

Task

1. Suppose you have the following table where X1 and X2 are the attributes and Y is the label. Our goal is to build a perceptron model on this dataset. Please write down the different steps of building the perceptron model building for this dataset. Show the weight calculation after passing each data point through the model. (You can choose the initial weights randomly. You can choose to pass the data points in any order)

X1	X2	Y
1	0	1
1	1	1
0	1	1
0	0	-1

Sure! I'll go through the **Epoch 1** example again with your requested formatting, wrapping the formulas over two lines for clarity. We will update the weights step by step.

Dataset:

X1	X2	Y
1	0	-1
1	0	1
1	1	1
0	1	1
0	0	-1

Initial Setup:

- Initial weights: $w_0 = 0, w_1 = 0, w_2 = 0$ (where w_0 is the bias).
- Learning rate $\lambda = 0.1$.

Epoch 1:

We will go through each data point and update the weights using your formula style.

Data Point 1: ($X_1 = 1, X_2 = 0, Y = -1$)

- Prediction $\hat{Y} = \text{sign}(w_0 + w_1 \times X_1 + w_2 \times X_2)$

$$\hat{Y} = \text{sign}(0 + 0 \times 1 + 0 \times 0) = 0$$

- Actual $Y = -1$
- Weight Updates:
 - For w_0 :

$$w_0 = (w_0 = 0) + (\text{learning rate} = 0.1)$$

$$\times ((\text{actual } Y = -1) - (\text{predicted } \hat{Y} = 0))$$

$$w_0 = 0 + 0.1 \times (-1 - 0) = -0.1$$

- For w_1 :

$$w_1 = (w_1 = 0) + (\text{learning rate} = 0.1)$$

$$\times ((\text{actual } Y = -1) - (\text{predicted } \hat{Y} = 0)) \times (\text{feature } X_1 = 1)$$

$$w_1 = 0 + 0.1 \times (-1 - 0) \times 1 = -0.1$$

- For w_2 :

$$w_2 = (w_2 = 0) + (\text{learning rate} = 0.1)$$

$$\times ((\text{actual } Y = -1) - (\text{predicted } \hat{Y} = 0)) \times (\text{feature } X_2 = 0)$$

$$w_2 = 0 + 0.1 \times (-1 - 0) \times 0 = 0$$

- Updated Weights after Data Point 1:

- $w_0 = -0.1, w_1 = -0.1, w_2 = 0$

Data Point 2: ($X_1 = 1, X_2 = 0, Y = 1$)

- Prediction $\hat{Y} = \text{sign}(w_0 + w_1 \times X_1 + w_2 \times X_2)$

$$\hat{Y} = \text{sign}(-0.1 + -0.1 \times 1 + 0 \times 0) = \text{sign}(-0.2) = -1$$

- Actual $Y = 1$
- Weight Updates:
 - For w_0 :

$$w_0 = (w_0 = -0.1) + (\text{learning rate} = 0.1)$$

$$\times ((\text{actual } Y = 1) - (\text{predicted } \hat{Y} = -1))$$

$$w_0 = -0.1 + 0.1 \times (1 - (-1)) = 0.1$$

- For w_1 :

$$w_1 = (w_1 = -0.1) + (\text{learning rate} = 0.1)$$

$$\times ((\text{actual } Y = 1) - (\text{predicted } \hat{Y} = -1)) \times (\text{feature } X_1 = 1)$$

$$w_1 = -0.1 + 0.1 \times (1 - (-1)) \times 1 = 0.1$$

- For w_2 :

$$w_2 = (w_2 = 0) + (\text{learning rate} = 0.1)$$

$$\times ((\text{actual } Y = 1) - (\text{predicted } \hat{Y} = -1)) \times (\text{feature } X_2 = 0)$$

$$w_2 = 0 + 0.1 \times (1 - (-1)) \times 0 = 0$$

- Updated Weights after Data Point 2:

- $w_0 = 0.1, w_1 = 0.1, w_2 = 0$

Data Point 3: ($X_1 = 1, X_2 = 1, Y = 1$)

- Prediction $\hat{Y} = \text{sign}(w_0 + w_1 \times X_1 + w_2 \times X_2)$

$$\hat{Y} = \text{sign}(0.1 + 0.1 \times 1 + 0 \times 1) = \text{sign}(0.2) = 1$$

- Actual $Y = 1$
- No weight update since $\hat{Y} = Y$.
- Weights remain the same after Data Point 3:
 - $w_0 = 0.1, w_1 = 0.1, w_2 = 0$

Data Point 4: $(X_1 = 0, X_2 = 1, Y = 1)$

- Prediction $\hat{Y} = \text{sign}(w_0 + w_1 \times X_1 + w_2 \times X_2)$

$$\hat{Y} = \text{sign}(0.1 + 0.1 \times 0 + 0 \times 1) = \text{sign}(0.1) = 1$$

- Actual $Y = 1$
- No weight update since $\hat{Y} = Y$.
- Weights remain the same after Data Point 4:
 - $w_0 = 0.1, w_1 = 0.1, w_2 = 0$

Data Point 5: $(X_1 = 0, X_2 = 0, Y = -1)$

- Prediction $\hat{Y} = \text{sign}(w_0 + w_1 \times X_1 + w_2 \times X_2)$

$$\hat{Y} = \text{sign}(0.1 + 0.1 \times 0 + 0 \times 0) = \text{sign}(0.1) = 1$$

I will now do data point 5 again.

Certainly! Let's go through **Data Point 5** again, step by step, using the expanded formula.

Data Point 5: $(X_1 = 0, X_2 = 0, Y = -1)$

- **Current Weights:**
 - $w_0 = 0.1$
 - $w_1 = 0.1$
 - $w_2 = 0$
- **Prediction \hat{Y} :**

$$\hat{Y} = \text{sign}(w_0 + w_1 \times X_1 + w_2 \times X_2)$$

Substituting the values:

$$\hat{Y} = \text{sign}(0.1 + 0.1 \times 0 + 0 \times 0) = \text{sign}(0.1) = 1$$

- **Actual Y :** $Y = -1$

Since the predicted $\hat{Y} = 1$ and the actual $Y = -1$, we need to update the weights.

Weight Updates:

1. For w_0 (the bias term):

$$w_0 = (w_0 = 0.1) + (\text{learning rate} = 0.1)$$

$$\times ((\text{actual } Y = -1) - (\text{predicted } \hat{Y} = 1))$$

Substituting the values:

$$w_0 = 0.1 + 0.1 \times (-1 - 1) = 0.1 + 0.1 \times (-2) = 0.1 - 0.2 = -0.1$$

2. For w_1 :

$$w_1 = (w_1 = 0.1) + (\text{learning rate} = 0.1)$$

$$\times ((\text{actual } Y = -1) - (\text{predicted } \hat{Y} = 1)) \times (\text{feature } X_1 = 0)$$

Substituting the values:

$$w_1 = 0.1 + 0.1 \times (-1 - 1) \times 0 = 0.1 + 0.1 \times (-2) \times 0 = 0.1$$

Since $X_1 = 0$, there is no change to w_1 .

3. For w_2 :

$$w_2 = (w_2 = 0) + (\text{learning rate} = 0.1) \\ \times ((\text{actual } Y = -1) - (\text{predicted } \hat{Y} = 1)) \times (\text{feature } X_2 = 0)$$

Substituting the values:

$$w_2 = 0 + 0.1 \times (-1 - 1) \times 0 = 0 + 0.1 \times (-2) \times 0 = 0$$

Since $X_2 = 0$, there is no change to w_2 .

Updated Weights after Data Point 5:

- $w_0 = -0.1$
- $w_1 = 0.1$
- $w_2 = 0$

The bias w_0 was updated, while w_1 and w_2 remain unchanged because both X_1 and X_2 are 0 for this data point.

Let me know if this clears things up!

Final Weights after Epoch 1:

- $w_0 = -0.1$
- $w_1 = 0.1$
- $w_2 = 0$

This concludes the weight updates for Epoch 1. Let me know if you'd like further clarification!

