## Maximum Likelihood

☐ Find parameters such that the likelihood functions achieves a maximum value, e.g. the values

$$\widehat{\mu}_{i} = \frac{\sum_{j=1}^{n} y_{ij}}{N_{i}}, \widehat{\sigma}^{2} = \frac{\sum_{j=1}^{n} (y_{ij} - \widehat{\mu}_{i})^{2}}{n-1}$$

are maximum likelihood estimates for  $\mathcal{L}\left(\mu, \sigma^2 \mid y\right)$ 

The value of  $\mathcal{L}(...|y)$  with respect to the maximum likelihood estimates is a measure of the correctness of a statistical model for data y

## Likelihood Ratio

Given likelihood estimates for two alternative hypothesis:

$$H_1 | \mu = \mu_1 : \mathcal{L}_1 (\mu_1, \sigma^2 | y_1, ..., y_n)$$

$$H_2 \mid \mu = \mu_2 : \mathcal{L}_1 (\mu_2, \sigma^2 \mid y_1, ..., y_n)$$

□ we can compute a relative likelihood

$$\frac{\mathscr{L}_{2}\left(\mu_{2},\sigma^{2}\mid y_{1},...,y_{n}\right)}{\sigma^{2}\left(\mu_{2},\sigma^{2}\mid y_{1},...,y_{n}\right)}$$

$$\mathcal{L}_1\left(\mu_1,\sigma^2\,|\,y_1,...,y_n\right)$$