MTH 300: Intro to Higher Mathematics - Fall 2022 Problem Set 1

Due Wednesday, August 24
Last Name, First Name

Problem 1. For each of the following sentences, determine if the sentence is a *statement*. For each *statement*, determine if the statement is true or false. Give a brief justification for each response.

1. The Cleveland Browns is a good football team. This is not a statement. Two people may disagree on the meaning of this statement and draw different conclusions.

2. The capitol of WV is Charleston. This is a statement because either WV's capital is Charleston or it isn't. This is a true statement.

3. $x \geq -200$. This is a not a statement because x is not defined and so the statement could be true or false depending on the value of x.

4. $-3 \geq -200$. This is a statement because there is no ambiguity. This is a true statement.

5. There exists a real number x such that $x^2 + 1 = 0$. This is a statement because all parts are clearly defined and there is no ambiguity. This is a false statement.

Proof. Here is where your proof/explanation goes! □

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Problem 2. For each of the following conditional statements identify the hypothesis and the conclusion.

1. If squares have three sides, then triangles have four sides. The hypothesis is squares have three sides and the conclusion is triangles have four sides.

2. If the sequence a is convergent, then a is bounded. The hypothesis is the sequence a is convergent, while the conclusion is that the sequence a is bounded.

3. If you have a GPA of 3.0, then you will graduate with honors. Hypothesis: you have a GPA of 3.0 Conclusion: you graduate with honors. The hypothesis is that you have a 3.0 GPA. The conclusion is you graduate from Marshall with honors.

4. If you do your homework, then you will pass your math class. The hypothesis is you finish your homework. The conclusion is your pass your math class.

Proof. Here is where your proof/explanation goes!

□

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Problem 3. (Text pg 12 #4) Determine which value(s) of *a* make each of the following conditional statements true and justify your choice:

1. If $a + 2 = 5$, then $8 < 5$. a is not equal to 3.

2. If $5 < 8$, then $a + 2 = 5$. a is equal to 3.

Proof. Here is where your proof/explanation goes! □

Problem 4. (Text pg 14 #8) Following are two theorems, one regarding cubic functions and one regarding x -intercepts. Using only these theorems and basic algebraic manipulations state any possible conclusions about the x -intercepts of the functions below, or explain why you don't have enough information to make a conclusion. Justify your conclusion clearly using the theorems.

Theorem 1. *If f is a cubic function of the form* $f(x) = x^3 - x + b$ *and* $b > 1$*, then the function f has exactly one x -intercept.*

Theorem 2. *If f and g are functions with* $g(x) = k \cdot f(x)$*, where k is a nonzero real number, then f and g have exactly the same x -intercepts.*

1. $f(x) = x^3 - x + 7$ The function f(x) has exactly one x intercept because the hypothesis of theorem 1 that b>1 is satisfied.

2. $g(x) = x^3 + x + 7$ We cannot make a logical deduction about how many x-intercepts of the function g(x) because Theorem 1 uses implies of propositional logic, not iff. If Theorem 1 had a converse that was proved true, a deduction could be made.

3. $h(x) = -x^3 + x - 5$ We cannot conclude anything.

Proof. Here is where your proof/explanation goes!

(d) $k(x) = 2x^3 + 2x + 3$ we cannot conclude anything.

(e) $r(x) = x^4 - x + 11$ we cannot conclude anything.

(f) $F(x) = 2x^3 - 2x + 7$ we can factor this to 2(x^3-x+3.5) to check against Theorem 1 and 3.5>1 satisfies the condition therefore this polynomial has exactly one intercept.

□

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Problem 5. Answer each of the following, with a brief sentence to justify.

1. Is the set of natural numbers closed under addition? Yes If you add any two natural numbers, you will always get a bigger natural number.

2. Is the set of natural numbers closed under subtraction? No, the set of natural numbers is not closed under subtraction because if you subtract a bigger natural number from a smaller natural number you will get a negative number.

3. Is the set of even integers closed under addition? Yes, the set of even integers is closed under addition because the sum of any number of even integers is an even integer.

4. Is the set of odd integers closed under subtraction? The set of odd numbers is not closed under subtraction, because every odd number is separated by an even distance. For example 3-1=2, which is even.

Proof. Here is where your proof/explanation goes! □