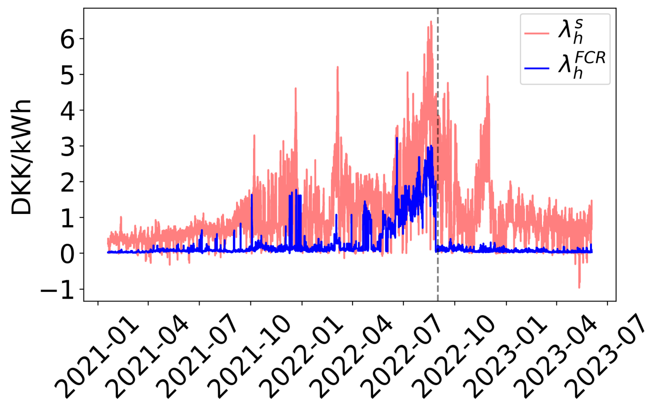

Fig. 10: FCR and spot prices in the period of January 2021 to July 2023 in the Danish bidding zone of DK1. In September 2022, FCR tenders included continental Europe.

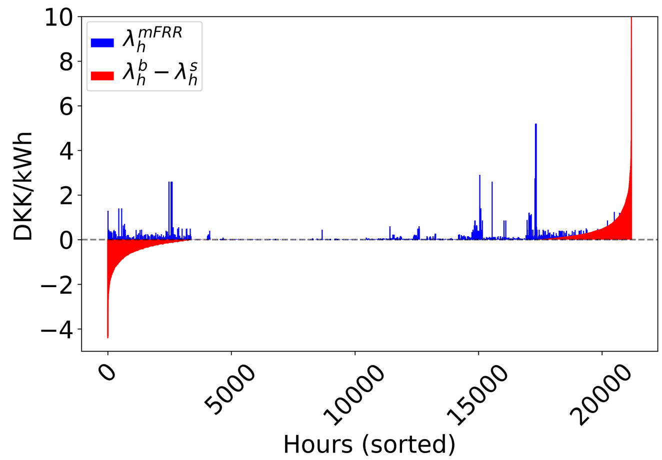

Fig. 11: Hourly mFRR prices and balancing price differentials in the time period of January 2021 to July 2023 in ascending order.

Online Appendix A: Market Prices

Fig. 10 shows the FCR (λ_h^{FCR}) and spot prices (λ_h^{s}) in DK1 in the period of January 2021 to July 2023. Notice how FCR prices went down after September 2022 where the Danish TSO opened up market participation from the entire continental Europe. Furthermore, FCR prices have been proportional to spot prices.

Fig. 11 shows the distribution of hourly balancing prices (λ_h^{b}) minus spot prices (referred to as balance price differentials) ordered from low to high in red. The mFRR prices ($\lambda_h^{\mathrm{mFRR}}$) in the same hours are shown in blue. When balance price differentials are below zero, down-regulation takes place, i.e., supply is greater than demand. When price differentials are above zero, up-regulation takes place, i.e., demand is higher than supply. The mFRR capacity is for up-regulation only, but flexible providers can both down- and up-regulate in the real-time balancing market if they choose. Here, however, we only consider the mFRR up-regulation capacity market where the zinc furnace is paid for both capacity as shown in blue in Fig. 11 and actual up-regulations as shown in red when the balance price differential is above zero.

Online Appendix B: Mixed-integer linear optimization for offering mFRR services

The mixed-integer linear optimization problem for the zinc furnace to optimally bid in the mFRR market reads as

$$\underset{\boldsymbol{p}^{\mathrm{r}},\boldsymbol{p}^{q,\mathrm{r}},\boldsymbol{\lambda}^{\mathrm{bid}},\boldsymbol{\Gamma}}{\text{Maximize}} \sum_{h=1}^{24} \lambda_h^{\mathrm{r}} p_h^{\mathrm{r}} + \Big(\sum_{h=1}^{24} \lambda_h^{\mathrm{b}} p_h^{\mathrm{b},\uparrow} - \sum_{h=1}^{24} \lambda_h^{\mathrm{b}} p_h^{\mathrm{b},\downarrow} - \sum_{h=1}^{24} \lambda^{\mathrm{p}} s_h\Big) \tag{9a}$$

$$\text{s.t. } (7),\ \forall h, \tag{9b}$$

$$p_h^{\mathrm{b},\downarrow} = p_h^{\mathrm{zu,b},\downarrow} + p_h^{\mathrm{zl,b},\downarrow},\ \forall h \tag{9c}$$

$$p_h^{\mathrm{b},\uparrow} = p_h^{\mathrm{zu,b},\uparrow} + p_h^{\mathrm{zl,b},\uparrow},\ \forall h \tag{9d}$$

$$T_{t+1}^{\mathrm{zu}} = T_t^{\mathrm{zu}} + dt \cdot \frac{1}{C^{\mathrm{zu}}}\Big(\frac{1}{R^{\mathrm{zuzl}}}(T_t^{\mathrm{zl}} - T_t^{\mathrm{zu}}) + \frac{1}{R^{\mathrm{wz}}}(T_t^{\mathrm{wu}} - T_t^{\mathrm{zu}})\Big), \quad \forall t < J-1 \tag{9e}$$

$$T_{t+1}^{\mathrm{zl}} = T_t^{\mathrm{zl}} + dt \cdot \frac{1}{C^{\mathrm{zl}}}\Big(\frac{1}{R^{\mathrm{zuzl}}}(T_t^{\mathrm{zu}} - T_t^{\mathrm{zl}}) + \frac{1}{R^{\mathrm{wz}}}(T_t^{\mathrm{wl}} - T_t^{\mathrm{zl}})\Big), \quad \forall t < J-1 \tag{9f}$$

$$T_{t+1}^{\mathrm{wu}} = T_t^{\mathrm{wu}} + dt \cdot \frac{1}{C^{\mathrm{wu}}}\Big((1-\mathbb{1}^{\mathrm{lid}})\frac{1}{R^{\mathrm{wua,off}}}(T^{\mathrm{a}} - T_t^{\mathrm{wu}}) + \mathbb{1}^{\mathrm{lid}}\frac{1}{R^{\mathrm{wua,on}}}(T^{\mathrm{a}} - T_t^{\mathrm{wu}}) + \frac{1}{R^{\mathrm{ww}}}(T_t^{\mathrm{wl}} - T_t^{\mathrm{wu}}) + \frac{1}{R^{\mathrm{wz}}}(T_t^{\mathrm{zu}} - T_t^{\mathrm{wu}}) + p_t^{\mathrm{u}}\Big), \quad \forall t < J-1 \tag{9g}$$

$$T_{t+1}^{\mathrm{wl}} = T_t^{\mathrm{wl}} + dt \cdot \frac{1}{C^{\mathrm{wl}}}\Big(\frac{1}{R^{\mathrm{wla}}}(T^{\mathrm{a}} - T_t^{\mathrm{wl}}) + \frac{1}{R^{\mathrm{ww}}}(T_t^{\mathrm{wu}} - T_t^{\mathrm{wl}}) + \frac{1}{R^{\mathrm{wz}}}(T_t^{\mathrm{zl}} - T_t^{\mathrm{wl}}) + p_t^{\mathrm{l}}\Big), \quad \forall t < J-1 \tag{9h}$$

$$T_{t+1}^{\mathrm{zu,\,Base}} = T_t^{\mathrm{zu,\,Base}} + dt \cdot \frac{1}{C^{\mathrm{zu}}}\Big(\frac{1}{R^{\mathrm{zuzl}}}(T_t^{\mathrm{zl,\,Base}} - T_t^{\mathrm{zu,\,Base}}) + \frac{1}{R^{\mathrm{wz}}}(T_t^{\mathrm{wu,\,Base}} - T_t^{\mathrm{zu,\,Base}})\Big), \quad \forall t < J-1 \tag{9i}$$

$$T_{t+1}^{\mathrm{zl,\,Base}} = T_t^{\mathrm{zl,\,Base}} + dt \cdot \frac{1}{C^{\mathrm{zl}}}\Big(\frac{1}{R^{\mathrm{zuzl}}}(T_t^{\mathrm{zu,\,Base}} - T_t^{\mathrm{zl,\,Base}}) + \frac{1}{R^{\mathrm{wz}}}(T_t^{\mathrm{wl,\,Base}} - T_t^{\mathrm{zl,\,Base}})\Big), \quad \forall t < J-1 \tag{9j}$$

$$T_{t+1}^{\mathrm{wu,\,Base}} = T_t^{\mathrm{wu,\,Base}} + dt \cdot \frac{1}{C^{\mathrm{wu}}}\Big((1-\mathbb{1}^{\mathrm{lid}})\frac{1}{R^{\mathrm{wua,off}}}(T^{\mathrm{a}} - T_t^{\mathrm{wu,\,Base}}) + \mathbb{1}^{\mathrm{lid}}\frac{1}{R^{\mathrm{wua,on}}}(T^{\mathrm{a}} - T_t^{\mathrm{wu,\,Base}}) + \frac{1}{R^{\mathrm{ww}}}(T_t^{\mathrm{wl,\,Base}} - T_t^{\mathrm{wu,\,Base}}) + \frac{1}{R^{\mathrm{wz}}}(T_t^{\mathrm{zu,\,Base}} - T_t^{\mathrm{wu,\,Base}}) + p_t^{\mathrm{u,\,Base}}\Big), \quad \forall t < J-1 \tag{9k}$$

$$T_{t+1}^{\mathrm{wl,\,Base}} = T_t^{\mathrm{wl,\,Base}} + dt \cdot \frac{1}{C^{\mathrm{wl}}}\Big(\frac{1}{R^{\mathrm{wla}}}(T^{\mathrm{a}} - T_t^{\mathrm{wl,\,Base}}) + \frac{1}{R^{\mathrm{ww}}}(T_t^{\mathrm{wu,\,Base}} - T_t^{\mathrm{wl,\,Base}})$$

$$+\frac{1}{R^{\mathrm{wz}}}(T_t^{\mathrm{zl,\ Base}}-T_t^{\mathrm{wl,\ Base}})+p_t^{\mathrm{l,\ Base}}\Big),\quad \forall t<J-1 \tag{9l}$$

$$\lambda_h^{\mathrm{bid}}-M(1-g_h)\leq\lambda_h^{\mathrm{b}}-\lambda_h^{\mathrm{s}}\leq\lambda_h^{\mathrm{bid}}+Mg_h,\ \forall h \tag{9m}$$

$$p_h^{\mathrm{b},\uparrow}\leq\phi_h\mathbb{1}^{\lambda_h^{\mathrm{b}}>\lambda_h^{\mathrm{s}}},\ \forall h \tag{9n}$$

$$p_h^{\mathrm{b},\uparrow}+s_h\geq\phi_h\mathbb{1}^{\lambda_h^{\mathrm{b}}>\lambda_h^{\mathrm{s}}},\ \forall h \tag{9o}$$

$$-g_hM\leq\phi_h\leq g_hM,\ \forall h \tag{9p}$$

$$-(1-g_h)M\leq\phi_h-p_h^{\mathrm{r},\uparrow}\leq(1-g_h)M,\forall h \tag{9q}$$

$$p_h^{\mathrm{q}}=P_h^{\mathrm{q,Base}}-p_h^{\mathrm{q,b},\uparrow}+p_h^{\mathrm{q,b},\downarrow},\ \forall h,q \tag{9r}$$

$$p_h^{\mathrm{q,b},\uparrow}\leq p_h^{\mathrm{r}}\mathbb{1}_h^{\lambda_h^{\mathrm{b}}>\lambda_h^{\mathrm{s}}},\ \forall h,q \tag{9s}$$

$$p_h^{\mathrm{q,b},\uparrow}\leq u_h^{\mathrm{q},\uparrow}\big(P_h^{\mathrm{q,Base}}-P^{\mathrm{q,Min}}\big),\ \forall h,q \tag{9t}$$

$$p_h^{\mathrm{q,b},\downarrow}\leq u_h^{\mathrm{q},\downarrow}\big(P^{\mathrm{q,Nom}}-P_h^{\mathrm{q,Base}}\big),\ \forall h,q \tag{9u}$$

$$P^{\mathrm{q,Min}}\leq p_h^{\mathrm{q}}\leq P^{\mathrm{q,Nom}},\ \forall h,q \tag{9v}$$

$$0\leq s_h^{\mathrm{q}}\leq P_h^{\mathrm{q,Base}},\ \forall h,q \tag{9w}$$

$$p_h^{\mathrm{q,b},\downarrow}\geq 0.10\ u_h^{\mathrm{q},\downarrow}\big(P^{\mathrm{q,Nom}}-P_h^{\mathrm{q,Base}}\big),\ \forall h,q \tag{9x}$$

$$u_{h-1}^{\mathrm{q},\uparrow}-u_h^{\mathrm{q},\uparrow}+y_h^{\mathrm{q},\uparrow}-z_h^{\mathrm{q},\uparrow}=0,\ \forall h>1,q, \tag{9y}$$

$$y_h^{\mathrm{q},\uparrow}+z_h^{\mathrm{q},\uparrow}\leq 1\ \forall h,q \tag{9z}$$

$$u_{h-1}^{\mathrm{q},\downarrow}-u_h^{\mathrm{q},\downarrow}+y_h^{\mathrm{q},\downarrow}-z_h^{\mathrm{q},\downarrow}=0,\ \forall h>1,q, \tag{9aa}$$

$$y_h^{\mathrm{q},\downarrow}+z_h^{\mathrm{q},\downarrow}\leq 1\ \forall h,q \tag{9ab}$$

$$u_h^{\mathrm{q},\uparrow}+u_h^{\mathrm{q},\downarrow}\leq 1\ \forall h,q \tag{9ac}$$

$$y_h^{\mathrm{q},\uparrow}+y_h^{\mathrm{q},\downarrow}\leq 1\ \forall h,q \tag{9ad}$$

$$z_h^{\mathrm{q},\uparrow}+z_h^{\downarrow}\leq 1\ \forall h,q \tag{9ae}$$

$$y_h^{\mathrm{q},\downarrow}\geq z_h^{\mathrm{q},\uparrow},\ \forall h,q \tag{9af}$$

$$\sum_{t=4(h-1)}^{4h}T_t^{\mathrm{q}}-T_t^{\mathrm{q,Base}}\geq\big(z_h^{\mathrm{q},\downarrow}-1\big)M,\ \forall h>1,q \tag{9ag}$$

$$\sum_{k=1}^{h}y_k^{\mathrm{q},\downarrow}\leq y_k^{\mathrm{q},\uparrow},\ \forall h,q. \tag{9ah}$$

The objective function (9a) maximizes the expected flexibility value of the zinc furnace.

Constraints (9b) represent the sum of zone power variables, i.e., total power, balancing power, and slack variable. The slack variable s_h represents power not delivered as promised. Constraints (9c)-(9d) represent sum of total balancing power as the sum of balancing power in both zones.

Aligned with (1), constraints (9e)-(9h) are the state-space model for the zinc and furnace wall temperature dynamics. Similarly, (9i)-(9l) include the baseline temperatures for the zinc and furnace wall, and model temperature dynamics for the baseline power. Recall in case the hour index h runs from 1 to 24, index t runs from 1 to $J=1440$.

Constraint (9m)-(9q) represent the McCormick relaxation of activation conditions for mFRR, i.e., that the zinc furnace should activate its capacity whenever there is a positive reserve bid, $p_h^r>0$, and when the regulation power bid is below the balancing price, $\lambda_h^{\mathrm{bid}}<\lambda_h^{\mathrm{b}}$, given that the balancing price is strictly greater than the spot price, $\lambda_h^{\mathrm{b}}>\lambda_h^{\mathrm{s}}$, i.e., whenever the system required balancing. See more details in [7].

Constraint (9r) sets the real-time power consumption p_h^{q} for both zones equal to the baseline power $P_h^{\mathrm{q,Base}}$ unless there is up-regulation $p_h^{\mathrm{q,b},\uparrow}$ or down-regulation $p_h^{\mathrm{q,b},\downarrow}$. Constraint (9s) ensures that up-regulation is zero when there is no need for up-regulation, and at the same time binds it to the reservation power. Constraint (9t) includes the binary variable $u_h^{\mathrm{q},\uparrow}$, indicating whether the zinc furnace is up-regulated in hour h. This constraint ensures that up-regulation is zero whenever $u_h^{\mathrm{q},\uparrow}=0$, and otherwise restricted to the maximum up-regulation service $P_h^{\mathrm{q,Base}}-P^{\mathrm{q,Min}}$ that can be provided. Note that $P^{\mathrm{q,Min}}$ is the minimum consumption level of the zinc furnace for zone q. Constraint (9u) works similarly for down-regulation. Note that the binary variable $u_h^{\mathrm{q},\downarrow}$ indicates whether down-regulation happens, whereas P^{qNom} is the nominal (maximum) consumption level of the zinc furnace. Constraint (9v) restricts the power consumption to lie within the minimum and nominal rates for each zone. Constraint (9w) binds the slack variable s_h^{q}, representing the service not delivered as promised. Constraint (9x) ensures that down-regulation is equal to at least 10% of the down-regulation capacity. Constraints (9y)-(9ae) define auxiliary binary variables $y_h^{\mathrm{q},\uparrow}$, $y_h^{\mathrm{q},\downarrow}$, $z_h^{\mathrm{q},\uparrow}$, and $z_h^{\mathrm{q},\downarrow}$, identifying transitions from/to up-regulation and down-regulation. During all hours with up-regulation, $y_h^{\mathrm{q},\uparrow}=1$. In the hour that up-regulation is stopped, $z_h^{\mathrm{q},\uparrow}$ is 1. There is a similar definition for $y_h^{\mathrm{q},\downarrow}$ and $z_h^{\mathrm{q},\downarrow}$ related to down-regulation. Constraints (9af)-(9ag) control the rebound behavior such that the rebound finishes when the temperature is below the baseline temperature. Note that M is a sufficiently big positive constant such that the zinc temperature is allowed to deviate from the baseline. Also, they ensure that the rebound happens right after up-regulation. Lastly, (9ah) ensures that up-regulation happens first. This makes sense since it impossible (or at least difficult) to anticipate potential up-regulation events in the power system. As such, it does not make sense to pre-cool (or pre-heat) a TCL in the context of mFRR.