Probability vs Likelihood

- We can extend the likelihood concept from single to multiple observations.
  - Given the probability of a single observation,

$$f\left(y \mid \mu, \sigma^2\right) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\mu)^2}{2\sigma^2}}$$

  - we calculate likelihood from a series of observations

$$\mathscr{L}\left(\mu, \sigma^2 \mid y_1, \ldots, y_n\right) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y_i-\mu)^2}{2\sigma^2}} = \left(\sqrt{2\pi\sigma^2}\right)^{-n} \mathbf{exp}\left\{\frac{1}{-2\sigma^2} \sum \left(y_i - \mu\right)^2\right\}$$

Maximum Likelihood

- Find parameters such that the likelihood functions achieves a maximum value, e.g. the values
$$\widehat{\mu}_i = \frac{\sum_{j=1}^{n} y_{ij}}{N_i}, \widehat{\sigma}^2 = \frac{\sum_{j=1}^{n} (y_{ij} - \widehat{\mu}_i)^2}{n-1}$$
are maximum likelihood estimates for $\mathscr{L}\left(\mu, \sigma^2 \,|\, y\right)$
- The value of $\mathscr{L}\left(\ldots \,|\, y\right)$ with respect to the maximum likelihood estimates is a measure of the correctness of a statistical model for data y