MTH 300: Intro to Higher Mathematics - Fall 2022 Problem Set 2

Due Monday, August 29
Last Name, First Name

Problem 1. Find the mathematical error in the "proof" shown below as you can and explain why the error is a problem. Rewrite the proof correcting the error and using good proof-writing technique.

Theorem 1. *If a and b are odd integers, then a − b is even.*

Proof. Let $a = 2k + 1$ and $b = 2k + 1$. Then $a - b = (2k + 1) - (2k + 1) = 0$. Since 0 is an even integer, the theorem holds. This proof implies that a=b. This is not part of the hypothesis, so the conclusion is not valid. This would be a proof to the claim If a and b are odd integers and a=b, then a-b is even.

The proof should state Let a=2k+1 and b=2m+1. Then a-b=(2k+1)-(2m+1)=2k-2m=2(k-m).

2(k-m) is an even integer and the theorem is proven. The conclusion is still true but the method and process and logical and communication and chain of steps of the proof is wrong.

Proof. Here is where your proof/explanation goes!

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Problem 2. Let $a,b \in Z$.
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1. Complete the following conditional statement so that if the hypothesis is true, so is the

   conclusion:

   If , then $a^2 + b^2$ is odd.

   If either a or b is an odd but not both a and b (exclusive or of a and b), then a^2+b^2 is odd.

2. Prove the completed statement from part (a). There are two possible cases: a is odd and b is even or a is even and b is odd.

Let us prove the first case holds. If a is odd it can be obtained as 2k+1 where k is a integer.

If b is even then then it can be obtained as $2m$ where m is an integer. If we symbolically manipulate a^2+b^2 we get $(2k+1)^2+(2m)^2$ and expanding we get $1+4k+4k^2+4m^2$. We can prove this can be obtained as $2n+1$ where n is an integer by subtracting 1 to get $4k+4k^2+4m^2$. We have now isolated $2n$. We factor out to get $2(2k+2k^2+2m^2)$. We add the one we subtracted earlier and now we have expressed a^2+b^2 in the form $2n+1$ where n represents $2k + 2k^2+2m^2$. Therefore the sum a^2+b^2 is an odd integer.

The same proof holds in the alternative case where a is even and b is odd because we can label b as a and a as a b. Swapping the values and the parity of the variables and b still results in an odd number because the operation of addition is commutative so if the sum of two expressions $x+y$ becomes $y+x$, the sum is invariant. In this case a^2+b^2 becomes b^2+a^2 the sum is invariant. If the sum is invariant, the parity of the sum is invariant. The theorem is proven.

Proof. Here is where your proof/explanation goes!

$\square$

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Problem 3. Given the following proof, identify the theorem being proved. Write the theorem as a conditional statement.

Proof. Let n be an odd integer. Then $n = 2k + 1$ for some $k \in Z$. Thus

$$\begin{aligned} 3n - 8 &= 3(2k + 1) - 8 \\ &= 6k + 3 - 8 \\ &= 6k - 5 \\ &= 2(3k - 3) + 1. \end{aligned}$$

Therefore $3n - 8$ is an odd integer.

□

Proof. Here is where your proof/explanation goes!
The theorem is that an odd integer n obtainable with an integer k as 2k+1 will always produce an odd integer by computing 3n-8. The hypothesis is that n is an odd integer obtainable as 2k+1. The conclusion is 3n-8 is an odd integer. If n is an odd integer, then 3n-8 is an integer.

□

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Problem 4. Practice with conjecture: Suppose $f(x) = e^{3x}$ for $x \in \mathbb{R}$. Find the first 5 derivatives of f. Identify the pattern and formulate a conjecture that appears to be true. Write your conjecture in the form: *If n is a natural number, then ....*

Proof. Here is where your proof/explanation goes!

If n is an a natural number the nth derivative of the mapping f(x)|->e^3x over the field of reals is 3^n e^(3 x) .

I used Table[D[E^(3 x), {x, n}], {n, 10}] to compute the first 10 derivatives in Mathematica as

{3E^(3 x), 9 E^(3 x), 27 E^(3 x), 81 E^(3 x), 243 E^(3 x),

729 E^(3 x), 2187 E^(3 x), 6561 E^(3 x), 19683 E^(3 x),

59049 E^(3 x)}. I then used FindSequenceFunction to find the pattern 3^n E^(3 x).

Problem 5. Your proof may only use the definitions of even and odd and basic facts about integers.

Theorem 2. *If x is an even integer, then* $3x^2 + 2x + 3$ *is an odd integer.*

Proof. Here is where your proof/explanation goes!

We can use the mapping x|->2k to transform the polynomial expression $3x^2 + 2x + 3$ into 3+6k+12k^2. We need to show that this meets the form of an odd integer. We can partition the expression into a smaller expression with a factor of 2 and 1: 2(1+3k+6k^2)+1. This is the form of an odd integer, and the theorem is proven.

$\square$