

# Probability vs Likelihood

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- Probability and likelihood are mathematically similar, but differ in the choice of free parameters:

- (Normal) Probability Density Function

$$f(y | \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y - \mu)^2}{2\sigma^2}}$$

- (Normal) Likelihood Function

$$\mathcal{L}(\mu, \sigma^2 | y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y - \mu)^2}{2\sigma^2}}$$

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- We we talk about p-values, we use the integral of the probability density function; with likelihood, we refer to the value of the probability function for a select value.

- (Normal) Cumulative Density Function

$$F_Y(y | \mu, \sigma^2) = \frac{1}{2} \left[ 1 + \operatorname{erf} \left( \frac{y - \mu}{\sqrt{2}\sigma} \right) \right] = P(Y \leq y)$$
$$\operatorname{erf}(y) = \frac{2}{\sqrt{\pi}} \int_0^y e^{-t^2} dt$$

- (Normal) Likelihood Function

$$\mathcal{L}(\mu, \sigma^2 | y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y - \mu)^2}{2\sigma^2}} = P_{\mu, \sigma^2}(Y = y)$$