Problem Set 10

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Exercise 1

Statement

Prove Theorem 5.31. Let Λ be a nonempty indexing set, let $\mathscr{A} = \{A_\alpha \mid \alpha \in \Lambda\}$ be an indexed family of sets, and let B be a set. Then

$$B \cup \left(\bigcap_{\alpha\in\Lambda} A_\alpha\right) = \bigcap_{\alpha\in\Lambda} (B \cup A_\alpha)$$

Proof at 3:20 pm

Let $b \in B \cup (\bigcap_{\alpha\in\Lambda} A_\alpha)$.

Then b∈B or b∈$(\bigcap_{\alpha\in\Lambda} A_\alpha)$. Then b∈B or $\boldsymbol{\forall_{\alpha,\alpha\in\Lambda}\, b \in A_\alpha}$.

We know $b \in \bigcap\limits_{\alpha\in\Lambda} B \cup A_\alpha$ if $\boldsymbol{\forall_{\alpha,\alpha\in\Lambda}\, b \in (B \cup A_\alpha)}$. We know this is true if $\boldsymbol{\forall_{\alpha,\alpha\in\Lambda}\, b \in B \text{ or } b \in A_\alpha}$. We assumed that b∈B or $\boldsymbol{\forall_{\alpha,\alpha\in\Lambda}\, b \in A_\alpha}$, so we can conclude that $b \in \bigcap\limits_{\alpha\in\Lambda} B \cup A_\alpha$. We have know proven that $B \cup (\bigcap_{\alpha\in\Lambda} A_\alpha) \subseteq \bigcap_{\alpha\in\Lambda} (B \cup A_\alpha)$. We will now prove $B \cup (\bigcap_{\alpha\in\Lambda} A_\alpha) \supseteq \bigcap_{\alpha\in\Lambda} (B \cup A_\alpha)$. Let $b \in \bigcap\limits_{\alpha\in\Lambda} (B \cup A_\alpha)$. Then $\boldsymbol{\forall_{\alpha,\alpha\in\Lambda}\, b \in (B \cup A_\alpha)}$. Then $\boldsymbol{\forall_{\alpha,\alpha\in\Lambda}\, b \in B \text{ or } b \in A_\alpha}$. Then $b \in B$ or $b \in (\bigcap_{\alpha\in\Lambda} A_\alpha)$. Therefore $b \in B \cup (\bigcap_{\alpha\in\Lambda} A_\alpha)$. Therefore $B \cup (\bigcap_{\alpha\in\Lambda} A_\alpha) \supseteq \bigcap_{\alpha\in\Lambda} (B \cup A_\alpha)$. We have proved that $B \cup (\bigcap_{\alpha\in\Lambda} A_\alpha) = \bigcap_{\alpha\in\Lambda} (B \cup A_\alpha)$.

Proof

Approach

We will prove that $B \cup (\bigcap_{\alpha\in\Lambda} A_\alpha) = \bigcap_{\alpha\in\Lambda} (B \cup A_\alpha)$ by first proving that $B \cup (\bigcap_{\alpha\in\Lambda} A_\alpha) \subseteq \bigcap_{\alpha\in\Lambda} (B \cup A_\alpha)$. We will then prove that $B \cup (\bigcap_{\alpha\in\Lambda} A_\alpha) \supseteq \bigcap_{\alpha\in\Lambda} (B \cup A_\alpha)$.

Let $b \in B \cup (\bigcap_{\alpha\in\Lambda} A_\alpha)$. We recall the definition of $B \cup (\bigcap_{\alpha\in\Lambda} A_\alpha)$.

Definition

Let Λ be a nonempty indexing set and let $\mathscr{A} = \{A_\alpha \mid \alpha \in \Lambda\}$ be an indexed family of sets. The intersection over $\mathcal{A}$ is the set of all elements that are in all of the sets A_α for each $\alpha\in\Lambda$. That is,

$$\bigcap_{\alpha\in\Lambda} A_\alpha = \{x \in U \mid \forall_{\alpha,\alpha\in\Lambda}\, x \in A_\alpha\}$$

We also have

$$\bigcap_{\alpha\in\Lambda} B \cup A_\alpha = \{x \in U \mid \forall_{\alpha,\alpha\in\Lambda}\, x \in (B \cup A_\alpha)\}$$

Using the definition

Since $b \in B \cup (\bigcap_{\alpha\in\Lambda} A_\alpha)$. b∈B or b $\in \bigcap_{\alpha\in\Lambda} A_\alpha$. The statement b $\in \bigcap_{\alpha\in\Lambda} A_\alpha$ implies $\forall_{\alpha,\alpha\in\Lambda}\, b \in A_\alpha$.

We will now look at the definition of the right hand side of$\{x \in U \mid \forall_{\alpha,\alpha\in\Lambda}\, x \in (B \cup A_\alpha)\}$. We know $x \in (B \cup A_\alpha)$ means that x∈B or $x \in A_\alpha$. Since $\forall_{\alpha,\alpha\in\Lambda}\, b \in A_\alpha$, b is an element of $B \cup A_\alpha$ for all α such that $\alpha\in\Lambda$. We have now shown that b $\in \bigcap_{\alpha\in\Lambda} A_\alpha \Rightarrow \forall_{\alpha,\alpha\in\Lambda}\, b \in (B \cup A_\alpha) \Rightarrow b \in \bigcap_{\alpha\in\Lambda} B \cup A_\alpha \Rightarrow B \cup (\bigcap_{\alpha\in\Lambda} A_\alpha) \subseteq \bigcap_{\alpha\in\Lambda} B \cup A_\alpha$

We use the fact that $\forall_{\alpha,\alpha\in\Lambda}\, b \in A_\alpha$ to conclude that b∈

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$$\int x\, dx + \sqrt{z}$$

$$\frac{x^2}{2} + \sqrt{z}$$

- Enter bulleted item text here.

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Enter text here. Enter formula for display in a separate cell below:

$$\int x\, dx + \sqrt{z}$$

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1.1. Enter numbered subitem text here.

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1.1.1. Enter subitem text here.

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Enter text here. Enter formula for numbered display in a separate cell below:

$$\int x\, dx + \sqrt{z} \tag{1}$$

Enter text here. Enter Wolfram Language program code below.

```
fun[x_]:=1
```

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```
DLLEXPORT int fun(WolframLibraryData libData, mreal A1, mreal *Res)
{
 mreal R0_0;
 mreal R0_1;
 R0_0 = A1;
 R0_1 = R0_0 * R0_0;
 *Res = R0_1;
 funStructCompile->WolframLibraryData_cleanUp(libData, 1);
 return 0;
}
```