4.1 Let Y be a random variable with p(y) given in the table below. [(This is an Entirety]

| y | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| p(y) | 0.4 | 0.3 | 0.2 | 0.1 |

{a} Give the distribution.

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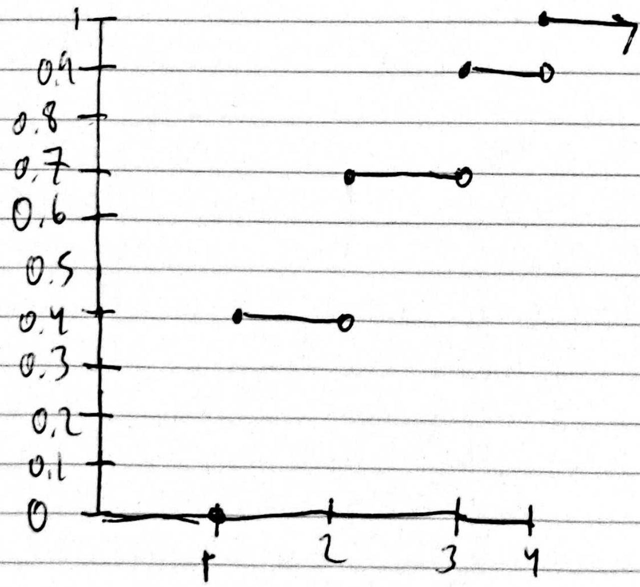
| y | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| p(y) | 0.4 | 0.3 | 0.2 | 0.1 |

{a} give the distribution function F(y). Be sure to specify the value of F(y) for all y $-\infty < y < \infty$

| y | $y < 1$ | $1 \le y < 2$ | $2 \le y < 3$ | $3 \le y < 4$ | $4 \le y < \infty$ |
|---|---|---|---|---|---|
| $F(y) = \sum p(y)$ | 0 | 0.4 | 0.7 | 0.9 | 1 |

{b} Sketch the distribution function in part {a}.



4.6 Consider a random variable with a geometric distribution (Section 3.5); that is,

$$p(y) = q^{y-1}p \quad y = 1, 2, 3, \ldots, \quad 0 < p < 1$$

{a} Show that $Y$ has a distribution function $F(y)$ such that $F(i) = 1 - q^i$ $i = 0, 1, 2\ldots$ and that, in general,

$$F(y) = \begin{cases} 0 & y < 0 \\ 1 - q^i & i \le y < i+1, \text{ for } i = 0, 1, 2, \ldots \end{cases}$$

$$\int q^{y-1} p \, dy = \frac{q^{y-1+1}}{y-1+1} p = \frac{q^y}{y}$$

The probability function for $Y$ is given by

$$p(y) = q^{y-1}p = (1-p)^{y-1}p = q^{y-1}(1-q) \quad y = 1, 2, 3\ldots,$$

$0 < p < 1$

This yields

$$p(0) = q^{0-1}p = (1-p)^{-1}p$$
$$p(1) = q^{1-1}p = q^0 p = p$$
$$p(2) = q^{2-1}p = q^1 p = qp$$
$$p(3) = q^{3-1}p = q^2 p$$
$$p(4) = q^{4-1}p = q^3 p$$

We have

$$F(0) = 1 - q^i \text{ when } i = 0 \text{ and } y = 0 \ge i = 0 \text{ and } y < 0+1$$
$$= 1 - q^0 = 1 - 1 = 0$$

This makes sense because the cumulative probability at 0 is 0.

$F(1) = 1 - q^1$ with $i \le y < i+1$ @ $i = 1$

We use the probabilities above to check this

We have $1 - q^1 = 1 - q^1 = 1 - q$ and

$$\sum_{n=1}^{1} p(n) = p(1) = p = 1 - q \text{ which equals } 1 - q$$

We will verify two more cases

LHS | RHS

$$\sum_{n=1}^{2} p(n) = p(1)+p(2) = p+qp$$

$$= (1-q)+(1-q)q$$

$$= 1-q+q-q^2$$

$$= 1-q^2$$ equal

$1-q^2$

$-(q-1)(q+1)$

$$\sum_{n=1}^{3} p(n) = p(1)+p(2)+p(3) = p+qp+q^2p =$$

$$(1-q)+q(1-q)+q^2(1-q)$$

$$\underbrace{1-q+q}_{\text{cancel}}\underbrace{-q^2+q^2}_{\text{cancel}}-q^3$$

$1-q^3 = 1-q^i$ @ $i=3$ $= 1-q^3$

If $F(y)$ is a distribution function, then

1. $F(-\infty) = \lim_{y \to -\infty} F(y) = 0$

$$\lim_{y \to -\infty} 1-q^y = 1-q^{-\infty} = 1-\frac{1}{q^\infty} =$$ (never mind) The piecewise definition makes it zero at ~~zero~~ negative inputs and below. $1-q^i$ only applies for positive numbers or zero

2. $\lim_{y \to \infty} F(y) = 1-q^\infty$ $q$ is a fraction between 0 and 1. Raising a fraction to infinity between 0 and 1 results in 0. for example $\left(\frac{1}{5}\right)^\infty$ assuming $0<\frac{1}{5}<1$ is 0.

3. $F(y)$ is a nondecreasing function of $y$. $1-q^i \le 1-q^{i+1}$

therefore $1-qq^i$ is $\ge 1-q^i$ $\quad 1-q^i \le 1-q^iq$

$q$ makes the term subtracted ~~bigger~~ smaller

4.16 Let $Y$ possess a density function $f(y) = \begin{cases} c(2-y) & 0 \le y \le 2 \\ 0 & \text{elsewhere} \end{cases}$

{a} Find $c$. we need to integrate from $-\infty$ to $\infty$ and solve for 1

$$\int_{-\infty}^{\infty} f(y)\,dy = 1 = \int_0^2 c(2-y)\,dy = \int_0^2 2c - yc\,dy = 2cy - \frac{y^2}{2}c\Big|_0^2$$

$$= \left(2c(2) - \frac{(2)^2}{2}c\right) - \left(2c(0) - \frac{(0)^2}{2}c\right) = \left(4c - \frac{4}{2}c\right) -$$

$$\left(0c - \frac{0}{2}c\right) = (4c - 2c) - (0c - 0c) = (2c) - 0c = 2c = 1$$

$$c = \frac{1}{2} \quad f(y) = \begin{cases} \frac{1}{2}(2-y) & 0 \le y \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

{b} Find $F(y) = \int f(y)\,dy = \begin{cases} \int \frac{1}{2}(2-y)\,dy & 0 \le y \le 2 \\ 0 & \text{elsewhere} \end{cases}$

$$= \begin{cases} \int 1 - \frac{y}{2} & 0 \le y \le 2 \\ 0 & \text{elsewhere} \end{cases} = \begin{cases} y - \frac{y^2}{2}\left(\frac{1}{2}\right) & 0 \le y \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

$$\boxed{F(y) = \begin{cases} y - \frac{y^2}{4} & 0 \le y \le 2 \\ 0 & \text{elsewhere} \end{cases}}$$

{c} Graph $f(y)$ and $F(y)$

plotted in Mathematica

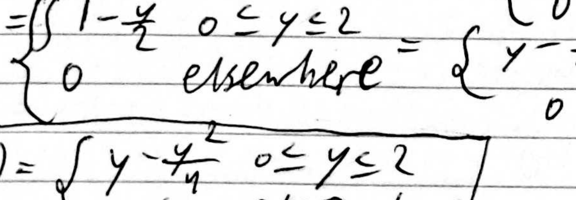
{d} Use $F(y)$ in part {b} to find $P(1 \le Y \le 2) = F(2) - F(1)$

$$F(2) = 2 - \frac{2^2}{4} = 2 - \frac{4}{4} = 1$$

$$F(1) = 1 - \frac{1^2}{4} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\boxed{\begin{aligned} P(1 \le Y \le 2) &= F(2) - F(1) \\ &= 1 - \frac{3}{4} = \frac{1}{4} \end{aligned}}$$

{e} Use geometry and the graph for $f(y)$ to calculate $P(1 \le Y \le 2)$




$$A = \frac{1}{2}bh = \frac{1}{2}\left(\frac{1}{2}\right)(1) = \frac{1}{4}$$

$$\boxed{P(1 \le Y \le 2) = \frac{1}{4}}$$

4.20 If, as in exercise 4.16, Y has density function

$$f(y) = \begin{cases} \frac{1}{2}(2-y) & 0 \le y \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

find the mean and variance of Y.

mean $= \mu = E(Y) = m^{(1)}(0)$

$$m(t) = \int_{-\infty}^{\infty} e^{tx} \tfrac{1}{2}(2-y)\,dy = \frac{-2t + e^{2t} - 1}{2t^2}$$

$$\mu_1 \text{ from } m^{(1)}(t) = \frac{1 - e^{2t} + t + e^{2t}t}{t^3}$$ I used Mathematica for this.

$$\mu_1' = m^{(1)}(0) = \frac{1 - e^{2t} + t + e^{2t}t}{t^3}\bigg|_{t=0} = \frac{2}{3}$$

$$m^{(2)}(t) = \frac{-3 + 3e^{2t} - 2t - 4e^{2t}t + 2e^{2t}t^2}{t^4}$$

$$\mu_2' = m^{(2)}(0) = m^{(2)}(t)\Big|_{t=0} = \frac{2}{3}$$

$$V(Y) = \mu_2' - \mu_1'^2 = \left(\frac{2}{3}\right) - \left(\frac{2}{3}\right)^2 = \frac{2}{3} - \frac{4}{9} = \frac{2}{9}$$

$$E(Y) = \mu_1' = \frac{2}{3}$$

4.51 done in Mathematica

4.52 done in Mathematica

4.73 done in Mathematica

4.88 done in Mathematica

4.90 done in Mathematica

4.94 done in Mathematica