

- **Course Title:** Engineering Cost Analysis & Economy (ENGR 222)
- **Session:** Fall 2024
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- **Class Time:** TR 9.30 AM-10.45 AM
- **Office hours:** TR 11.00 AM-12.30 PM



# **Annual Worth Techniques to Evaluate and Select Alternatives**

## Alternatives usually have the following cash flow estimates

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**Initial investment,  $P$**  – First cost of an asset

**Salvage value,  $S$**  – Estimated value of asset at end of useful life

**Annual amount,  $A$**  – Cash flows associated with asset, such as annual operating cost (AOC), etc.

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Relationship between AW, PW and FW

$$AW = PW(A/P, i\%, n) = FW(A/F, i\%, n)$$

$n$  is years for equal-service comparison (value of LCM or specified study period)

**AW for one life cycle is the *same for all* life cycles!!**

Example 1. An asset has a first cost of \$20,000, an annual operating cost of \$8000 and a salvage value of \$5000 after 3 years.  
Calculate the AW for one and two life cycles at  $i = 10\%$

10%		Compound Interest Factors							10%
Single Payment			Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find <i>F</i> Given <i>P</i> <i>F/P</i>	Find <i>P</i> Given <i>F</i> <i>P/F</i>	Find <i>A</i> Given <i>F</i> <i>A/F</i>	Find <i>A</i> Given <i>P</i> <i>A/P</i>	Find <i>F</i> Given <i>A</i> <i>F/A</i>	Find <i>P</i> Given <i>A</i> <i>P/A</i>	Find <i>A</i> Given <i>G</i> <i>A/G</i>	Find <i>P</i> Given <i>G</i> <i>P/G</i>	<i>n</i>
1	1.100	.9091	1.0000	1.1000	1.000	0.909	0	0	1
2	1.210	.8264	.4762	.5762	2.100	1.736	0.476	0.826	2
3	1.331	.7513	.3021	.4021	3.310	2.487	0.937	2.329	3
4	1.464	.6830	.2155	.3155	4.641	3.170	1.381	4.378	4
5	1.611	.6209	.1638	.2638	6.105	3.791	1.810	6.862	5
6	1.772	.5645	.1296	.2296	7.716	4.355	2.224	9.684	6
7	1.949	.5132	.1054	.2054	9.487	4.868	2.622	12.763	7
8	2.144	.4665	.0874	.1874	11.436	5.335	3.004	16.029	8
9	2.358	.4241	.0736	.1736	13.579	5.759	3.372	19.421	9
10	2.594	.3855	.0627	.1627	15.937	6.145	3.725	22.891	10

## Capital Recovery and AW

Capital recovery (CR) is the **equivalent annual amount** that an asset, process, or system must earn each year to just **recover the first cost and a stated rate of return** over its expected life. Salvage value is considered when calculating CR.

$$CR = -P(A/P, i\%, n) + S(A/F, i\%, n)$$

$$AW = CR + A$$

**Example 2.** Humana Hospital Corporation installed a new MRI machine at a cost of \$750,000 this year in its medical professional clinic in Cedar Park. This state-of-the-art system is expected to be used for 5 years and then sold for \$125,000. Humana uses a return requirement of 25% per year for all of its medical diagnostic equipment. As a bioengineering student currently serving a coop semester on the management staff of Humana Corporation in Louisville, Kentucky, you are asked to determine the minimum revenue required each year to realize the expected recovery and return.

- (a) What is your answer?
- (b) If the AOC is expected to be \$80,000 per year, what is the total revenue required to provide for recovery of capital, the 25% return, and the annual expenses?

25%		Compound Interest Factors								25%
		Single Payment		Uniform Payment Series				Arithmetic Gradient		
		Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
		Find <i>F</i> Given <i>P</i> <i>F/P</i>	Find <i>P</i> Given <i>F</i> <i>P/F</i>	Find <i>A</i> Given <i>F</i> <i>A/F</i>	Find <i>A</i> Given <i>P</i> <i>A/P</i>	Find <i>F</i> Given <i>A</i> <i>F/A</i>	Find <i>P</i> Given <i>A</i> <i>P/A</i>	Find <i>A</i> Given <i>G</i> <i>A/G</i>	Find <i>P</i> Given <i>G</i> <i>P/G</i>	<i>n</i>
1		1.250	.8000	1.0000	1.2500	1.000	0.800	0	0	1
2		1.563	.6400	.4444	.6944	2.250	1.440	0.444	0.640	2
3		1.953	.5120	.2623	.5123	3.813	1.952	0.852	1.664	3
4		2.441	.4096	.1734	.4234	5.766	2.362	1.225	2.893	4
5		3.052	.3277	.1218	.3718	8.207	2.689	1.563	4.204	5
6		3.815	.2621	.0888	.3388	11.259	2.951	1.868	5.514	6
7		4.768	.2097	.0663	.3163	15.073	3.161	2.142	6.773	7
8		5.960	.1678	.0504	.3004	19.842	3.329	2.387	7.947	8
9		7.451	.1342	.0388	.2888	25.802	3.463	2.605	9.021	9
10		9.313	.1074	.0301	.2801	33.253	3.571	2.797	9.987	10
11		11.642	.0859	.0235	.2735	42.566	3.656	2.966	10.846	11
12		14.552	.0687	.0184	.2684	54.208	3.725	3.115	11.602	12
13		18.190	.0550	.0145	.2645	68.760	3.780	3.244	12.262	13
14		22.737	.0440	.0115	.2615	86.949	3.824	3.356	12.833	14
15		28.422	.0352	.00912	.2591	109.687	3.859	3.453	13.326	15



# Evaluation of Alternatives

**One alternative:** If  $AW \geq 0$ , the requested MARR is met or exceeded and the alternative is economically justified.

**Two or more alternatives:** Select the alternative with the  $AW$  that is **numerically largest**, that is, less negative or more positive. This indicates a lower  $AW$  of cost for cost alternatives or a larger  $AW$  of net cash flows for revenue alternatives.

**Not necessary to use LCM for different life alternatives**

Example 3. A company is considering two machines. Machine X has a first cost of \$30,000, AOC of \$18,000, and S of \$7000 after 4 years.

Machine Y will cost \$50,000 with an AOC of \$16,000 and S of \$9000 after 6 years.

Which machine should the company select at an interest rate of 12% per year?

12%		Compound Interest Factors								12%
		Single Payment		Uniform Payment Series				Arithmetic Gradient		
		Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
		Find <i>F</i> Given <i>P</i> <i>F/P</i>	Find <i>P</i> Given <i>F</i> <i>P/F</i>	Find <i>A</i> Given <i>F</i> <i>A/F</i>	Find <i>A</i> Given <i>P</i> <i>A/P</i>	Find <i>F</i> Given <i>A</i> <i>F/A</i>	Find <i>P</i> Given <i>A</i> <i>P/A</i>	Find <i>A</i> Given <i>G</i> <i>A/G</i>	Find <i>P</i> Given <i>G</i> <i>P/G</i>	<i>n</i>
1		1.120	.8929	1.0000	1.1200	1.000	0.893	0	0	1
2		1.254	.7972	.4717	.5917	2.120	1.690	0.472	0.797	2
3		1.405	.7118	.2963	.4163	3.374	2.402	0.925	2.221	3
4		1.574	.6355	.2092	.3292	4.779	3.037	1.359	4.127	4
5		1.762	.5674	.1574	.2774	6.353	3.605	1.775	6.397	5
6		1.974	.5066	.1232	.2432	8.115	4.111	2.172	8.930	6
7		2.211	.4523	.0991	.2191	10.089	4.564	2.551	11.644	7
8		2.476	.4039	.0813	.2013	12.300	4.968	2.913	14.471	8
9		2.773	.3606	.0677	.1877	14.776	5.328	3.257	17.356	9
10		3.106	.3220	.0570	.1770	17.549	5.650	3.585	20.254	10
11		3.479	.2875	.0484	.1684	20.655	5.938	3.895	23.129	11
12		3.896	.2567	.0414	.1614	24.133	6.194	4.190	25.952	12
13		4.363	.2292	.0357	.1557	28.029	6.424	4.468	28.702	13
14		4.887	.2046	.0309	.1509	32.393	6.628	4.732	31.362	14
15		5.474	.1827	.0268	.1468	37.280	6.811	4.980	33.920	15

**Example 4.** Two methods can be used for producing solar panels for electric power generation. Method 1 will have an initial cost of \$550,000, an annual operating cost of \$160,000 per year, and \$125,000 salvage value after its three-year life. Method 2 will cost \$830,000 with an annual operating cost of \$120,000, and a \$240,000 salvage value after its five-year life. The company has asked you to determine which method is better, but it Wants the analysis done over a three-year planning period. The salvage value of Method 2 will be 35% higher after three years than it is after five years. If the company's minimum attractive rate of return is 10% per year, which method should the company select?

10%		Compound Interest Factors							10%
Single Payment			Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find <i>F</i> Given <i>P</i> <i>F/P</i>	Find <i>P</i> Given <i>F</i> <i>P/F</i>	Find <i>A</i> Given <i>F</i> <i>A/F</i>	Find <i>A</i> Given <i>P</i> <i>A/P</i>	Find <i>F</i> Given <i>A</i> <i>F/A</i>	Find <i>P</i> Given <i>A</i> <i>P/A</i>	Find <i>A</i> Given <i>G</i> <i>A/G</i>	Find <i>P</i> Given <i>G</i> <i>P/G</i>	
<i>n</i>									<i>n</i>
1	1.100	.9091	1.0000	1.1000	1.000	0.909	0	0	1
2	1.210	.8264	.4762	.5762	2.100	1.736	0.476	0.826	2
3	1.331	.7513	.3021	.4021	3.310	2.487	0.937	2.329	3
4	1.464	.6830	.2155	.3155	4.641	3.170	1.381	4.378	4
5	1.611	.6209	.1638	.2638	6.105	3.791	1.810	6.862	5
6	1.772	.5645	.1296	.2296	7.716	4.355	2.224	9.684	6
7	1.949	.5132	.1054	.2054	9.487	4.868	2.622	12.763	7
8	2.144	.4665	.0874	.1874	11.436	5.335	3.004	16.029	8
9	2.358	.4241	.0736	.1736	13.579	5.759	3.372	19.421	9
10	2.594	.3855	.0627	.1627	15.937	6.145	3.725	22.891	10
11	2.853	.3505	.0540	.1540	18.531	6.495	4.064	26.396	11
12	3.138	.3186	.0468	.1468	21.384	6.814	4.388	29.901	12
13	3.452	.2897	.0408	.1408	24.523	7.103	4.699	33.377	13
14	3.797	.2633	.0357	.1357	27.975	7.367	4.996	36.801	14
15	4.177	.2394	.0315	.1315	31.772	7.606	5.279	40.152	15

**Example 5.**

You have two machines under consideration for an improved automated wrapping process for Snickers Fun Size candy bars as detailed below.

- (a) Using an AW analysis, determine which should be selected at  $i = 15\%$  per year.

<b>Machine</b>	<b>C</b>	<b>D</b>
First cost, \$	−40,000	−65,000
Annual cost, \$/year	−10,000	−12,000
Salvage value, \$	12,000	25,000
Life, years	3	6

15%		Compound Interest Factors							15%
Single Payment			Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find <i>F</i> Given <i>P</i> <i>F/P</i>	Find <i>P</i> Given <i>F</i> <i>P/F</i>	Find <i>A</i> Given <i>F</i> <i>A/F</i>	Find <i>A</i> Given <i>P</i> <i>A/P</i>	Find <i>F</i> Given <i>A</i> <i>F/A</i>	Find <i>P</i> Given <i>A</i> <i>P/A</i>	Find <i>A</i> Given <i>G</i> <i>A/G</i>	Find <i>P</i> Given <i>G</i> <i>P/G</i>	<i>n</i>
1	1.150	.8696	1.0000	1.1500	1.000	0.870	0	0	1
2	1.322	.7561	.4651	.6151	2.150	1.626	0.465	0.756	2
3	1.521	.6575	.2880	.4380	3.472	2.283	0.907	2.071	3
4	1.749	.5718	.2003	.3503	4.993	2.855	1.326	3.786	4
5	2.011	.4972	.1483	.2983	6.742	3.352	1.723	5.775	5
6	2.313	.4323	.1142	.2642	8.754	3.784	2.097	7.937	6
7	2.660	.3759	.0904	.2404	11.067	4.160	2.450	10.192	7
8	3.059	.3269	.0729	.2229	13.727	4.487	2.781	12.481	8
9	3.518	.2843	.0596	.2096	16.786	4.772	3.092	14.755	9
10	4.046	.2472	.0493	.1993	20.304	5.019	3.383	16.979	10
11	4.652	.2149	.0411	.1911	24.349	5.234	3.655	19.129	11
12	5.350	.1869	.0345	.1845	29.002	5.421	3.908	21.185	12
13	6.153	.1625	.0291	.1791	34.352	5.583	4.144	23.135	13
14	7.076	.1413	.0247	.1747	40.505	5.724	4.362	24.972	14
15	8.137	.1229	.0210	.1710	47.580	5.847	4.565	26.693	15