Bidding stochastic demand for ancillary services: the P90 rule

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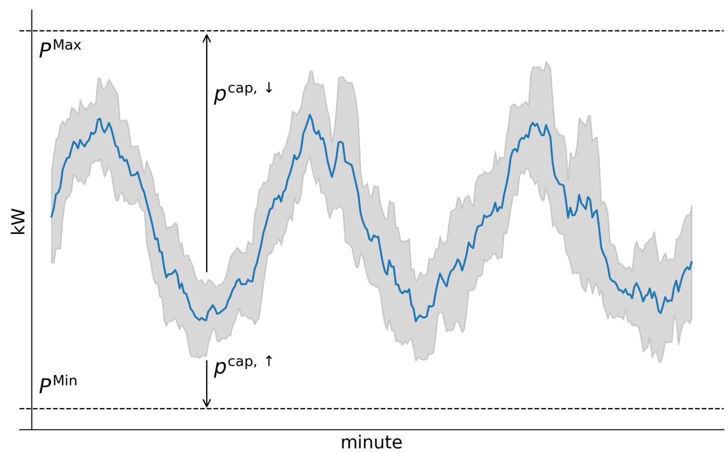

Figure 1: Uncertain power consumption of flexible demand portfolio, denoted by $p_m^{\mathrm{B}}(\xi)$. The available capacity for bidding flexibility for a given time instance is illustrated as $p^{\mathrm{cap},\downarrow}$ for down-regulation and $p^{\mathrm{cap},\uparrow}$ for up-regulation. P^{Max} and P^{Min} is the maximum and minimum power consumption of the portfolio, respectively.

Introduction

Abbreviations:

- **C**hance **C**onstraint: **CC**
- **J**oint **C**hance **C**onstraint: **JCC**
- **D**istributionally **R**obust **J**oint **c**hance **c**onstraint: **DRJCC**

Sets:

- $\mathcal{H} = \{1, 2, \dots 24\}$
- $\mathcal{M} = \{1, 2, \dots 1440\}$
- $\mathcal{M}_h = \{h \times 60 + m \mid m \in \{0, 1, 2, \dots, 59\}\}$

Definitions:

Definition 1 (Energinet's P90 rule [1])**.** *This means, that the participant's prognosis, which must be approved by Energinet, evaluates that the probability is 10% that the sold capacity is not available. This entails that there is a 90%*

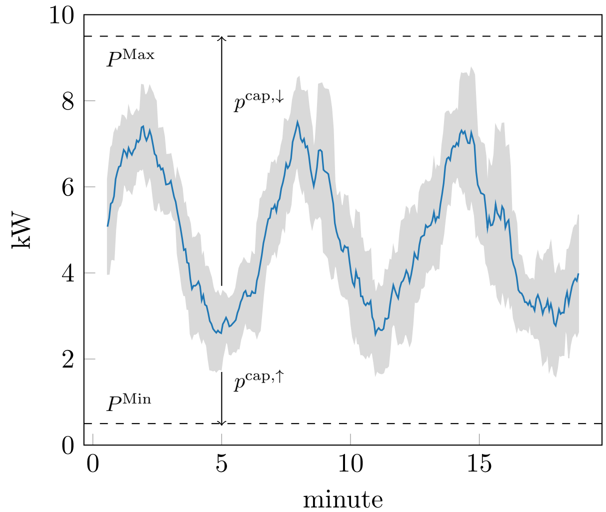

Figure 2: Uncertain power consumption of flexible demand portfolio, denoted by $p_m^{\mathrm{B}}(\xi)$. The available capacity for bidding flexibility for a given time instance is illustrated as $p^{\mathrm{cap},\downarrow}$ for down-regulation and $p^{\mathrm{cap},\uparrow}$ for up-regulation. P^{Max} and P^{Min} is the maximum and minimum power consumption of the portfolio, respectively.

chance that the sold capacity or more is available. This is when the prognosis is assumed to be correct.

The probability is then also 10%, that the entire sold capacity is not available. If this were to happen, it does not entail that the sold capacity is not available at all, however just that a part of the total capacity is not available. The available part will with high probability be close to the sold capacity.

Problem 1 optimizes reserve capacity for a flexible demand (Figure 2) such that there is at least a $1 - \epsilon$ probability of fulfilling the P90 requirement from Energinet.

$$\max_{p_h^{\mathrm{cap},\uparrow}, p_h^{\mathrm{cap},\downarrow}} \quad \sum_h \left(\lambda_h p_h^{\mathrm{cap},,\uparrow} + \lambda_h p_h^{\mathrm{cap},\downarrow} \right) \tag{1}$$

$$\text{s.t.} \quad \mathbb{P}\left(p_h^{\mathrm{cap},\uparrow} \leq P_m^{\mathrm{B}}(\xi) - P^{\mathrm{Min}}, \quad \forall h \in \mathcal{H}, \forall m \in \mathcal{M}_h \right) \geq 1 - \epsilon \tag{2}$$

$$\mathbb{P}\left(p_h^{\mathrm{cap},\downarrow} \leq P^{\mathrm{Max}} - P_m^{\mathrm{B}}(\xi), \quad \forall h \in \mathcal{H}, \forall m \in \mathcal{M}_h \right) \geq 1 - \epsilon \tag{3}$$

$$p_h^{\mathrm{cap},(.)} \leq P^{\mathrm{Max}}, \qquad \forall h \in \mathcal{H} \tag{4}$$

$$p_h^{\mathrm{cap},(.)} \geq 0, \qquad \forall h \in \mathcal{H} \tag{5}$$

Here, we note that Problem 1 can be simplified by assuming symmetric flexibility, i.e., $p_h^{\mathrm{cap},\downarrow} = p_h^{\mathrm{cap},\uparrow} = p_h^{\mathrm{cap}}$. Then the two JCCs are essentially the same since the uncertain power consumption, $P_h^{\mathrm{B}}(\xi)$, is simply set to $P_h^{\mathrm{B}}(\xi) \triangleq \min\{P_h^{\mathrm{B}}(\xi) - P^{\mathrm{Min}}, P^{\mathrm{Max}} - P_h^{\mathrm{B}}(\xi)\}$.

Problem 1 can be simplified further by only considering each minute within that hour within one JCC (and omitting trivial constraints for bounding capacity for cleanness):

$$\max_{p_h^{\mathrm{cap}}} \quad \sum_h \lambda_h p_h^{\mathrm{cap}} \tag{6}$$

$$\text{s.t.} \quad \mathbb{P}\left(p_h^{\mathrm{cap}} \leq P_m^{\mathrm{B}}(\xi), \quad \forall m \in \mathcal{M}_h\right) \geq 1 - \epsilon, \qquad \forall h \in \mathcal{H} \tag{7}$$

Problem 6 is now decomposed by hour and each hour can be solved in parallel to obtain p_h^{cap}, $\forall h \in \mathcal{H}$.

Doing so, the h index can be removed and Problem 6 contains one JCC:

$$\max_{p^{\mathrm{cap}}} \quad \lambda p^{\mathrm{cap}} \tag{8}$$

$$\text{s.t.} \quad \mathbb{P}\left(p^{\mathrm{cap}} \leq P_m^{\mathrm{B}}(\xi), \quad \forall m \in \mathcal{M}_h\right) \geq 1 - \epsilon \tag{9}$$

The uncertainty, ξ, is seen in Figure 2 on the realized power consumption, $p_m^{\mathrm{cap}}(\xi)$. To solve Problem 8, we need to create a sample set $p_m^{\mathrm{cap}}(\xi)$ governed by $\mathbb{P}$ from either:

1. Historical data
2. A predictive forecast (e.g., quantile or probabilistic)

We are interested in two methods for approximating the JCC in Problem 8: (i) CVaR approximation and (ii) ALSO-X [2].

CVaR

$$\max_{p^{\mathrm{cap}}} \quad \lambda p^{\mathrm{cap}} \tag{10}$$

$$\text{s.t.} \quad \mathbb{P}\text{-CVaR}_{1-\epsilon}\left(p^{\mathrm{cap}} \leq P_m^{\mathrm{B}}(\xi), \quad \forall m \in \mathcal{M}_h\right) \tag{11}$$

ALSO-X

$$\max_{p^{\mathrm{cap}}} \quad \lambda p^{\mathrm{cap}} \tag{12}$$

$$\text{s.t.} \quad \text{ALSO-X}_{1-\epsilon}\left(p^{\mathrm{cap}} \leq P_m^{\mathrm{B}}(\xi), \quad \forall m \in \mathcal{M}_h\right) \tag{13}$$

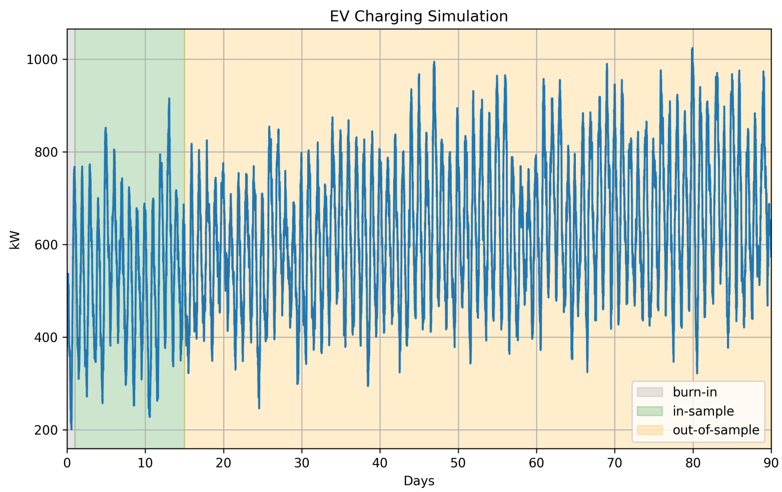

Figure 3: .

DRJCC

In this section, we take a different approach. Converting Problem 8 to its distributionally robust counterpart, we get:

$$\max_{p^{\mathrm{cap}}} \quad \lambda p^{\mathrm{cap}} \tag{14}$$

$$\text{s.t.} \quad \min_{\mathbb{P} \in \mathcal{P}} \mathbb{P}\left(p^{\mathrm{cap}} \leq P_m^{\mathrm{B}}(\xi), \quad \forall m \in \mathcal{M}_h\right) \geq 1 - \epsilon \tag{15}$$

It is still possible to use CVaR and ALSO-X for Problem 14 as it yields tractable, linear formulations [4] for CVaR and ALSO-X [3].

Write something

Number of samples required

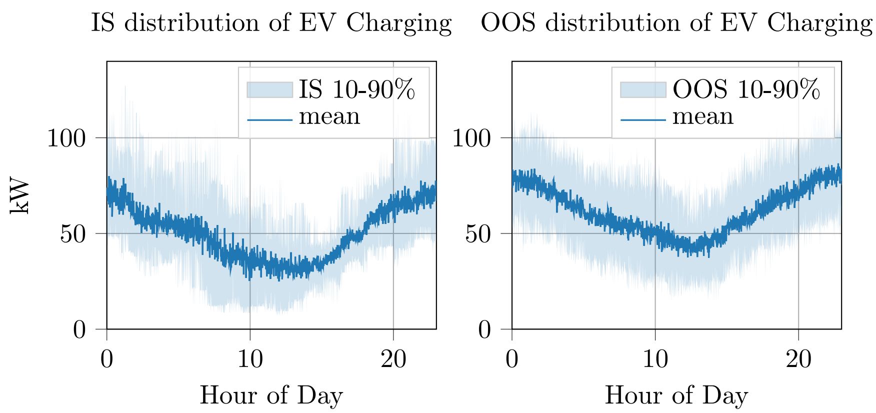

Figure 4: some caption.

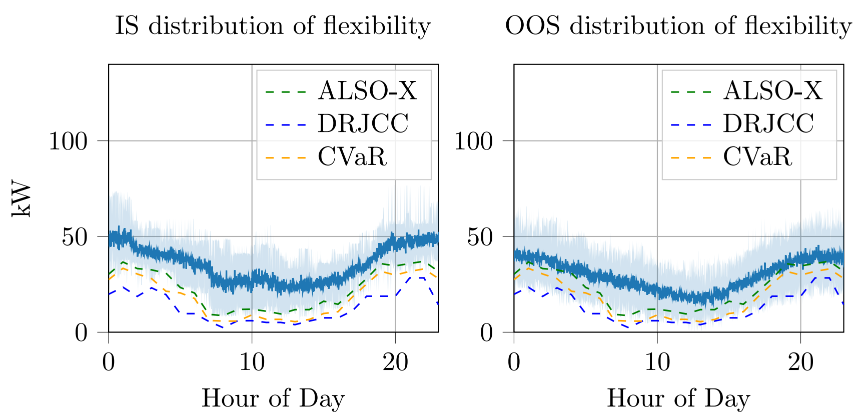

Figure 5: .

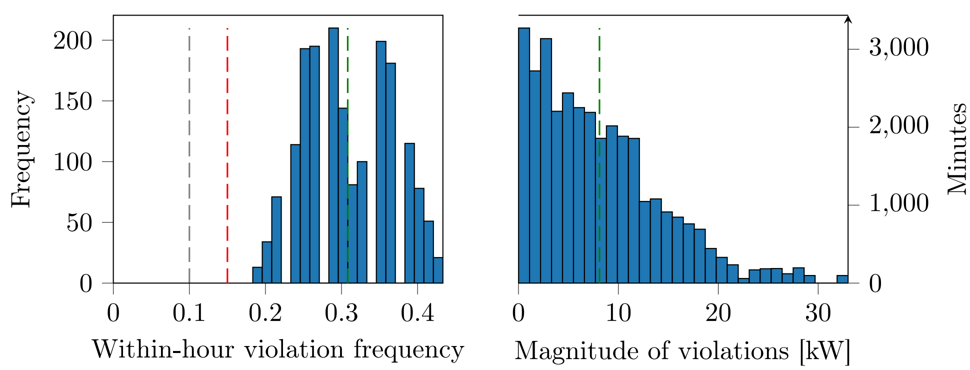

Figure 6: .

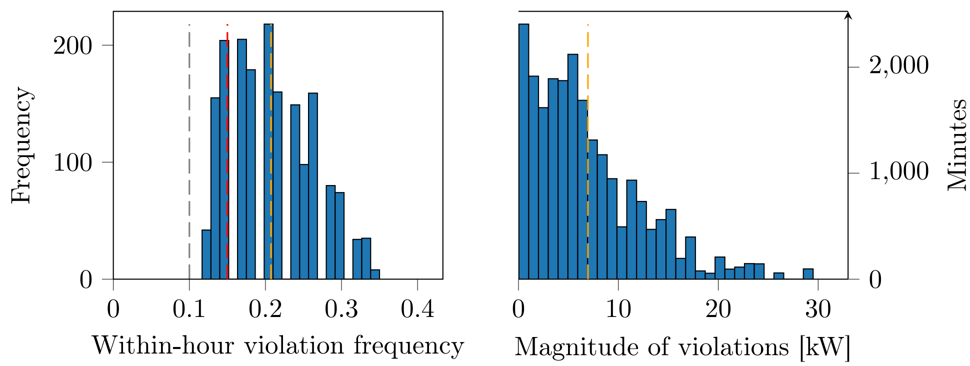

Figure 7: .

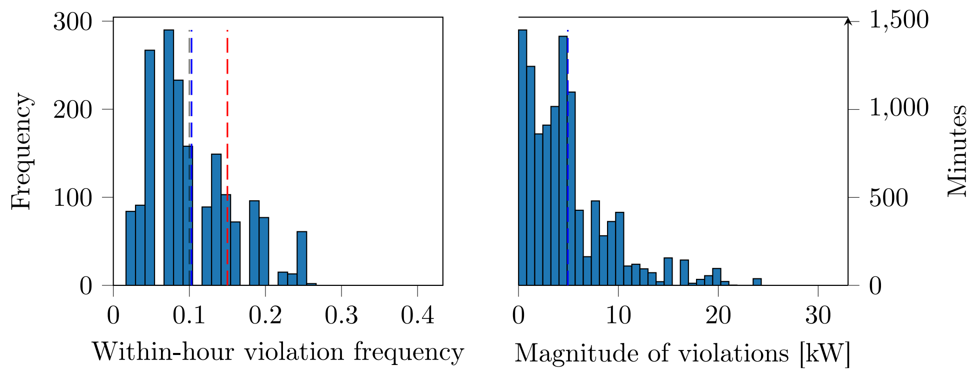

Figure 8: .

Bibliography

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[2] Nan Jiang and Weijun Xie. "ALSO-X and ALSO-X+: Better convex approximations for chance constrained programs". In: *Operations Research* 70.6 (2022), pp. 3581–3600.

[3] Nan Jiang and Weijun Xie. "ALSO-X#: Better Convex Approximations for Distributionally Robust Chance Constrained Programs". In: *arXiv preprint arXiv:2302.01737* (2023).

[4] Christos Ordoudis et al. "Energy and reserve dispatch with distributionally robust joint chance constraints". In: *Operations Research Letters* 49.3 (2021), pp. 291–299.