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$$\int x\,dx + \sqrt{z}$$

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Problem Set 6

Problem 1

Theorem 1. Let x, y, z $\in \mathbb{Z}$. If x does not divide y times z, then x does not divide z.

1. Explain why proving this theorem using a direct proof method is challenging. Use 1-2 complete sentences.

Here's why we can't use the direct proof. We can't formulate the statement x does not y×z with a finite number of logical quantifiers, because there are countably infinite integers.

Let x, y, and z be integers. Let x not divide y×z. Proving that x does not divide z follows from the statement x does not divide y×z is challenging because the integers are infinitely large, specifically countably infinite.

To prove the theorem if x does not divide y×z directly, x does not divide z, we would have to use the definition of x does not divide y×z, known as there does not exist an integer H_1 such that $y \times z = x \times H_1$. $y \times z = x \times H_1$ ($\exists_{H_1 \in \mathbb{Z}}\, y \times z = x \times H_1$). This means , $y \times z \neq x \times H_1$.

2. Carefully write the contrapositive of the theorem.

We will prove the theorem through the contrapositive proof technique. The contrapositive is that given a set of integers x, y, z, if x divides z then x divides y×z.

3. Prove the theorem by contrapositive.

Proof

Since x divides z, there exists an integer H_1 such that $z = x \times H_1$ ($\exists_{H_1 \in \mathbb{Z}}\, z = x \times H_1$).

Let x, y, and z be integers. Let x divide z. Then there exists an integer H_1 such that z equals x times H_1 ($\exists_{H_1 \in \mathbb{Z}}\, z = x \times H_1$). Our goal is to prove that x divides y×z. If x divides y×z, then there exists an integer H_2 such that y times z is equal to x times H_2 ($\exists_{H_2 \in \mathbb{Z}}\, y \times z = x \times H_2$). We substitute x times H_1 for z in the expression y×z to get $y \times x \times H_1$

We want $y \times x \times H_1$ to be in the form $x \times H_2$. H_2 is the integer $y \times H_1$. The group of integers with the operation of multiplication is closed under the operation of multiplication, so H_2 is an integer.

We have proved that there exists an integer H_2 such that y times z is equal to x times H_2 ($\exists_{H_2 \in \mathbb{Z}}\, y \times z = x \times H_2$). We have proved x|y×z from x|z. This proves the original theorem because the contrapositive is true.

Problem 2

Problem 2. Prove: Let a, b $\in \mathbb{Z}$. Then a b is odd if and only if a is odd and b is odd.

Proof

We will prove the iff statement by proving a b is odd if a is odd and b is odd and a is odd and b is odd if a b is odd.

If a is odd, then $\exists_{b_1 \in \mathbb{Z}}\, a = 2 b_1 + 1$. If b is odd then $\exists_{b_2 \in \mathbb{Z}}\, b = b_2 + 1$. We then see $a \times b = (2 b_1 + 1)(2 b_2 + 1) = 4 b_1 b_2 + 2 b_1 + 2 b_2 + 1 = 2 (b_1 b_2 + b_1 + b_2) + 1$. We have proved $\exists_{b_3 \in \mathbb{Z}}\, a \times b = 2 b_3 + 1$. To be specific, $b_3 = b_1 b_2 + b_1 + b_2$. We have finished proving if a is an odd integer and b is odd integer, then a×b is an odd integer.

Furthermore, if a×b is odd, $\exists_{b_4 \in \mathbb{Z}}\, a \times b = 2 b_4 + 1$. We aim to prove that a is odd and b is odd. We will show that allowing a or b to be even leads to a logical contradiction. If a is even, $\exists_{b_5 \in \mathbb{Z}}\, a = 2 b_5$. If b is even $\exists_{b_6 \in \mathbb{Z}}\, b = 2 b_6$. Then $a \times b = (2 b_5) \times (2 b_6)$. Then $a \times b = 4 b_5 b_6$. Then $4 b_5 b_6 = 2 b_4 + 1$. The only set of values that satisfies this equation has at least one rational number for b_4, b_5, b_6. For example, if b_5 is 1 and b_4 is 1, then the value of b_6 that makes $4 b_5 b_6 = 2 b_4 + 1$ or $4 (1) b_6 = 2 (1) + 1 = 4 b_6 = 3$ is b_6=4/3. This leads to a logical contradiction because we assumed b_6 is an integer and 4/3 is not an integer. We assumed that b_5 or b_6 is an integer such that $a = 2 b_5$ and $b = 2 b_6$ which led to nonsense. Therefore a and b must always be odd. We have shown that if a×b is odd, a and b must be odd.

We have proven a×b is odd iff a is odd and b is odd.

Problem 3

Problem 3: Prove: There exists x∈ℝ such that $x^2 - 8 < 2 x$.

Proof

We will find an instance of x∈ℝ such that $x^2 - 8 < 2 x$ to prove $\exists_{x, x \in \mathbb{R}}\, x^2 - 8 < 2 x$. We have x=0. The number 0 is a real number and $0^2 - 8$=-8 and -8<2(0) and -8<0, so the inequality holds. We have proven $\exists_{x, x \in \mathbb{R}}\, x^2 - 8 < 2 x$!

Problem 4

Problem 3: Prove: There exists x, y∈ℝ such that $\sqrt{x^2 + y^2} \le 2 x y$.

Proof

We will find an instance of x, y∈ℝ such that $\sqrt{x^2 + y^2} \le 2 x y$ to prove $\exists_{x, x \in \mathbb{R}}\, \exists_{y, y \in \mathbb{R}}\, \sqrt{x^2 + y^2} \le 2 x y$. We have x=0 and y=0. The number 0 is a real number. We see $\sqrt{(0)^2 + (0)^2} \le 2 (0) (0)$. Next $\sqrt{0} \le 0$. Next $0 \le 0$, so the inequality holds with x=0 and y=0. We have proven $\exists_{x, x \in \mathbb{R}}\, \exists_{y, y \in \mathbb{R}}\, \sqrt{x^2 + y^2} \le 2 x y$!

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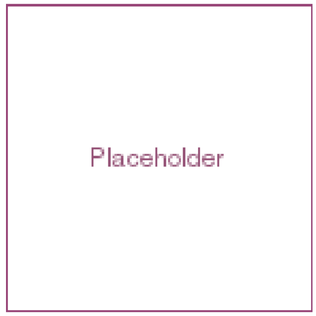

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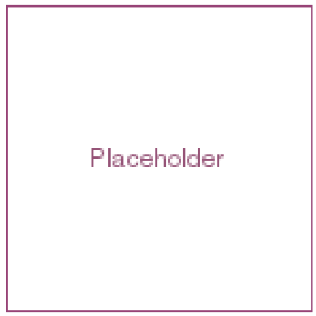

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Corollary 1. *Enter corollary text here.*

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Definition 1. *Enter definition text here.*

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Proposition 1. *Enter proposition text here.*

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Solution 1. Enter solution text here.

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Proof 1. Enter proof text here.

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