Probability vs Likelihood

- We we talk about p-values, we use the integral of the probability density function; with likelihood, we refer to the value of the probability function for a select value.
  - (Normal) Cumulative Density Function

$$F_Y\left(y\,|\,\mu,\sigma^2\right) = \frac{1}{2}\left[1 + erf\left(\frac{y-\mu}{\sqrt{2}\sigma}\right)\right] = P(Y \le y)$$

$$erf\left(y\right) = \frac{2}{\sqrt{\pi}}\int_0^y e^{-t^2}dt$$

  - (Normal) Likelihood Function

$$\mathscr{L}\left(\mu,\sigma^2\,|\,y\right) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(y-\mu)^2}{2\sigma^2}} = P_{\mu,\sigma^2}(Y = y)$$

Probability vs Likelihood

- We can extend the likelihood concept from single to multiple observations.
    - Given the probability of a single observation,

$$f\left(y \mid \mu, \sigma^2\right) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\mu)^2}{2\sigma^2}}$$

    - we calculate likelihood from a series of observations

$$\mathscr{L}\left(\mu, \sigma^2 \mid y_1, \ldots, y_n\right) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y_i-\mu)^2}{2\sigma^2}} = \left(\sqrt{2\pi\sigma^2}\right)^{-n} \textbf{exp} \left\{ \frac{1}{-2\sigma^2} \sum \left(y_i - \mu\right)^2 \right\}$$