



Estimation and Learning in Aerospace

2022/2023 exam project

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Project tasks

The project deals with the dynamics of a multirotor UAV and involves the following two tasks.

Task I

- System identification of longitudinal dynamics from simulated data for a given input

Task II

- Optimisation of experiment design to maximise model accuracy

A small quadrotor



Weight: 270g

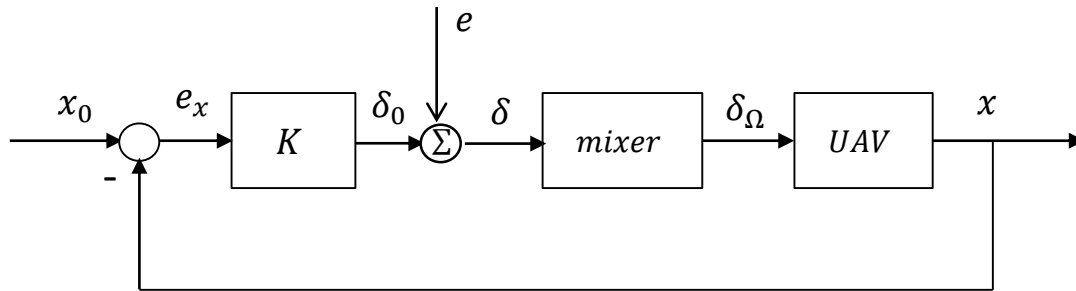
Dimensions: 20 x 20 x 4 cm

Diameter (motor-to-motor distance): 16cm

Hovering time: 7'30''



Identification experiments and I/O estimation data



Decoupled dynamics

➤ *Longitudinal dynamics*

- ❑ Input: δ_{lon} / Outputs: q, a_x
- ❑ Known time delay of 4 sampling intervals.

Linearised model for longitudinal dynamics

State equation

$$\begin{bmatrix} \dot{u} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_q & -g \\ M_u & M_q & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} X_\delta \\ M_\delta \\ 0 \end{bmatrix} \delta_{lon}$$

u Longitudinal (body) velocity [m/s]

q Pitch rate [rad/s]

θ Pitch angle [rad]

a_x Longitudinal (body) acceleration [m/s²]

δ_{lon} Pitch moment [normalised to -1, +1 range]

Output equation

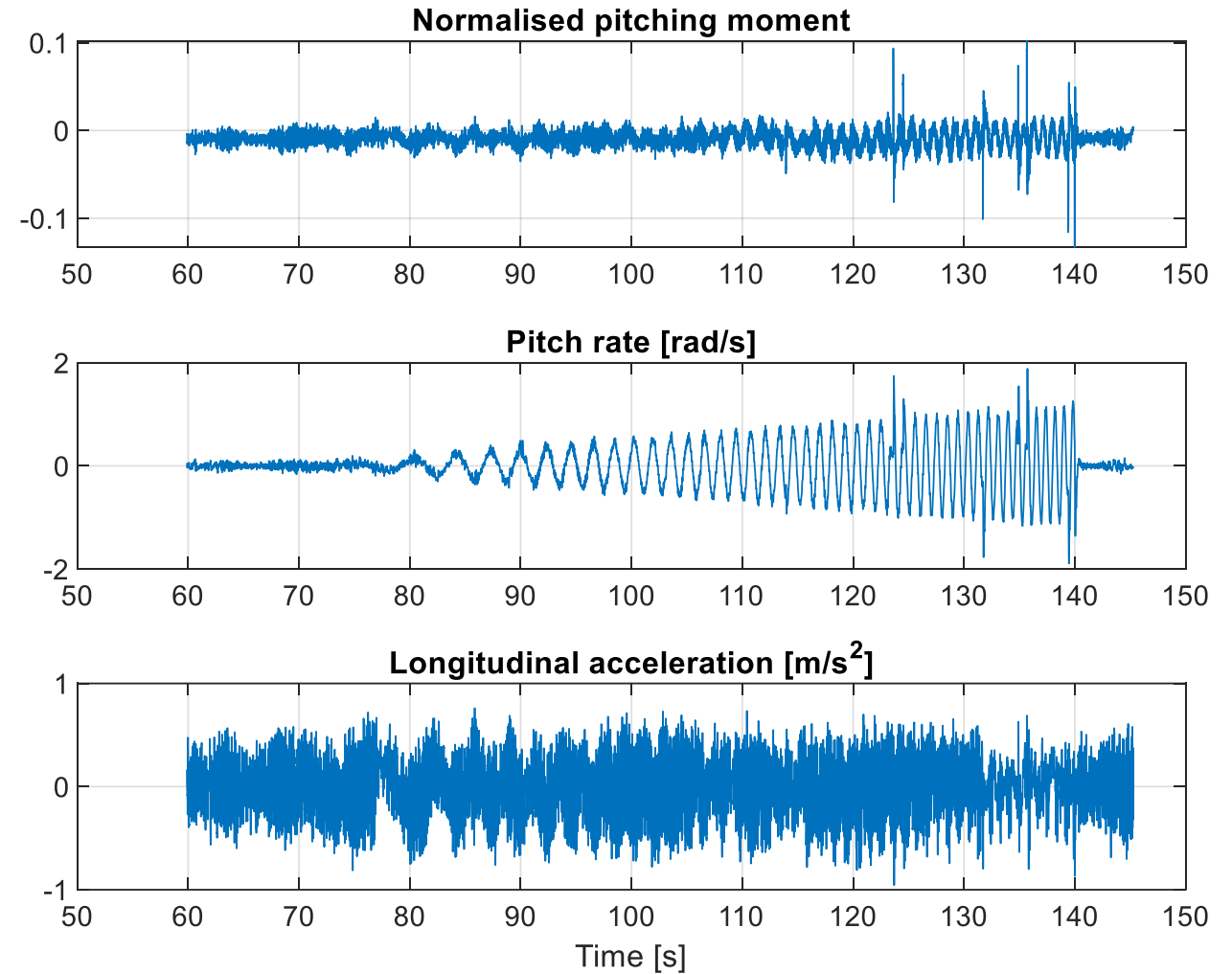
$$\begin{bmatrix} q \\ a_x \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ X_u & X_q & 0 \end{bmatrix} \begin{bmatrix} u \\ q \\ \phi \end{bmatrix} + \begin{bmatrix} 0 \\ X_\delta \end{bmatrix} \delta_{lon}$$

Model parameters: $\Theta = \begin{bmatrix} X_u \\ X_q \\ M_u \\ M_q \\ X_\delta \\ M_\delta \end{bmatrix}$

+ delay (computational time, sensor readings, ...)

Longitudinal dynamics: excitation signal

- Experiments are carried out under position and attitude feedback, in order to guarantee safe operation.
- Baseline excitation input is a long duration sweep: excite low-frequency translational dynamics



Task 1: model identification

Use the simulator and the provided baseline excitation to:

- Identify a grey-box model for the longitudinal dynamics of the multirotor (input: **total pitching moment**; outputs: **pitch rate** and **longitudinal acceleration**) using the **response** to the **sweep** provided in the ExcitationM.mat file
- **Assess the uncertainty** of the identified model

Check if uncertainty measure through the Fisher Matrix can be computed from the Sensitivity.

Note that the open loop dynamics is *unstable*

Task 2: optimisation of experiment design

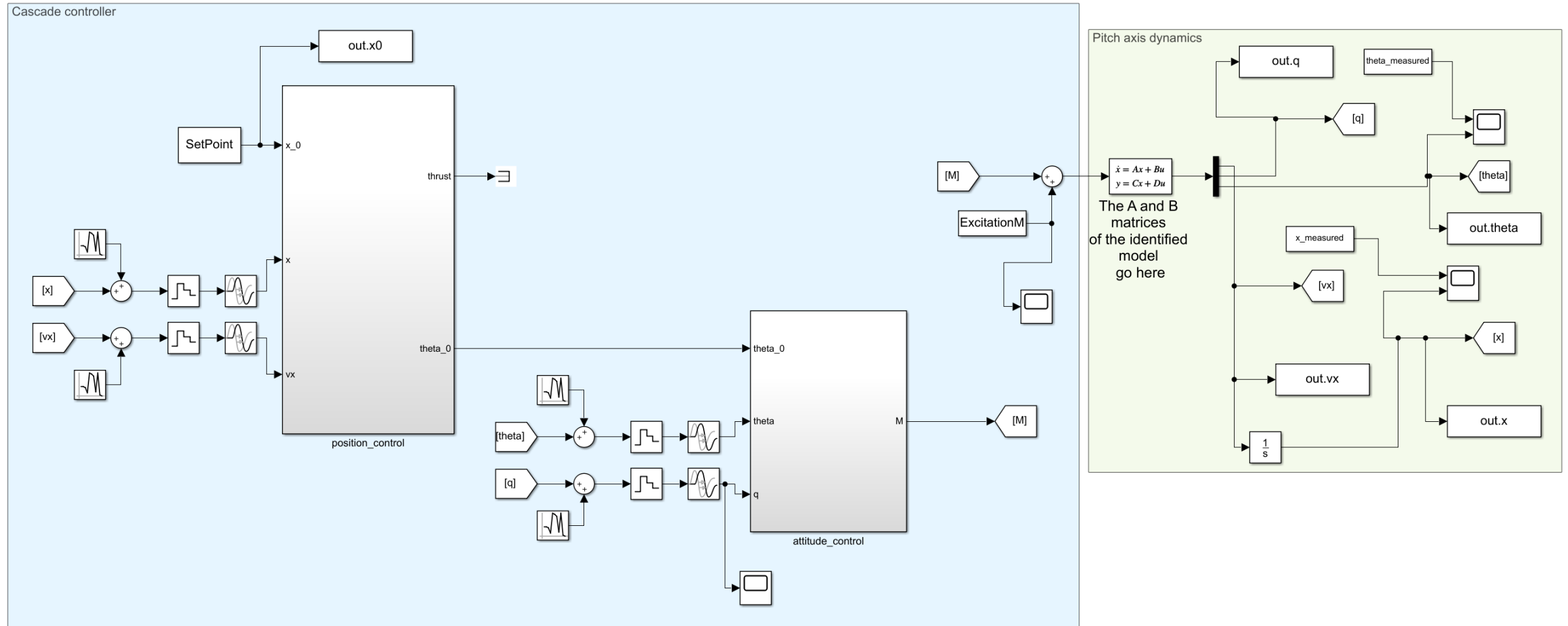
- The identified model can be used to re-design the experiment so as to optimise the accuracy of the model
- Therefore the second task of the project consists in
 - Defining and parameterising a class of inputs for the experiment (see also the attached document)
 - Defining a performance metric for model quality
 - Optimising the input class to maximise the quality of the obtainable model.
 - Evaluating the quality of the optimal obtainable model.

An initial identified model is provided to be able to run the simulator, the parameters are given in the script `main_quad_ANTX.m` which runs the simulator

The simulator is representative of the local linear closed-loop dynamics of the drone and combines the identified model with a complete model of the full attitude and position control system.

It includes the following files:

- `main_quad_ANTX.m` (which calls `parameters_controller.m`): set up the parameters and run the simulation
- `ExcitationM.mat` contains the vectors of time and normalised pitching moment corresponding to the baseline excitation
- `Simulator_Single_Axis.slx` the Simulink model corresponding to the closed-loop longitudinal dynamics



- Model identified using data generated with the provided excitation input:
 - Estimated parameters
 - Associated uncertainties
- Formulation of the experiment design problem
- Implementation of the solution
- Results:
 - Optimised excitation
 - Identified model (estimated parameters and uncertainties)