- **Course Title:** Engineering Cost Analysis & Economy (ENGR 222)
- **Session:** Fall 2024
- **Instructor:** Sudipta Chowdhury (chowdhurys@marshall.edu)
- **Class Time: TR 9.30 AM-10.45 AM**
- **Office hours: TR 11.00 AM-12.30 PM**

Single Payment Formulas (F/P and P/F)

- The most fundamental equation in engineering economy is the one that determines the amount of money F accumulated after n years (or periods) from a single present worth P, with interest compounded one time per year (or period).
- Recall that compound interest refers to interest paid on top of interest. Therefore, if an amount P is invested at time $t = 0$, the amount F_1 accumulated 1 year hence at an interest rate of i percent per year will be

$$\boldsymbol{F_1 = P + Pi = P(1 + i)}$$

- The amount F_2 accumulated 2 years at an interest rate of i percent per year will be

$$\boldsymbol{F_2 = F_1 + F_1 i = P(1 + i) + P(1 + i)i = P(1 + i)^2}$$

- By mathematical induction, the formula can be generalized for n years to

$$\boldsymbol{F = P(1 + i)^n}$$

$$\boldsymbol{F = P(1 + i)^n}$$

- $\boldsymbol{(1 + i)^n}$ is called a factor and is known as the single-payment compound amount factor (SPCAF) or F/P factor
- This is the conversion factor that yields the future amount F of an initial amount P after n years at interest rate i

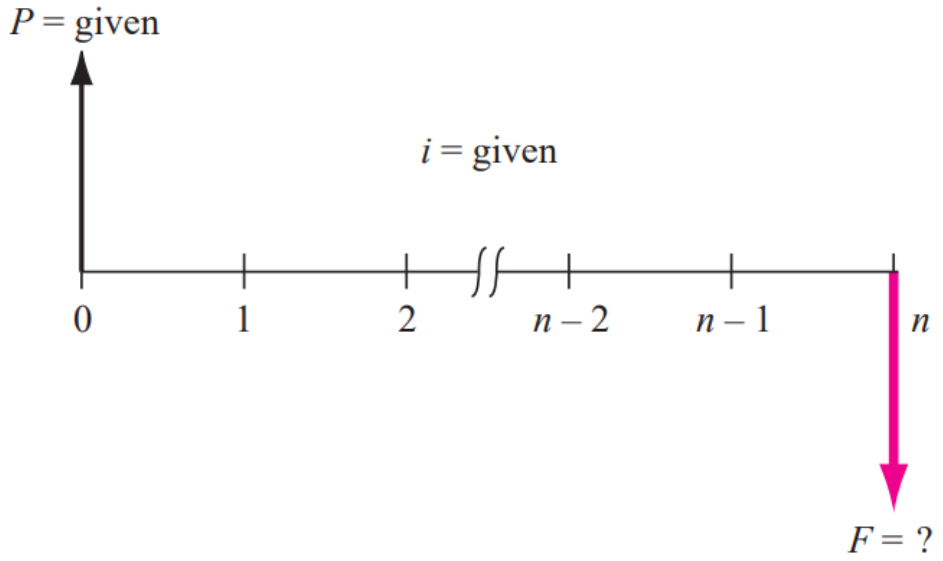


Figure: Cash flow diagrams for single-payment factors: find F

$$\boldsymbol{F = P(1 + i)^n}$$

- The same equation can be used to determine the P value for a stated amount F.

$$\boldsymbol{P = F\left[\frac{1}{(1 + i)^n}\right]}$$

- The expression in brackets is known as the single-payment present worth factor (SPPWF), or the P/F factor.
- This expression determines the present worth P of a given future amount F after n years at interest rate i.

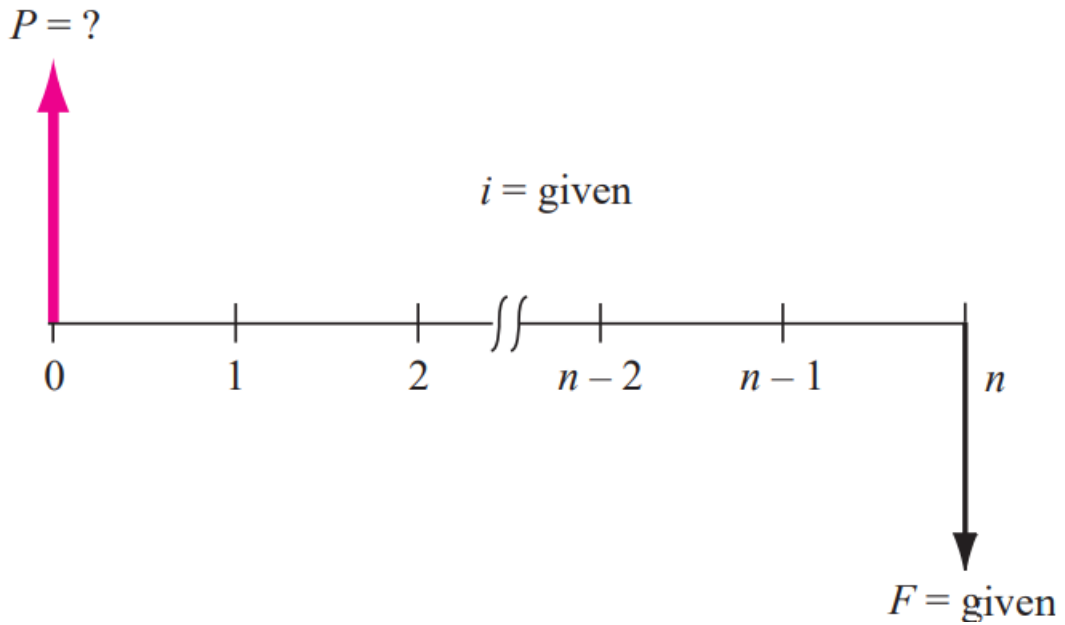


Figure: Cash flow diagrams for single-payment factors: find P

The two factors derived here (F/P and P/F) are for single payments; that is, they are used to find the present or future amount when only one payment or receipt is involved

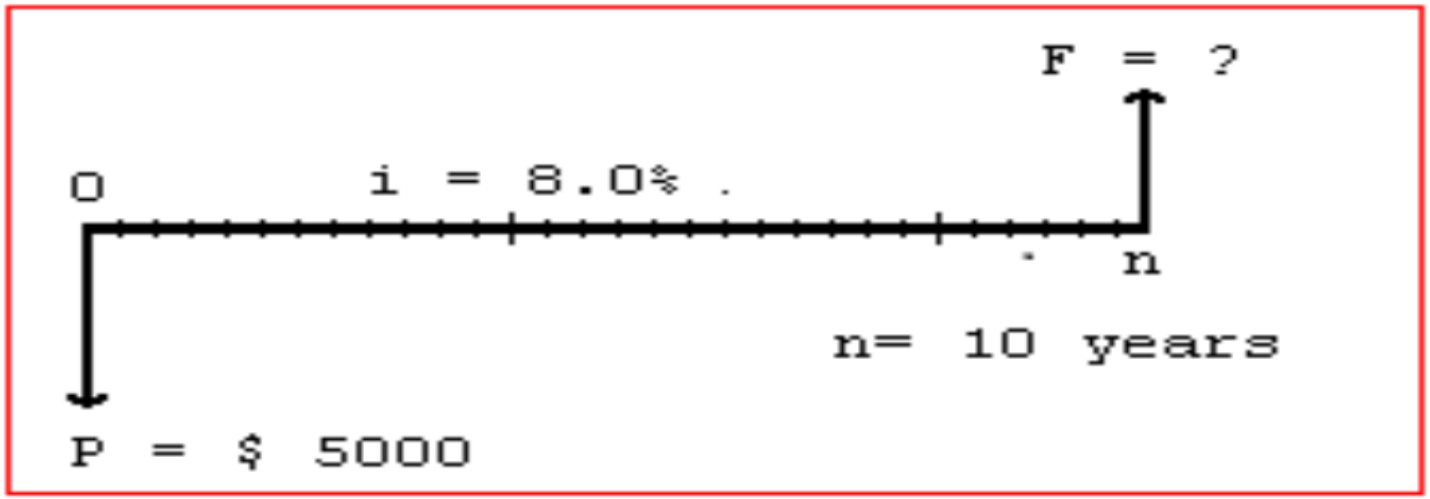
Standard Notation for Analysis

- It is always in the general form $(X/Y, i, n)$. The letter X represents what is sought, while the letter Y represents what is given.
- For example,F/P means find F when given P.
- (F/P,10%,30) represents the factor that is used to calculate the future amount F accumulated in 30 periods if the interest rate is 10% per period. The P is given.
- (P/F,10%,30) represents the factor that is used to calculate the present worth P of a given future amount F after 30 years at interest rate 10%. The F is given.

Example. 1: A person deposits $5000 into an account which pays interest at a rate of 8% per year. The amount in the account after 10 years is closest to:

(A) $2,792 (B) $9,000 (C) $10,795 (D) $12,165

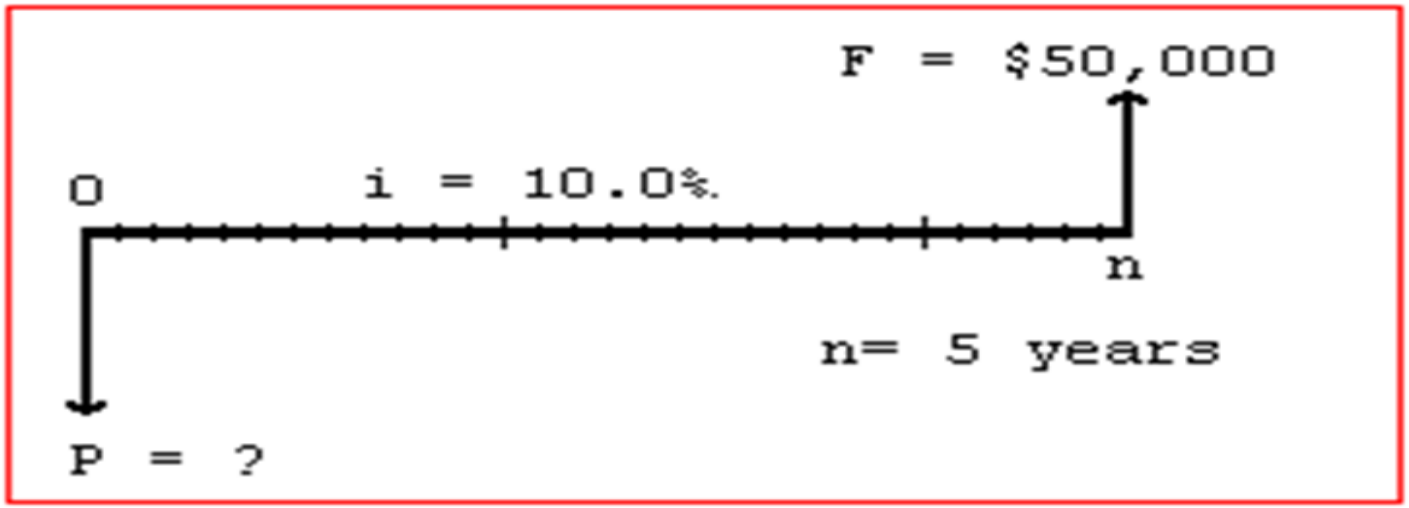
The cash flow diagram is:



Example. 2: A small company wants to make a single deposit now so it will have enough money to purchase a backhoe costing $50,000 five years from now. If the account will earn interest of 10% per year, the amount that must be deposited now is nearest to:

(A) $10,000 (B) $ 31,050 (C) $ 33,250 (D) $319,160

The cash flow diagram is:



- Till now, we have used the factor formula to calculate P/F or F/P
- You can also use the tables
- They are easy to use and reduces the potential of wrong calculation

Using Tables

Example. 3: A small company wants to make a single deposit now so it will have enough money to purchase a backhoe costing $50,000 five years from now. If the account will earn interest of 10% per year, the amount that must be deposited now is nearest to:

10% Compound Interest Factors 10%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
n	Compound Amount Factor Find *F* Given *P* *F/P*	Present Worth Factor Find *P* Given *F* *P/F*	Sinking Fund Factor Find *A* Given *F* *A/F*	Capital Recovery Factor Find *A* Given *P* *A/P*	Compound Amount Factor Find *F* Given *A* *F/A*	Present Worth Factor Find *P* Given *A* *P/A*	Gradient Uniform Series Find *A* Given *G* *A/G*	Gradient Present Worth Find *P* Given *G* *P/G*	*n*
1	1.100	.9091	1.0000	1.1000	1.000	0.909	0	0	1
2	1.210	.8264	.4762	.5762	2.100	1.736	0.476	0.826	2
3	1.331	.7513	.3021	.4021	3.310	2.487	0.937	2.329	3
4	1.464	.6830	.2155	.3155	4.641	3.170	1.381	4.378	4
5	1.611	.6209	.1638	.2638	6.105	3.791	1.810	6.862	5
6	1.772	.5645	.1296	.2296	7.716	4.355	2.224	9.684	6
7	1.949	.5132	.1054	.2054	9.487	4.868	2.622	12.763	7
8	2.144	.4665	.0874	.1874	11.436	5.335	3.004	16.029	8
9	2.358	.4241	.0736	.1736	13.579	5.759	3.372	19.421	9
10	2.594	.3855	.0627	.1627	15.937	6.145	3.725	22.891	10

Example. 4: Sandy, a manufacturing engineer, just received a year-end bonus of $10,000 that will be invested immediately. With the expectation of earning at the rate of 8% per year, Sandy hopes to take the entire amount out in exactly 20 years to pay for a family vacation. Find the amount of funds that will be available in 20 years by using hand solution by applying the factor formula and tabulated value.

8%	Compound Interest Factors					
	Single Payment		Uniform Payment Series			
n	Compound Amount Factor Find *F* Given *P* *F/P*	Present Worth Factor Find *P* Given *F* *P/F*	Sinking Fund Factor Find *A* Given *F* *A/F*	Capital Recovery Factor Find *A* Given *P* *A/P*	Compound Amount Factor Find *F* Given *A* *F/A*	Present Worth Factor Find *P* Given *A* *P/A*
1	1.080	.9259	1.0000	1.0800	1.000	0.926
2	1.166	.8573	.4808	.5608	2.080	1.783
3	1.260	.7938	.3080	.3880	3.246	2.577
4	1.360	.7350	.2219	.3019	4.506	3.312
5	1.469	.6806	.1705	.2505	5.867	3.993
6	1.587	.6302	.1363	.2163	7.336	4.623
7	1.714	.5835	.1121	.1921	8.923	5.206
8	1.851	.5403	.0940	.1740	10.637	5.747
9	1.999	.5002	.0801	.1601	12.488	6.247
10	2.159	.4632	.0690	.1490	14.487	6.710
11	2.332	.4289	.0601	.1401	16.645	7.139
12	2.518	.3971	.0527	.1327	18.977	7.536
13	2.720	.3677	.0465	.1265	21.495	7.904
14	2.937	.3405	.0413	.1213	24.215	8.244
15	3.172	.3152	.0368	.1168	27.152	8.559
16	3.426	.2919	.0330	.1130	30.324	8.851
17	3.700	.2703	.0296	.1096	33.750	9.122
18	3.996	.2502	.0267	.1067	37.450	9.372
19	4.316	.2317	.0241	.1041	41.446	9.604
20	4.661	.2145	.0219	.1019	45.762	9.818

Example. 5: The Department of Traffic Security of a city is considering the purchase of a new drone for aerial surveillance of traffic on its most congested streets. A similar purchase 4 years ago cost $1,200,000. At an interest rate of 7% per year, what is the equivalent value today of the previous $1,200,000 expenditure?

7%	Compound Interest Factors								7%
	Single Payment		Uniform Payment Series				Arithmetic Gradient		
n	Compound Amount Factor Find *F* Given *P* *F/P*	Present Worth Factor Find *P* Given *F* *P/F*	Sinking Fund Factor Find *A* Given *F* *A/F*	Capital Recovery Factor Find *A* Given *P* *A/P*	Compound Amount Factor Find *F* Given *A* *F/A*	Present Worth Factor Find *P* Given *A* *P/A*	Gradient Uniform Series Find *A* Given *G* *A/G*	Gradient Present Worth Find *P* Given *G* *P/G*	n
1	1.070	.9346	1.0000	1.0700	1.000	0.935	0	0	1
2	1.145	.8734	.4831	.5531	2.070	1.808	0.483	0.873	2
3	1.225	.8163	.3111	.3811	3.215	2.624	0.955	2.506	3
4	1.311	.7629	.2252	.2952	4.440	3.387	1.416	4.795	4
5	1.403	.7130	.1739	.2439	5.751	4.100	1.865	7.647	5
6	1.501	.6663	.1398	.2098	7.153	4.767	2.303	10.978	6
7	1.606	.6227	.1156	.1856	8.654	5.389	2.730	14.715	7
8	1.718	.5820	.0975	.1675	10.260	5.971	3.147	18.789	8
9	1.838	.5439	.0835	.1535	11.978	6.515	3.552	23.140	9
10	1.967	.5083	.0724	.1424	13.816	7.024	3.946	27.716	10

Example. 6: Electric car maker Gentech signed a $75 million contract with Power Systems, Inc. to automate a major part of its assembly line system. If Power Systems will be paid 2 years from now, when the systems are ready, determine the present worth of the contract at 18% per year interest.

18%	Compound Interest Factors								18%
	Single Payment		Uniform Payment Series				Arithmetic Gradient		
n	Compound Amount Factor Find *F* Given *P* *F/P*	Present Worth Factor Find *P* Given *F* *P/F*	Sinking Fund Factor Find *A* Given *F* *A/F*	Capital Recovery Factor Find *A* Given *P* *A/P*	Compound Amount Factor Find *F* Given *A* *F/A*	Present Worth Factor Find *P* Given *A* *P/A*	Gradient Uniform Series Find *A* Given *G* *A/G*	Gradient Present Worth Find *P* Given *G* *P/G*	n
1	1.180	.8475	1.0000	1.1800	1.000	0.847	0	0	1
2	1.392	.7182	.4587	.6387	2.180	1.566	0.459	0.718	2
3	1.643	.6086	.2799	.4599	3.572	2.174	0.890	1.935	3
4	1.939	.5158	.1917	.3717	5.215	2.690	1.295	3.483	4
5	2.288	.4371	.1398	.3198	7.154	3.127	1.673	5.231	5
6	2.700	.3704	.1059	.2859	9.442	3.498	2.025	7.083	6
7	3.185	.3139	.0824	.2624	12.142	3.812	2.353	8.967	7
8	3.759	.2660	.0652	.2452	15.327	4.078	2.656	10.829	8
9	4.435	.2255	.0524	.2324	19.086	4.303	2.936	12.633	9
10	5.234	.1911	.0425	.2225	23.521	4.494	3.194	14.352	10

Example. 7: Loadstar Sensors is a company that makes load/ force sensors based on capacitive sensing technology. For a major plant expansion project, the company wants to have $30 million 5 years from now. If the company already has $15 million in an investment account for the expansion, how much more must the company add to the account now so that it will have the $30 million 5 years from now? The funds earn interest at the rate of 10% per year.

10% Compound Interest Factors 10%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
n	Compound Amount Factor Find *F* Given *P* *F/P*	Present Worth Factor Find *P* Given *F* *P/F*	Sinking Fund Factor Find *A* Given *F* *A/F*	Capital Recovery Factor Find *A* Given *P* *A/P*	Compound Amount Factor Find *F* Given *A* *F/A*	Present Worth Factor Find *P* Given *A* *P/A*	Gradient Uniform Series Find *A* Given *G* *A/G*	Gradient Present Worth Find *P* Given *G* *P/G*	n
1	1.100	.9091	1.0000	1.1000	1.000	0.909	0	0	1
2	1.210	.8264	.4762	.5762	2.100	1.736	0.476	0.826	2
3	1.331	.7513	.3021	.4021	3.310	2.487	0.937	2.329	3
4	1.464	.6830	.2155	.3155	4.641	3.170	1.381	4.378	4
5	1.611	.6209	.1638	.2638	6.105	3.791	1.810	6.862	5
6	1.772	.5645	.1296	.2296	7.716	4.355	2.224	9.684	6
7	1.949	.5132	.1054	.2054	9.487	4.868	2.622	12.763	7
8	2.144	.4665	.0874	.1874	11.436	5.335	3.004	16.029	8
9	2.358	.4241	.0736	.1736	13.579	5.759	3.372	19.421	9
10	2.594	.3855	.0627	.1627	15.937	6.145	3.725	22.891	10

Factor Values for Untabulated i or n

Determine the value for (F/P, 8.3%,10)

Formula: $\mathrm{F} = (1 + 0.083)^{10} = 2.2197$ ⟵ **OK**

Interpolation: 8% ------ 2.1589

8.3% ------ y

9% ------ 2.3674

$$y = 2.1589 + [(8.3 - 8.0)/(9.0 - 8.0)][2.3674 - 2.1589]$$

$$= 2.2215$$ ⟵ **(Too high)**

Absolute Error $= \mathbf{2.2215 - 2.2197 = 0.0018}$

Factor Values for Untabulated i or n

8%	Compound Interest Factors								8%
	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	*F/P*	*P/F*	*A/F*	*A/P*	*F/A*	*P/A*	*A/G*	*P/G*	*n*
1	1.080	.9259	1.0000	1.0800	1.000	0.926	0	0	1
2	1.166	.8573	.4808	.5608	2.080	1.783	0.481	0.857	2
3	1.260	.7938	.3080	.3880	3.246	2.577	0.949	2.445	3
4	1.360	.7350	.2219	.3019	4.506	3.312	1.404	4.650	4
5	1.469	.6806	.1705	.2505	5.867	3.993	1.846	7.372	5
6	1.587	.6302	.1363	.2163	7.336	4.623	2.276	10.523	6
7	1.714	.5835	.1121	.1921	8.923	5.206	2.694	14.024	7
8	1.851	.5403	.0940	.1740	10.637	5.747	3.099	17.806	8
9	1.999	.5002	.0801	.1601	12.488	6.247	3.491	21.808	9
10	2.159	.4632	.0690	.1490	14.487	6.710	3.871	25.977	10

9%	Compound Interest Factors								9%
	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
	Find *F* Given *P*	Find *P* Given *F*	Find *A* Given *F*	Find *A* Given *P*	Find *F* Given *A*	Find *P* Given *A*	Find *A* Given *G*	Find *P* Given *G*	
n	*F/P*	*P/F*	*A/F*	*A/P*	*F/A*	*P/A*	*A/G*	*P/G*	*n*
1	1.090	.9174	1.0000	1.0900	1.000	0.917	0	0	1
2	1.188	.8417	.4785	.5685	2.090	1.759	0.478	0.842	2
3	1.295	.7722	.3051	.3951	3.278	2.531	0.943	2.386	3
4	1.412	.7084	.2187	.3087	4.573	3.240	1.393	4.511	4
5	1.539	.6499	.1671	.2571	5.985	3.890	1.828	7.111	5
6	1.677	.5963	.1329	.2229	7.523	4.486	2.250	10.092	6
7	1.828	.5470	.1087	.1987	9.200	5.033	2.657	13.375	7
8	1.993	.5019	.0907	.1807	11.028	5.535	3.051	16.888	8
9	2.172	.4604	.0768	.1668	13.021	5.995	3.431	20.571	9
10	2.367	.4224	.0658	.1558	15.193	6.418	3.798	24.373	10

QUESTIONS?