LEVEL H

H 1-20 : Review up to G

The review includes the four operations of positive and negative numbers/ fractions, simplifying algebraic expressions, solving linear equations.

H 21-40 : Literal Equations

Literal equations are very similar to linear equations, except that numbers are replaced by letters. Note that all algebraic rules are preserved.

Solve the following equations for x.

$$a = b - x \qquad \boxed{x = -a + b}$$

$$a = x - 3 \qquad \boxed{\begin{aligned} a + 3 &= x \\ x &= a + 3 \end{aligned}}$$

$$ax - c = b \qquad \boxed{\begin{aligned} ax &= b + c \\ x &= \frac{b+c}{a} \end{aligned}}$$

Reduce the answers when necessary.

$$(a+b)x = b + a \qquad \boxed{\begin{aligned} x &= \frac{b+a}{a+b} \\ &= \frac{a+b}{a+b} \\ &= 1 \end{aligned}}$$

$$(a-b)x = -a + b \qquad \boxed{\begin{aligned} x &= \frac{-a+b}{a-b} \\ &= \frac{-(a-b)}{(a-b)} \\ &= -1 \end{aligned}}$$

Collect all terms with x on the LHS, "take out" x

$$ax = b + cx \qquad \boxed{\begin{aligned} ax - cx &= b \\ (a-c)x &= b \\ x &= \frac{b}{a-c} \end{aligned}}$$

$$-ax = b + x \qquad \boxed{\begin{aligned} -ax - x &= b \\ (-a-1)x &= b \\ x &= \frac{b}{-a-1} \quad \left[-\frac{b}{a+1}\right] \end{aligned}}$$

When there are fractions, multiply both sides of the equation by the LCM of the denominator(s).

$$\frac{x}{a} - \frac{x}{b} = c \qquad \boxed{\begin{aligned} \left(\frac{x}{a} - \frac{x}{b}\right) \times ab &= c \times ab \\ bx - ax &= abc \\ (b-a)x &= abc \\ x &= \frac{abc}{-a+b} \end{aligned}}$$

$$\frac{x}{a} = b + 3x \qquad \boxed{\begin{aligned} \frac{x}{a} \times a &= (b+3x) \times a \\ x &= ab + 3ax \\ x - 3ax &= ab \\ (1-3a)x &= ab \\ x &= \frac{ab}{-3a+1} \end{aligned}}$$

Note: Have your instructor carefully guide you through the word problems on H36-40.

H 41-90 : Simultaneous Equations in 2 Variables

Elimination Method

1) Decide which variable (x or y) to eliminate first.

2) Multiply the equations so that the variable has the same coefficient in both equations.

3) Add/ subtract both equations to completely remove the chosen variable.

4) Solve for the remaining variable, and subsequently the variable that was eliminated.

$$\begin{cases} x - 2 = 3(y+2) & \cdots ① \\ 2y + 5 = x - 2(y-1) & \cdots ② \end{cases}$$

① becomes $\quad x - 3y = 8 \quad \cdots ①'$

② becomes $\quad -x + 4y = -3 \quad \cdots ②'$

①′ + ②′ $\quad : \quad y = 5$

Substituting $y = 5$ into equation ①′,

$$x = 8 + 3(5) = 23$$

$$\begin{cases} \dfrac{x-3}{5} = \dfrac{y-7}{2} & \cdots ① \\ 7x = 3y & \cdots ② \end{cases}$$

① × 10 $\quad : 2x - 6 = 5y - 35$

$\qquad 2x - 5y = -29 \quad \cdots ①'$

② becomes $7x - 3y = 0 \quad \cdots ②'$

①′ × 3 $\quad : 6x - 15y = -87 \quad \cdots ③$

②′ × 5 $\quad : 35x - 15y = 0 \quad \cdots ④$

④ − ③ $\quad : 29x = 87$

$\qquad x = 3$

Substituting $x = 3$ into equation ②,

$$\begin{aligned} 3y &= 7(3) \\ y &= 7 \end{aligned}$$

Substitution Method

Express one variable (explicitly) in terms of another. Substitute into the other equation and solve.

$$\begin{cases} y = -2x + 3 & \cdots ① \\ 2x - 3y = 7 & \cdots ② \end{cases}$$

Substituting ① into ②,

$$\begin{aligned} 2x - 3(-2x+3) &= 7 \\ 2x + 6x - 9 &= 7 \\ 8x &= 16 \\ x &= 2 \end{aligned}$$

Substituting $x = 2$ into equation ①,

$$\begin{aligned} y &= -2(2) + 3 \\ &= -1 \end{aligned}$$

H 91-110 : Simultaneous Equations in > 3 Variables

To solve simultaneous equations in 3 variables, first eliminate one of the variables to give two equations in 2 variables, and then solve as H41-90. *The same concept applies to equations in 4 variables.*

$$\begin{cases} x+y+z=13 & \cdots ① \\ x+2y-z=7 & \cdots ② \\ 3x-y+z=23 & \cdots ③ \end{cases}$$

$$\begin{array}{l} ①+②: \\ ②+③: \end{array} \begin{cases} 2x+3y=20 & \cdots ④ \\ 4x+y=30 & \cdots ⑤ \end{cases}$$

$$④\times 2 \quad : 4x+6y=40 \cdots ④'$$

$$⑤-④': \quad -5y=-10$$

$$y=2$$

Substituting $y=2$ into equation ④,

$$2x+6=20$$

$$2x=14$$

$$x=7$$

Substituting $x=7,\ y=2$ into equation ①,

$$7+2+z=13$$

$$z=4$$

H 111-120 : Applications of Equations

This particular set contains challenging word problems that require higher order thinking. Make sure to go through the problems carefully with a knowledgeable instructor.

H 121-140 : Inequalities

Inequalities can (almost) be treated the same way as linear equations. **Note:** When multiplying or dividing both sides of an inequality by a negative number, **reverse the inequality symbol**.

$$\begin{aligned} 4x-6 &> 7x+9 \\ 4x-7x &> 9+6 \\ -3x &> 15 \\ x &< -5 \end{aligned} \qquad \begin{aligned} 7x-3(2x+7) &\geq 2 \\ 7x-6x-21 &\geq 2 \\ x &\geq 23 \end{aligned}$$

When there are two (or more) inequalities, solve them graphically.

$$\begin{cases} x > -2 \cdots ① \\ x < 5 \quad \cdots ② \end{cases}$$

The interval of x which satisfies both inequalities is ▭.

Ans. $-2 < x < 5$

$$\begin{cases} x < -2 \cdots ① \\ x > 5 \quad \cdots ② \end{cases}$$

No value of x is common to both inequalities.

Ans. No solution

Example:

$$\begin{cases} 3x+4 < 5x+8 & \cdots ① \\ 4x-8 \leq x+1 & \cdots ② \end{cases}$$

① becomes $\quad -2x < 4$

$$x > -2$$

② becomes $\quad 3x \leq 9$

$$x \leq 3$$

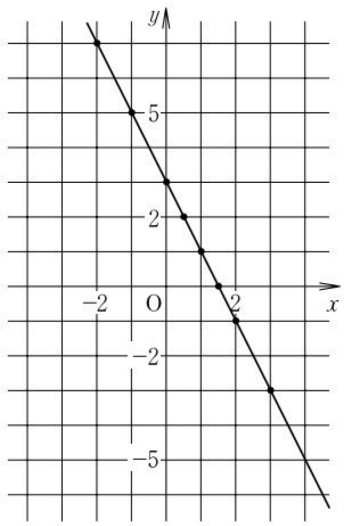
Ans. $-2 < x \leq 3$

H 141-180 : Functions & Graphs

When graphing a function, construct a table of points (x, y).

$y=-2x+3$

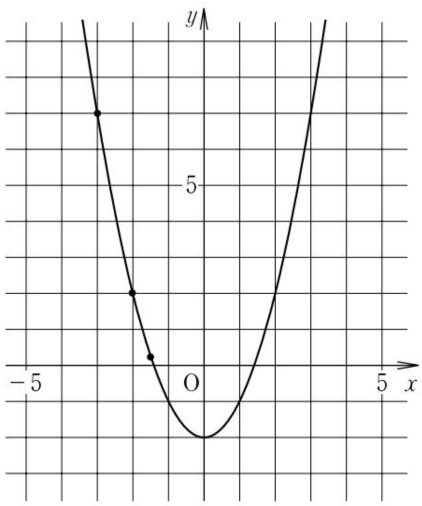
x	y
−2	7
−1	5
0	3
0.5	2
1	1
1.5	0
2	−1
3	−3



Note: The line above represents a **linear function**.

$y=x^2-2$

x	y
−3	7
−2	2
$-\frac{3}{2}$	$\frac{1}{4}$
−1	−1
0	−2
1	−1
$\frac{3}{2}$	$\frac{1}{4}$
2	2
3	7



Note: The curve represents a **quadratic function**.

Where the relation between x and y can be expressed as $y = mx + b$ ($m \neq 0$), the relation is called a **linear function**. The graph of a linear function is a straight line.

m = gradient , b = y-intercept

$y = 3x + 1$

Gradient: 3 y-intercept: 1

$y = -\frac{3}{2}x + 5$

Gradient: $-\frac{3}{2}$ y-intercept: 5

The gradient of a line that passes through two points (a, b) and (c, d) can be found as follows:

$$\text{Gradient} = \frac{d-b}{c-a}$$

How to determine the equation of a line

Method 1: First determine the gradient of the line, then substitute any point to find the y-intercept

Determine the equation of a line that passes through $(5, 3)$ and $(7, 6)$.

The gradient is $\frac{6-3}{7-5} = \frac{3}{2}$

Let $y = \frac{3}{2}x + b \quad \cdots ①$

Substituting $x = 5$ and $y = 3$ into ①,

$$3 = \frac{15}{2} + b$$

$$b = -\frac{9}{2}$$

Therefore $y = \frac{3}{2}x - \frac{9}{2}$

Method 2: Substitute the points into the equation of the line and solve simultaneous equations

Determine the equation of a line that passes through $(2, 3)$ and $(4, 7)$.

Let the equation of the line be $y = mx + b \quad \cdots ①$

Since the line passes through $(2, 3)$,

$$3 = 2m + b \quad \cdots ②$$

Since the line passes through $(4, 7)$,

$$7 = 4m + b \quad \cdots ③$$

Solving ② and ③, we find

$$m = 2 \quad \text{and} \quad b = -1$$

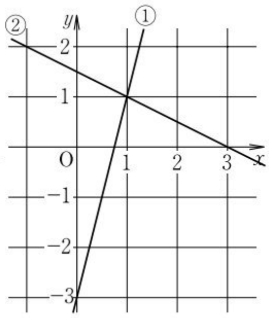
Substituting the values of m and b into ① ,

$$y = 2x - 1$$

The solution of the simultaneous equations corresponds to the point of intersection of the two lines whose equations are given.

① $y = 4x - 3$

② $y = -\frac{1}{2}x + \frac{3}{2}$



The point of intersection is $(1, 1)$

The equation $x = k$ is a line that passes through (k, 0) and is parallel to the y-axis.

The equation $y = m$ is a line that passes through (0, m) and is parallel to the x-axis.

Theorem: Two lines are parallel if and only if their gradients are equal.

H 181-200 : Simplifying Monomials & Polynomials

1) Determine the sign of the answer.

2) Calculate the numbers in front of the letters.

3) Calculate the powers of each variable.

$$(-2a^2b)(-3a^5b^3) = 6a^7b^4$$

$$5x(-yz^2)^3 = -5x \cdot y^3z^6 = -5xy^3z^6$$

$$4a^3(-2b^2c)^4 = 4a^3 \cdot 16b^8c^4 = 64a^3b^8c^4$$

$$-3a^2b^3\left(-\frac{2}{3}bc^2\right)^3 = 3a^2b^3 \cdot \frac{8}{27}b^3c^6 = \frac{8}{9}a^2b^6c^6$$

Reduce when dividing expressions.

$$12a^2x^5y^4 \div (3ax^2y)^2 = \frac{12a^2x^5y^4}{9a^2x^4y^2} = \frac{4xy^2}{3}$$

$$\frac{2}{3}ab^3 \div \left(-\frac{1}{6}ab\right)^2 = \frac{2}{3}ab^3 \div \left(\frac{1}{36}a^2b^2\right) = \frac{2ab^3}{3} \times \frac{36}{a^2b^2} = \frac{24b}{a}$$

The distributive property

$$3a(a-4b) = 3a^2 - 12ab$$

$$\begin{aligned} 2x(x+4) - 3x(2x-5) &= 2x^2 + 8x - 6x^2 + 15x \\ &= -4x^2 + 23x \end{aligned}$$

$$\begin{aligned} \frac{1}{2a}(8a^3 - 12a^2 + 6a) &= \frac{8a^3}{2a} - \frac{12a^2}{2a} + \frac{6a}{2a} \\ &= 4a^2 - 6a + 3 \end{aligned}$$

$$\begin{aligned} \frac{-10x^2y + 6x^2y^2 - 2xy}{-2xy} &= \frac{-10x^2y}{-2xy} + \frac{6x^2y^2}{-2xy} + \frac{-2xy}{-2xy} \\ &= 5x - 3xy + 1 \end{aligned}$$