Vergelijking Gauss-Legendre, Clenshaw-Curtis en Romberg integratie

Alexander Boucquey, Peter Coppens, Simon Dirckx, Cedric Picron

1 Exercise 1

Figure 1 contains a plot of all of the test functions. We calculate the solutions of the integral analytically as:

$$\frac{1}{21}x^{21}, \quad e^x, \quad \frac{1}{2}\sqrt{\pi}\mathrm{erf}(x), \tag{1}$$

$$\frac{1}{4}tan^{-1}(4x), \quad \sqrt{\pi}\mathrm{erf}(\frac{1}{x}) + e^{-\frac{1}{x^2}}x, \quad \frac{1}{4}x^4\mathrm{sign}(x) \tag{2}$$

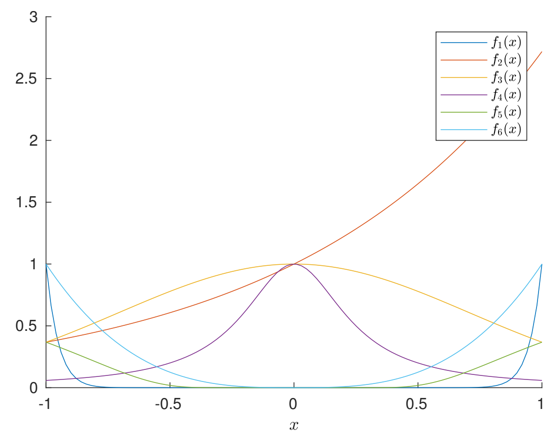

Figure 1: The functions that we will be integrating.

2 Exercise 2

We use the code provided in the paper as suggested. Calculating the analytical integral of $f_5(x)$ proved difficult, as the integral has a discontinuity in $x = 0$. This is solved using the fact that the function is even, so we integrate over $[0, 1]$ and multiply the solution by 2.

Figure 2 shows the relative error. We notice that it is very similar to the solutions in the paper.

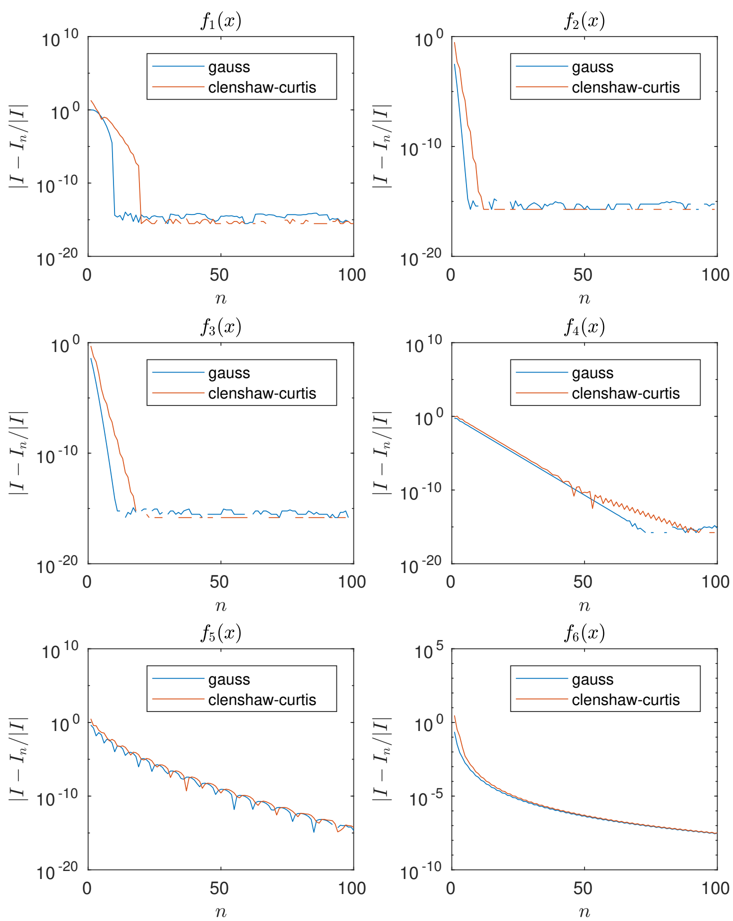

Figure 2: The relative error for each function and each method.

We also calculate the numerical cost to get to a precision of 7 digits.
f_1: cost – gauss: 10 – clenshaw-curtis: 18
f_2: cost – gauss: 4 – clenshaw-curtis: 6
f_3: cost – gauss: 6 – clenshaw-curtis: 9
f_4: cost – gauss: 33 – clenshaw-curtis: 35
f_5: cost – gauss: 34 – clenshaw-curtis: 31
f_6: cost – gauss: 73 – clenshaw-curtis: 75
We notice that for f_5 clenshaw-curtis requires less points. For everything else clenshaw-curtis requires some more points, but only for f_1 it needs about twice as many. As predicted by the paper.