Networks and Random Processes

Problem sheet 2 – Part 2

Sheet counts $60/100$ homework marks, [x] indicates weight of the question.
Please put solutions in my pigeon hole or give them to me by **Friday, 28.10.2016, 2pm**.

2.4 Dorogovtsev-Mendes-Samukhin model [12]

Consider the following generalization of the Barabási-Albert model. Starting with $m_0 = 5$ connected nodes, in each timestep a node j is added and linked to $m = 5$ existing distinct nodes according to the probability (to be adapted to avoid double edges)

$$\pi_{j\leftrightarrow i} = \frac{k_0 + k_i}{\sum_{i\in V(t)}(k_i + k_0)} , \quad k_0 \in \mathbb{N}_0 .$$

Simulate the model for three different values of $k_0 = 0, 2, 4$ to generate graphs of size $N = |V| = 1000$, with 20 independent realizations in each case.

(a) Plot the degree distribution in a double logarithmic plot for a single realization and for all 100. For each k_0 compare to the power law with exponent $-3 - k_0/m$.

(b) Compute $k_{nn}(k) = \mathbb{E}\Big[\sum_{i\in V} k_{nn,i}\delta_{k_i,k}\Big/\sum_{i\in V}\delta_{k_i,k}\Big]$ where $k_{nn,i} = \frac{1}{k_i}\sum_{j\in V} a_{ij}k_j$, and decide whether the graphs are typically uncorrelated or (dis-)assortative.

(c) Plot the spectrum of the adjacency matrix A using all realizations with a kernel density estimate, and compare it to the Wigner semi-circle law.

2.5 Erdős Rényi random graphs [12]

Consider the Erdős Rényi random graph model and simulate at least 20 realizations of $\mathcal{G}_{N,p}$ graphs with $p = p_N = z/N$, $z = 0.1, 0.2, \ldots, 3.0$ for $N = 100$ and $N = 1000$.

(a) Plot the expected size of the largest two components against z for both values of N.

(b) For $N = 1000$ plot the expected local clustering coefficient $\mathbb{E}[\langle C_i\rangle]$ against z.

(c) Consider $z = 0.5, 1.5, 5$ and 10. Plot the spectrum of the adjacency matrix A using all realizations with a kernel density estimate, and compare it to the Wigner semi-circle law.