Homework 6

STA 445 Probability and Statistics I

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4.16

In[110]:=

$$f = y \mapsto \begin{cases} \frac{1}{2}(2-y) & 0 \le y \le 2 \\ 0 & !\,(0 \le y \le 2) \end{cases}$$

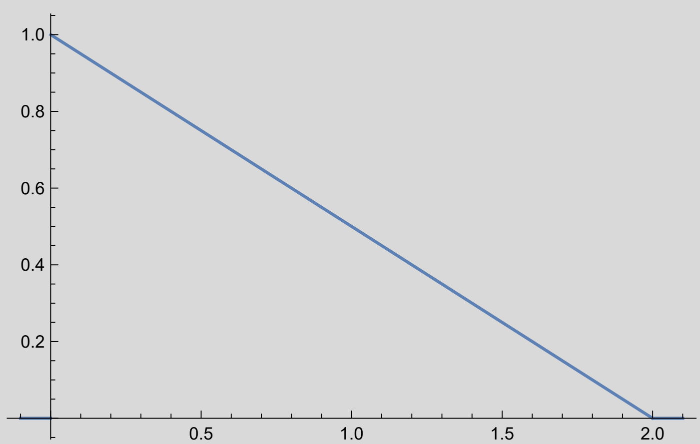
Out[110]=

$$\text{Function}\left[y, \begin{cases} \frac{2-y}{2} & 0 \le y \le 2 \\ 0 & !\,0 \le y \le 2 \end{cases}\right]$$

In[111]:=

$$\text{Plot}\left[f[y], \left\{y, \frac{-1}{10}, 2 + \frac{1}{10}\right\}\right]$$

Out[111]=



In[113]:=

$$F = y \mapsto \begin{cases} y - \frac{y^2}{4} & 0 \le y \le 2 \\ 0 & !\,(0 \le y \le 2) \end{cases}$$

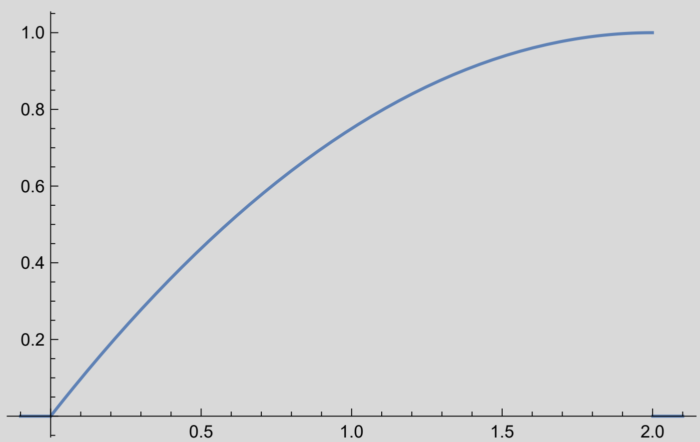
Out[113]=

$$\text{Function}\left[y, \begin{cases} y - \frac{y^2}{4} & 0 \le y \le 2 \\ 0 & !\,0 \le y \le 2 \end{cases}\right]$$

In[114]:=

$$\text{Plot}\left[F[y], \left\{y, \frac{-1}{10}, 2 + \frac{1}{10}\right\}\right]$$

Out[114]=



In[115]:=

```
F[2] - F[1]
```

Out[115]=

$$\frac{1}{4}$$

In[117]:=

```
f[2]
```

Out[117]=

```
0
```

In[118]:=

```
f[1]
```

Out[118]=

$$\frac{1}{2}$$

In[119]:=

$$\int_{-\infty}^{\infty} e^{(t\,x)}\, f[x]\, dx$$

Out[119]=

$$\frac{-1 + e^{2\,t} - 2\,t}{2\,t^2}$$

In[120]:=

$$\int_{-\infty}^{\infty} e^{(t\,x)}\, f[x]\, dx \;//\; \text{TraditionalForm}$$

Out[120]//TraditionalForm=

$$\frac{-2\,t + e^{2\,t} - 1}{2\,t^2}$$

In[121]:=

$$m = t \mapsto \int_{-\infty}^{\infty} e^{(t\,x)}\, f[x]\, dx$$

Out[121]=

$$\text{Function}\left[t,\ \int_{-\infty}^{\infty} e^{t\,x}\, f[x]\, dx\right]$$

In[122]:=

```
m'[0]
```

Out[122]=

$$\frac{2}{3}$$

In[123]:=

```
m[t]
```

Out[123]=

$$\frac{-1 + e^{2\,t} - 2\,t}{2\,t^2}$$

In[124]:=

```
m'[t]
```

Out[124]=

$$\frac{1 - e^{2\,t} + t + e^{2\,t}\,t}{t^3}$$

In[125]:=

```
m''[t]
```

Out[125]=

$$\frac{-3+3\,e^{2\,t}-2\,t-4\,e^{2\,t}\,t+2\,e^{2\,t}\,t^2}{t^4}$$

In[126]:=

```
m''[0]
```

Out[126]=

$$\frac{2}{3}$$

In[127]:=

```
m''[0] - m'[0]
```

Out[127]=

```
0
```

In[128]:=

```
m''[0] - m'[0]^2
```

Out[128]=

$$\frac{2}{9}$$

4.51

In[129]:=

```
cycleTimeDistribution =
 QuantityDistribution[UniformDistribution[{50, 70}], "Minutes"]
```

Out[129]=

```
QuantityDistribution[UniformDistribution[{50, 70}], min]
```

In[130]:=

```
Probability[c > 65 min  c > 55 min , c ≈ cycleTimeDistribution]
```

Out[130]=

$$\frac{1}{3}$$

4.52

mean

In[142]:=

```
Mean[cycleTimeDistribution]
```

Out[142]=

60 min

In[143]:=

```
Expectation[c, c ≈ cycleTimeDistribution]
```

Out[143]=

60 min

variance

In[146]:=

```
Variance[cycleTimeDistribution]
Variance[cycleTimeDistribution] // N
```

Out[146]=

$\frac{100}{3}\ \text{min}^2$

Out[147]=

$33.3333\ \text{min}^2$

In[148]:=

```
Expectation[(c - Mean[cycleTimeDistribution])^2, c ≈ cycleTimeDistribution]
Expectation[(c - Mean[cycleTimeDistribution])^2, c ≈ cycleTimeDistribution] // N
```

Out[148]=

$\frac{100}{3}\ \text{min}^2$

Out[149]=

$33.3333\ \text{min}^2$

4.73

In[150]:=

```
widthOfBoltsOfFabricDistribution =
 QuantityDistribution[NormalDistribution[950, 10], "Millimeters"]
```

Out[150]=

```
QuantityDistribution[NormalDistribution[950, 10], mm]
```

a

In[152]:=

```
Probability[947 mm ≤ p ≤ 958 mm, p ≈ widthOfBoltsOfFabricDistribution]
```

Out[152]=

$$\frac{1}{2}\left(\mathrm{Erf}\left[\frac{3}{10\sqrt{2}}\right]+\mathrm{Erf}\left[\frac{2\sqrt{2}}{5}\right]\right)$$

In[153]:=

```
NProbability[947 mm ≤ p ≤ 958 mm,
 p ≈ widthOfBoltsOfFabricDistribution, WorkingPrecision → 400]
```

Out[153]=

```
0.4060560236055559517309971913499218279633868735178010394564937353215478260301
 65522004980301806851643233292831613774072040925609448685459982549718237136500
 29912739891202927664519604694743372749597683541839395077105983118814539745509
 69904726925768965606310273435023002100114601273098692009238072944551348652718
 24703703356031087501074744992469181964402168665373797663657424865997019215444
 5006876323254397
```

b

In[154]:=

```
InverseCDF[widthOfBoltsOfFabricDistribution, 0.8531]
```

Out[154]=

```
960.498 mm
```

In[155]:=

```
InverseCDF[widthOfBoltsOfFabricDistribution, 0.8531] // FullForm
```

Out[155]//FullForm=

```
Quantity[960.4982190962642`, "Millimeters"]
```

4.88

In[157]:=
```
earthquakeMagnitudes = ExponentialDistribution[2.4]
```
Out[157]=
```
ExponentialDistribution[2.4]
```

a

In[158]:=
```
Probability[m > 3.0, m ≈ earthquakeMagnitudes]
```
Out[158]=
```
0.000746586
```

b

In[159]:=
```
Probability[2.0 < m < 3.0, m ≈ earthquakeMagnitudes]
```
Out[159]=
```
0.00748316
```

4.90

In[162]:=
```
Probability[m > 5.0, m ≈ earthquakeMagnitudes]
```
Out[162]=

6.14421×10^{-6}

In[163]:=
```
Probability[c ≥ 1,
 c ≈ BinomialDistribution[10, Probability[m > 5.0, m ≈ earthquakeMagnitudes]]]
```
Out[163]=
```
0.0000614404
```

4.94

a

In[165]:=

```
carbonMonoxide =
 QuantityDistribution[ExponentialDistribution[3.6], "PartsPerMillion"]
```

Out[165]=

```
QuantityDistribution[ExponentialDistribution[3.6], ppm]
```

In[167]:=

```
Probability[p > 9 ppm , p ≈ carbonMonoxide]
```

Out[167]=

8.48904×10^{-15}

b

In[168]:=

```
newCarbonMonoxide =
 QuantityDistribution[ExponentialDistribution[2.5], "PartsPerMillion"]
```

Out[168]=

```
QuantityDistribution[ExponentialDistribution[2.5], ppm]
```

In[170]:=

```
Probability[n ≥ 9 ppm , n ≈ newCarbonMonoxide]
```

Out[170]=

1.6919×10^{-10}