Proof by Contradiction Example

- Proposition :
  $\sqrt{2}$ is irrational

- Proof :
  Proof is by contraction. Assume $\sqrt{2}$ is rational. Then
  $\sqrt{2} = \frac{p}{q}$,
  such that p and q are integers that have no common factors.

Proof by Contradiction Example

- Given $\sqrt{2} = \dfrac{p}{q}$,
  $p^2 = 2q^2$, and p is even. Then q is odd.
- Since p is even, $p = 2k$.
  So $p^2 = 4k^2 = 2q^2$ and $q^2 = 2k^2$. Then q is even.
- q cannot be both odd and even, therefore $\sqrt{2}$ cannot be rational