Problem Set 5

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$$\int x\,dx + \sqrt{z}$$

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27 September 2022

Problem Set 5

Problem 1 Proof

Let n be an even integer. Since n is even, two divides n. Since 2|n, there exists an integer H_1 such that n equals two times H_1 ($\exists_{H_1 \in \mathbb{Z}}\ n = 2\,H_1$). We aim to prove that $4 \mid (n^2 - 6\,n + 8)$. We will do this by demonstrating that there exists an integer H_2 such that $\left(\exists_{H_2 \in \mathbb{Z}}\ n^2 - 6\,n + 8 = 4\,H_2\right)$. We will replace n with $2\,H_1$ in the expression n^2-6n+8 to get $(2\,H_1)^2 - 6\,(2\,H_1) + 8$. We distribute the exponent and parentheses to get $4\,H_1^2 - 12\,H_1 + 8$, which equals $4\,H_2$. We are left with $4\,H_1^2 - 12\,H_1 + 8 = 4\,H_2$. We can factor out a 4 to obtain the following form: $4\left(H_1^2 - 3\,H_1 + 2 = H_2\right)$. We have showed that there exists an integer H_2 of the form $H_1^2 - 3\,H_1 + 2$. We know this form will always be an integer because the multiplying and adding integers always ends with another integer. Since there exists an $\left(\exists_{H_2 \in \mathbb{Z}}\ n^2 - 6\,n + 8 = 4\,H_2\right)$, (specifically of the form $H_2 = H_1^2 - 3\,H_1 + 2$), 4 divides $n^2 - 6\,n + 8$. We have proven $4 \mid n^2 - 6\,n + 8$!!!

Problem 2 Proof

Let a, b, and c be integers with the condition that a is not zero. Suppose a divides b (a|b). Since a|b, there exists ar $b = a \times H_1$ ($\exists_{H_1 \in \mathbb{Z}}\ b = a \times H_1$). We aim to show that a divides b times c. We can show this by demonstrating that there exists an integer H_2 such that $b \times c = a \times H_2$ ($\exists_{H_2 \in \mathbb{Z}}\ b \times c = a \times H_2$). We can use the equation $b = a \times H_1$ and substitute for b into $b \times c = a \times H_2$ to get $(a \times H_1) \times c = a \times H_2$. We can divide both sides by a as $\frac{(a \times H_1) \times c}{a} = \frac{a \times H_2}{a}$. We can be sure that this is valid because we know a is not zero so we will not ever divide by zero. We now end up with $H_1 \times c = H_2$. We have showed that the form of the integer H_2 that makes the statement $b \times c = a \times H_2$ true has the form $H_1 \times c$. The number H_2 will be an integer because the product of two integers, in this case H_1 and c ($H_1 \times c$) is always an integer. Now that we have demonstrated that there exists an integer H_2 such that $b \times c = a \times H_2$, the statement that a divides b times c follows. We have proven a|(b× c)!!!

Problem 3 Proof

Let a and b be integers. Suppose a is congruent to 3 modulo 7 (a≡3 mod 7). Suppose b is congruent to 6 modulo 7 (b≡6 mod 7). Since a≡3 mod 7, 7 divides a-3. Since 7|a-3, there exists an integer H_1 such that $a = 3 + 7\,H_1$ ($\exists_{H_1 \in \mathbb{Z}}\ a = 3 + 7\,H_1$). Since b≡6 mod 7, 7 divides b-6. Since 7|b-6, there exists an integer H_2 such that $b = 6 + 7\,H_2$. ($\exists_{H_2 \in \mathbb{Z}}\ b = 6 + 7\,H_2$). We add the expressions a and b to get $a + b = (3 + 7\,H_1) + (6 + 7\,H_2)$. We get $a + b = 7\,H_1 + 7\,H_2 + 9 = 7\,H_1 + 7\,H_2 + 7 + 2 = 7\,(H_1 + H_2 + 1) + 2$. We can make H_3 have the value $H_1 + H_2 + 1$, and we see that there exists an integer H_3 such that $a + b = 7\,H_3 + 2$. We are certain that H_3 is an integer because the integers are closed under addition and multiplication. Since there exists an integer H_3 such that $a + b = 7\,H_3 + 2$, 7 divides (a+b)-2.. Since 7|((a+b)-2, a+b is congruent to 2 modulo 7. Therefore a+b≡2 mod 7. Proven! QED

Problem 4 Proof

Let a, b, and c be integers. Let n be a natural number, aka a positive integer. Suppose a is congruent to b modulo n (a≡b mod n). Suppose b is congruent to c modulo n (b≡c mod n). Since a≡b mod n, n divides a-b (n|a-b). Since n|a-b, there exists an integer H_1 such that $a - b = n$ ($\exists_{H_1 \in \mathbb{Z}}\ a - b = n \times H_1$). Since b≡c mod n, n divides b-c (n|b-c). Since n|b-c, there exists an integer H_2 such that $b - c = n \times H_2$ ($\exists_{H_2 \in \mathbb{Z}}\ b - c = n \times H_2$). We add expressions a-b and b-c to get a-c, which is equal to $n \times H_1 + n \times H_2$, which is equal to $n \times (H_1 + H_2)$. Since $a - c = n \times (H_1 + H_2)$, n divides a-c. Since n|a-c, a is congruent to c modulo n. We have proved a≡c (mod n).

Problem 5 Proof

Let a, b, c, and d be integers. Let n be a natural number, aka a positive integer. Suppose a is congruent to b modulo n (a≡b mod n). Suppose c is congruent to d modulo n (c≡d mod n). Since a≡b mod n, n divides a-b (n|a-b). Since n|a-b, there exists an integer H_1 such that $a - b = n$ ($\exists_{H_1 \in \mathbb{Z}}\ a - b = n \times H_1$). Since c≡d mod n, n divides c-d (n|c-d). Since n|c-d, there exists an integer H_2 such that $c - d = n \times H_2$ ($\exists_{H_2 \in \mathbb{Z}}\ c - d = n \times H_2$). We add expressions a-b and c-d to get (a+c)-(b+d), which is equal to $n \times H_1 + n \times H_2$, which is equal to $n \times (H_1 + H_2)$. Since $(a + c) - (b + d) = n \times (H_1 + H_2)$, n divides (a+c)-(b+d). Since n|(a+c)-(b+d), (a+c) is congruent to b+d modulo n. We have proved (a+c)≡ (b+d) (mod n).

$$\int x\,dx + \sqrt{z}$$

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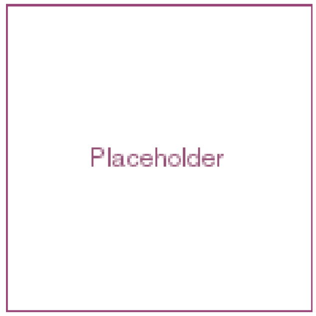

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