Data Mining

Lecture 4

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Revising the Fundamental Concepts of Probability

Probability is the measure of the likelihood or chance that a particular event will occur. Probability is defined as a the numerical value that represents how likely an event is to occur, typically ranging from 0 (impossible) to 1(certain).

Random Experiment : Experiment (action) whose result is uncertain (cannot be predicted with certainty) before it is performed.

Trial : Single performance of the random experiment.

Outcome : Result of a trial.

Sample Space S : The set of all possible outcomes of a random experiment

Event : The subset of the sample space S (to which a probability can be assigned)

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Traditional definitions of the probability of an event A

Classical: P(A) $= \dfrac{\text{\# of possible outcomes for event } A}{\text{\# of possible outcomes for space } S}$

Relative Frequency: P(A) $= \lim_{N \to \infty} \dfrac{\text{\# of occurrences of event } A}{N \text{ (total \# of trials)}}$

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Example: Die-rolling events

Rolling a die is a random experiment. An outcome can be any number from 1 to 6.
Sample space = {1, 2, 3, 4, 5, 6}.
Some possible examples of events are

1. A = { an even number showing up} = {2, 4, 6} (3 outcomes)
2. B = { a number greater than 5 shows up} = {6} (single outcome)
3. C = { 2 shows up} = {2} (single outcome)
4. D = {a number greater than 7 shows up} = { } (no outcome)
5. E = {2 and 4 shows up} = { } (no outcome)
6. F = {2 or 4 shows up} = { 2, 4} (2 outcomes)
7. G = { a number from 1 to 6 shows up} = {1, 2, 3, 4, 5, 6} (all outcomes)

Thus if 2 showed up as a result of rolling a die, we say that the events A, C, F and G have all occurred.

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Let's work out their probabilities

1. A = { an even number showing up} = {2, 4, 6} (3 outcomes)
   P (an even number showing up) = 3 / 6 = 1 /2

2. B = { a number greater than 5 shows up} = {6} (single outcome)
   P(a number greater than 5 shows up) = 1 / 6

3. C = { 2 shows up} = {2} (single outcome)
   P (2 shows up) = 1 / 6

4. D = {a number greater than 7 shows up} = { } (no outcome)
   P (a number greater than 7 shows up) = 0 / 6 = 0
   Impossible events have 0 probability.

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Let's work out their probabilities

5. E = {2 and 4 shows up} = { } (no outcome)
   P (2 and 4 shows up) = 0 / 6 = 0
   Impossible events have 0 probability

6. F = {2 or 4 shows up} = { 2, 4} (2 outcomes)
   P (2 or 4 shows up} 2 / 6 = 1 / 3

7. G = { a number from 1 to 6 shows up} = {1, 2, 3, 4, 5, 6} (all outcomes)
   P (a number from 1 to 6 shows up) = 6 / 6 = 1
   Sure events have the probability 1.

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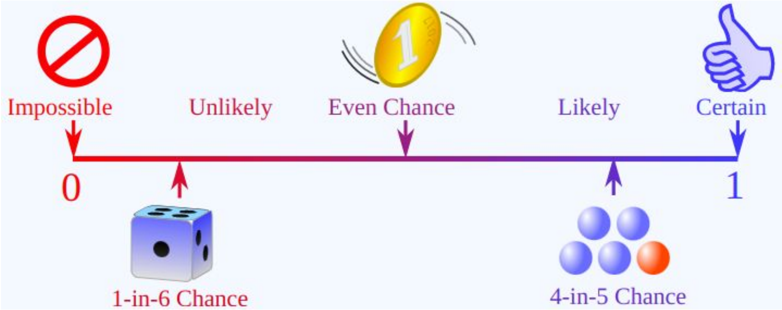
We can show probability on a probability line.



(image source: math is fun website)

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Example 2: Let’s consider another example. Two coins are tossed.
Let A be the event “two heads are obtained” and
B be the event “one head and one tail is obtained”.
Find P(A), P(B).

The sample space S = {HH, HT, TH, TT}

A = {HH}, hence P(A) = 1/4

B = {HT, TH}, hence P(B) = 2/4 = ½
Please notice that the outcomes HT and TH are different.

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Example 3: A marble is drawn at random from a bag containing 3 red, 2 blue, 5 green and 1 yellow marble. What is the probability that it is not green?

We can work out the probability that the marble is green: P (marble is green) = 5 / 11
P (marble is NOT green) = 1 - P (marble is green) = 1 - (5/11) = 6 / 11.

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Probability of the complement of an event. Let A be an event. The complement of the event is denoted as $\overline{A}$. The probability of the complement of an event A can be calculated in the following way

$$P(\overline{A}) = 1 - P(A)$$

Two events are complementary if they cannot occur at the same time and they make up the whole sample space.

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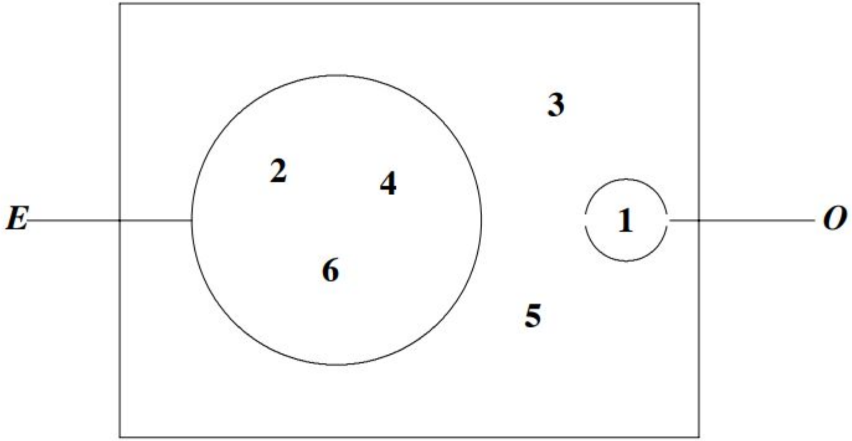
Mutually exclusive or **disjoint** events.

A set of events $A_1, A_2, .. A_n$ is said to be mutually exclusive or disjoint if the intersection of A_i and A_j is empty for all $i \neq j$ i.e. at most one event can occur (if one occurs, any other event cannot occur)

Let's assume a die is rolled.
The event E = "obtaining an even number"
The event O = "obtaining a 1"



As we can see the events E and O are mutually Exclusive as they do not intersect.

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For any two events A and B,
$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$

If the events A and B are **mutually exclusive**, then
$P(A \cup B) = P(A) + P(B)$

Let's assume we rolled a die. We want to calculate the probability of the event "die shows an even face or a die shows a number greater than 3".

Let us denote A as the event "die shows even face". A = {2, 4, 6}. $P(A) = 3/6 = 1/2$
Let us denote B as the event "die shows a number greater than 3". B = {4, 5, 6}. $P(B) = 3/6 = 1/2$

The intersection of A and B is not empty. A and B are not mutually exclusive events.
A intersection B = {4, 6}. $P(A \text{ and } B) = 2/6 = 1/3$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = (1/2) + (1/2) - (1/3)$

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Conditional probability: The probability of an event A under the condition that the event B has already occurred. It is denoted as P(A|B).

How to calculate conditional probability?

Let's look at an example - A lecture on a topic of public health is held and 300 people attend. They are classified in the following way:

Gender	Doctors	Nurses	Total
Female	90	90	180
Male	100	20	120
Total	190	110	300

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If one person is selected at random, find the following probabilities:

(a) P(a doctor is chosen);
(b) P(a female is chosen);
(c) P(a nurse is chosen);
(d) P(a male is chosen);
(e) P(a female nurse is chosen);
(f) P(a male doctor is chosen).

Gender	Doctors	Nurses	Total
Female	90	90	180
Male	100	20	120
Total	190	110	300

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Solution:

a) The number of doctors is 190 and the total number of people is 300, so P(doctor) = 190 / 300

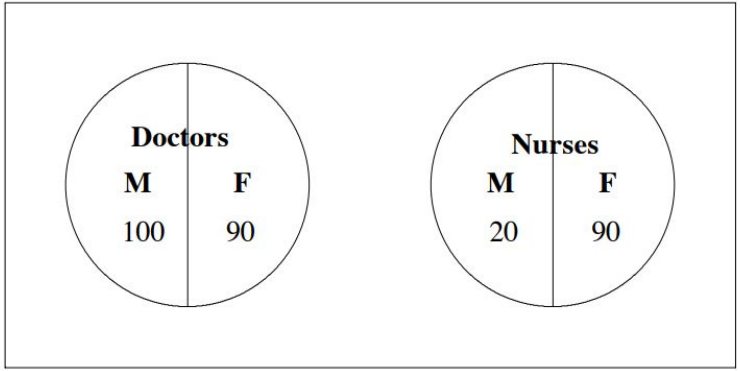
b) P(female) = 180 / 300

c) P (male) = 120 / 300

d) P(nurse) = 110 / 300

e) There are 90 female nurses, therefore P(female ∩ nurse) = 90 / 300

f) P(male doctor) = P(male ∩ doctor) = 100 / 300 .



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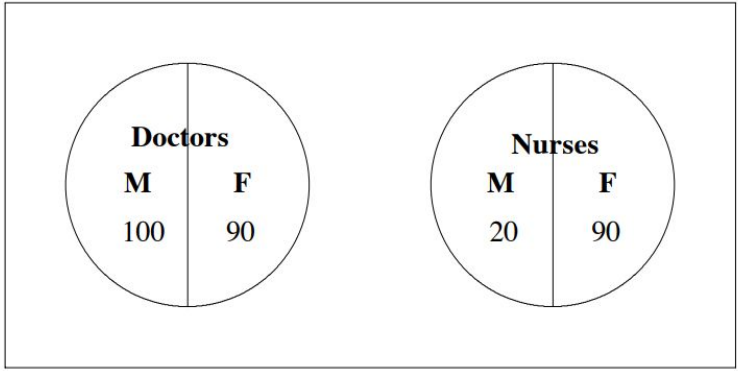
Now suppose you are given the information that a female is chosen and you wish to find the probability that she is a nurse. This is what we call **conditional probability**.



We want the probability that the person chosen is a nurse, subject to the condition that we know she is female.

The notation used for this is:

P(nurse | female)

and the notation is read as '*the probability of the person chosen being a nurse, given that she is female*'.

Since there are 180 females and of these 90 are nurses, the required probability is 90 / 180 = 1 / 2

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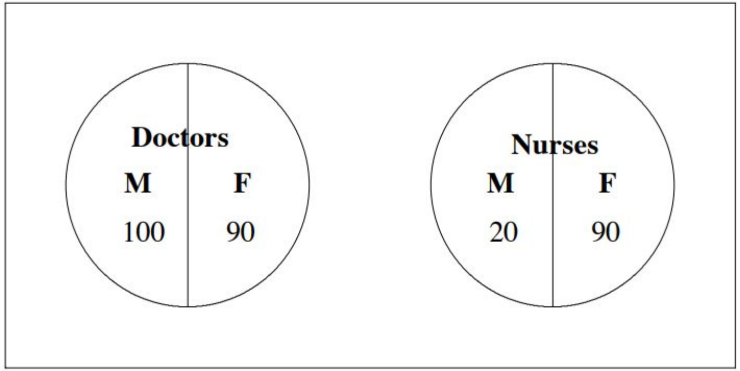
$P(\text{nurse} \mid \text{female}) = 90 / 180$
$= (90 / 300) / (180 / 300)$
$= P(\text{nurse} \cap \text{female}) / P(\text{female})$



The conditional probability of A given the event B has occurred is defined by

$$P(A \mid B) = P(A \cap B) / P(B) \text{ , provided that } P(B) \neq 0.$$

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Hence if A and B are any two events with probabilities greater than 0, then

$$P(A \cap B) = P(A \mid B).P(B)$$
$$= P(B \mid A).P(A),$$

as $P(B \cap A) = P(A \cap B)$

In conditional probability the order matters

$P(A \mid B)$ is not the same as $P(B \mid A)$.

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Independence: Two events A and B are said to be independent if and only if $P(A \mid B) = P(A)$, that is, when the conditional probability of A given B is the same as the probability of A.

In other words if the chance of a given outcome of the event A remains the same, irrespective of whether or not another event B has occurred, then the events A and B are said to be independent.

Tasks

A. For the previous example, calculate the conditional probabilities:

1. P(female | nurse),
2. P(doctor | male),
3. P(male | doctor).

B. Consider the experiment: tossing a coin thrice. Assume A is the event that “at least two heads show up”. Write down the Sample Space, Event, and the probability of the event A i.e. P(A).