MTH 300: Intro to Higher Mathematics - Fall 2022

Problem Set 4

Due Monday, September 12

Last Name, First Name

Burbery, peter

Problem 1. For each statement below, explain why it is true (one or two sentences should be enough) or give a *counterexample*, that is, a specific example showing why the statement is false.

1. For every $x \in \mathbb{R}$, $x^2 + 1 \geq 1$.

2. For every $x \in \mathbb{N}$, $\sqrt[3]{x}$ is irrational.

3. For every $x \in \mathbb{R}$, $x > 0$ or $x < 0$.

Proof. Here is where your proof/explanation goes! ☐

For every $x \in \mathbb{R}$, $x^2+1 \geq 1$ is true.
$\forall_x, \cancel{x} \in \mathbb{R}$ $x^2+1 \geq 1$ is also true.

~~we~~ ~~put~~ we manipulate to get $\cancel{x}^2 \geq 0$. This statement is true because we would have to have ~~an~~ a complex number with a non zero imaginary part greater than 0 or less than 0 and a zero real part to make $x^2+1 \geq 1$ false. The domain is real numbers not imaginary numbers, so the statement is true.

The statement for every natural number x, the cube root of x is irrational is false for all perfect cubes for example 1 because $\sqrt[3]{1}=1$ is not irrational.

The number 0 is a real number, but its not less than 0 or greater than 0, so the statement for every real number x, $x > 0$ or $x < 0$, is false.

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Problem 2. For each statement below, give a specific example showing that the statement is true, or explain why there is no such example, thereby making the statement false.

1. There exists $x \in \mathbb{R}$ such that $x^2 + 2x + 4 = 0$.

2. There exist two irrational numbers x and y such that xy is rational.

3. There exists a positive integer n such that $3n < 3$.

Proof. Here is where your proof/explanation goes! ☐

The statement "There exists a real number x such that $x^2+2x+4=0$ is zero" is false because the roots of the polynomial x^2+2x+4 are $i\sqrt{3}-1$ and $-i\sqrt{3}-1$ or in trig form $2\left[\cos\left(-\frac{2\pi}{3}\right)+i\sin\left(-\frac{2\pi}{3}\right)\right]$ and $2\left[\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)\right]$ or in exponential form $\cancel{2e^{(2i\pi)/3}}$ $2e^{-(2i\pi)/3}$ and $2e^{(2i\pi)/3}$. ~~As these~~ ~~or~~ There are no real roots therefore the statement is false.

The statement there exist two irrational numbers x and y such that xy is rational is true. For example $\sqrt{2}\cdot\sqrt{2}=2$. $\sqrt{2}$ is irrational and 2 is rational.

The statement $3n<3$ is ~~equivalent~~ equivalent to $n<1$. There are no positive integers less than 1 so the statement "There exists a positive integer n such that $3n<3$" is false.

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Problem 3. Define a new logical connective Δ by the truth table:

p	q	$p\Delta q$
T	T	F
T	F	F
F	T	F
F	F	T

1. Prove that $\neg p \equiv p\Delta p$ using a truth table.

2. Find a compound statement containing only p, q, parentheses, and Δ (some or all may appear more than once) that is logically equivalent to $p \vee q$. Prove your assertion with an appropriate truth table.

Proof. Here is where your proof/explanation goes! □

p	q	$p \Delta \neg p$	$p \Delta q$	$p \Delta p$	$\neg p \equiv p \Delta p$
T	T	F	F	F	T
F	T	T	F	T	T
T	F	F	F	F	T
F	F	T	T	T	T

$P \vee Q \equiv \overline{P \Delta Q} \equiv \neg (P \Delta Q)$

$P \vee Q \equiv (P \Delta Q) \Delta (P \Delta Q)$

P	Q	$P \Delta Q$	$(P \Delta Q) \Delta (P \Delta Q)$	$P \vee Q$	$(P\Delta Q)\Delta(P\Delta Q) \equiv P \vee Q$
T	T	F	T	T	T
F	T	F	T	T	T
T	F	F	T	T	T
F	F	T	F	F	T

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Problem 4. For each of the following quantified statements, write the statement in symbols, negate the statement in symbols, and write the negation in words.

1. For each $x \in \mathbb{Z}$, $x \geq 3$ or $x \leq 2$.

2. There exists $y \in \mathbb{R}$ such that $y + 4 > 8$ and $y \leq 4$.

3. For each $x \in \mathbb{R}$, there exists $y \in \mathbb{R}$ such that $x \neq y$ and $x - y \in \mathbb{Z}$.

Proof. Here is where your proof/explanation goes! □

1. For all integers x, $x \geq 3$ or $x \leq 2$.

$\forall_{x, x \in \mathbb{Z}} \; x \geq 3 \vee x \leq 2$

$\exists_{x, x \in \mathbb{Z}} \; x < 3 \wedge x > 2$

$\exists_{x, x \in \mathbb{Z}} \; 2 < x < 3$

There exists an integer x such that x is greater than 2 and less than 3.

2. $\exists_{y, y \in \mathbb{R}} \; (y + 4 > 8) \wedge (y \leq 4)$

$\forall_{y, y \in \mathbb{R}} \; (y + 4 \leq 8) \vee (y > 4)$

For all ~~y~~ real numbers y, $y + 4 \leq 8$ or $y > 4$.

3. $\forall_{x, x \in \mathbb{R}} \; \exists_{y, y \in \mathbb{R}} \; ((x \neq y) \wedge (x - y \in \mathbb{Z}))$

$\exists_{x, x \in \mathbb{R}} \; \forall_{y, y \in \mathbb{R}} \; ((x = y) \vee (x - y \notin \mathbb{Z}))$

There exists a real number x such that for all real numbers y such that $x = y$ or $x - y$ is not an integer.

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Problem 5. Prove: For every integer a, if $4 \mid (a-1)$, then $4 \mid (a^2-1)$.

Proof. Here is where your proof/explanation goes! □

Assume a is an integer, and $4|(a-1)$.

Then the statement $(a-1) = 4K1$ for some integer $K1$ follows from the definition of divisibility.

~~then the~~ The statement $4|(a^2-1)$ ~~implies~~ is true iff $a^2-1 = 4J$ for some integer J.

~~and~~ This is algebraically the same as $(a-1)(a+1) = 4J$ for some integer J.

The statement $(a-1) = 4K1$ and $(a-1) = \frac{4J}{(a+1)}$ are both true when $K1$ represents ~~an~~ the integer $\frac{4J}{(a+1)}$.

Therefore, $4|(a^2-1)$.