CS-203: Design and Analysis of Algorithms
CS-603: Advanced Algorithms

Assignment II

Course Instructor: Dr. Dibyendu Roy

Due: Dec 20, 2023, 11:59 pm

Instructions: Answers must be handwritten. Clearly justify your answers. Upload the scanned copy of your handwritten answers. Write name and roll number on the top of every page. The file name of your code will be YOUR ROLL-NO.pdf

1. Give an $O(n^2)$-time algorithm to find the longest monotonically increasing subsequence of a sequence of n numbers.

2. Consider a modification to the activity-selection problem in which each activity a_i has, in addition to a start and finish time, a value v_i. The objective is no longer to maximize the number of activities scheduled, but instead to maximize the total value of the activities scheduled. That is, we wish to choose a set A of compatible activities such that $\sum_{a \in A} v_a$ is maximized. Give a polynomial-time algorithm for this problem.

3. What is an optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci numbers?

   a:1 b:1 c:2 d:3 e:5 f:8 g:13 h:21

   Can you generalize your answer to find the optimal code when the frequencies are the first n Fibonacci numbers?

4. The **diameter** of a tree $T = (V, E)$ is defined as $\max_{u,v \in V} d(u, v)$, that is, the largest of all shortest-path distances in the tree. Give an efficient algorithm to compute the diameter of a tree, and analyze the running time of your algorithm.

5. Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is $(5, 10, 3, 12, 5, 50, 6)$. Also, write an algorithm to print the optimal parenthesization of a matrix-chain product whose sequence is given.

6. Give an $O(n)$-time dynamic-programming algorithm to compute the n^{th} Fibonacci number. Draw the subproblem graph. How many vertices and edges are in the graph?

7. Give a dynamic-programming solution to the $0 - 1$ knapsack problem that runs in $O(nW)$ time, where n is the number of items and W is the maximum weight of items that the thief can put in his knapsack.