Exercises

Scaling

- Consider square with left-bottom corner at (2, 2) and right-top corner at (6, 6) apply the transformation which makes its size half.

  Give the Final Coordinates

Scaling Example Answer

- **Example**: - Consider square with left-bottom corner at (2, 2) and right-top corner at (6, 6) apply the transformation which makes its size half.
- As we want size half so value of scale factor are $s_x = 0.5, s_y = 0.5$ and Coordinates of square are [A (2, 2), B (6, 2), C (6, 6), D (2, 6)].
- $P' = S \cdot P$
- $P' = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} 2 & 6 & 6 & 2 \\ 2 & 2 & 6 & 6 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} 2 & 6 & 6 & 2 \\ 2 & 2 & 6 & 6 \end{bmatrix}$
- $P' = \begin{bmatrix} 1 & 3 & 3 & 1 \\ 1 & 1 & 3 & 3 \end{bmatrix}$
- Final coordinate after scaling are:

  [A′ (1, 1), B′ (3, 1), C′ (3, 3), D′ (1, 3)]

Multiple Translations Example

- **Example:** Obtain the final coordinates after two translations on point $p(2,3)$ with translation vector $(4,3)$ and $(-1,2)$ respectively.

Multiple Translations Example

- **Example:** Obtain the final coordinates after two translations on point $p(2,3)$ with translation vector $(4,3)$ and $(-1,2)$ respectively.
- $P' = T\left(t_{x1} + t_{x2}, t_{y1} + t_{y2}\right) \cdot P$
- $P' = \begin{bmatrix} 1 & 0 & t_{x1} + t_{x2} \\ 0 & 1 & t_{y1} + t_{y2} \\ 0 & 0 & 1 \end{bmatrix} \cdot P = \begin{bmatrix} 1 & 0 & 4 + (-1) \\ 0 & 1 & 3 + 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$
- $P' = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 1 \end{bmatrix}$
- Final Coordinates after translations are $p'(5,8)$.

Multiple Rotations Example

- **Example:** Obtain the final coordinates after two rotations on point $p(6,9)$ with rotation angles are 30^o and 60^o respectively.

Multiple Rotations Example

- **Example:** Obtain the final coordinates after two rotations on point $p(6,9)$ with rotation angles are 30^o and 60^o respectively.
- $P' = R(\theta_1+\theta_2) \cdot P$
- $P' = \begin{bmatrix} cos(\theta_1+\theta_2) & -sin(\theta_1+\theta_2) & 0 \\ sin(\theta_1+\theta_2) & cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot P$
- $P' = \begin{bmatrix} cos(30+60) & -sin(30+60) & 0 \\ sin(30+60) & cos(30+60) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot P$
- $P' = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 9 \\ 1 \end{bmatrix} = \begin{bmatrix} -9 \\ 6 \\ 1 \end{bmatrix}$
- Final Coordinates after rotations are $p'(-9, 6)$.

Multiple Scaling Example

- **Example:** Obtain the final coordinates after two scaling on line pq $[p(2,2),\ q(8,8)]$ with scaling factors are $(2,2)$ and $(3,3)$ respectively.

Multiple Scaling Example

- **Example:** Obtain the final coordinates after two scaling on line pq $[p(2,2),\ q(8,8)]$ with scaling factors are $(2,2)$ and $(3,3)$ respectively.
- $P' = S(s_{x1} \cdot s_{x2}, s_{y1} \cdot s_{y2}) \cdot P$
- $P' = \begin{bmatrix} s_{x1} \cdot s_{x2} & 0 & 0 \\ 0 & s_{y1} \cdot s_{y2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot P = \begin{bmatrix} 2 \cdot 3 & 0 & 0 \\ 0 & 2 \cdot 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot P$
- $P' = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 8 \\ 2 & 8 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 48 \\ 12 & 48 \\ 1 & 1 \end{bmatrix}$
- Final Coordinates after rotations are $p'(12, 12)$ and $q'(48, 48)$.

General Pivot-Point Rotation Example

- **Example**: - Locate the new position of the triangle ABC

  [A (5, 4), B (8, 3), C (8, 8)] after its rotation by 90º clockwise about the centroid.

General Pivot-Point Rotation Example

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- Pivot point is centroid of the triangle so:

- $x_r = \frac{5+8+8}{3} = 7\ ,\ \ y_r = \frac{4+3+8}{3} = 5$

- As rotation is clockwise we will take $\theta = -90^{\circ}$.

- $P' = R_{(x_r,\, y_r, \theta)} \cdot P$

- $P' = \begin{bmatrix} \cos\theta & -\sin\theta & x_r(1-\cos\theta) + y_r\sin\theta \\ \sin\theta & \cos\theta & y_r(1-\cos\theta) - x_r\sin\theta \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 8 & 8 \\ 4 & 3 & 8 \\ 1 & 1 & 1 \end{bmatrix}$

Contd.

- P'

$$= \begin{bmatrix} \cos(-90) & -\sin(-90) & 7(1-\cos(-90)) + 5\sin(-90) \\ \sin(-90) & \cos(-90) & 5(1-\cos(-90)) - 7\sin(-90) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 8 & 8 \\ 4 & 3 & 8 \\ 1 & 1 & 1 \end{bmatrix}$$

- $P' = \begin{bmatrix} 0 & 1 & 7(1-0) - 5(1) \\ -1 & 0 & 5(1-0) + 7(1) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 8 & 8 \\ 4 & 3 & 8 \\ 1 & 1 & 1 \end{bmatrix}$

- $P' = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 12 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 8 & 8 \\ 4 & 3 & 8 \\ 1 & 1 & 1 \end{bmatrix}$

Contd.

- $P' = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 12 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 8 & 8 \\ 4 & 3 & 8 \\ 1 & 1 & 1 \end{bmatrix}$
- $P' = \begin{bmatrix} 6 & 5 & 10 \\ 7 & 4 & 4 \\ 1 & 1 & 1 \end{bmatrix}$
- Final coordinates after rotation are [A′ (6, 7), B′ (5, 4), C′ (10, 4)].

General Fixed-Point Scaling Example

- **Example**: - Consider square with left-bottom corner at (2, 2) and right-top corner at (6, 6) apply the transformation which makes its size half such that its center remains same.

General Fixed-Point Scaling Example

- **Example**: - Consider square with left-bottom corner at (2, 2) and right-top corner at (6, 6) apply the transformation which makes its size half such that its center remains same.
- Fixed point is center of square so:
- $x_f = \frac{6+2}{2}\ , \quad y_f = \frac{6+2}{2}$
- As we want size half so value of scale factor are $s_x = 0.5, s_y = 0.5$ and Coordinates of square are [A (2, 2), B (6, 2), C (6, 6), D (2, 6)].
- $P' = \ S\left(x_f, y_f, s_x, s_y\right) \cdot P$

Contd.

- $P' = S(x_f, y_f, s_x, s_y) \cdot P$

- $P' = \begin{bmatrix} s_x & 0 & x_f(1 - s_x) \\ 0 & s_y & y_f(1 - s_y) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 6 & 6 & 2 \\ 2 & 2 & 6 & 6 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

- $P' = \begin{bmatrix} 0.5 & 0 & 4(1 - 0.5) \\ 0 & 0.5 & 4(1 - 0.5) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 6 & 6 & 2 \\ 2 & 2 & 6 & 6 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

- $P' = \begin{bmatrix} 0.5 & 0 & 2 \\ 0 & 0.5 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 6 & 6 & 2 \\ 2 & 2 & 6 & 6 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

Contd.

- $P' = \begin{bmatrix} 0.5 & 0 & 2 \\ 0 & 0.5 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 6 & 6 & 2 \\ 2 & 2 & 6 & 6 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

- $P' = \begin{bmatrix} 3 & 5 & 5 & 3 \\ 3 & 3 & 5 & 5 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

- Final coordinate after scaling are:

  [A′ (3, 3), B′ (5, 3), C′ (5, 5), D′ (3, 5)]

Reflection Example

- **Example**: - Find the coordinates after reflection of the triangle

  [A (10, 10), B (15, 15), C (20, 10)] about x axis.

Reflection Example

- **Example**: - Find the coordinates after reflection of the triangle

  [A (10, 10), B (15, 15), C (20, 10)] about x axis.

- $P' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 & 15 & 20 \\ 10 & 15 & 10 \\ 1 & 1 & 1 \end{bmatrix}$

- $P' = \begin{bmatrix} 10 & 15 & 20 \\ -10 & -15 & -10 \\ 1 & 1 & 1 \end{bmatrix}$

- Final coordinate after reflection are:

  [A′ (10, -10), B′ (15, -15), C′ (20, -10)]

- DID you READ about SHEAR in X and Y directions
  - If not do so , use the revision slides to refresh on the same