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1711
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$N = 8+6 = 14$

1. $A(-1, 0, 1)$

$B(0, 1, -1)$

$C(1, 0, -1)$

a) $\overline{AB}(x_B - x_A, y_B - y_A, z_B - z_A)$

$\overline{AB}(0+1, 1-0, -1-1)$

$\overline{AB}(1, 1, -2)$

$\overline{AC}(x_C - x_A, y_C - y_A, z_C - z_A)$

$\overline{AC}(1+1, 0-0, -1, -1)$

$\overline{AC}(2, 0, -2)$

$\overline{BC}(x_C - x_B, y_C - y_B, z_C - z_B)$

$\overline{BC}(1-0, 0-1, -1+1)$

$\overline{BC}(1, -1, 0)$

$AB = |\overline{AB}| = \sqrt{1^2+2^2+(-2)^2} = \sqrt{1+1+4} = \sqrt{6}$

$BC = |\overline{BC}| = \sqrt{1^2+(-1)^2+0^2} = \sqrt{1+1} = \sqrt{2}$

$AC = |\overline{AC}| = \sqrt{2^2+0^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$\overline{AB}, \overline{AC}$ coliniari $\Leftrightarrow \frac{1}{2} = \frac{1}{0} = \frac{-2}{-2}$ FALS $\Rightarrow$

$\Rightarrow \overline{AB}, \overline{AC}$ nu sunt coliniari $\Rightarrow$ punctele A, B, C formează un triunghi

$$\cos A = \frac{\overline{AB} \cdot \overline{AC}}{|\overline{AB}| \, |\overline{AC}|} = \frac{1 \cdot 2 + 1 \cdot 0 + (-2)(-2)}{\sqrt{6} \cdot 2\sqrt{2}} = \frac{2+4}{4\sqrt{3}} =$$

$$= \frac{\sqrt{3}/3}{2\sqrt{3}} = \frac{3\sqrt{3}}{2 \cdot 3} = \frac{\sqrt{3}}{2}$$

$$A = \arccos \frac{\sqrt{3}}{2}$$

$$A = \frac{\pi}{6}$$

$$m(\sphericalangle A) = 30^\circ$$

b) $$S_{ABC} = \frac{|\overline{AB} \times \overline{AC}|}{2} = \frac{|\overline{AB}| |\overline{AC}| \cdot \sin A}{2} =$$

$$= \frac{2\sqrt{2} \cdot \sqrt{6} \cdot \frac{1}{\cancel{2}}}{\cancel{2}} = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$\sin A = \sin \frac{\pi}{6} = \frac{1}{2}$$

c) $AB: \dfrac{x - x_A}{x_B - x_A} = \dfrac{y - y_A}{y_B - y_A} = \dfrac{z - z_A}{z_B - z_A}$

$$AB: \frac{x+1}{0+1} = \frac{y+0}{1-0} = \frac{z-1}{-1-1}$$

$$AB: \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{-2}$$

$$(ABC): \begin{vmatrix} x - x_A & y - y_A & z - z_A \\ x_B - x_A & y_B - y_A & z_B - z_A \\ x_C - x_A & y_C - y_A & z_C - z_A \end{vmatrix} = 0 \iff$$

$$\iff \begin{vmatrix} x+1 & y-0 & z-1 \\ 0+1 & 1-0 & -1-1 \\ 1+1 & 0-0 & -1-1 \end{vmatrix} = 0 \iff \begin{vmatrix} x+1 & y & z-1 \\ 1 & 1 & -2 \\ 2 & 0 & -2 \end{vmatrix} = 0 \iff$$

$$\iff -2(x+1) - 4y - 2(z-1) + 2y = 0$$

$$-2x \cancel{-2} - 2y - 2z \cancel{+2} = 0$$

$$-2(x+y+z) = 0$$

$x + y + z = 0$ ecuația planului (ABC)

2. a) $y' = x^{14} - \sin x + e^x + \dfrac{1}{x^2 - 14^2} + 1$

$$y = \int \left(x^{14} - \sin x + e^x + \frac{1}{x^2 - 14^2} + 1 \right) dx$$

$$= \frac{x^{15}}{15} - (-\cos x) + e^x + \frac{1}{2 \cdot 14} \ln \left| \frac{x - 14}{x + 14} \right| + x + C$$

$$= \frac{x^{15}}{15} + \cos x + e^x + \frac{1}{28} \ln \left| \frac{x - 14}{x + 14} \right| + x + C$$

b) $(x + 14) y' = y - 14$

$$y' = \frac{y - 14}{x + 14}$$

$$\frac{dy}{dx} = \frac{y - 14}{x + 14}$$

$$\frac{dy}{y - 14} = \frac{dx}{x + 14}$$

$$\int \frac{dy}{y - 14} = \int \frac{dx}{x + 14}$$

$$\ln |y - 14| = \ln |x + 14| + \ln C$$

$$y - 14 = C(x + 14)$$

$$y = C(x + 14) + 14$$

c) $14 \cdot y' + \frac{1}{x} y + 2x^2 \cdot y^{15} = 0$

$$14 y' + \frac{1}{x} y = -2x^2 y^{15} \quad \Big| \cdot \frac{1}{y^{15}}$$

$$14 \cdot \frac{y'}{y^{15}} + \frac{1}{x} \cdot \frac{1}{y^{14}} = -2x^2$$

$$z = y^{1-15} = y^{-14} \Rightarrow z' = -14 \cdot y^{-15} \cdot y'$$

$$-z' = 14 y^{-15} \cdot y'$$

$$-z' + \frac{1}{x} \cdot z = -2x^2$$

$$-z' + \frac{1}{x} z = 0$$

$$-z' = -\frac{z}{x}$$

$$\frac{dz}{dx} = \frac{z}{x}$$

$$\frac{dz}{z} = \frac{dx}{x}$$

$$\int \frac{dz}{z} = \int \frac{dx}{x}$$

$$\ln|z| = \ln|x| + c$$

$$z_0 = c \cdot x$$

$$z_p = c|x| \cdot x$$

$$-(C(x)\cdot x)' + \frac{1}{x}\, c(x)\, x = -2x^2$$

$$-C'(x)\cdot x - \cancel{C(x)} + \cancel{c(x)} = -2x^2$$

$$C'(x) = +2x$$

$$\int C'(x)\,dx = \int +2x\,dx$$

$$C(x) = +2\cdot\frac{x^2}{2} = +x^2$$

$$z_p = +x^2\cdot x = +x^3$$

$$z = z_0 + z_p = C\cdot x + x^3$$

$$y^{-14} = C\cdot x + x^3$$

d) $$y'' - 3y' - 10y = -10\cdot 14x^2 - 6\cdot 14x + 2\cdot 14$$

$$y'' - 3y' - 10y = -140x^2 - 84x + 28$$

$$y'' - 3y' - 10y = 0$$

$$\lambda^2 - 3\lambda - 10 = 0$$

$$\Delta = 9 + 40 = 49$$

$$\lambda_1 = \frac{3-7}{2} = -\frac{4}{2} = -2 \Rightarrow y_1 = e^{-2x}$$

$$\lambda_2 = \frac{3+7}{2} = \frac{10}{2} = 5 \Rightarrow y_2 = e^{5x}$$

$$y_0 = C_1 e^{-2x} + C_2 e^{5x}$$

$$f(x) = -140x^2 - 84x + 28$$

$$\Rightarrow y_0 = Ax^2 + Bx + C$$

$$y_p' = 2Ax + B$$

$$y_p'' = 2A$$

$$y_p'' - 3y_p' - 10y_p = -140x^2 - 84x + 28$$

$$2A - 3(2Ax + B) - 10(Ax^2 + Bx + C) = -140x^2 - 84x + 28$$

$$2A - \underline{6Ax} - 3B - 10Ax^2 \underline{- 10Bx} - 10C = -140x^2 - 84x + 28$$

$$\begin{cases} -10A = -140 \quad \Rightarrow \boxed{A = 14} \\ -6A - 10B = -84 \\ 2A - 3B - 10C = 28 \end{cases}$$

$$-6 \cdot 14 - 10B = -84 \qquad 2 \cdot 14 - 10C = 28$$

$$-84 - 10B = -84 \qquad 28 - 10C = 28$$

$$\boxed{B = 0} \qquad \boxed{C = 0}$$

$$y_p = 14x^2$$

$$y = y_0 + y_p = C_1 e^{-2x} + C_2 \cdot e^{5x} + 14x^2$$