches on” and generates enzyme at its maximal rate. We can then express as a dimensionless variable as , hence:

We can prepare the function for integration by substituting :

We then integrate both sides:

Considering the fact that , we conclude that Hence:

Or:

Which fit the form:

with and .

#### B: Low concentration of Inducers

For data set B, when the inducers are introduced at low concentration, only a fraction of bacteria is activated, with this fraction slowly growing. Hence we assume that is a linear function of with and being the rate at which activation “spreads” through the population. Hence we again start with the equation:

However, we substitute as a function of t

Again using a dimensionless substitution, this time for as $z = $ We arrive at

Which we can rearrange in the form of

Which we can use an integration factor of which we can multiply both sides of the equation and integrate:

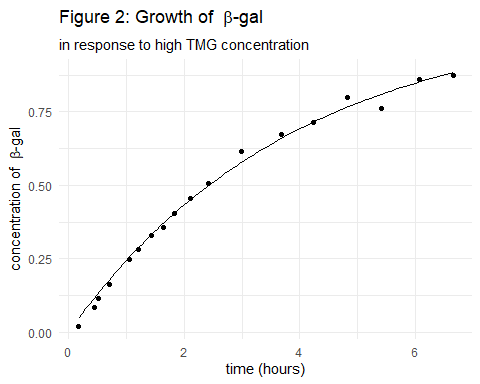
We realize that this equation fits the form at large t, but does not fit the initial condition of , hence we adding the term $A e^{-} $ with being a free constant (not to be confused with from data set A), which we adjust to to satisfy this initial condition, resulting in:

Now we replace and $ $ with their physical counterparts:

Which fit the form $z = B(-1 + + e^{-}) $ with and .

### Fitting function on Data set A

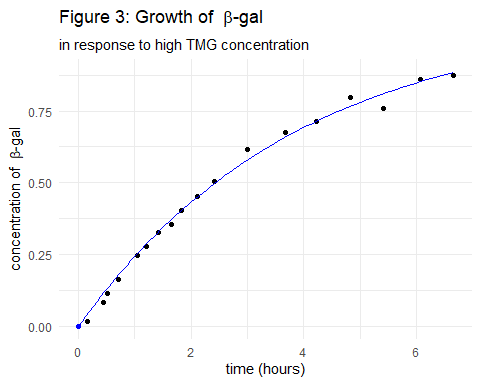
From the derived equation A: , we set the two parameters $A = $ and so the equation is of the form:



|  | data | values |
| --- | --- | --- |
| ampl | dataset A | 1.080 |
| tau | dataset A | 3.908 |

With the coefficients calculated, the nonlinear fitting of A results in the function:

With A = 1.08 and = 3.91 We observe that at , , which is valid in its physical meaning, as there would be no -gal before any inducers are introduced. Hence, the data point is included into the data and the same model is fitted again.



|  | data | values |
| --- | --- | --- |
| 1 | 1.0797195971604 | 3.907721 |
| ampl | with (0,0) | 1.080000 |
| tau | with (0,0) | 3.908000 |

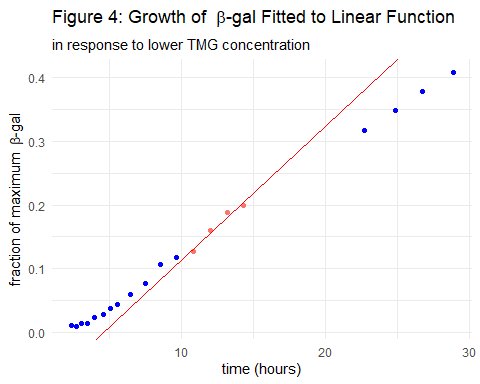
##   
## Formula: a\_q3\_df$concentration ~ growth\_a(t, ampl, tau)  
##   
## Parameters:  
## Estimate Std. Error t value Pr(>|t|)   
## ampl 1.07972 0.03931 27.47 1.59e-15 \*\*\*  
## tau 3.90772 0.24889 15.70 1.50e-11 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.02143 on 17 degrees of freedom  
##   
## Number of iterations to convergence: 5   
## Achieved convergence tolerance: 1.242e-06

We see that here the two fitting parameters are identical before and after is included. This is reasonable according to the theory since there would be no -gal production before any inducer is introduced. Mathematically, . The fitting returns a residual standard error of RSE = 0.0214; the mean standard error returns as MSE = 4.11^{-4}. The two measures of error are both quite small, hence we conclude that the model is a good fit for the data.

### Fitting function on Data set B

First we start by fitting a straight line over the range $10 t $

## (Intercept) b\_q2$t   
## -0.09474840 0.02094596

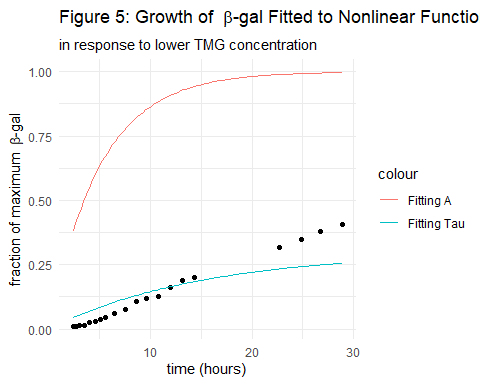


##   
## Call:  
## lm(formula = b\_q2$concentration ~ b\_q2$t)  
##   
## Residuals:  
## 1 2 3 4   
## -0.004175 0.002961 0.007005 -0.005791   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## (Intercept) -0.094748 0.036016 -2.631 0.119   
## b\_q2$t 0.020946 0.002847 7.356 0.018 \*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.007376 on 2 degrees of freedom  
## Multiple R-squared: 0.9644, Adjusted R-squared: 0.9465   
## F-statistic: 54.11 on 1 and 2 DF, p-value: 0.01798

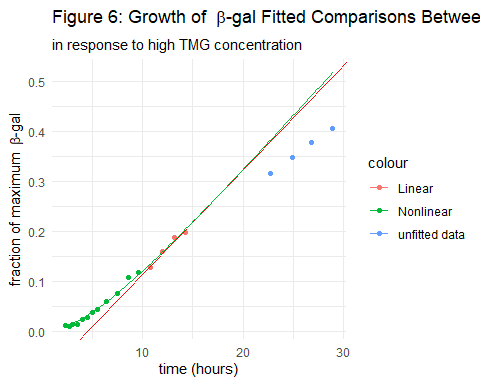
## Warning: 'newdata' had 20 rows but variables found have 4 rows

## # A tibble: 2 × 2  
## MSE data   
## <dbl> <chr>   
## 1 0.0000272 10<= t <= 15   
## 2 0.0176 entire dataset

As shown in figure 4, the straight line (in red) is fitted over the data between 10 and 15 hours, over which the line fits quite well. the Mean Standard Error (MSE) for the model is much smaller within the range of 10<= t <= 15 with a MSE of 2.7199507^{-5}, but not well over the entire data set with a MSE of 0.0175584. Near the low end of t,



While attempting to fit data set A’s function () over data set B, we realize that it is impossible to fit. If we were to fit the function around the range , the data point would inevitably exceed the value for , which is impossible since defines the theoretical maximum amount of -gal that can exist in the system, as it is defined as . Hence the amount of -gal should only asymptotically approach the value for . If we try to find a value for , the value would be unreasonably high to the point where the function is irrelevant to the data (See Figure 5). Hence we need to use the different function derived earlier to fit data set B.



##   
## Formula: b\_q4\_df$concentration ~ growth\_b4(t, beta, tau)  
##   
## Parameters:  
## Estimate Std. Error t value Pr(>|t|)   
## beta 0.11755 0.02959 3.973 0.00139 \*\*   
## tau 5.34585 0.89590 5.967 3.44e-05 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 0.005333 on 14 degrees of freedom  
##   
## Number of iterations to convergence: 6   
## Achieved convergence tolerance: 2.284e-06

As shown in figure 6, the two fitted functions both perform well with respect to the data they are fitted to (i.e.  for nonlinear function in green, and for linear function in red). The two also converges in the range around . This behavior is accounted for mathematically, as we consider the nonlinear function $ z(t) = B(-1 + + e^{-})$. If we take the limit as t approaches infinity:

The function becomes linear with the slope defined by (question B.5). The mathematical explanation can be complemented by an interpretation of the physical meaning of each term of the fitted function, which can explain the shape of the curve which is parabolic at small and linear at large . This interpretation can be done with a Taylor Expansion of the function:

From which we get

## Dicussion & Conclusion

## Appendix:

Complete Data:

| t | concentration |
| --- | --- |
| 0.170 | 0.0190 |
| 0.443 | 0.0855 |
| 0.511 | 0.1160 |
| 0.716 | 0.1640 |
| 1.060 | 0.2470 |
| 1.200 | 0.2800 |
| 1.430 | 0.3280 |
| 1.650 | 0.3560 |
| 1.830 | 0.4040 |
| 2.110 | 0.4540 |
| 2.420 | 0.5060 |
| 3.000 | 0.6150 |
| 3.680 | 0.6750 |
| 4.230 | 0.7130 |
| 4.830 | 0.7980 |
| 5.410 | 0.7600 |
| 6.060 | 0.8600 |
| 6.660 | 0.8740 |

| t | concentration |
| --- | --- |
| 2.38 | 0.01090 |
| 2.72 | 0.00934 |
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