Bidding stochastic demand for ancillary services: the P90 rule

Peter A. Vistar Gade

November 29, 2023

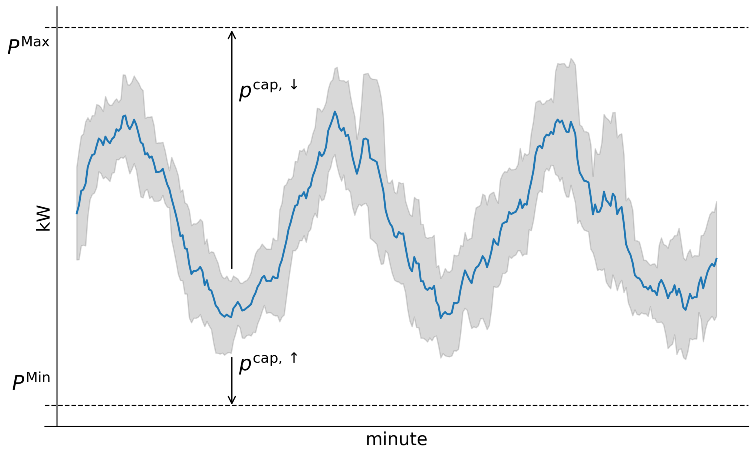

Figure 1: Uncertain power consumption of flexible demand portfolio, denoted by $p_m^{\mathrm{B}}(\xi)$. The available capacity for bidding flexibility for a given time instance is illustrated as $p^{\mathrm{cap},\downarrow}$ for down-regulation and $p^{\mathrm{cap},\uparrow}$ for up-regulation. P^{Max} and P^{Min} is the maximum and minimum power consumption of the portfolio, respectively.

Introduction

Abbreviations:

- **C**hance **C**onstraint: **CC**
- **J**oint **C**hance **C**onstraint: **JCC**
- **D**istributionally **R**obust **J**oint **c**hance **c**onstraint: **DRJCC**

Sets:

- $\mathcal{H} = \{1, 2, \ldots 24\}$
- $\mathcal{M} = \{1, 2, \ldots 1440\}$
- $\mathcal{M}_h = \{h \times 60 + m \mid m \in \{0, 1, 2, \ldots, 59\}\}$

Definitions:

Definition 1 (Energinet's P90 rule [1])**.** *This means, that the participant's prognosis, which must be approved by Energinet, evaluates that the probability is 10% that the sold capacity is not available. This entails that there is a 90%*

chance that the sold capacity or more is available. This is when the prognosis is assumed to be correct.

The probability is then also 10%, that the entire sold capacity is not available. If this were to happen, it does not entail that the sold capacity is not available at all, however just that a part of the total capacity is not available. The available part will with high probability be close to the sold capacity.

Problem 1 optimizes reserve capacity for a flexible demand (Figure 1) such that there is at least a $1-\epsilon$ probability of fulfilling the P90 requirement from Energinet.

$$
\begin{aligned}
\max_{p_h^{\text{cap},\uparrow}, p_h^{\text{cap},\downarrow}} \quad & \sum_h \left(\lambda_h p_h^{\text{cap},,\uparrow} + \lambda_h p_h^{\text{cap},\downarrow} \right) && (1)\\
\text{s.t.} \quad & \mathbb{P}\left(p_h^{\text{cap},\uparrow} \leq P_m^{\text{B}}(\xi) - P^{\text{Min}}, \quad \forall h \in \mathcal{H}, \forall m \in \mathcal{M}_h \right) \geq 1-\epsilon && (2)\\
& \mathbb{P}\left(p_h^{\text{cap},\downarrow} \leq P^{\text{Max}} - P_m^{\text{B}}(\xi), \quad \forall h \in \mathcal{H}, \forall m \in \mathcal{M}_h \right) \geq 1-\epsilon && (3)\\
& p_h^{\text{cap},(.)} \leq P^{\text{Max}}, \qquad \forall h \in \mathcal{H} && (4)\\
& p_h^{\text{cap},(.)} \geq 0, \qquad \forall h \in \mathcal{H} && (5)
\end{aligned}
$$

Here, we note that Problem 1 can be simplified by assuming symmetric flexibility, i.e., $p_h^{\text{cap},\downarrow} = p_h^{\text{cap},\uparrow} = p_h^{\text{cap}}$. Then the two JCCs are essentially the same since the uncertain power consumption, $P_h^{\text{B}}(\xi)$, is simply set to $P_h^{\text{B}}(\xi) \triangleq \min\{P_h^{\text{B}}(\xi) - P^{\text{Min}}, P^{\text{Max}} - P_h^{\text{B}}(\xi)\}$.

Problem 1 can be simplified further by only considering each minute within that hour within one JCC (and omitting trivial constraints for bounding capacity for cleanness):

$$
\begin{aligned}
\max_{p_h^{\text{cap}}} \quad & \sum_h \lambda_h p_h^{\text{cap}} && (6)\\
\text{s.t.} \quad & \mathbb{P}\left(p_h^{\text{cap}} \leq P_m^{\text{B}}(\xi), \quad \forall m \in \mathcal{M}_h \right) \geq 1-\epsilon, \qquad \forall h \in \mathcal{H} && (7)
\end{aligned}
$$

Problem 6 is now decomposed by hour and each hour can be solved in parallel to obtain p_h^{cap}, $\forall h \in \mathcal{H}$.

Doing so, the h index can be removed and Problem 6 contains one JCC:

$$
\begin{aligned}
\max_{p^{\text{cap}}} \quad & \lambda p^{\text{cap}} && (8)\\
\text{s.t.} \quad & \mathbb{P}\left(p^{\text{cap}} \leq P_m^{\text{B}}(\xi), \quad \forall m \in \mathcal{M}_h \right) \geq 1-\epsilon && (9)
\end{aligned}
$$

The uncertainty, ξ, is seen in Figure 1 on the realized power consumption, $p_m^{\text{cap}}(\xi)$. To solve Problem 8, we need to create a sample set $p_m^{\text{cap}}(\xi)$ governed by $\mathbb{P}$ from either:

1. Historical data

2. A predictive forecast (e.g., quantile or probabilistic)

We are interested in two methods for approximating the JCC in Problem 8: (i) CVaR approximation and (ii) ALSO-X [2].

CVaR

$$\max_{p^{\mathrm{cap}}} \quad \lambda p^{\mathrm{cap}} \tag{10}$$

$$\text{s.t.} \quad \mathbb{P}\text{-CVaR}_{1-\epsilon}\left(p^{\mathrm{cap}} \leq P_m^{\mathrm{B}}(\xi), \quad \forall m \in \mathcal{M}_h\right) \tag{11}$$

ALSO-X

$$\max_{p^{\mathrm{cap}}} \quad \lambda p^{\mathrm{cap}} \tag{12}$$

$$\text{s.t.} \quad \text{ALSO-X}_{1-\epsilon}\left(p^{\mathrm{cap}} \leq P_m^{\mathrm{B}}(\xi), \quad \forall m \in \mathcal{M}_h\right) \tag{13}$$

DRJCC

In this section, we take a different approach. Converting Problem 8 to its distributionally robust counterpart, we get:

$$\max_{p^{\mathrm{cap}}} \quad \lambda p^{\mathrm{cap}} \tag{14}$$

$$\text{s.t.} \quad \min_{\mathbb{P} \in \mathcal{P}} \mathbb{P}\left(p^{\mathrm{cap}} \leq P_m^{\mathrm{B}}(\xi), \quad \forall m \in \mathcal{M}_h\right) \geq 1-\epsilon \tag{15}$$

It is still possible to use CVaR and ALSO-X for Problem 14 as it yields tractable, linear formulations [4] for CVaR and ALSO-X [3].

Number of samples required

Bibliography

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[2] Nan Jiang and Weijun Xie. "ALSO-X and ALSO-X+: Better convex approximations for chance constrained programs". In: *Operations Research* 70.6 (2022), pp. 3581–3600.

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[4] Christos Ordoudis et al. "Energy and reserve dispatch with distributionally robust joint chance constraints". In: *Operations Research Letters* 49.3 (2021), pp. 291–299.