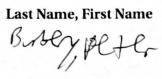
# MTH 300: Intro to Higher Mathematics - Fall 2022 Problem Set 4 Due Monday, September 12



**Problem 1**. For each statement below, explain why it is true (one or two sentences should be enough) or give a *counterexample*, that is, a specific example showing why the statement is false.

- 1. For every  $x \in \mathbb{R}$ ,  $x^2 + 1 \ge 1$ .
- 2. For every  $x \in \mathbb{N}$ ,  $\sqrt[3]{x}$  is irrational.
- 3. For every  $x \in \mathbb{R}$ , x > 0 or x < 0.

<i>Proof.</i> Here is where your proof/explanation goes!
For every $x \in \mathbb{R}, x^2 +  z $ is the.
we say we manipulate to get x20 this statement is true seemse we would have to have as a complex number with a non zero imaginary pat greate than a or less than a and a new real part to make x2 + 121 table. The domain is well himself not imaginary numbers, so the statement is true.
The statement for every natural number x the cose root of x is irrational is false for all perfect coses for example I because the all the number of is a real number, but its not
1855 Them O or greatly then O, so the statement for levery real num slow.  X, X, ZO or XCO, S false.

### **Problem Set 4**

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**Problem 2**. For each statement below, give a specific example showing that the statement is true, or explain why there is no such example, thereby making the statement false.

- 1. There exists  $x \in \mathbb{R}$  such that  $x^2 + 2x + 4 = 0$ .
- 2. There exist two irrational numbers x and y such that xy is rational.
- 3. There exists a positive integer n such that 3n < 3.

<i>Proof.</i> Here is where your proof/explanation goes!
The startment cethere exists a real number x
such that x2+ 2x+4=00 15 200" is false
bleause the vots of the polynomial x 4 Caty
at ist3-1 and -ist3-1 or in trig form
2 (cos(-2)+ isin(-25)) and 2 [cos(25)+isin(25)]
2[cos(-la)+isin(-la)] and 2[cos(27)+isin(2)] or in exponential form [2e(2)71)03 and (2i7)/3
2e the statement is false.
The statement the exist two wational rumbers x and y such that xy is brational is the for example $\sqrt{2}$ , $\sqrt{2}=2$ $\sqrt{2}$ is rational and $2$ is tational
The statement 3n < 3 is egiglisted Egisvalent to n<1. There are no positive integers less than 1 so the state ment "There exists a positive integer n  with that 3n < 3 n is take

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**Problem 3**. Define a new logical connective  $\Delta$  by the truth table:

		1
p	q	$p\Delta q$
Т	Т	F
T	F	F
F	Т	F
F	F	T

- 1. Prove that  $\neg p \equiv p\Delta p$  using a truth table.
- 2. Find a compound statement containing only p,q, parentheses, and  $\Delta$  (some or all may appear more than once) that is logically equivalent to  $p \vee q$ . Prove your assertion with an appropriate truth table.

### **Problem Set 4**

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**Problem 4**. For each of the following quantified statements, write the statement in symbols, negate the statement in symbols, and write the negation in words.

- 1. For each  $x \in \mathbb{Z}$ ,  $x \ge 3$  or  $x \le 2$ .
- 2. There exists  $y \in \mathbb{R}$  such that y + 4 > 8 and  $y \le 4$ .
- 3. For each  $x \in \mathbb{R}$ , there exists  $y \in \mathbb{R}$  such that  $x \neq y$  and  $x y \in \mathbb{Z}$ .

<i>Proof.</i> Here is where your proof/explanation goes!	
1. For all integers x, xZ3 or x<2.	<u></u>
$\forall x \in \mathbb{Z} \times \mathbb{Z} \exists Y x \in \mathbb{Z}$	
$\frac{x, x \in \mathcal{I}}{x, x \in \mathcal{I}} \times <3 \land x > 2$	
3 <sub>x,x</sub> & 2 2 c x < 3	
Theelxists an integer X such that x is greatly than 2 and less than 3.	
2. 7 (4+4>8) 1 (y < 4) There exist	25. Ser X
Y, YER (y+ 458) V (y > y) such that to Such that	cau .y. x=y
3 For all yorlar numbers y 4+4=8 or 4>4	ガッキアか
$\forall x \in \mathbb{R}  \forall y \in \mathbb{R}  (x \neq y) \land (x = y) \lor (x = y) \lor (x = y) \lor (x = y) \lor (x = y) )$	an intlylr

# MTH 300: Intro to Higher Mathematics - Fall 2022

**Problem 5**. Prove: For every integer a, if  $4 \mid (a-1)$ , then  $4 \mid (a^2-1)$ .

**Problem Set 4** 

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<i>Proof.</i> Here is where your proof/explanation goes!
Assume a is an integer, and 416-1).

Then the startement (9-1)= 4Kl for some integer Kl follows from the definition of divisibility that the The startment 4 (92-1) Complete is the iff 921=4J for some integer J.

This is algebrai carry the same as CA-1)(a+D=4) for some integel J.

The statement (a-1) = 4/1 and (a-1)=45 and (

Theefore, 41 (a2-1)