Problem Set 10

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Exercise 4

Let Λ be a nonempty indexing set, let $\mathcal{A} = \{A_\alpha \mid \alpha \in \Lambda\}$ be an indexed family of sets, and let B be a set. Use the results of Theorem 5.30 and Theorem 5.31 to prove each of the following:

$(\bigcap_{\alpha\in\Lambda} A_\alpha) - B = \bigcap_{\alpha\in\Lambda} (A_\alpha - B)$

Proof

We will prove the equality by proving $(\bigcap_{\alpha\in\Lambda} A_\alpha) - B \subseteq \bigcap_{\alpha\in\Lambda} (A_\alpha - B)$ and $(\bigcap_{\alpha\in\Lambda} A_\alpha) - B \supseteq \bigcap_{\alpha\in\Lambda} (A_\alpha - B)$.

Let $b \in (\bigcap_{\alpha\in\Lambda} A_\alpha) - B$. Then b $\in$ $(\bigcap_{\alpha\in\Lambda} A_\alpha)$. Then by the definition of the intersection over an indexed family of sets, $\forall_{\alpha,\alpha\in\Lambda}\ b \in A_\alpha$. We also know b∉B. Then $b \in A_\alpha\ - B\ \forall_{\alpha,\alpha\in\Lambda}$. Then since $b \in A_\alpha\ - B\ \forall_{\alpha,\alpha\in\Lambda}$, b∈ $\bigcap_{\alpha\in\Lambda} (A_\alpha - B)$. This proves $(\bigcap_{\alpha\in\Lambda} A_\alpha) - B \subseteq \bigcap_{\alpha\in\Lambda} (A_\alpha - B)$.

We will prove $(\bigcap_{\alpha\in\Lambda} A_\alpha) - B \supseteq \bigcap_{\alpha\in\Lambda} (A_\alpha - B)$. Let b∈$\bigcap_{\alpha\in\Lambda} (A_\alpha - B)$. Then $b \in A_\alpha$ and b∉B $\forall_{\alpha,\alpha\in\Lambda}$. Then $b \in A_\alpha\ \forall_{\alpha,\alpha\in\Lambda}$. Then $b \in (\bigcap_{\alpha\in\Lambda} A_\alpha)$ and b∉B. Then b∈$(\bigcap_{\alpha\in\Lambda} A_\alpha) - B$. This proves $(\bigcap_{\alpha\in\Lambda} A_\alpha) - B \supseteq \bigcap_{\alpha\in\Lambda} (A_\alpha - B)$.

Since $(\bigcap_{\alpha\in\Lambda} A_\alpha) - B \subseteq \bigcap_{\alpha\in\Lambda} (A_\alpha - B)$ and $(\bigcap_{\alpha\in\Lambda} A_\alpha) - B \supseteq \bigcap_{\alpha\in\Lambda} (A_\alpha - B)$, $(\bigcap_{\alpha\in\Lambda} A_\alpha) - B = \bigcap_{\alpha\in\Lambda} (A_\alpha - B)$.

Exercise 5

Statement

Prove Theorem 5.31. Let Λ be a nonempty indexing set, let $\mathcal{A} = \{A_\alpha \mid \alpha \in \Lambda\}$ be an indexed family of sets, and let B be a set. Then

$$\boldsymbol{B} \cup \left(\bigcap_{\alpha\in\Lambda} \boldsymbol{A}_\alpha\right) = \bigcap_{\alpha\in\Lambda} (\boldsymbol{B} \cup \boldsymbol{A}_\alpha)$$

Proof

Let $b \in B \cup (\bigcap_{\alpha\in\Lambda} A_\alpha)$.

Then b∈B or b∈$(\bigcap_{\alpha\in\Lambda} A_\alpha)$. Then b∈B or for all α that are elements of Λ, $b \in A_\alpha(\boldsymbol{\forall_{\alpha,\alpha\in\Lambda}\ b \in A_\alpha})$.

We assumed that b∈B or $\boldsymbol{\forall_{\alpha,\alpha\in\Lambda}\ b\in A_{\alpha}}$, so we can conclude that $\boldsymbol{\forall_{\alpha,\alpha\in\Lambda}\ b\in(B\cup A_{\alpha})}$. Then $b\in\bigcap_{\alpha\in\Lambda}B\cup A_{\alpha}$. We have know proven that $B\cup(\bigcap_{\alpha\in\Lambda}A_{\alpha})\subseteq\bigcap_{\alpha\in\Lambda}(B\cup A_{\alpha})$. We will now prove $B\cup(\bigcap_{\alpha\in\Lambda}A_{\alpha})\supseteq\bigcap_{\alpha\in\Lambda}(B\cup A_{\alpha})$. Let $b\in\bigcap_{\alpha\in\Lambda}(B\cup A_{\alpha})$. Then $\boldsymbol{\forall_{\alpha,\alpha\in\Lambda}\ b\in(B\cup A_{\alpha})}$. Then $\boldsymbol{\forall_{\alpha,\alpha\in\Lambda}\ b\in B}$ **or** $\boldsymbol{b\in A_{\alpha}}$. Then $b\in B$ or $b\in(\bigcap_{\alpha\in\Lambda}A_{\alpha})$. Therefore $b\in B\cup(\bigcap_{\alpha\in\Lambda}A_{\alpha})$. Therefore $B\cup(\bigcap_{\alpha\in\Lambda}A_{\alpha})\supseteq\bigcap_{\alpha\in\Lambda}(B\cup A_{\alpha})$. We have proved that $B\cup(\bigcap_{\alpha\in\Lambda}A_{\alpha})=\bigcap_{\alpha\in\Lambda}(B\cup A_{\alpha})$.