- **Course Title:** Engineering Cost Analysis & Economy (ENGR 222)
- **Session:** Fall 2024
- **Instructor:** Sudipta Chowdhury (chowdhurys@marshall.edu)
- **Class Time: TR 9.30 AM-10.45 AM**
- **Office hours: TR 11.00 AM-12.30 PM**

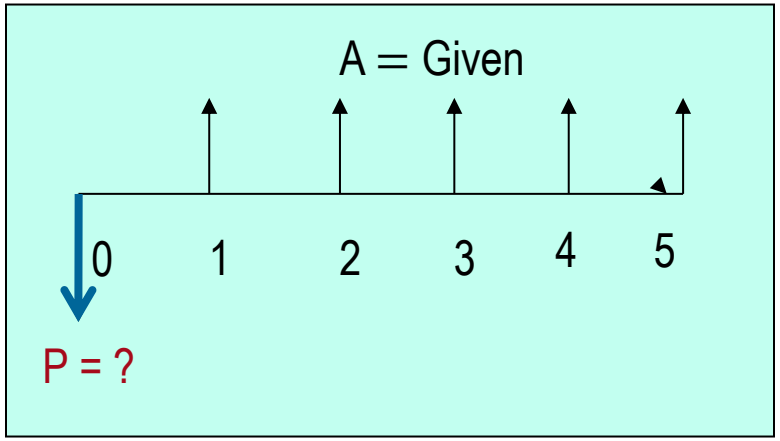
Uniform Series Formulas (P/A, A/P, F/A and A/F)

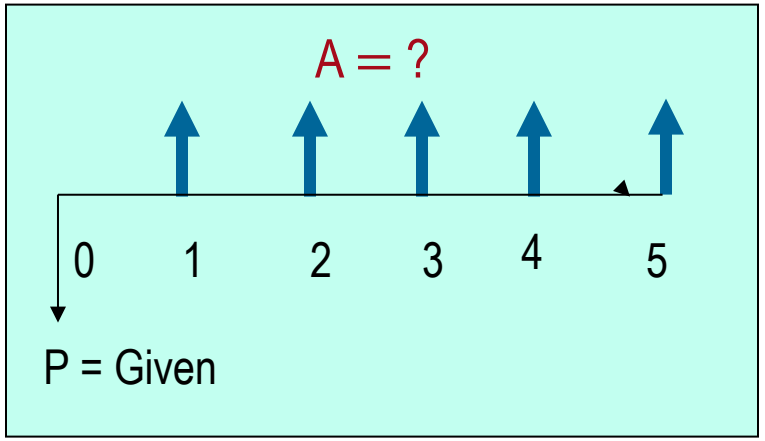
The uniform series factors that involve P and A are derived as follows:

(1) Cash flow occurs in *consecutive* interest periods

(2) Cash flow amount is *same* in each interest period

The cash flow diagrams are:





$P = A(P/A,i,n)$ ⟵ Standard Factor Notation ⟶ $A = P(A/P,i,n)$

Note: P is one period *Ahead* of first A value

- Suppose that there is a series of "n" uniform payments, uniform in amount and uniformly spaced, such as a payment every year.
- Let "A" be the amount of each uniform payment.
- Let "P" be a single amount equivalent to the series

The two equations that relate P and A are as follows.

$$P = A(P/A, i, n) = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

The factor $\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$ is called the uniform series present worth factor

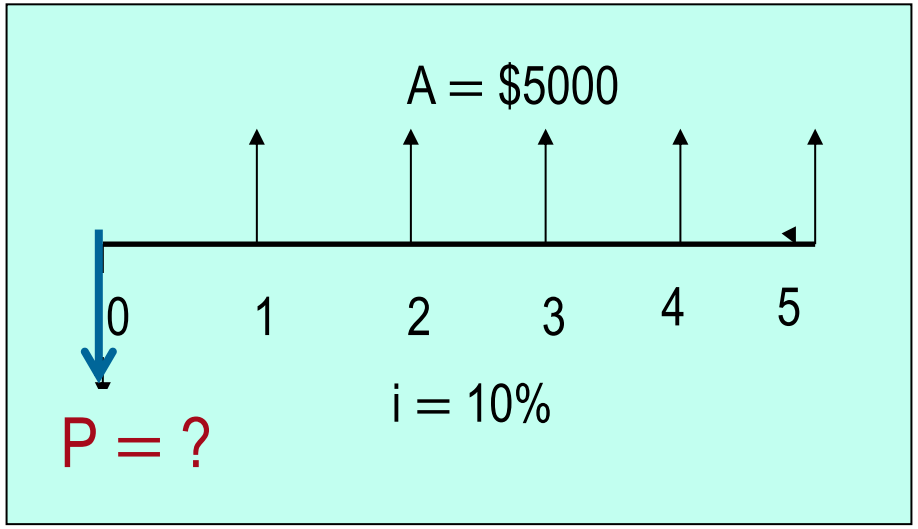
$$A = P(A/P, i, n) = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

The factor $\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$ is called the capital recovery factor

Example 1. A chemical engineer believes that by modifying the structure of a certain water treatment polymer, his company would earn an extra $5000 per year. At an interest rate of 10% per year, how much could the company afford to spend now to just break even over a 5 year project period?

(A) $11,170 **(B) 13,640** **(C) $15,300** **(D) $18,950**

The cash flow diagram is as



Example 2. Suppose that $30,000 is borrowed today at 12% interest. The loan is to be repaid by uniform annual payments for 5 years, beginning 1 year from now. Calculate the annual payment. Use the table values.

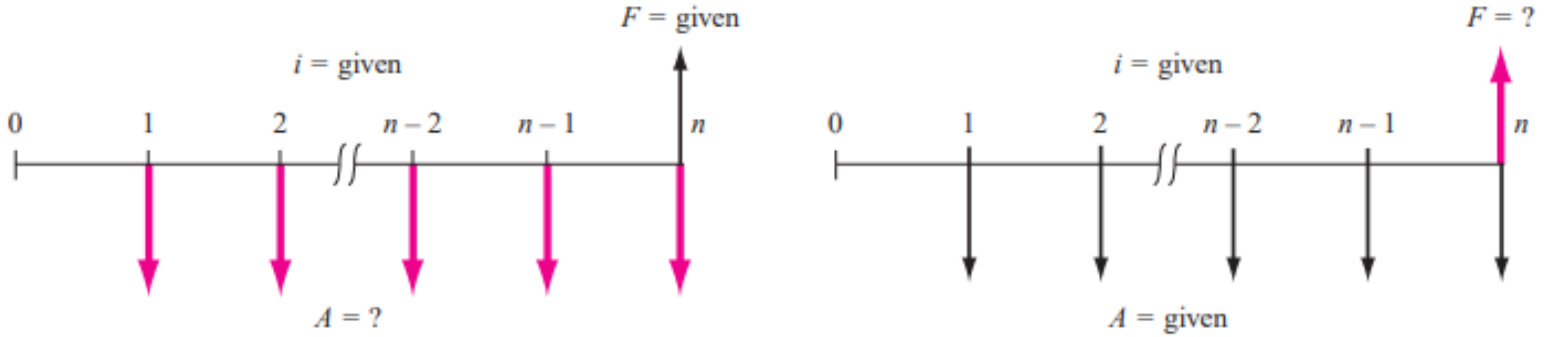
12%	Compound Interest Factors								12%
	Single Payment		Uniform Payment Series				Arithmetic Gradient		
n	Compound Amount Factor Find *F* Given *P* *F/P*	Present Worth Factor Find *P* Given *F* *P/F*	Sinking Fund Factor Find *A* Given *F* *A/F*	Capital Recovery Factor Find *A* Given *P* *A/P*	Compound Amount Factor Find *F* Given *A* *F/A*	Present Worth Factor Find *P* Given *A* *P/A*	Gradient Uniform Series Find *A* Given *G* *A/G*	Gradient Present Worth Find *P* Given *G* *P/G*	n
1	1.120	.8929	1.0000	1.1200	1.000	0.893	0	0	1
2	1.254	.7972	.4717	.5917	2.120	1.690	0.472	0.797	2
3	1.405	.7118	.2963	.4163	3.374	2.402	0.925	2.221	3
4	1.574	.6355	.2092	.3292	4.779	3.037	1.359	4.127	4
5	1.762	.5674	.1574	.2774	6.353	3.605	1.775	6.397	5

The uniform series factors that involve F and A are derived as follows:

(1) Cash flow occurs in *consecutive* interest periods

(2) Last cash flow occurs in *same* period as F

Cash flow diagrams are:



A = F(A/F,i,n) ⟵ Standard Factor Notation ⟶ F = A(F/A,i,n)

Note: F takes place in the *same* period as last A

- Suppose that there is a series of "n" uniform payments, uniform in amount and uniformly spaced, such as a payment every year.
- Let "A" be the amount of each uniform payment.
- Let "F" be a future, single amount equivalent to the series, with "F" occurring at the same time as the last "A" payment.

Then the relationship between F and A is:

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

The factor $\left[\frac{(1+i)^n - 1}{i}\right]$ is called the uniform series compound amount factor

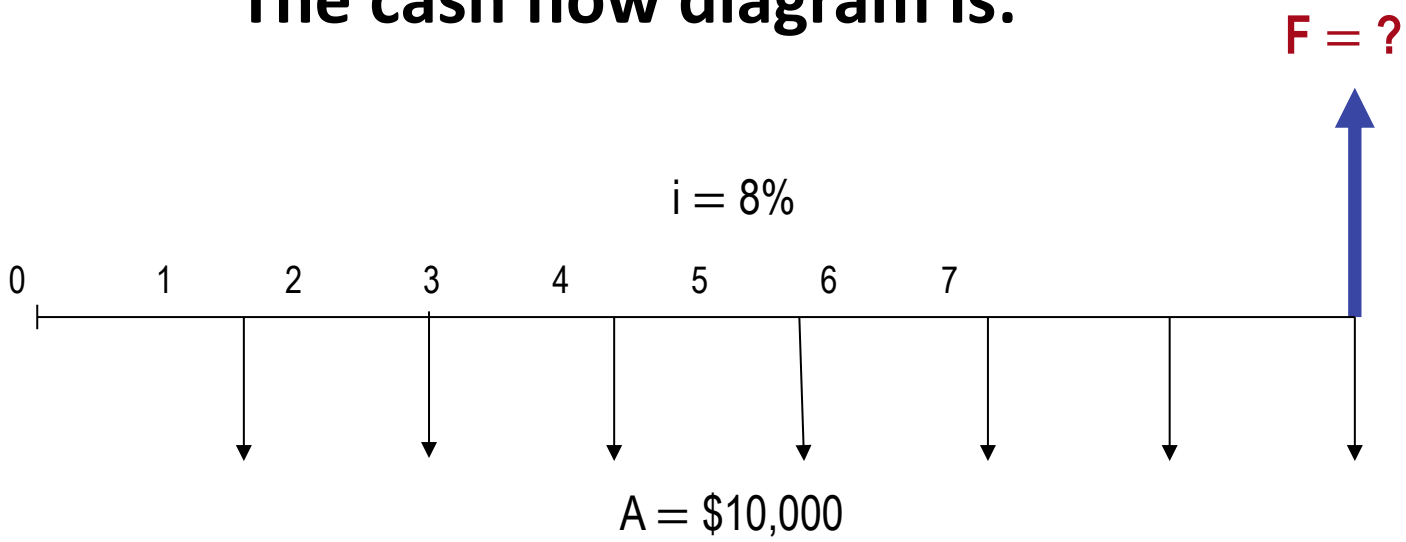
$$A = F\left[\frac{i}{(1+i)^n - 1}\right]$$

The factor $\left[\frac{i}{(1+i)^n - 1}\right]$ is called the sinking fund factor

Example 3. An industrial engineer made a modification to a chip manufacturing process that will save her company $10,000 per year. At an interest rate of 8% per year, how much will the savings amount to in 7 years?

(A) $45,300 (B) $68,500 (C) $89,228 (D) $151,500

The cash flow diagram is:



Example 4. Determine the size of your investment account 30 years from now (when you plan to retire) if you deposit $12,000 each year, beginning 1 year from now, and the account earns interest at a rate of 10% per year.

10%	Compound Interest Factors					
	Single Payment		Uniform Payment Series			
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*
n	*F/P*	*P/F*	*A/F*	*A/P*	*F/A*	*P/A*
1	1.100	.9091	1.0000	1.1000	1.000	0.909
2	1.210	.8264	.4762	.5762	2.100	1.736
3	1.331	.7513	.3021	.4021	3.310	2.487
4	1.464	.6830	.2155	.3155	4.641	3.170
5	1.611	.6209	.1638	.2638	6.105	3.791
6	1.772	.5645	.1296	.2296	7.716	4.355
7	1.949	.5132	.1054	.2054	9.487	4.868
8	2.144	.4665	.0874	.1874	11.436	5.335
9	2.358	.4241	.0736	.1736	13.579	5.759
10	2.594	.3855	.0627	.1627	15.937	6.145
11	2.853	.3505	.0540	.1540	18.531	6.495
12	3.138	.3186	.0468	.1468	21.384	6.814
13	3.452	.2897	.0408	.1408	24.523	7.103
14	3.797	.2633	.0357	.1357	27.975	7.367
15	4.177	.2394	.0315	.1315	31.772	7.606
16	4.595	.2176	.0278	.1278	35.950	7.824
17	5.054	.1978	.0247	.1247	40.545	8.022
18	5.560	.1799	.0219	.1219	45.599	8.201
19	6.116	.1635	.0195	.1195	51.159	8.365
20	6.728	.1486	.0175	.1175	57.275	8.514
21	7.400	.1351	.0156	.1156	64.003	8.649
22	8.140	.1228	.0140	.1140	71.403	8.772
23	8.954	.1117	.0126	.1126	79.543	8.883
24	9.850	.1015	.0113	.1113	88.497	8.985
25	10.835	.0923	.0102	.1102	98.347	9.077
26	11.918	.0839	.00916	.1092	109.182	9.161
27	13.110	.0763	.00826	.1083	121.100	9.237
28	14.421	.0693	.00745	.1075	134.210	9.307
29	15.863	.0630	.00673	.1067	148.631	9.370
30	17.449	.0573	.00608	.1061	164.494	9.427

Example 5. The Public Service Board (PSB) awarded two contracts worth a combined $3.07 million to increase the depth of a retention basin and reconstruct a spillway that was severely damaged in a flood 2 years ago. The PSB president stated that, surprisingly, the bids were $1,150,000 lower than PSB engineers estimated. If the projects are assumed to have a 20-year life, what is the annual worth of the savings at an interest rate of 5% per year?

5% Compound Interest Factors

	Single Payment		Uniform Payment Series			
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*
n	*F/P*	*P/F*	*A/F*	*A/P*	*F/A*	*P/A*
1	1.050	.9524	1.0000	1.0500	1.000	0.952
2	1.102	.9070	.4878	.5378	2.050	1.859
3	1.158	.8638	.3172	.3672	3.152	2.723
4	1.216	.8227	.2320	.2820	4.310	3.546
5	1.276	.7835	.1810	.2310	5.526	4.329
6	1.340	.7462	.1470	.1970	6.802	5.076
7	1.407	.7107	.1228	.1728	8.142	5.786
8	1.477	.6768	.1047	.1547	9.549	6.463
9	1.551	.6446	.0907	.1407	11.027	7.108
10	1.629	.6139	.0795	.1295	12.578	7.722
11	1.710	.5847	.0704	.1204	14.207	8.306
12	1.796	.5568	.0628	.1128	15.917	8.863
13	1.886	.5303	.0565	.1065	17.713	9.394
14	1.980	.5051	.0510	.1010	19.599	9.899
15	2.079	.4810	.0463	.0963	21.579	10.380
16	2.183	.4581	.0423	.0923	23.657	10.838
17	2.292	.4363	.0387	.0887	25.840	11.274
18	2.407	.4155	.0355	.0855	28.132	11.690
19	2.527	.3957	.0327	.0827	30.539	12.085
20	2.653	.3769	.0302	.0802	33.066	12.462

Example 6. Formasa Plastics has major fabrication plants in Texas and Hong Kong. The president wants to know the equivalent future worth of $1 million capital investments each year for 8 years, starting 1 year from now. Formasa capital earns at a rate of 15% per year.

15%	Compound Interest Factors								15%
	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	*F/P*	*P/F*	*A/F*	*A/P*	*F/A*	*P/A*	*A/G*	*P/G*	*n*
1	1.150	.8696	1.0000	1.1500	1.000	0.870	0	0	1
2	1.322	.7561	.4651	.6151	2.150	1.626	0.465	0.756	2
3	1.521	.6575	.2880	.4380	3.472	2.283	0.907	2.071	3
4	1.749	.5718	.2003	.3503	4.993	2.855	1.326	3.786	4
5	2.011	.4972	.1483	.2983	6.742	3.352	1.723	5.775	5
6	2.313	.4323	.1142	.2642	8.754	3.784	2.097	7.937	6
7	2.660	.3759	.0904	.2404	11.067	4.160	2.450	10.192	7
8	3.059	.3269	.0729	.2229	13.727	4.487	2.781	12.481	8
9	3.518	.2843	.0596	.2096	16.786	4.772	3.092	14.755	9
10	4.046	.2472	.0493	.1993	20.304	5.019	3.383	16.979	10

QUESTIONS?