STA 445/545 – Homework #2 – Due at the beginning of class on Wednesday 9/14/2022

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1. Use this page as a cover letter for your completed homework assignment.
2. Add all produced figures and necessary explanations/comments.
3. Start each new problem on a new Page.
4. Due at the beginning of class on Wednesday 9/14 (5:00 PM)
5. Late submission will be penalized by 10% reduction per day after the deadline.
6. HW#2 weighs 30 points: I will select three questions to grade in detail based on accuracy and proficiency. Each graded problem weighs 8 points. The rest of the questions will be marked based on completion, effort, or attempt, and will weigh 6 points
7. **Please answer each question as directed.**

Complete and solve the following exercises:

2.2, 2.4, 2.6, 2.9, 2.10, 2.13, 2.15, 2.28, 2.29 & 2.32.

Best Wishes & Good Luck ☺

Exercise 2.2

Suppose that A and B are two events.

Write ~~down~~ expressions involving unions, intersections, complements that describe the following:

a. Both events occur.

$A \cap B$

b. At least one occurs

$A \cup B$

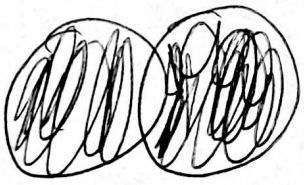
c. Neither occurs

$\overline{A \cup B} = \bar{A} \cap \bar{B}$

d Exactly one occurs

~~A B~~ $(A \cap \bar{B}) \cup (\bar{A} \cap B)$

Note this is the symmetric difference of A and B.

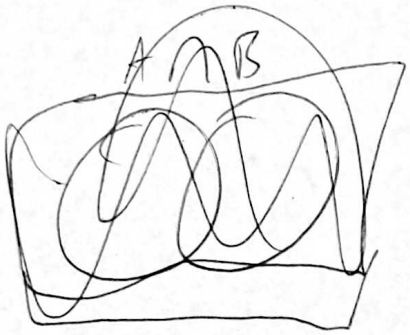


The symmetric difference is related to Boolean function xor/exclusive or/ $\oplus$.

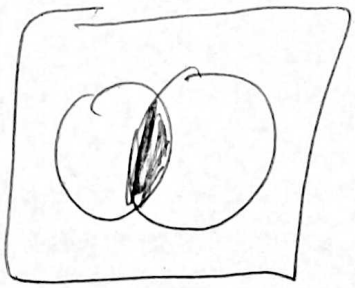
Exercise 2.4

If A and B are two sets, draw Venn diagrams to verify the following:

a: $A = (A \cap B) \cup (A \cap \bar{B})$

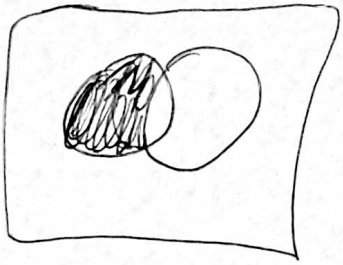


$A \cap B$

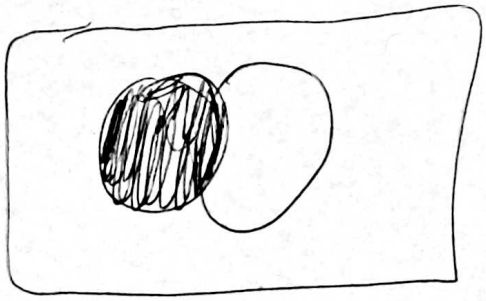


$\bar{B}$

$A \cap \bar{B}$



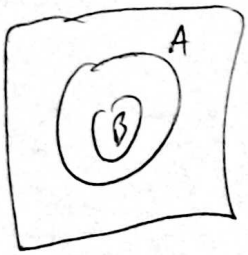
$(A \cap B) \cup (A \cap \bar{B})$

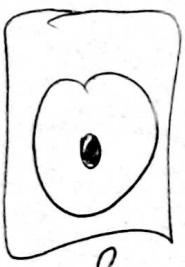


This is all of A.

b. If $B \subset A$ then $A = B \cup (A \cap \bar{B})$

This is all of A





B

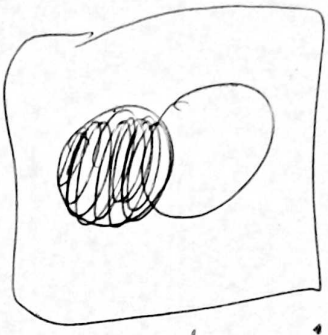
$\bar{B}$

$A \cap \bar{B}$

$B \cup (A \cap \bar{B})$

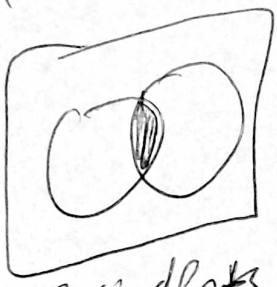
Exercise 2.6

60 students in the box



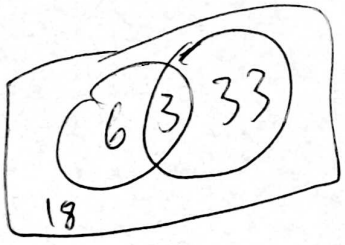
9 students in region A which represents living off-campus

36 students in region B which represents undergraduate students



3 students in $A \cap B$ which represents undergraduate students on campus

we have



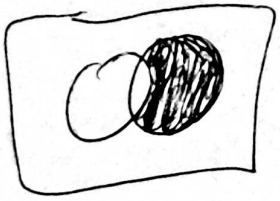
a. undergraduate students, living off campus, or both.

$A \cup B$

$|A \cup B| = 6 + 3 + 33 = 42$

b undergraduates living on campus

$A \cap \bar{B}$ B $\bar{A} \cap B$ undergraduate



33 students on campus.

$|\bar{A} \cap B| = 33$

c graduate students living on-campus.

$\overline{A \cup B} = \bar{A} \cap \bar{B}$ by De morgans law

$|\bar{A} \cap \bar{B}| = 18$

18 students on campus graduate

Exercise 2.9

Every person's blood type is A, B, AB, or O.
In addition, each individual has the Rhesus (Rh) factor (+)
or does not (−). A medical technicians records
a persons blood type and Rh factor.
List the sample for this experiment.

~~Sample Space = Ω~~

SAMPLESPACE = {A+, A−, B+, B−, AB+, AB−, O+, O−}.

Exercise

2.10 The proportions of blood phenotypes A, B, AB, and O, in the populations of all Caucasians in the United States are approximately 0.41, 0.10, 0.04, and 0.45, respectively. A single Caucasian is chosen at random from the population.

a. List the sample space for this experiment.

SAMPLESPACE = {A, B, AB, O}

b. Make use of the information given to assign probabilities to each of the simple events.

P(A) = 0.41

P(B) = 0.10

P(AB) = 0.04

P(O) = 0.45

c. What is the probability that the person chosen at random has either type A or type AB blood?

P(A or AB) = P(A) + P(AB) = 0.41 + 0.04 = 0.45

P(A or AB) = 0.45

2.13 Americans can be quite suspicious, especially when it comes to government conspiracies. On the question of whether the U.S. Air Force has withheld proof of the existence of intelligent life on other planets, the proportions of Americans with varying opinions are given in the table.

Opinion	Proportion
Very likely	0.24
Somewhat likely	0.24
Unlikely	0.40
Other	0.12

Suppose that one American is selected and his or her opinion is recorded.

a. What are the simple events for this experiment?

SIMPLE EVENTS = {"Very likely", "Somewhat likely", "Unlikely", "Other"}

b Are the simple events that you gave in part (a) all equally likely? If not, what are the probabilities that should be assigned to each?

No, the simple events are not all equally likely.

The probabilities are as follows

p("very likely") = 0.24

p("somewhat likely") = 0.24

p("unlikely") = 0.40

p("other") = 0.12

Exercise 2.15

An oil prospecting firm hits oil or gas on 10% of its drillings. If the firm drills two wells, the four possible simple events and three of their associated probabilities are given in the following table. Find the probability that the company will hit oil or gas.

a. on the first drilling and miss on the second.

b. on at least one of the two drillings.

Simple Event	outcome of First Drilling	outcome of second Drilling	Probability
E1	Hit(oil or gas)	Hit(oil or gas)	0.01
E2	Hit	Miss	0.09
E3	Miss	Hit	0.09
E4	Miss	Miss	0.81

a

a $P(E2) = 1 - P(E1) - P(E3) - P(E4)$

$= 1 - 0.01 - 0.09 - 0.81 = 0.09$

$P(E2) = 0.09$

b At least one hit occurs in E1, E2, and E3.

$P(\text{at least one hit}) = P(E1) + P(E2) + P(E3)$

$= 0.01 + 0.09 + 0.09$

$= 0.19$

b The probability of at least one hit is 0.19.

Exercise 2.28

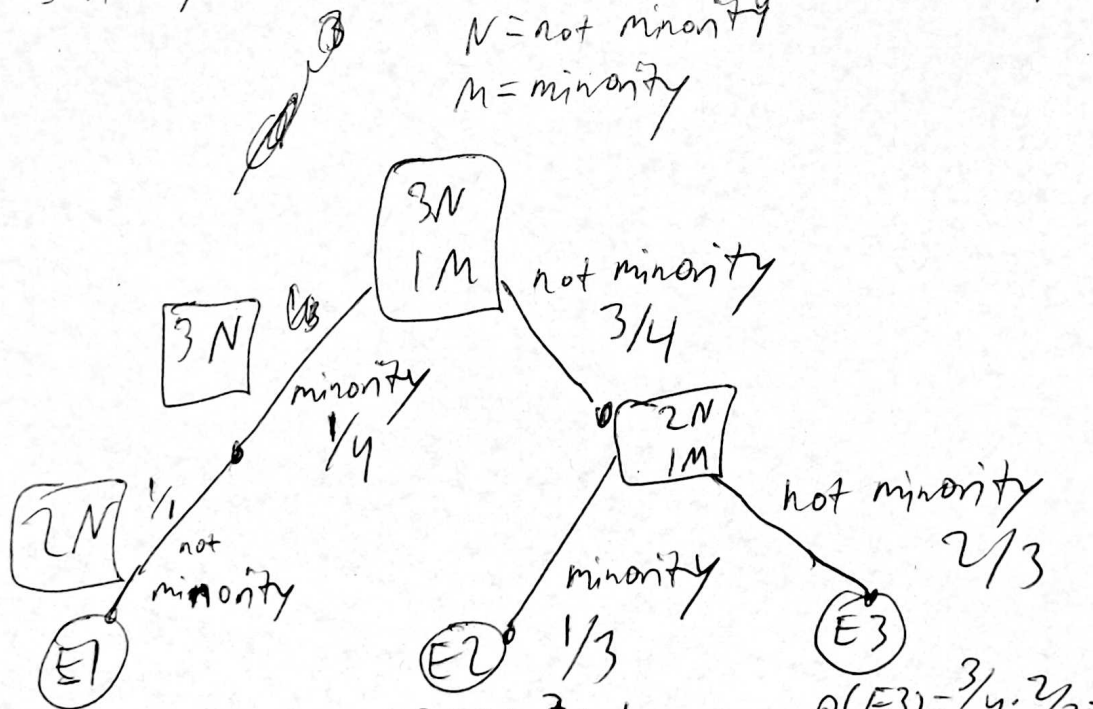
Four equally qualified people apply for two identical positions in a company. One and only one applicant is a member of a minority group. The positions are filled by choosing two of the applicants at random.

a. List the possible outcomes for this experiment.

	The first applicant chosen	The second applicant chosen	$P(E_n)$
E1	minority	not minority	1/4
E2	not minority	minority	1/4
E3	not minority	not minority	1/2

b Assign reasonable outcomes to the sample points.

N = not minority
M = minority



$P(E1) = 1/4 \cdot 1/1 = 1/4$

$P(E2) = 3/4 \cdot 1/3 = 1/4$

$P(E3) = 3/4 \cdot 2/3 = 6/12 = 1/2$

c Find the probability that the ~~minority applicant is selected~~ group applicant from the minority group is selected for a position.

$P(\text{minority}) = P(E1) + P(E2) = 1/4 + 1/4 = 2/4 = 1/2$ ✓ 1/2

Exercise 2.29

Two additional jurors are needed to complete a jury for a criminal trial. There are six prospective jurors, two women and four men.

Two jurors are randomly selected from the six available.

a Define the experiment and describe one sample point. Assume that you need describe only the two jurors chosen and not the order in which they were selected.

2M for 2 men

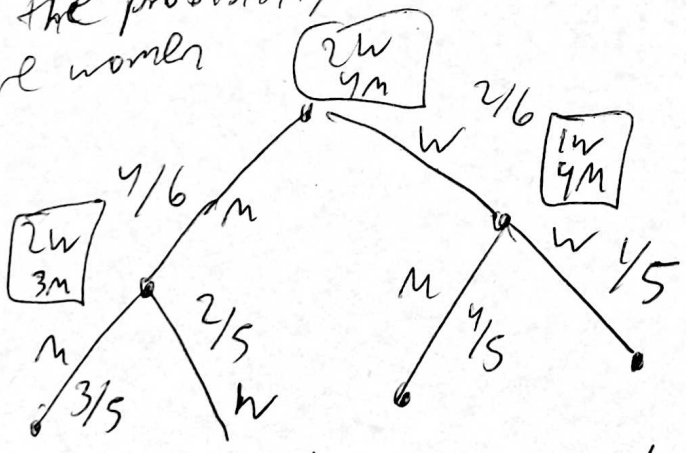
1M 1w for 1 man 1 woman

2w for 2 women

b List the sample space associated with this experiment.

SAMPLESPACE = {2M, 1M1W, 2w}

c What is the probability that both of the jurors selected are women



Now we multiply the right side of the tree to get

$2/6 \cdot 1/5 = 2/30 = 1/15$

The probability that both of the jurors selected are women is 1/15.

Exercise 2.32

Patients arriving at a hospital outpatient clinic can select one of three stations for service. Suppose that physicians are assigned randomly to the stations and that the patients therefore have no station preference. Three patients arrive at the clinic and their selection of stations is observed.

a. List the sample points for this experiment.

1 represents favorite station
2 represents second station preference
3 represents third station preference

The sample space is $\{\_, \_, \_\}$ where the first pat represents the station a patient is assigned to. For example $\{2,3,1\}$ means the first patient gets their 2nd preference, the 2nd patient gets their 3rd preference and the 3rd patient gets their 1st preference.

The sample points that make up the sample space are

$\{1,2,3\}\ \{1,3,2\}\ \{2,1,3\}\ \{2,3,1\}\ \{3,1,2\}\ \{3,2,1\}$

b. Let A be the event that each