Bidding stochastic demand for ancillary services: the P90 rule

Peter A. Vistar Gade

January 19, 2024

Equations

Sets:

- $\mathcal{H} = \{1, 2, \dots 24\}$
- $\mathcal{M} = \{1, 2, \dots 1440\}$
- $\mathcal{M}_h = \{h \times 60 + m \mid m \in \{0, 1, 2, \dots, 59\}\}$
- $\mathcal{I} = \{1, 2, \dots \text{ Number of samples}\}$

Definitions:

Definition 1 (Energinet's P90 rule [2])**.** *This means, that the participant's prognosis, which must be approved by Energinet, evaluates that the probability is 10% that the sold capacity is not available. This entails that there is a 90% chance that the sold capacity or more is available. This is when the prognosis is assumed to be correct.*

The probability is then also 10%, that the entire sold capacity is not available. If this were to happen, it does not entail that the sold capacity is not available at all, however just that a part of the total capacity is not available. The available part will with high probability be close to the sold capacity.

Simple optimization problem where bids, p_h^{cap}, must be adhere to uncertain available flexibility, $P_m^{\text{B}}(\xi)$.

$$\max_{p_h^{\text{cap}}} \quad \sum_h \lambda_h p_h^{\text{cap}} \tag{1a}$$

$$\text{s.t.} \quad \mathbb{P}\left(p_h^{\text{cap}} \leq P_m^{\text{B}}(\xi), \quad \forall m \in \mathcal{M}_h, \forall h \in \mathcal{H}\right) \geq 1 - \epsilon \tag{1b}$$

DRJCC

Converting (1) to DRJCC for distributions within some ambiguity set defined by Wasserstein distance θ:

$$\max_{p_h^{\text{cap}}} \quad \sum_h \lambda_h p_h^{\text{cap}} \tag{2a}$$

$$\text{s.t.} \quad \mathbb{P}\left(p_h^{\text{cap}} \leq P_m^{\text{B}}(\xi), \quad \forall m \in \mathcal{M}_h, \forall h \in \mathcal{H}\right) \geq 1 - \epsilon \quad \forall \mathbb{P} \in \mathcal{F}(\theta) \tag{2b}$$

A tractable formulation of (2) is given by [1, Proposition 2]:

$$
\begin{aligned}
\max_{p_h^{\text{cap}}, q_i, s_i \geq 0, t} \quad & \sum_h \lambda_h p_h^{\text{cap}} && \text{(3a)} \\
\text{s.t.} \quad & \epsilon |\mathcal{I}| t - \sum_{i \in \mathcal{I}} s_i \geq \theta |\mathcal{I}| && \text{(3b)} \\
& P_m^{\text{B}}(\xi_i) - p_h^{\text{cap}} + M \cdot q_i \geq t - s_i, \quad \forall m \in \mathcal{M}_h, \forall h \in \mathcal{H}, \forall i \in \mathcal{I} && \text{(3c)} \\
& M(1 - q_i) \geq t - s_i && \text{(3d)} \\
& q_i \in \{0, 1\}, \quad \forall i \in \mathcal{I} && \text{(3e)}
\end{aligned}
$$

Let the set of flexible demands be denoted by $d \in \{\mathcal{D}\}$. Then TSO profit is described by the following bi-level problem:

$$
\begin{aligned}
\max_{\epsilon, \theta} \quad & \sum_h \lambda_h p_{h,d}^{\text{cap}} - \sum_h \sum_d \frac{1}{|\mathcal{I}|} \sum_i \nu_{h,i,d} && \text{(4a)} \\
\text{s.t.} \quad & \text{Problem (3)}, \quad \forall d && \text{(4b)} \\
& \nu_{h,i,d} = \left(p_{h,d}^{\text{cap}} - P_m^{\text{B}}(\xi_i) \right)^+, \quad \forall i, \forall h, \forall d && \text{(4c)}
\end{aligned}
$$

ALSO-X

Inspired by [3, 4], the reformulation of (1) is:

$$
\begin{aligned}
\max_{p_h^{\text{cap}}, y_{m,i}} \quad & \sum_h \lambda_h p_h^{\text{cap}} && \text{(5)} \\
\text{s.t.} \quad & -(1 - y_{m,i})M \leq p_h^{\text{cap}} - P_m^{\text{B}}(\xi_i) \leq y_{m,i} M, \quad \forall m \in \mathcal{M}_h, \forall h \in \mathcal{H}, \forall i \in \mathcal{I} && \text{(6)} \\
& \sum_{i \in \mathcal{I}, m \in \mathcal{H}} y_{m,i} \leq q && \text{(7)} \\
& y_{m,i} \in \{0, 1\}, \quad \forall m \in \mathcal{M}_h, \forall h \in \mathcal{H}, \forall i \in \mathcal{I} && \text{(8)}
\end{aligned}
$$

Here, $q = \lfloor \epsilon \cdot |\mathcal{I}| \cdot |\mathcal{M}| \rfloor$ and $y_{m,i}$ is a binary variable indicating an overbid in minute m and sample i. The ALSO-X algorithm as listed in Algorithm 1 iteratively conducts an exponential search for the best value of q such that the frequency of violations is less than ϵ by solving a relaxed version of (5) with respect to $y_{m,i}$. Thus, ALSO-X does not consider the magnitude of violation - as opposed to CVaR - but only the frequency of violations. It is therefore expected that ALSO-X is less conservative than CVaR which was also the motivation behind its incarnation [3].

Algorithm 1 ALSO-X

Require: Relax the integrality of y Input: Stopping tolerance parameter δ

1: $q \leftarrow 0, \bar{q} \leftarrow \lceil -\epsilon \rceil$

2: **while** $q \neq \bar{q}$ **do**

3: Set $q_a \leftarrow (q + \bar{q})/2$ and retrieve Θ^* as an optimal solution to (4).

4: Set $q \leftarrow q_a$ if $P(y^* = 0) \geq 1 - \epsilon$; otherwise, $\bar{q} \leftarrow q$

5: **end while**Output: A feasible solution of model (5).

CVaR

First, let $\mathcal{I}$ denote the number of samples for the empirical distribution. The CVaR formulation of the JCC in (1) is then [3]:

$$\max_{p_h^{\text{cap}}, \beta \geq 0, \zeta_{m,i}} \quad \sum_h \lambda_h p_h^{\text{cap}} \tag{9}$$

$$\text{s.t.} \quad p_h^{\text{cap}} - P_m^{\text{B}}(\xi_i) \leq \zeta_{m,i}, \quad \forall m \in \mathcal{M}_h, \forall h \in \mathcal{H}, \forall i \in \mathcal{I} \tag{10}$$

$$\frac{1}{|\mathcal{M}| \cdot |\mathcal{I}|} \sum_{i \in \mathcal{I}, m \in \mathcal{H}} \zeta_{m,i} - (1 - \epsilon) \cdot \beta \leq 0 \tag{11}$$

$$\beta \leq \zeta_{m,i}, \quad \forall m \in \mathcal{M}_h, \forall h \in \mathcal{H}, \forall i \in \mathcal{I} \tag{12}$$

The CVaR approximation in (9) is a simple LP. The JCC is now approximated such that expected value of the ϵ worst samples and minutes is attenuated.

Bibliography

[1] Zhi Chen, Daniel Kuhn, and Wolfram Wiesemann. "Data-driven chance constrained programs over Wasserstein balls". In: *Operations Research* (2022).

[2] "Energinet: Prequalification and Test". In: *Energinet* (2023). Accessed: 2023-11-24. URL: `https://en.energinet.dk/electricity/ancillary-services/prequalification-and-test/`.

[3] Nan Jiang and Weijun Xie. "ALSO-X and ALSO-X+: Better convex approximations for chance constrained programs". In: *Operations Research* 70.6 (2022), pp. 3581–3600.

[4] Álvaro Porras et al. "Integrating Automatic and Manual Reserves in Optimal Power Flow via Chance Constraints". In: *arXiv preprint arXiv:2303.05412* (2023).