Occupancy Grid Maps

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Learning Objectives

- Define occupancy grid
- Handling noisy data by using Bayesian updates

- Issue with the Bayesian Probability Update
- Present a solution utilizing log odds
- Bayesian log odds update derivation

- Examples

What is an occupancy grid?

- Discretized fine grain grid map of space
- Occupancy by <u>static objects</u>
  - Trees, buildings, curbs, walls

(cost)

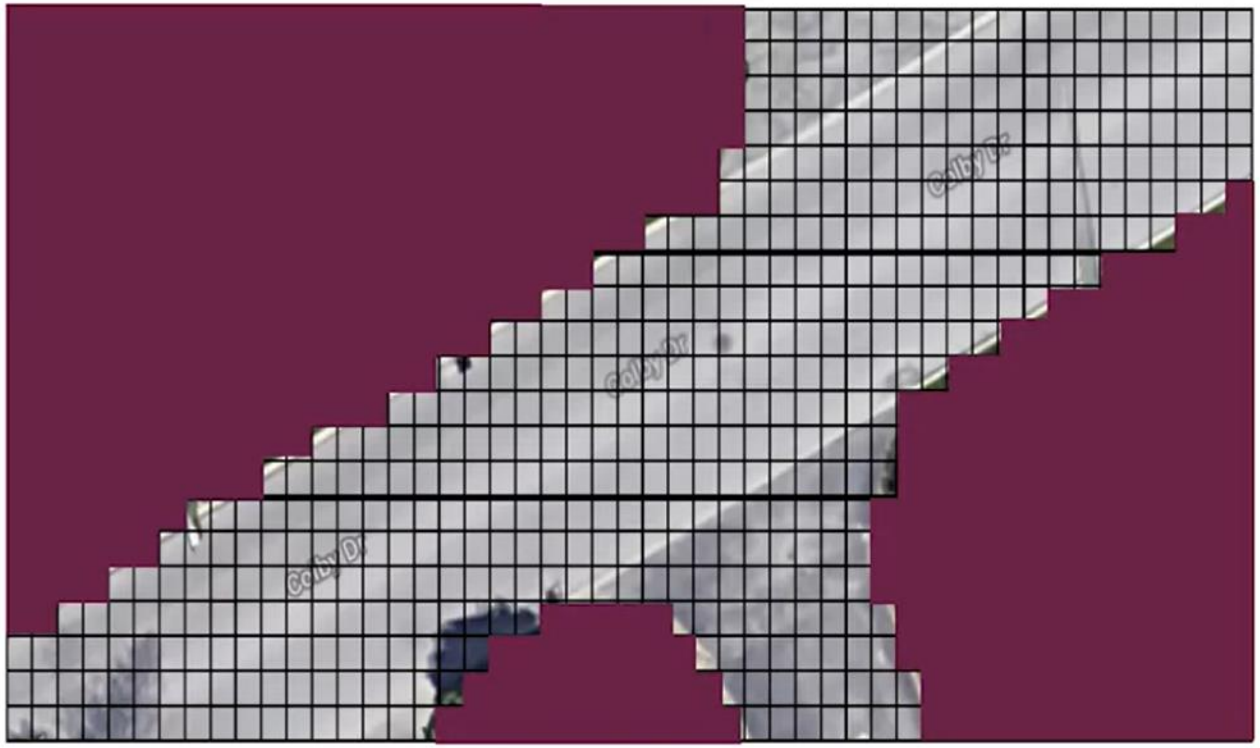
measurements

- A belief map is built

vehicle location

$$bel_t(m_i) = p(m_i|z, x)$$

- Threshold of certainty will be used to establish occupancy



Courtesy: Motion Planning for Self-Driving Cars, Probabilistic Robotics

Assumptions

1. Either occupied or free

$$m_i \in \{0, 1\}$$

2. The world is static

3. Cells are independent of each other

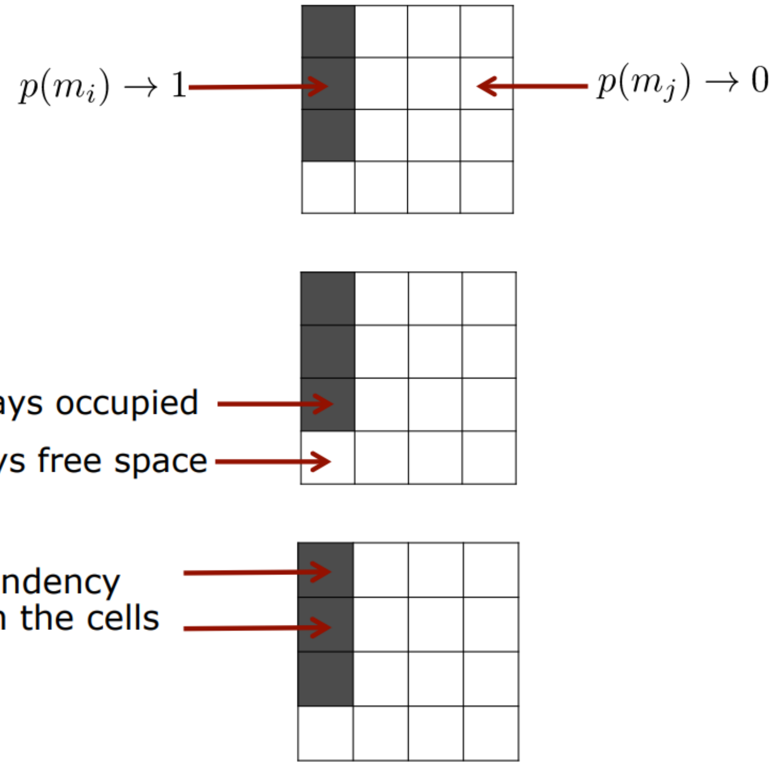
$$p(m) = \prod p(m_i)$$

map

individual cell

"Joint Distribution"

$$p\left(M = \begin{array}{cc} 1 & 2 \\ \blacksquare & \blacksquare \\ \blacksquare & \square \\ 3 & 4 \end{array}\right) = p(m_1 = 1) \cdot p(m_2 = 1) \cdot p(m_3 = 1) \cdot p(m_4 = 0)$$



Handling noisy data by Bayesian Updates

- To improve robustness multiple timesteps are used to produce the current map

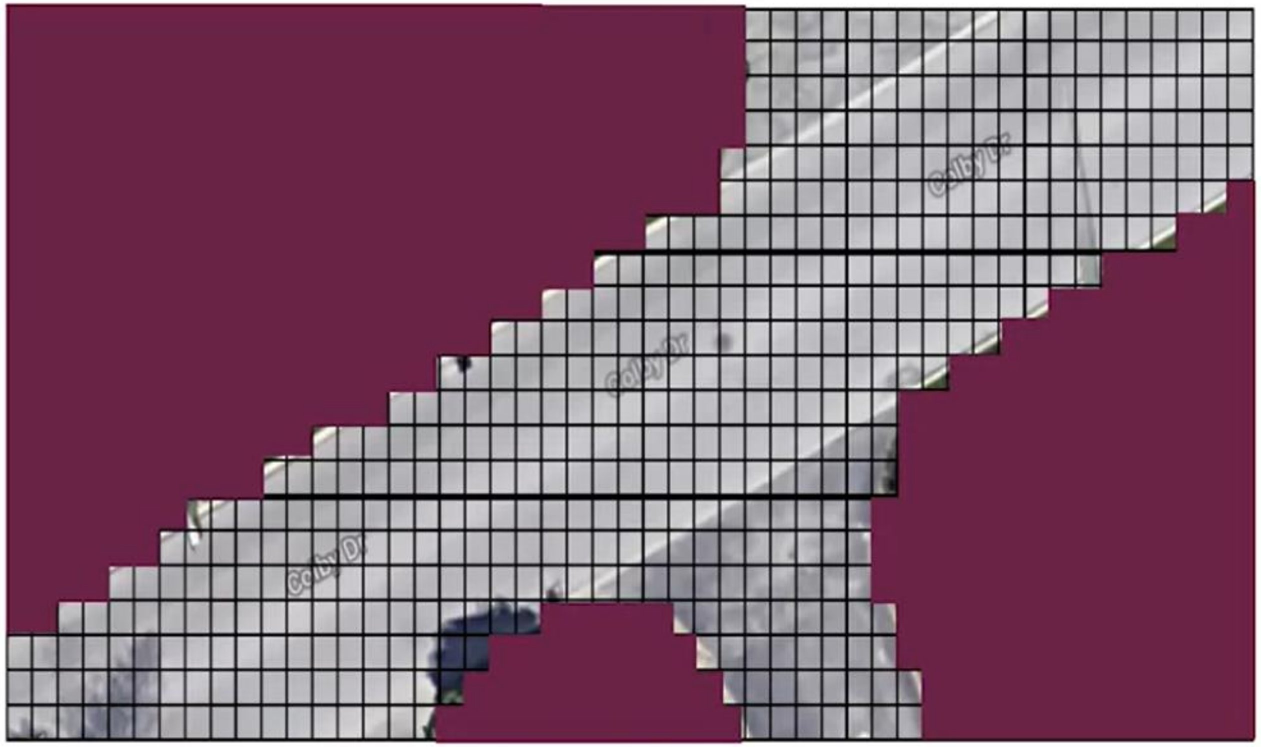
$$bel_t(m_i) = p(m_i|z, x)$$

$$bel_t(m_i) = p(m_i|(z, x)_{1:t})$$

- Bayes' theorem is applied at each update step for each cell

$$bel_t(m_i) = \underset{\text{normalizing constant}}{n}\,\overbrace{p(y_t|m_i)}^{\text{current measurement}}\,\underbrace{bel_{t-1}(m_i)}_{\text{previous belief map}}$$

(Markov)



Issue with the Standard Bayesian Update

- Update a single unoccupied grid cell

$$bel_t(m_i) = np(y_t|m_i)bel_{t-1}(m_i)$$

0.000000008 ! 0.000012 0.000638

- Multiplication of numbers close to zero is hard for computers
  - An inefficient way to perform belief updates
  - Significant rounding errors of very small floating point numbers lead to instabilities

- Store the log odds ratio rather than probability

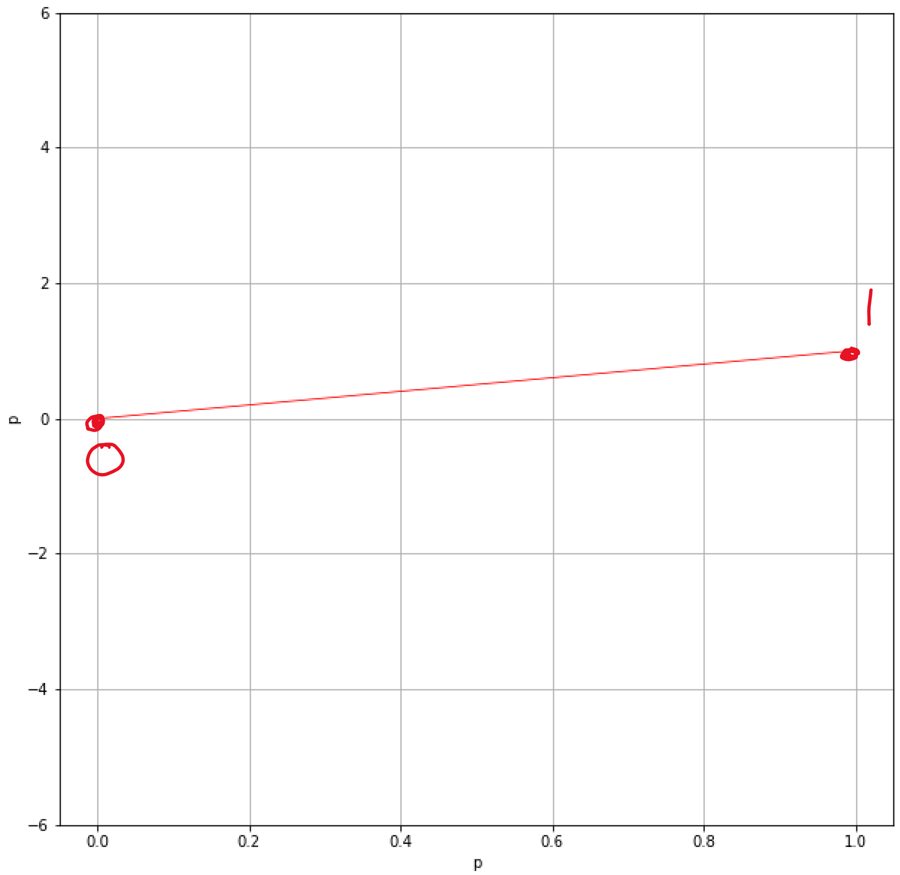
$$bel_t(m) \rightarrow (-\infty, \infty)$$

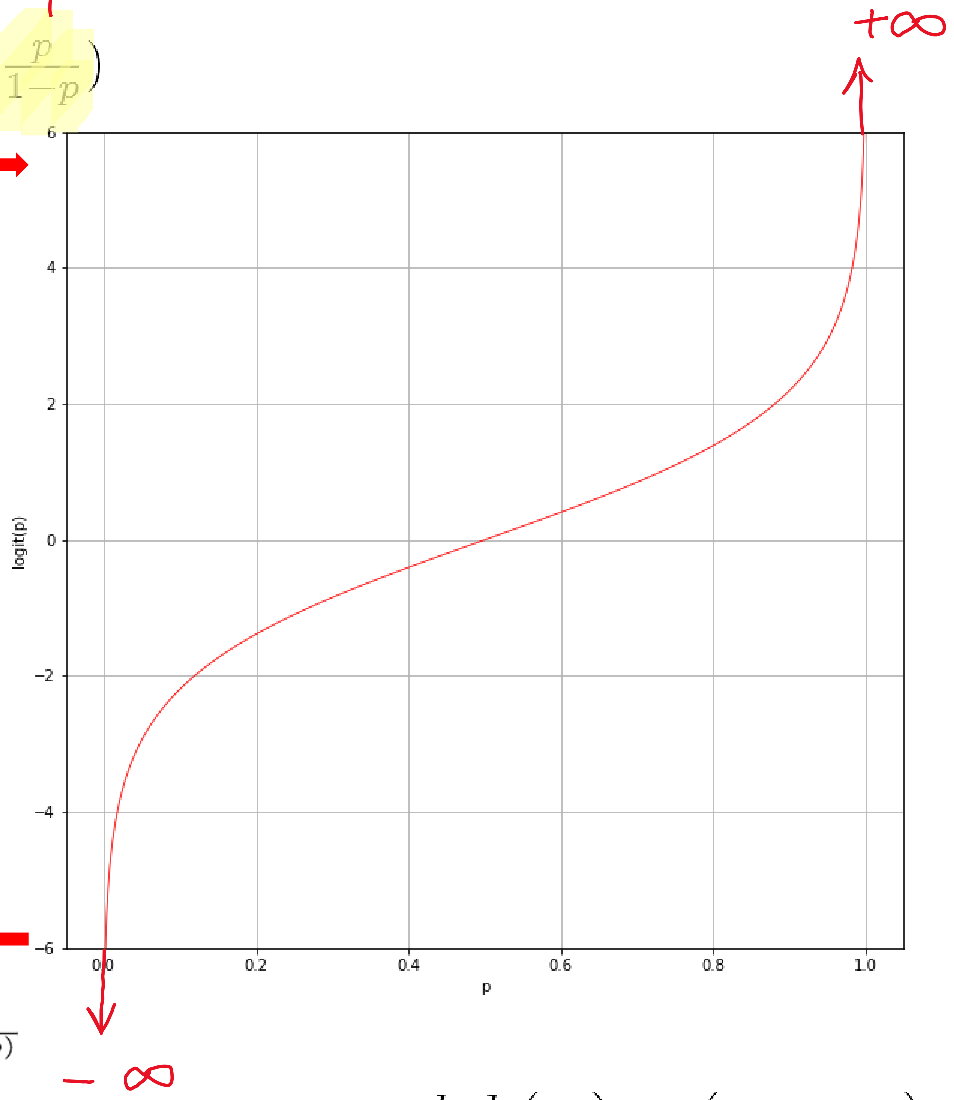
(0, 1)

Logit Function

Odds ratio

$$logit(p) = log(\frac{p}{1-p})$$





$$p = \frac{e^{logit(p)}}{1+e^{logit(p)}}$$

$$bel_t(m) \to (-\infty, \infty)$$

Bayesian Log Odds Single Cell Update Derivation

$$p(m_i \mid z_{1:t}, x_{1:t})$$

$$\overset{\text{Markov}}{=} \frac{p(z_t \mid m_i, x_t)\, p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

Bayesian Log Odds Single Cell Update Derivation

$$p(m_i \mid z_{1:t}, x_{1:t}) \stackrel{\text{Bayes rule}}{=} \frac{p(z_t \mid m_i, \cancel{z_{1:t-1}}, x_{1:t})\; p(m_i \mid z_{1:t-1}, x_{1:t})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$\stackrel{\text{Markov}}{=} \frac{p(z_t \mid m_i, x_t)\; p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

probability of getting z_t given the cell state at all previous measurements

probability of getting z_t given all measurements up to $t-1$.

probability the cell is occupied given all measurements to $t-1$

Bayesian Log Odds Single Cell Update Derivation

$$p(m_i \mid z_{1:t}, x_{1:t}) \overset{\text{Bayes rule}}{=} \frac{p(z_t \mid m_i, z_{1:t-1}, x_{1:t})\, p(m_i \mid z_{1:t-1}, x_{1:t})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$\overset{\text{Markov}}{=} \frac{p(z_t \mid m_i, x_t)\, p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$\overset{\text{Bayes rule}}{=} \frac{p(m_i \mid z_t, x_t)\, p(z_t \mid x_t)\, p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(m_i \mid x_t)\, p(z_t \mid z_{1:t-1}, x_{1:t})}$$

Bayesian Log Odds Single Cell Update Derivation

$$p(m_i \mid z_{1:t}, x_{1:t}) \overset{\text{Bayes rule}}{=} \frac{p(z_t \mid m_i, z_{1:t-1}, x_{1:t})\, p(m_i \mid z_{1:t-1}, x_{1:t})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$\overset{\text{Markov}}{=} \frac{p(z_t \mid m_i, x_t)\, p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$\overset{\text{Bayes rule}}{=} \frac{p(m_i \mid z_t, x_t)\, p(z_t \mid x_t)\, p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(m_i \mid \cancel{x_t})\, p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$\overset{\text{Markov}}{=} \frac{p(m_i \mid z_t, x_t)\, p(z_t \mid x_t)\, p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(m_i)\, p(z_t \mid z_{1:t-1}, x_{1:t})}$$

Bayesian Log Odds Single Cell Update Derivation

$$p(m_i \mid z_{1:t}, x_{1:t}) \stackrel{\text{Bayes rule}}{=} \frac{p(z_t \mid m_i, z_{1:t-1}, x_{1:t})\, p(m_i \mid z_{1:t-1}, x_{1:t})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$\stackrel{\text{Markov}}{=} \frac{p(z_t \mid m_i, x_t)\, p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$\stackrel{\text{Bayes rule}}{=} \frac{p(m_i \mid z_t, x_t)\, p(z_t \mid x_t)\, p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(m_i \mid x_t)\, p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$p(\neg \text{event}) = 1 - p(\text{event})$

$$\stackrel{\text{Markov}}{=} \frac{p(m_i \mid z_t, x_t)\, p(z_t \mid x_t)\, p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(m_i)\, p(z_t \mid z_{1:t-1}, x_{1:t})} \quad \ldots 1$$

$$p(\neg m_i \mid z_{1:t}, x_{1:t}) \stackrel{\text{the same}}{=} \frac{p(\neg m_i \mid z_t, x_t)\, p(z_t \mid x_t)\, p(\neg m_i \mid z_{1:t-1}, x_{1:t-1})}{p(\neg m_i)\, p(z_t \mid z_{1:t-1}, x_{1:t})} \quad \ldots 2$$

Bayesian Log Odds Single Cell Update Derivation

$$\frac{p(m_i \mid z_{1:t}, x_{1:t})}{p(\neg m_i \mid z_{1:t}, x_{1:t})} = \frac{\dfrac{p(m_i|z_t,x_t)\, \cancel{p(z_t|x_t)}\, p(m_i|z_{1:t-1},x_{1:t-1})}{p(m_i)\, \cancel{p(z_t|z_{1:t-1},x_{1:t})}}}{\dfrac{p(\neg m_i|z_t,x_t)\, \cancel{p(z_t|x_t)}\, p(\neg m_i|z_{1:t-1},x_{1:t-1})}{p(\neg m_i)\, \cancel{p(z_t|z_{1:t-1},x_{1:t})}}}$$

$$\frac{p(m_i \mid z_{1:t}, x_{1:t})}{1 - p(m_i \mid z_{1:t}, x_{1:t})} = \underbrace{\frac{p(m_i \mid z_t, x_t)}{1 - p(m_i \mid z_t, x_t)}}_{\text{uses } z_t \text{ (current measurement)}} \underbrace{\frac{p(m_i \mid z_{1:t-1}, x_{1:t-1})}{1 - p(m_i \mid z_{1:t-1}, x_{1:t-1})}}_{\text{recursive term}} \underbrace{\frac{1 - p(m_i)}{p(m_i)}}_{\text{prior}}$$

log both sides

$$\log\left(\frac{p(m_i \mid z_{1:t}, x_{1:t})}{1 - p(m_i \mid z_{1:t}, x_{1:t})}\right) = \log\left(\frac{p(m_i \mid z_t, x_t)}{1 - p(m_i \mid z_t, x_t)} \frac{p(m_i \mid z_{1:t-1}, x_{1:t-1})}{1 - p(m_i \mid z_{1:t-1}, x_{1:t-1})} \frac{1 - p(m_i)}{p(m_i)}\right)$$

Bayesian Log Odds Single Cell Update Derivation

$$\log\left(\frac{p(m_i \mid z_{1:t}, x_{1:t})}{1-p(m_i \mid z_{1:t}, x_{1:t})}\right) = \log\left(\frac{p(m_i \mid z_t, x_t)}{1-p(m_i \mid z_t, x_t)}\frac{p(m_i \mid z_{1:t-1}, x_{1:t-1})}{1-p(m_i \mid z_{1:t-1}, x_{1:t-1})}\frac{1-p(m_i)}{p(m_i)}\right)$$

$$= \operatorname{logit}(p(m_i \mid z_t, x_t)) + \operatorname{logit}(p(m_i \mid z_{1:t}, x_{1:t-1}) - \operatorname{logit}(p(m_i))$$

$$l(m_i \mid z_{1:t}, x_{1:t}) = l(m_i \mid z_t, x_t) + l(m_i \mid z_{1:t-1}, x_{1:t-1}) - l(m_i)$$

Bayesian Log Odds Single Cell Update Derivation

$$\begin{aligned} &l(m_i \mid z_{1:t}, x_{1:t}) \\ &= \quad l(m_i \mid z_t, x_t) \quad + l(m_i \mid z_{1:t-1}, x_{1:t-1}) - l(m_i) \end{aligned}$$

Short-hand notation

$$\boxed{l_{t,i} = logit(p(m_i|y_t) + l_{t-1,i} - l_{0,i})}$$

Inverse_Sensor_Model

- Numerically Stable
- Computationally Efficient

Bayesian Log Odds Single Cell Update Derivation

occupancy_grid_mapping($\{l_{t-1,i}\}, x_t, z_t$):

for all cells m_i *do*
    if m_i *in perceptual field of* z_t *then*
        $l_{t,i} = l_{t-1,i} + \text{inv\_sensor\_model}(m_i, x_t, z_t) - l_0$
    else
        $l_{t,i} = l_{t-1,i}$
    endif
endfor
return $\{l_{t,i}\}$

highly efficient, we only have to compute sums

Inverse Measurement Model

- So far we have only seen the following measurement model:

$$p(z_t|m_i)$$

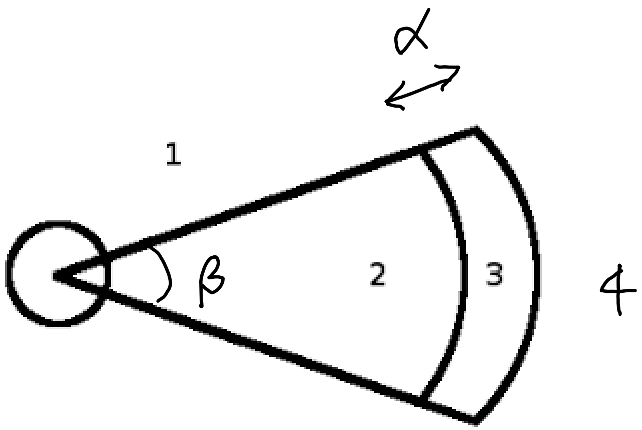
- But…

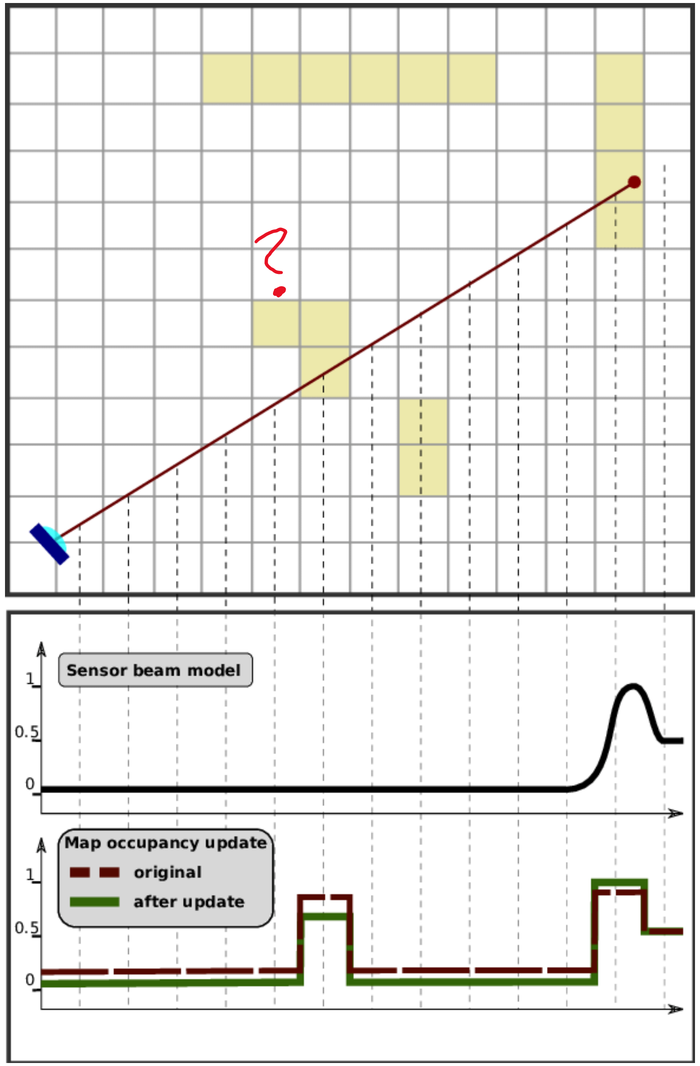
$$l_{t,i} = logit(p(m_i|\underset{z_t}{\cancel{z_t}})) + l_{t-1,i} - l_{0,i})$$

- Bresenham's Line Algorithm (Ray Tracing)

Inverse Measurement Model

- For LIDARs:





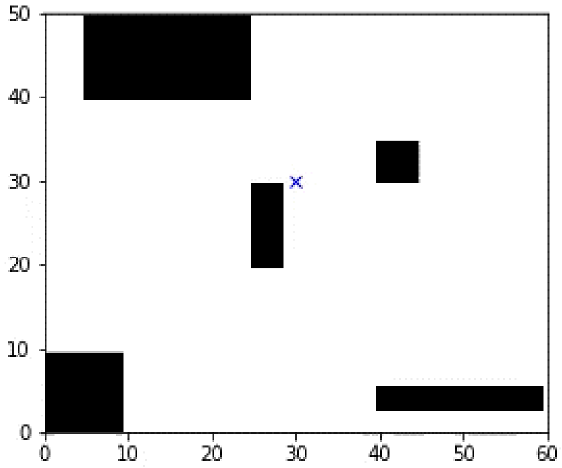
$1 \,\&\, 4 : \quad p(m_i) = 0.5$

$2 : \quad p(m_i) = 0$

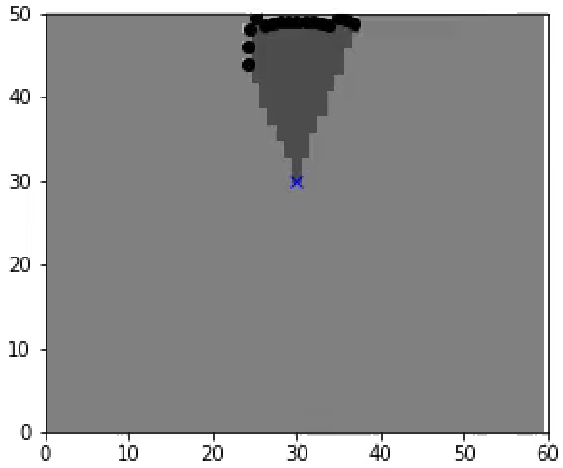
$3 : \quad p(m_i) = 1$

Courtesy: Probabilistic Robotics, Henrik Andreasson

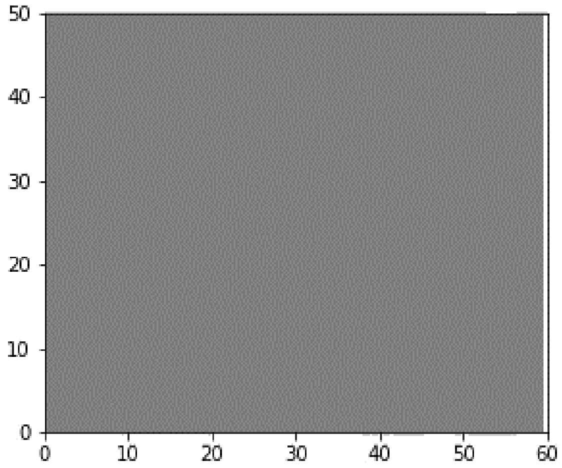
Jupyter Notebook Example



Ground Truth

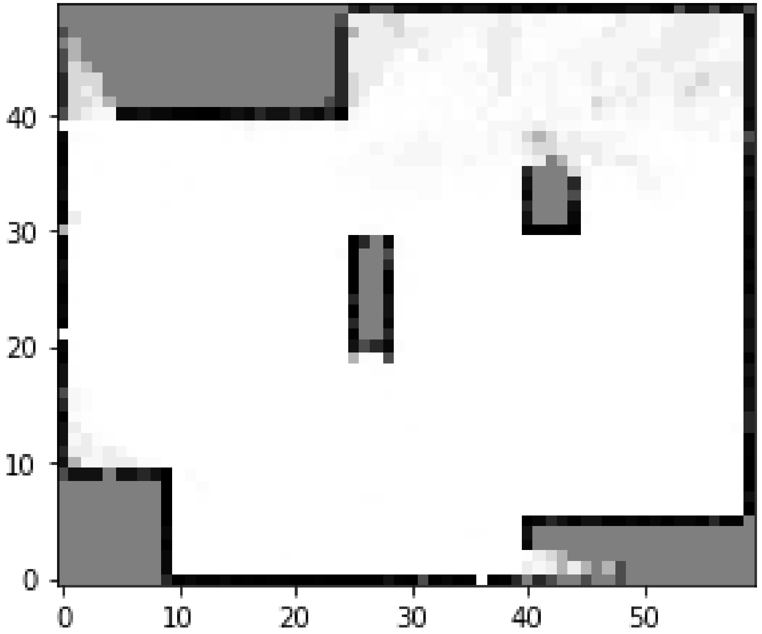


Sensor Model



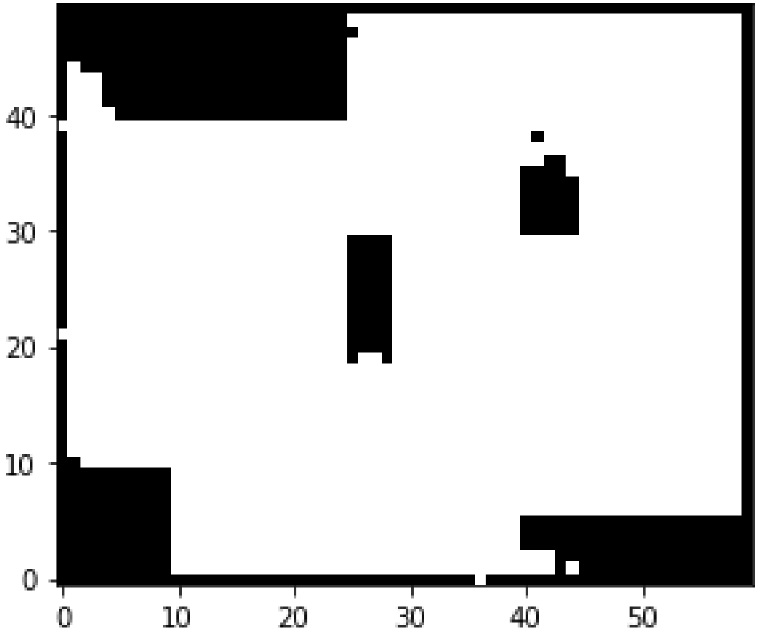
Belief Map

Jupyter Notebook Example



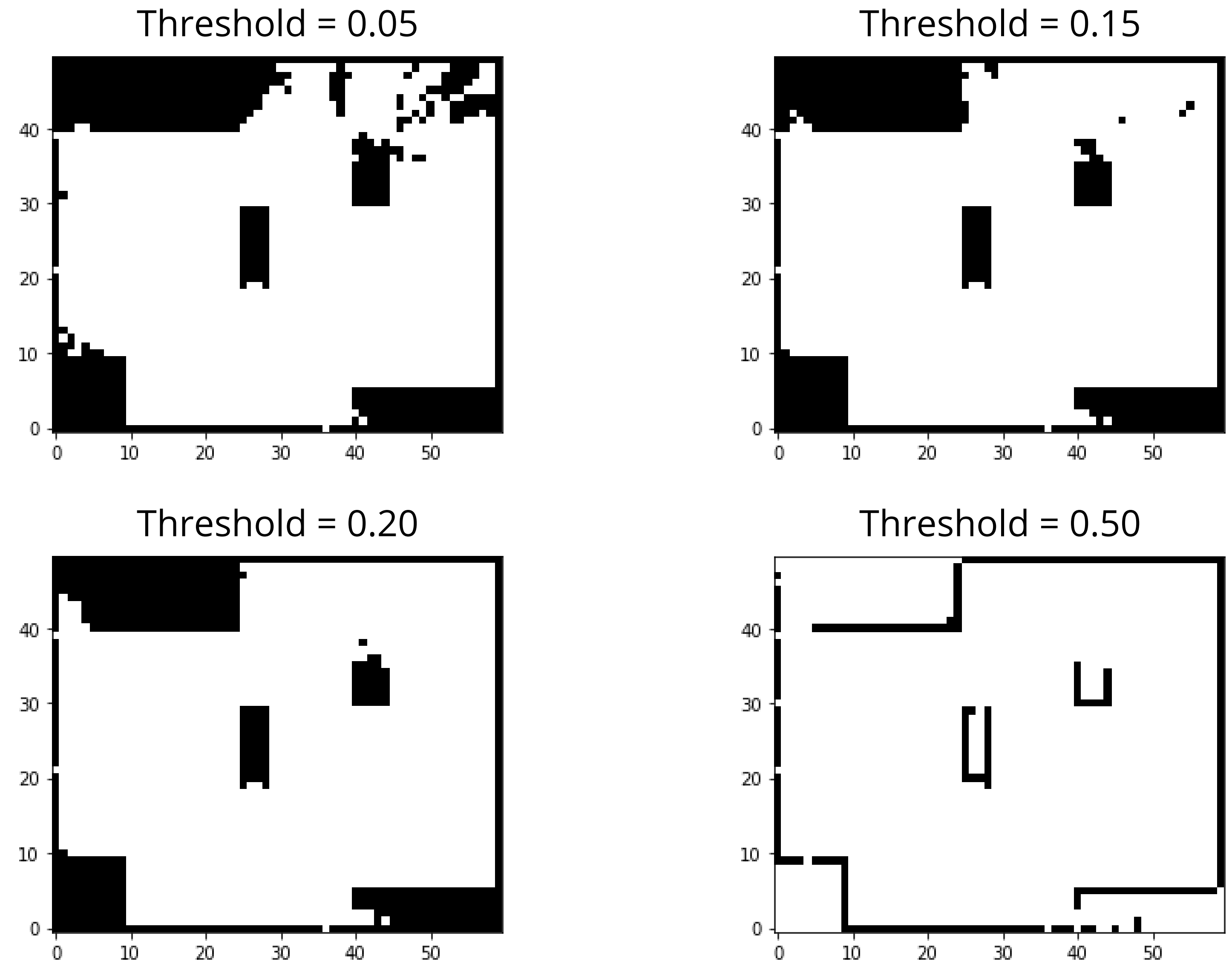
Belief Map

Threshold Filter



Binary Occupancy Grid

Jupyter Notebook Example



References

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Thank you!

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