

The best way to probe CP violation in the lepton sector is with long-baseline accelerator neutrino experiments in the appearance mode. I will show that it is possible to discover CP violation with disappearance experiments only, by combining JUNO for electron neutrinos and DUNE or Hyper-Kamiokande for muon neutrinos. While the maximum sensitivity to discover CP is quite modest, some values of  $\delta$  may be disfavored by  $> 3\sigma$  depending on the true value of  $\delta$ .

Neutrino oscillation experiments will be entering the precision era in the next decade. Correctly estimating the confidence intervals from data for the oscillation parameters requires very large Monte Carlo data sets involving calculating the oscillation probabilities in matter many, many times. In this talk, I will leverage past work to present a new, fast, precise technique for calculating neutrino oscillation probabilities in matter optimized for long-baseline neutrino oscillations called NuFast. I will also present recent results for atmospheric and solar neutrinos.

# CP-Violation with Neutrino Disappearance and NuFast

Peter B. Denton

Oxford

January 15, 2026

2309.03262 PRL

2405.02400 and 2511.04735 with S. Parke  
[github.com/PeterDenton/NuFast-LBL](https://github.com/PeterDenton/NuFast-LBL)  
[github.com/PeterDenton/NuFast-Earth](https://github.com/PeterDenton/NuFast-Earth)

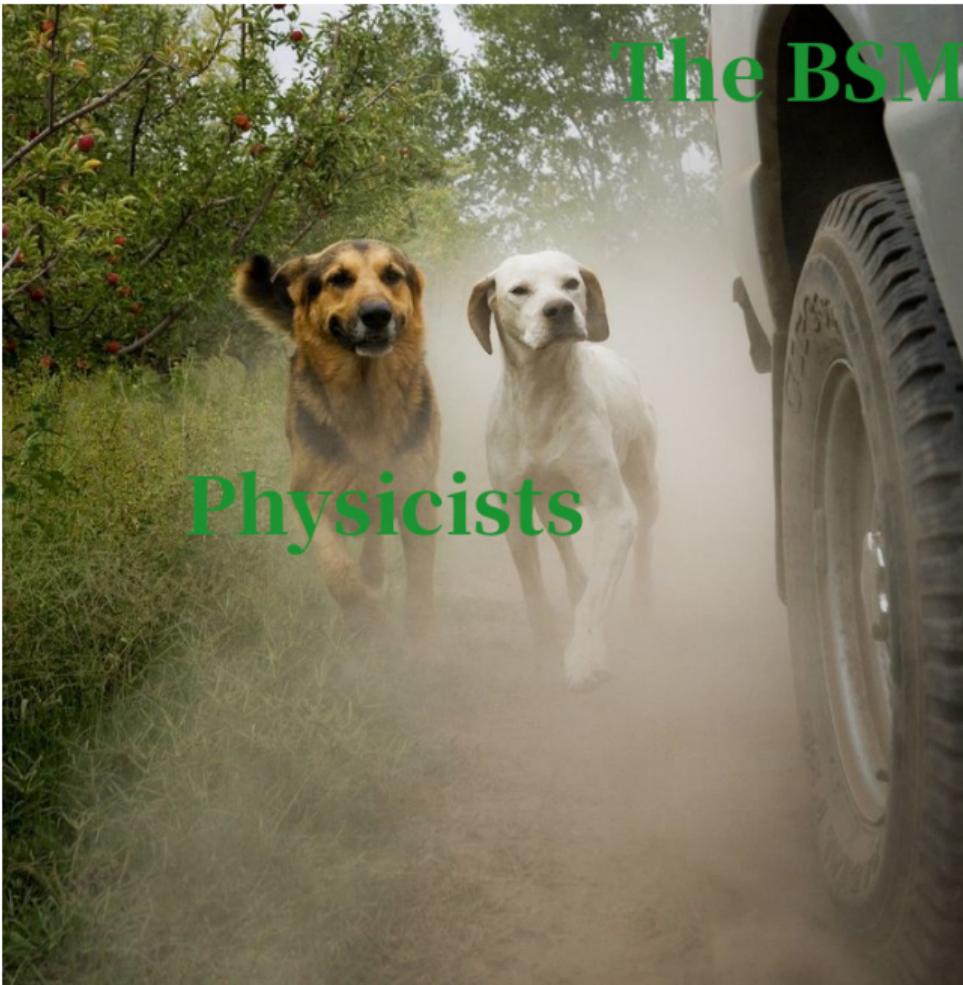


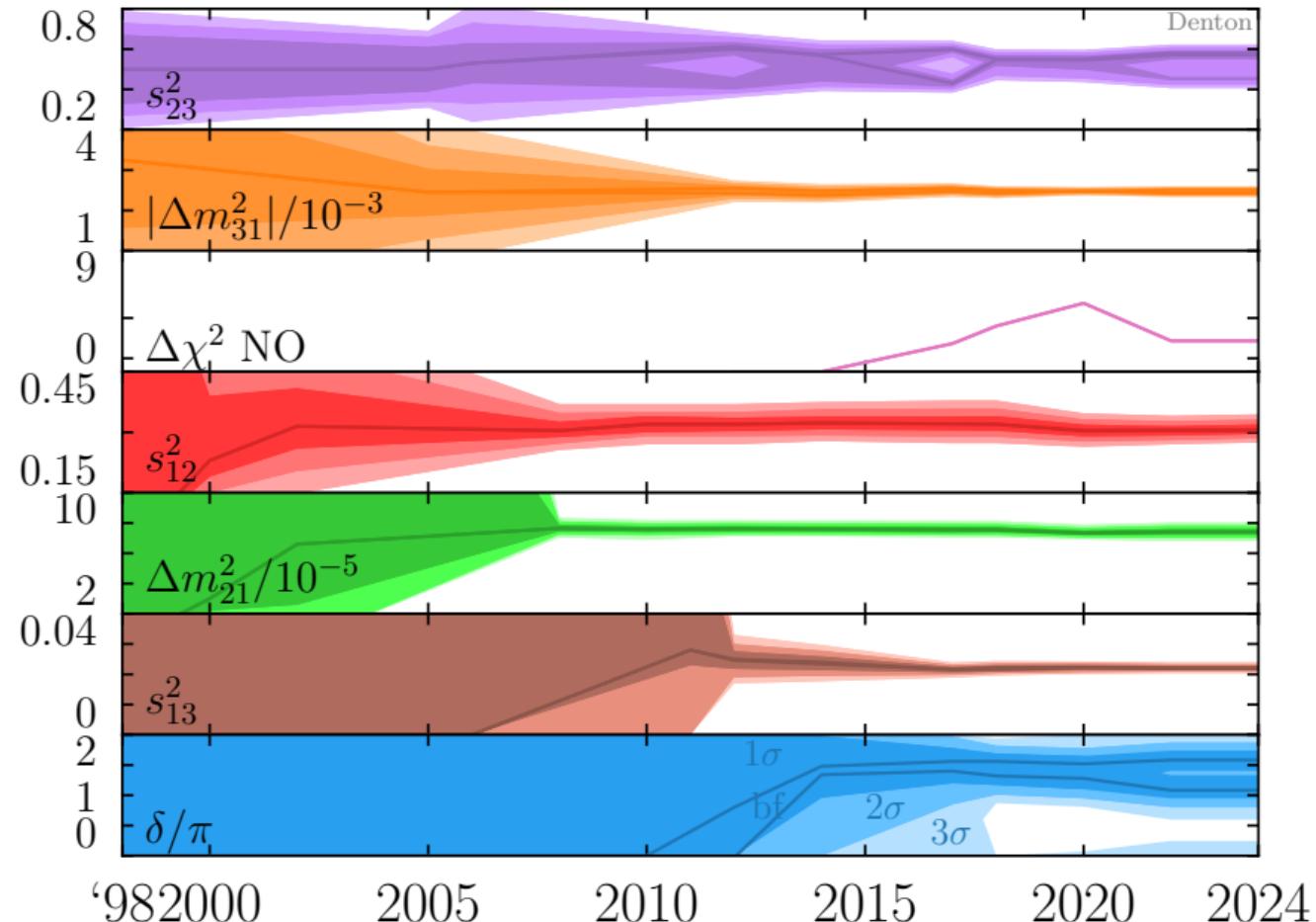
**Brookhaven**  
National Laboratory



The BSM

Physicists





# Four Known Unknown in Particle Physics: All Neutrinos

Atmospheric mass ordering

$\theta_{23}$  octant

Complex phase

Absolute mass scale

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# Why is CPV interesting?

# $\delta$ and CP Violation

$$J_{CP} = s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23} \sin \delta$$

C. Jarlskog [PRL 55, 1039 \(1985\)](#)



# $\delta$ and CP Violation

$$J_{CP} = s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23} \sin \delta$$

C. Jarlskog [PRL 55, 1039 \(1985\)](#)



1. Strong interaction: no observed EDM  $\Rightarrow$  CP (nearly) **conserved**

$$\frac{\bar{\theta}}{2\pi} < 10^{-11}$$

J. Pendlebury, et al. [1509.04411](#)

2. Quark mass matrix: non-zero but **small** CP violation

$$\frac{|J_{CKM}|}{J_{\max}} = 3 \times 10^{-4}$$

CKMfitter [1501.05013](#)

3. Lepton mass matrix: ?

$$\frac{|J_{PMNS}|}{J_{\max}} < 0.34$$

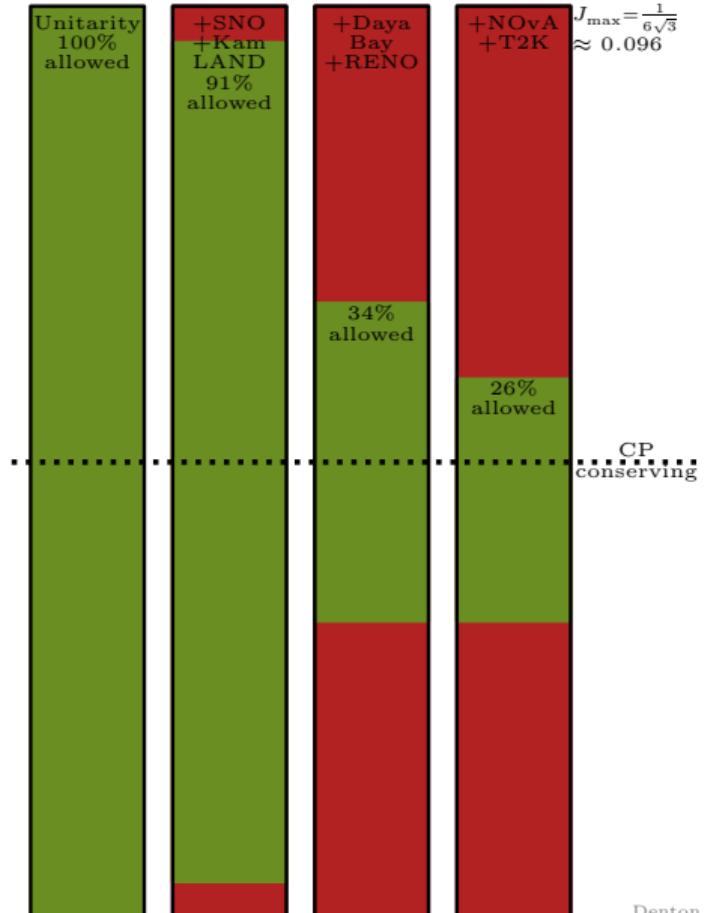
PBD, J. Gehrlein, R. Peses [2008.01110](#)

$$J_{\max} = \frac{1}{6\sqrt{3}} \approx 0.096$$

# $\delta, J$ : Current Status

Maximal CP violation is already ruled out:

1.  $\theta_{12} \neq 45^\circ$  at  $\sim 15\sigma$
2.  $\theta_{13} \neq \tan^{-1} \frac{1}{\sqrt{2}} \approx 35^\circ$  at many (100)  $\sigma$
3.  $\theta_{23} = 45^\circ$  allowed at  $\sim 1\sigma$
4.  $|\sin \delta| = 1$  allowed



## When $\delta$ and When $J$ ?

If the goal is **CP violation** the Jarlskog invariant should be used

however

If the goal is **measuring the parameters** one must use  $\delta$

Given  $\theta_{12}$ ,  $\theta_{13}$ ,  $\theta_{23}$ , and  $J$ , I can't determine the sign of  $\cos \delta$  which is physical

e.g.  $P(\nu_\mu \rightarrow \nu_\mu)$  depends on  $\cos \delta$

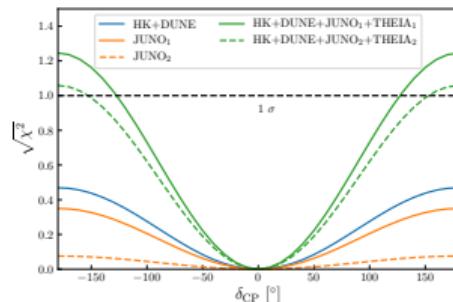
# Other Non-standard CPV Probes

1. Some information in solar due to loops in elastic scattering

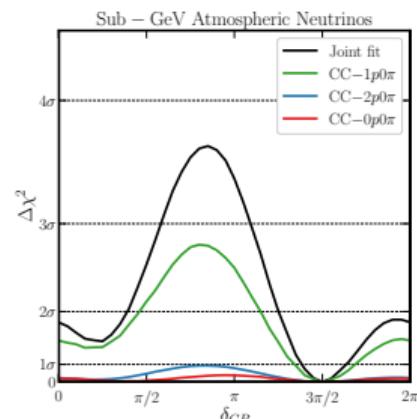
V. Brdar, X-J. Xu [2306.03160](#)  
K. Kelly, et al. [2407.03174](#) requires 3k Borexinos

2. Sub-GeV  $\rightarrow$  sub-100 MeV atmospheric neutrinos

K. Kelly, et al. [1904.02751](#)  
See also e.g. A. Suliga, J. Beacom [2306.11090](#)

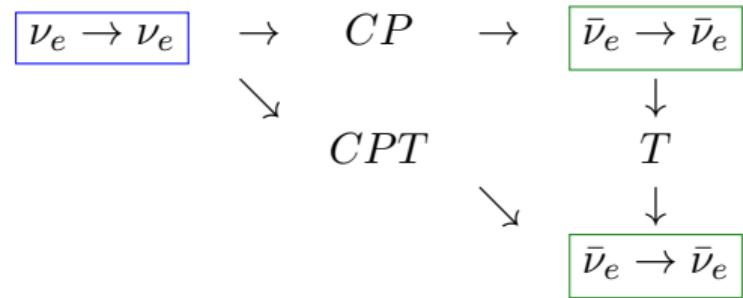


Solar (no systematics)



Atmospheric neutrinos at DUNE

# CP, T: Disappearance

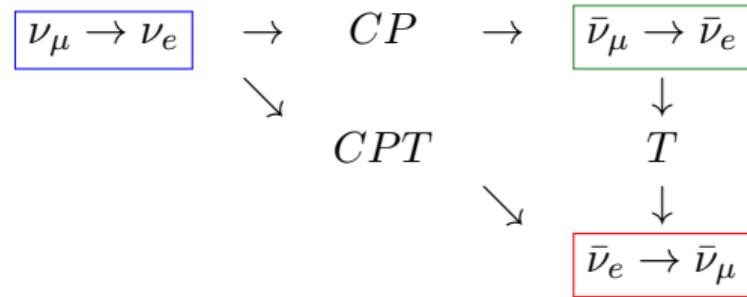


Disappearance measurements are even eigenstates of  $CP$

$$CP[P(\nu_e \rightarrow \nu_e)] = P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \stackrel{CPT}{=} P(\nu_e \rightarrow \nu_e)$$

Assume that CPT is a good symmetry

# CP, T: Appearance



Appearance measurements are not eigenstates of  $CP$

# Appearance, Disappearance, and CP

# Appearance and Disappearance, CP Even and CP Odd Terms

**Disappearance:**

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\alpha) &= 1 - 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2 \Delta_{21} \\ &\quad - 4|U_{\alpha 1}|^2|U_{\alpha 3}|^2 \sin^2 \Delta_{31} \\ &\quad - 4|U_{\alpha 2}|^2|U_{\alpha 3}|^2 \sin^2 \Delta_{32} \\ &= P_{\alpha\alpha}^{CP+} \end{aligned}$$

$$\Delta_{ij} \equiv \Delta m_{ij}^2 L / 4E$$

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**Appearance:**

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) &= -4\Re[U_{\alpha 1} U_{\beta 1}^* U_{\alpha 2}^* U_{\beta 2}] \sin^2 \Delta_{21} \\ &\quad - 4\Re[U_{\alpha 1} U_{\beta 1}^* U_{\alpha 3}^* U_{\beta 3}] \sin^2 \Delta_{31} \\ &\quad - 4\Re[U_{\alpha 3} U_{\beta 3}^* U_{\alpha 2}^* U_{\beta 2}] \sin^2 \Delta_{32} \\ &\quad \pm 8J_{CP} \sin \Delta_{21} \sin \Delta_{31} \sin \Delta_{32} \\ &= P_{\alpha \beta}^{CP+} + P_{\alpha \beta}^{CP-} \end{aligned}$$

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Sign depends on  $\alpha, \beta$

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3. Any CPV or  $\delta$  dependence in disappearance is in  $\nu_\mu$  not  $\nu_e$  [Confusing/False]

$$U_{\text{PDG}} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Can reparameterize so that any row/column is “simple”

PBD, R. Pestes [2006.09384](#)

# Correct Statements

- ▶ Appearance is the best way to measure  $\delta$  and CPV
  - ... given known oscillation parameters, systematics, and realistic experiments
  - ▶ Probes mostly  $\sin \delta$  not  $\cos \delta$
  - ▶ Don't need both  $\nu$  and  $\bar{\nu}$  (but systematics)
- ▶ Disappearance can measure  $\delta$ 
  - ▶ CPV can be discovered with only disappearance measurements
  - ▶ Probes mostly  $\cos \delta$  not  $\sin \delta$
  - ▶ Requires measurements of two flavors
  - ▶ "Works through unitarity" (as do nearly all oscillation measurements)

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$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - 4 \sum_{i>j} C_{ij}^\alpha \sin^2 \Delta_{ij}$$

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$$C_{ij}^\alpha = |U_{\alpha i}|^2 |U_{\alpha j}|^2$$

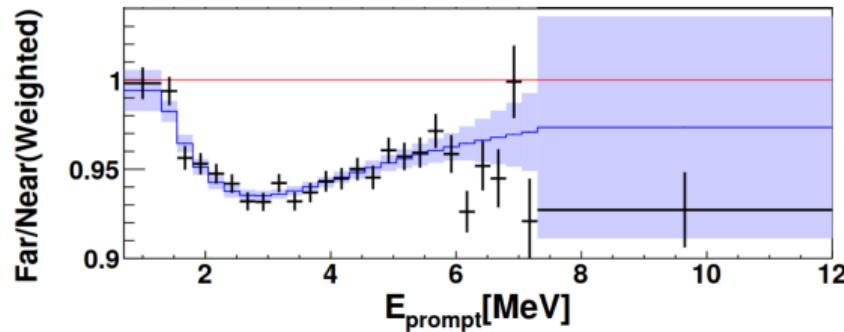
$$|U_{\alpha i}| = \left( \frac{C_{ij}^\alpha C_{ik}^\alpha}{C_{jk}^\alpha} \right)^{1/4}$$

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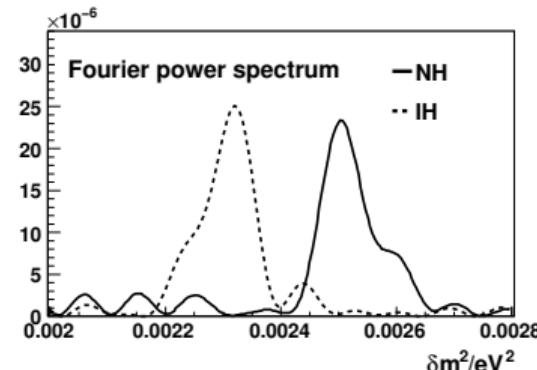
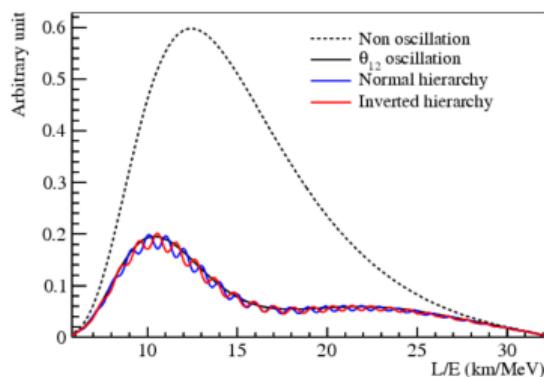
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L. Zhan, et al. 0807.3203

Oxford: January 15, 2026 16/52

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- ▶ Any row can be “simple” (e.g.  $c_{12}c_{13}$ ,  $s_{12}c_{13}$ , ...)  $\Rightarrow$  no one row is ever enough
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6. If we determine  $\cos \delta \neq \pm 1$   $\Rightarrow$  CP is violated!

## Direct Analytic Calculation

Disappearance experiments measure various  $|U_{\alpha i}|^2$  terms

Suppose 4 are measured:  $|U_{e2}|^2, |U_{e3}|^2, |U_{\mu 2}|^2, |U_{\mu 3}|^2$

Actually this gives all 9 magnitudes by unitarity

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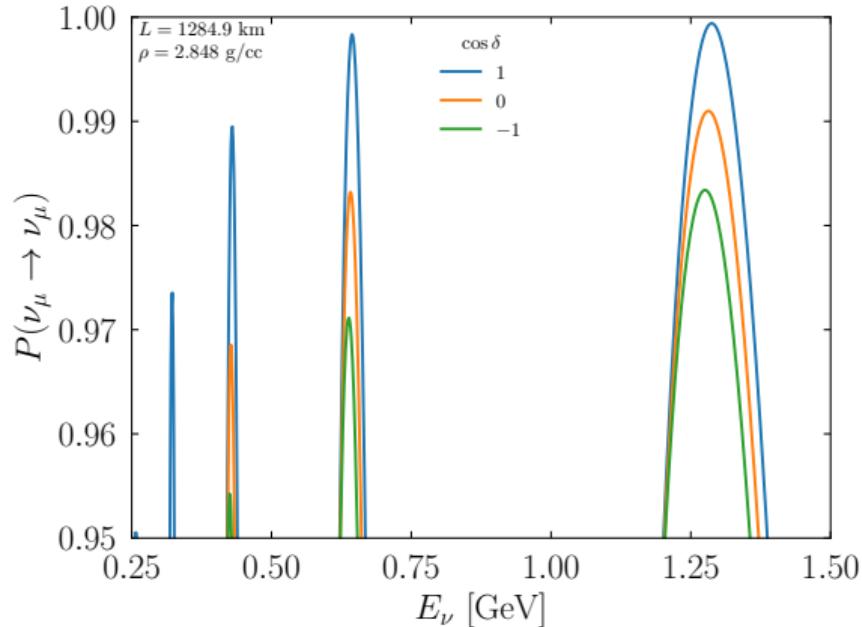
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$$J_{CP}^2 = |U_{e2}|^2 |U_{\mu 2}|^2 |U_{e3}|^2 |U_{\mu 3}|^2 - \frac{1}{4} (1 - |U_{e2}|^2 - |U_{\mu 2}|^2 - |U_{e3}|^2 - |U_{\mu 3}|^2 + |U_{e2}|^2 |U_{\mu 3}|^2 + |U_{e3}|^2 |U_{\mu 2}|^2)^2$$

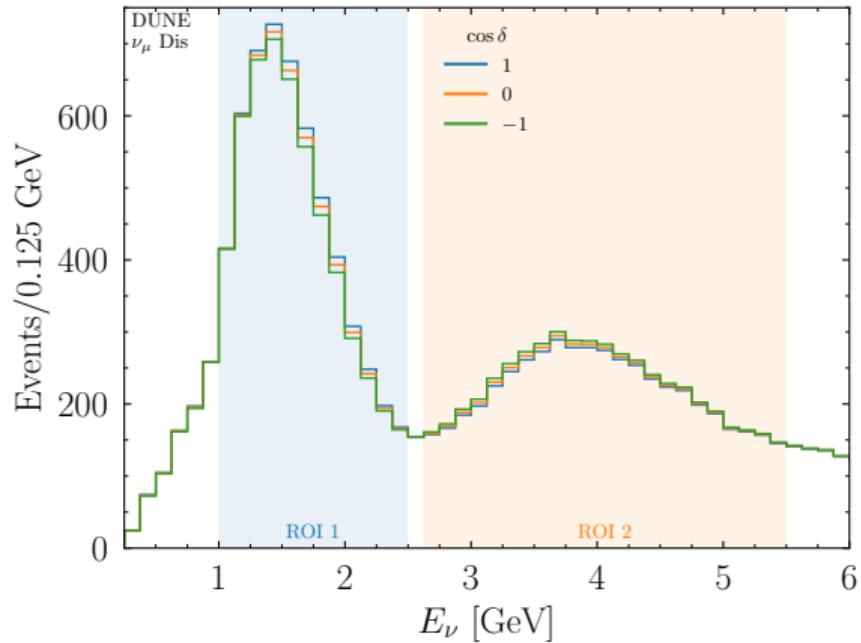
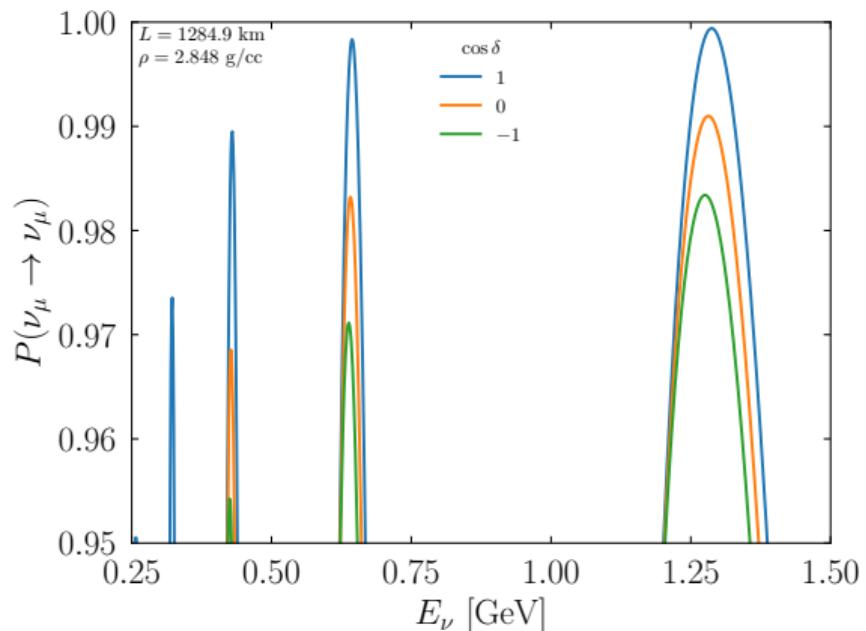
Disappearance can tell us if CP is violated,  
but not if Nature prefers  $\nu$ 's or  $\bar{\nu}$ 's

Can show that if any one  $|U_{\alpha i}|^2 = 0 \Rightarrow J = 0$

# Where is $|U_{\mu 2}|^2$ ?



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	$\cos \delta$	ROI 1	ROI 2
6.5 yrs $\nu_\mu$ rates	1	5506	5038
	0	5418	5115
	-1	5334	5193

## Approximate Size of $|U_{\mu 2}|^2$ Signal: (21) Sector

- There is no  $\delta$  information in  $|U_{\mu 1}|^2 + |U_{\mu 2}|^2$  (sum of  $\Delta_{31}$  and  $\Delta_{32}$  terms)

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**PBD**, J. Gehrlein [2302.08513](#)

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- ▶ This term is

$$\begin{aligned} P_{\mu\mu} &\supset -4c_{23}^2 (s_{12}^2 c_{12}^2 + s_{23} c_{23} s_{13} \sin 2\theta_{12} \cos 2\theta_{12} \cos \delta) \sin^2 \Delta_{21} \\ &\approx -2 (0.21 + 0.03 \cos \delta) \left( \frac{\pi}{33} \right)^2 \end{aligned}$$
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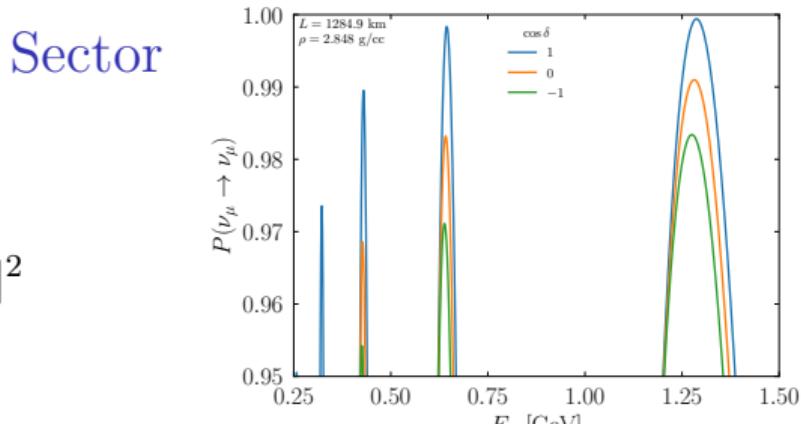
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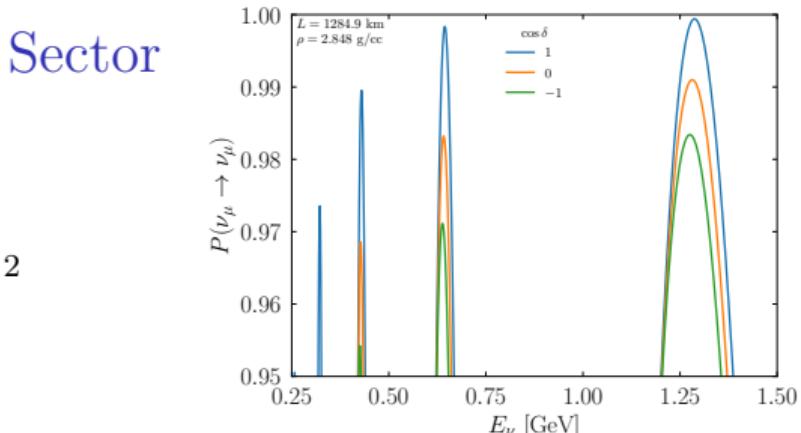
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Sign is wrong

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Magnitude is  $\sim 16$  too small

## Matter Effects Matter: (21) Sector

- ▶ Let's start at

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- ▶ Solar splitting in matter modified by

$$\frac{\Delta m_{21}^2 \rightarrow \Delta m_{21}^2 S_\odot}{S_\odot \approx \sqrt{(\cos 2\theta_{12} - c_{13}^2 a / \Delta m_{21}^2)^2 + \sin^2 2\theta_{12}}} \approx 3.4$$

at  $E = 1.3$  GeV  
PBD, S. Parke [1902.07185](#)

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- ▶ Mixing angle is modified

$$\cos 2\theta_{12} = 0.37 \rightarrow \frac{\cos 2\theta_{12} - c_{13}^2 a / \Delta m_{21}^2}{\mathcal{S}_\odot} \approx -0.96 < 0$$

$$a \propto \rho E$$

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- ▶ So the sign is swapped

$$a \propto \rho E$$

$$\sin 2\theta_{12} \cos 2\theta_{12} = 0.35 \rightarrow -0.26$$

## Matter Effects Matter: (21) Sector

- ▶ Let's start at

$$P_{\mu\mu} \supset -4c_{23}^2 (s_{12}^2 c_{12}^2 + s_{23} c_{23} s_{13} \sin 2\theta_{12} \cos 2\theta_{12} \cos \delta) \sin^2 \Delta_{21}$$

- ▶ Solar splitting in matter modified by

$$\frac{\Delta m_{21}^2}{\mathcal{S}_\odot} \rightarrow \Delta m_{21}^2 \mathcal{S}_\odot$$
$$\mathcal{S}_\odot \approx \sqrt{(\cos 2\theta_{12} - c_{13}^2 a / \Delta m_{21}^2)^2 + \sin^2 2\theta_{12}} \approx 3.4$$

at  $E = 1.3$  GeV

PBD, S. Parke [1902.07185](#)

- ▶ Mixing angle is modified

$$\cos 2\theta_{12} = 0.37 \rightarrow \frac{\cos 2\theta_{12} - c_{13}^2 a / \Delta m_{21}^2}{\mathcal{S}_\odot} \approx -0.96 < 0$$

- ▶ So the sign is swapped

$$a \propto \rho E$$

$$\sin 2\theta_{12} \cos 2\theta_{12} = 0.35 \rightarrow -0.26$$

- ▶ Also  $s_{13}$  increases in matter  $\sim 15\%$ : total effect is  $0.004 \cos \delta$

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- ▶ This gets us **half** of the effect, and the correct sign

# Matter Effects Matter: (32) Sector

- ▶  $\frac{\Delta m_{\mu\mu}^2 L}{4E}$  in matter at  $P_{\max}$  is  $\sim \pi$

H. Nunokawa, S. Parke, R. Funchal [hep-ph/0503283](#)  
PBD, S. Parke [2401.10326](#)

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- ▶ Total is  $\approx 0.008\cos\delta$  which agrees with numerical calculation

$\sim 1\%$  effect

Matter effect shifts the numbers  
Numerous subtle effects in play

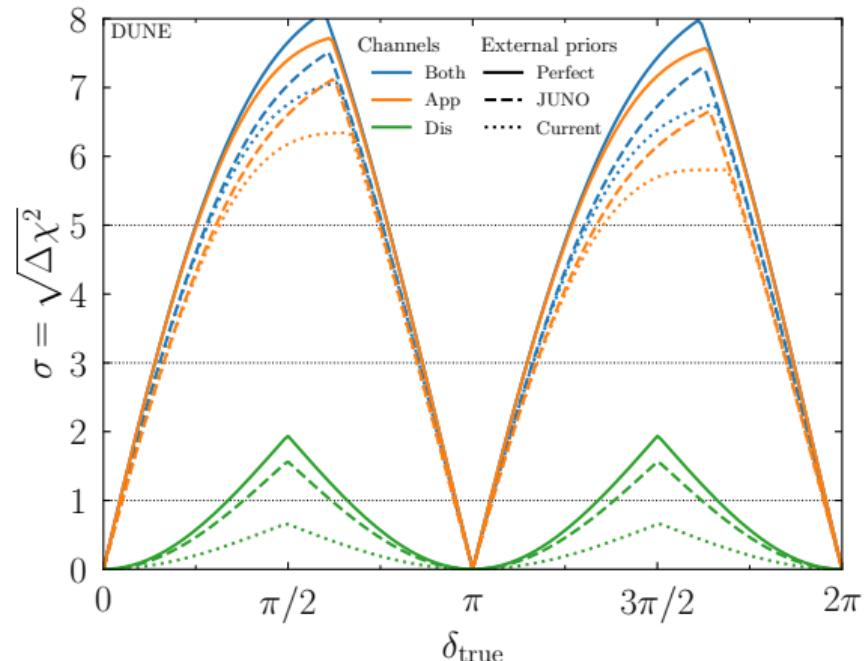
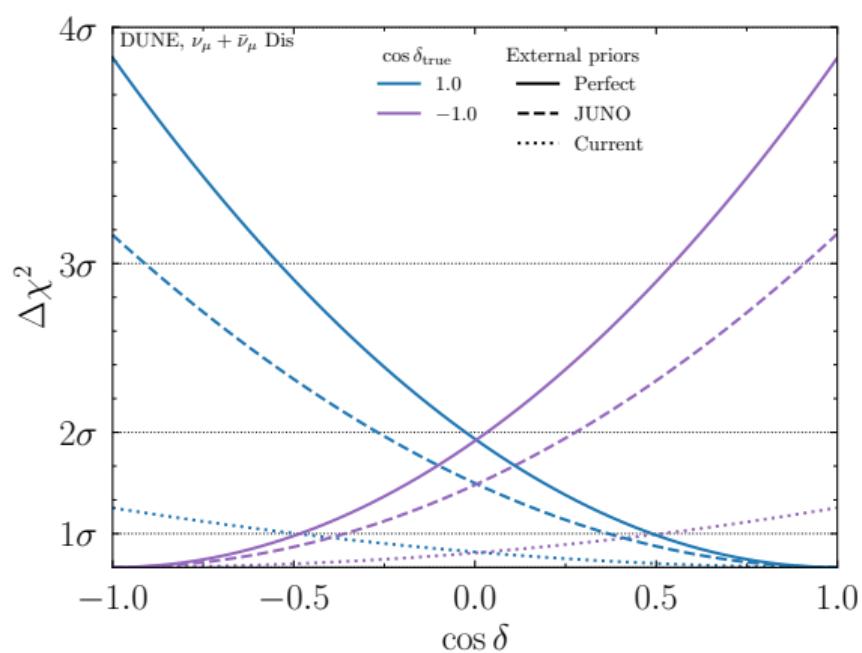
# Numerical Studies

Inputs are *only*:

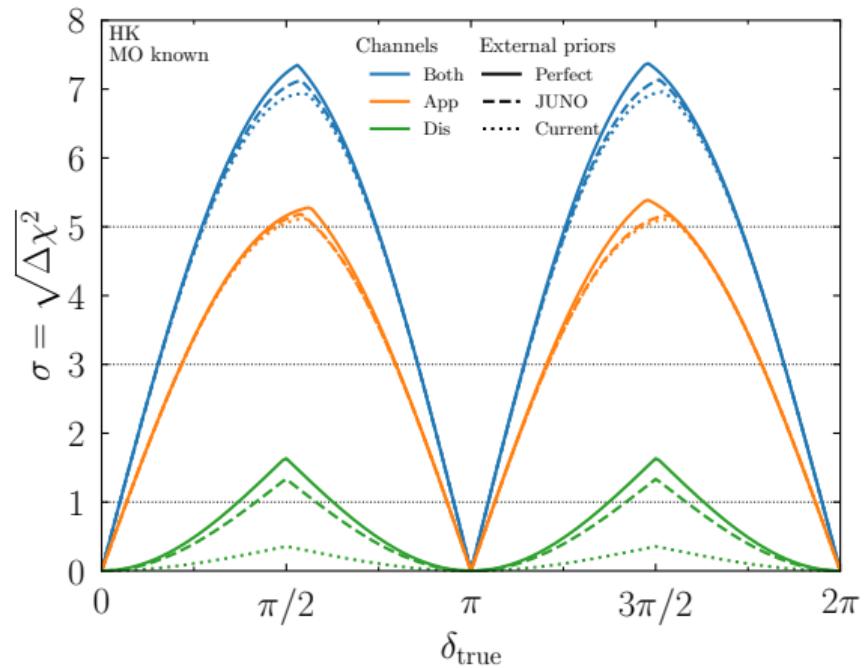
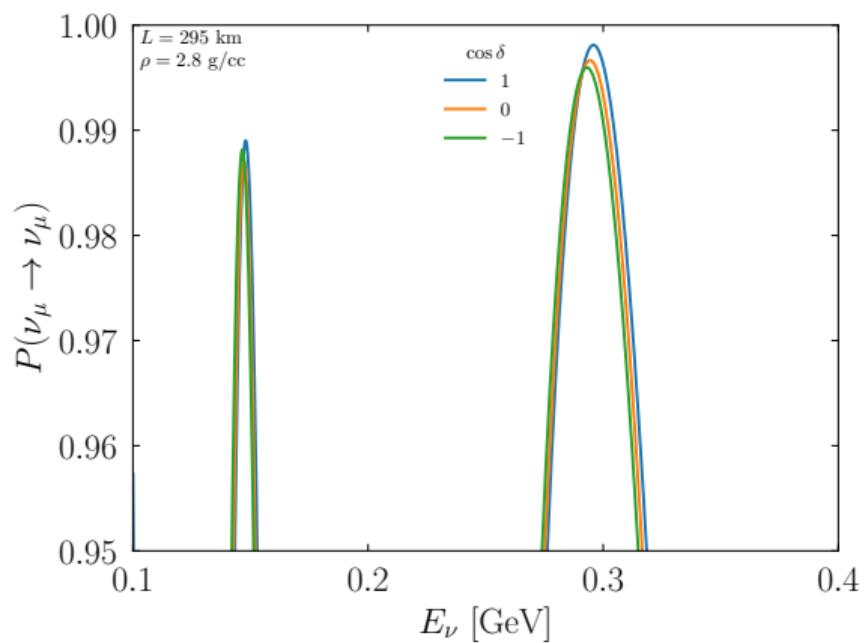
- ▶ Daya Bay data for  $\theta_{13}$  1809.02261
- ▶ KamLAND data for  $\theta_{12}$  and  $\Delta m_{21}^2$  1303.4667
- ▶ JUNO 6 yrs precision sensitivity on  $\theta_{12}$ ,  $\Delta m_{21}^2$ ,  $\Delta m_{31}^2$  2204.13249
- ▶ DUNE 6.5+6.5 yrs disappearance channels sensitivity only 2103.04797

Also looked at varying JUNO's and DUNE's runtime, and at HK

# JUNO and DUNE Disappearance Sensitivities



# JUNO and HK Disappearance Sensitivities



# NuFast-LBL

A fast code for long-baseline  
neutrino oscillation probabilities in matter

[github.com/PeterDenton/NuFast-LBL](https://github.com/PeterDenton/NuFast-LBL)

# NuFast-Earth

A fast code for neutrino oscillations through the Earth from the  
atmosphere, the Sun, or a supernova

[github.com/PeterDenton/NuFast-Earth](https://github.com/PeterDenton/NuFast-Earth)

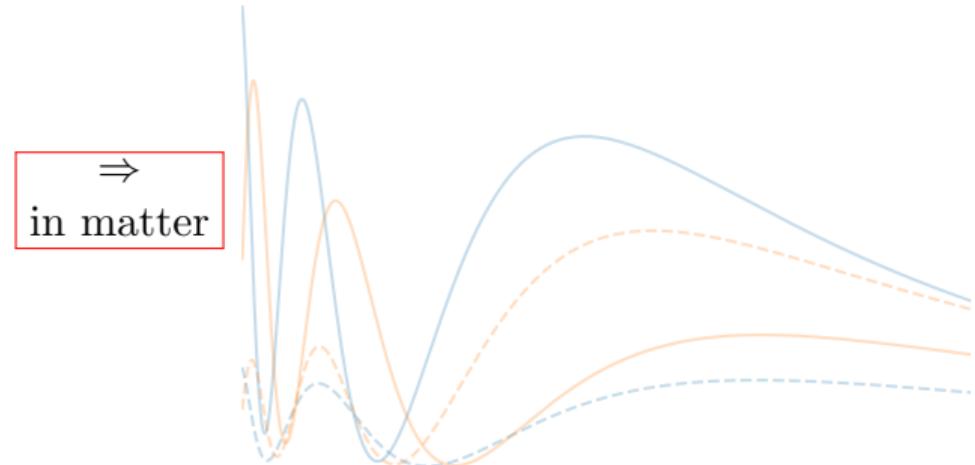
[2405.02400](https://doi.org/10.4248/2405.02400) & [2511.04735](https://doi.org/10.4248/2511.04735) with S. Parke

# The Problem

Fundamental parameters

$$\begin{aligned} \Delta m_{21}^2, \Delta m_{31}^2 \\ s_{23}^2, s_{13}^2, s_{12}^2 \\ \delta \end{aligned}$$

Physical observables



# Many Approaches

Solve the Schrödinger equation

$$i\frac{d}{dt}|\nu\rangle = H(t)|\nu\rangle$$

If  $H(t) = H$  (constant density)

$$\mathcal{A}(\nu_\alpha \rightarrow \nu_\beta) = [e^{-iHL}]_{\beta\alpha} \quad P = |\mathcal{A}|^2$$

Exponential requires computing eigenvalues and eigenvectors of  $H$

# Many Approaches

## Modify vacuum probabilities

- ▶ Get the eigenvalues by solving the cubic

Cardano 1545  
V. Barger, et al. PRD 22 (1980) 2718

- ▶ Get the eigenvectors

H. Zaglauer, K. Schwarzer Z.Phys. C40 (1988) 273  
K. Kimura, A. Takamura, H. Yokomakura hep-ph/0205295  
PBD, S. Parke, X. Zhang 1907.02534  
A. Abdulahi, S. Parke 2212.12565

# Fermilab computing experts bolster NOvA evidence, 1 million cores consumed

July 3, 2018 | Marcia Teckenbrock

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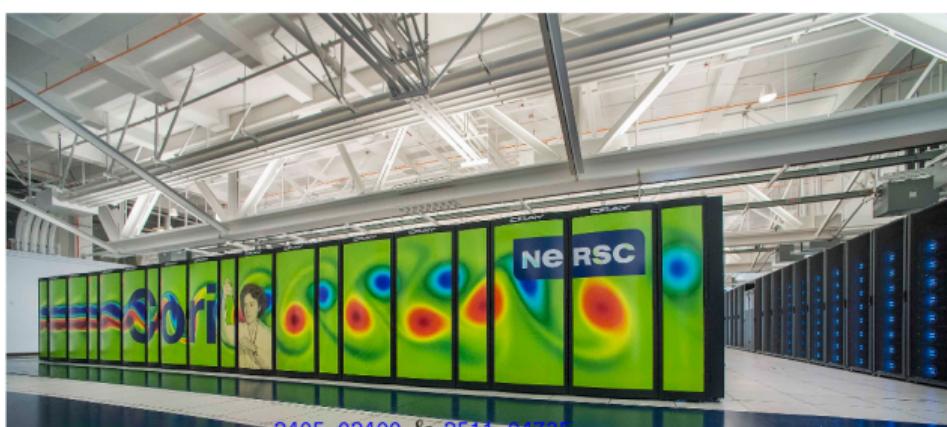
## Array

How do you arrive at the physical laws of the universe when you're given experimental data on a renegade particle that interacts so rarely with matter, it can cruise through light-years of lead? You call on the power of advanced computing.

The NOvA neutrino experiment, in collaboration with the Department of Energy's Scientific Discovery through Advanced Computing (SciDAC-4) program and the HEPCloud program at DOE's Fermi National Accelerator Laboratory, was able to perform the largest-scale analysis ever to support the [recent evidence of antineutrino oscillation](#), a phenomenon that may hold clues to how our universe evolved.

Using Cori, the newest supercomputer at the [National Energy Research Scientific Computing Center \(NERSC\)](#), located at Lawrence Berkeley National Laboratory, NOvA used over 1 million computing cores, or CPUs, between May 14 and 15 and over a short timeframe one week later. This is the largest number of CPUs ever used concurrently over this duration — about 54 hours — for a single high-energy physics experiment. This unprecedented amount of computing enabled scientists to carry out some of the most complicated techniques used in neutrino physics, allowing them to dig deeper into the seldom seen interactions of neutrinos. This Cori allocation was more than 400 times the amount of Fermilab computing allocated to the NOvA experiment and 50 times the total computing capacity at Fermilab allocated for all of its rare-physics experiments. A continuation of the analysis was performed on NERSC's Cori and Edison supercomputers one week later. In total, [nearly 35 million core-hours were consumed by NOvA](#) in the 54-hour period. Executing the same analysis on a single desktop computer would take 4,000 years.

[FNAL Newsroom](#)



2405.02400 & 2511.04735

# Monte-Carlo Estimates of Statistical Significances

Wilks' theorem is often wrong

At each point in parameter space, simulate the experiment many times

“many” means  $\gg 1/p$  for a desired  $p$ -value

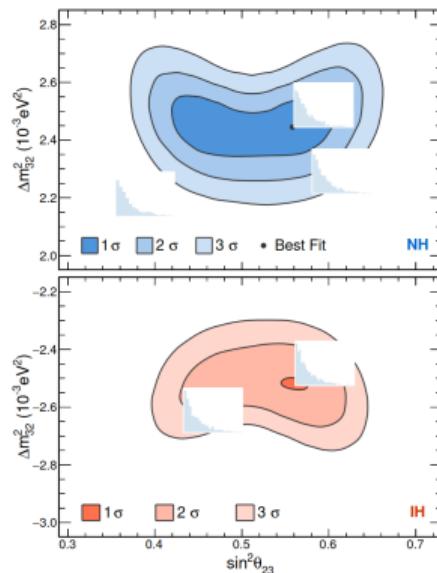
This is sometimes called Feldman-Cousins

G. Feldman, R. Cousins [physics/9711021](#)  
This isn't actually what was novel in the FC paper

Study found most of  
the time was spent  
computing probabilities

NOvA/T2K are  $\sim 3\sigma$  experiments,  
but DUNE/HK will be  $\gtrsim 5\sigma$  experiments!

DUNE sensitivities require computing  
the probabilities “a zillion times”



# How to Achieve Speed

## 1. Avoid costly operations

- ▶ `sin`, `cos` (and inverse functions) are very slow
- ▶ `sqrt` is quite slow, but not as bad as trigs
- ▶ Division is slower than multiplication ( $0.2x$  may be faster than  $x/5$ )

## 2. Avoid complex numbers as possible

## 3. Reduce repeated calculations

- ▶ Compute  $\frac{L}{4E}$  in the correct units once
- ▶ Compute each of the three  $\sin \frac{\Delta m_{ij}^2 L}{4E}$  once

## 4. Don't perform unnecessary linear algebra computations

All of these are compiler dependent

# Optimal Structure of the Probability for Long-Baseline

1. Amplitude requires four trig functions of kinematic variables ( $\Delta m_{ij}^2 L / 4E$ ) ✗
2. Writing the probabilities out requires three trig functions ✓
3. Disappearance structure is straightforward:

$$P_{\alpha\alpha} = 1 - 4 \sum_{i>j} |V_{\alpha i}|^2 |V_{\alpha j}|^2 \sin^2 \frac{\Delta\lambda_{ij} L}{4E}$$

$H$  in matter has eigenvalues  $\lambda_i$  and eigenvectors  $V_{\alpha i}$

# Optimal Structure of the Probability for Long-Baseline

4. Appearance structure:

$T$  conserving:

$$P_{\mu e}^{TC} = 2 \sum_{i>j} (|V_{\tau k}|^2 - |V_{\mu i}|^2 |V_{ej}|^2 - |V_{\mu j}|^2 |V_{ei}|^2) \sin^2 \frac{\Delta \lambda_{ij} L}{4E}$$

Fun fact:

$$\begin{aligned} & 2\Re(V_{\alpha i} V_{\beta j}^* V_{\alpha j}^* V_{\beta i}) \\ &= |V_{\alpha k}|^2 |V_{\beta k}|^2 - |V_{\alpha i}|^2 |V_{\beta i}|^2 - |V_{\alpha j}|^2 |V_{\beta j}|^2 \\ &= |V_{\gamma k}|^2 - |V_{\alpha i}|^2 |V_{\beta j}|^2 - |V_{\alpha j}|^2 |V_{\beta i}|^2 \end{aligned}$$

$T$  violating:

$$P_{\mu e}^{TV} = -8J \frac{\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2}{\Delta \lambda_{21} \Delta \lambda_{31} \Delta \lambda_{32}} \sin \frac{\Delta \lambda_{21} L}{4E} \sin \frac{\Delta \lambda_{31} L}{4E} \sin \frac{\Delta \lambda_{32} L}{4E}$$

Leverages NHS identity:  
V. Naumov [IJMP 1992](#)

P. Harrison, W. Scott [hep-ph/9912435](#)

# Account for Matter in Long-Baseline

1. Need the eigenvalues  $\lambda_i$
2. For eigenvectors, naively need  $\Re(V_{\alpha i} V_{\beta j}^* V_{\alpha j}^* V_{\beta i})$
3. Given our form, need only the  $|V_{\alpha i}|^2$  and  $J$ 
  - ▶ Don't need any phase information of the eigenvectors!

Leverages PBD, S. Parke, X. Zhang [1907.02534](#)

4. Can compute the  $|V_{\alpha i}|^2$  from the  $\lambda_i$  and submatrix eigenvalues (requires only a square root) using Eigenvector-Eigenvalue Identity

$$|V_{\alpha i}|^2 = \frac{\prod_{k=1}^{n-1} (\lambda_i - \xi_k^\alpha)}{\prod_{k=1; k \neq i}^n (\lambda_i - \lambda_k)}$$

See e.g. PBD, S. Parke, T. Tao, X. Zhang [1908.03795](#)  
Can actually avoid the  $\sqrt{-}$  in practice

## Eigenvalues are Hard

The eigenvalues in matter  $\lambda_i$  depend on  $S$ :

$$S = \cos \left\{ \frac{1}{3} \cos^{-1} \left[ \frac{2A^3 - 9AB + 27C}{2(A^2 - 3B)^{3/2}} \right] \right\}$$

where

$$A = \sum \lambda_i = \Delta m_{21}^2 + \Delta m_{31}^2 + a$$

$$B = \sum_{i>j} \lambda_i \lambda_j = \Delta m_{21}^2 \Delta m_{31}^2 + a[\Delta m_{21}^2(1 - |U_{e2}|^2) + \Delta m_{31}^2(1 - |U_{e3}|^2)]$$

$$C = \prod \lambda_i = a \Delta m_{21}^2 \Delta m_{31}^2 |U_{e1}|^2$$

# Approximate Eigenvalues

1. Instead, approximate one eigenvalue
  - ▶  $\lambda_3$  is best because is never parametrically small and easy to approximate
2. From DMP:

$$\lambda_3 \approx \Delta m_{31}^2 + \frac{1}{2} \Delta m_{ee}^2 \left( x - 1 + \sqrt{(1-x)^2 + 4x s_{13}^2} \right)$$

$$x \equiv \frac{a}{\Delta m_{ee}^2} \quad \Delta m_{ee}^2 \equiv \Delta m_{31}^2 - s_{12}^2 \Delta m_{21}^2$$

H. Minakata, S. Parke [1505.01826](#)  
**PBD**, H. Minakata, S. Parke [1604.08167](#)  
H. Nunokawa, S. Parke, R. Funchal [hep-ph/0503283](#)

3. Get other two eigenvalues by picking two of  $A, B, C$  conditions

Requires one more  $\sqrt{-}$

# The Approximation

- ▶ This is the only approximation used in the entire approach
- ▶ In vacuum the approximation returns to the correct value

Many approximations in the literature are not correct in vacuum limit  
See G. Barenboim, PBD, S. Parke, C. Ternes [1902.00517](#)

- ▶ In fact can iteratively improve  $\lambda_3$  with rapid convergence via Newton's method:

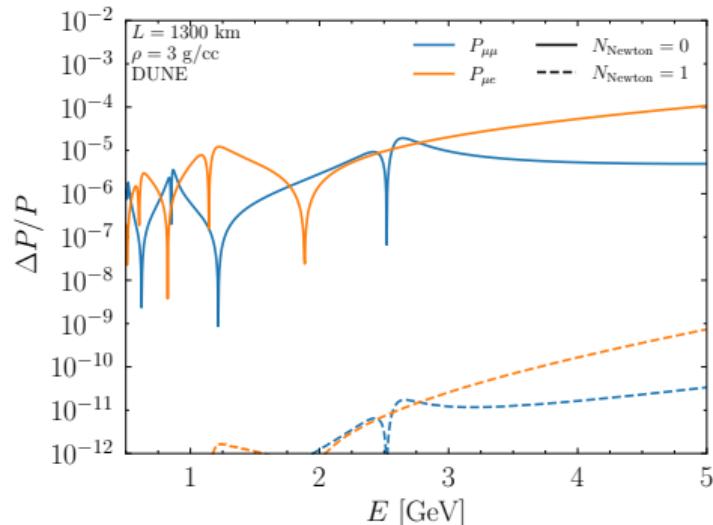
$$\lambda_3 \rightarrow \lambda_3 - \frac{X(\lambda_3)}{X'(\lambda_3)}$$

$$X(\lambda) = \lambda^3 - A\lambda^2 + B\lambda - C = 0$$

- ▶ Precision improvement starts at  $10^{-5}$  for the first step
- ▶ The improvement is quadratic thereafter
- ▶ One line of code, just loop as many times as desired

# NuFast-LBL Precision

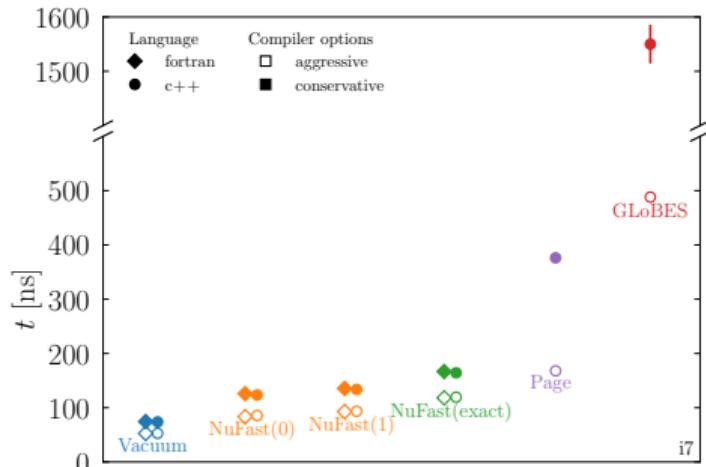
Is this approximation okay?  
DUNE requires  $\lesssim 1\%$  level precision



Slightly better for HK  
 $\sim 10^{-10}$  for JUNO  
See backups

# NuFast-LBL Speed

Is this algorithm fast?  
How does it compare?



- ▶ “Conservative” = default, `-O0`
- ▶ “Aggressive” = `-Ofast` and `-ffast-math`
- ▶ Some variation expected due to architecture

See also J. Page [2309.06900](#)  
and P. Huber, et al. [hep-ph/0701187](#)

# NuFast-LBL: The Code

- ▶ Code is on github: [github.com/PeterDenton/NuFast-LBL](https://github.com/PeterDenton/NuFast-LBL)
- ▶ Implementations in c++ and f90

---

```
// NuFast.cpp
// Calculate the probabilities:
Probability_Matter_LBL(s12sq, s13sq, s23sq, delta,
    Dmsq21, Dmsq31, L, E, rho, Ye, N_Newton,
    &probs_returned);
// Print out the probabilities to terminal
for (int alpha = 0; alpha < 3; alpha++)
{
    for (int beta = 0; beta < 3; beta++)
    {
        printf("%d %d %g\n", alpha, beta,
            probs_returned[alpha][beta]);
    } // beta, 3
} // alpha, 3
```

---

- ▶  $\bar{\nu}$ :  $E < 0$ ; IO:  $\Delta m_{31}^2 < 0$
- ▶ Folder called **Benchmarks** to make the plots and speed tests in the paper
- ▶ Used in **NuOscillator**, **MaCh3**, **GUNDAM**, and theory papers
- ▶ Speed up of  $\gtrsim 5\times$  over state-of-the-art in realistic usage

# Optimal Structure of the Probability for Atmospherics

1. Cannot apply the previous tricks through the Earth
2. Must compute the amplitude in Each layer

$$\prod_j \mathcal{A}_j$$

# Optimal Structure of the Probability for Atmospherics

3. Work in the tilde basis with  $\theta_{23}$  and  $\delta$  pulled out

- ▶ Can shift  $\delta$  from  $\theta_{13}$  to  $\theta_{23}$

$$\begin{pmatrix} 1 & & \\ c_{23} & s_{23} & \\ -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & s_{13}e^{-i\delta} \\ -s_{13}e^{i\delta} & c_{13} \end{pmatrix} R_{12} \rightarrow \begin{pmatrix} 1 & & \\ c_{23} & s_{23}e^{i\delta} & \\ -s_{23}e^{-i\delta} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & s_{13} \\ -s_{13} & c_{13} \end{pmatrix} R_{12}$$

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►  $U_{23}$  commutes with the matter potential

$$(2E)H_f = U_{23}(\theta_{23}, \delta) \left[ R_{13}R_{12} \begin{pmatrix} 0 & & \\ & \Delta m_{23}^2 & \\ & & \Delta m_{31}^2 \end{pmatrix} R_{12}^T R_{13}^T + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \right] U_{23}^\dagger(\theta_{23}, \delta)$$

$$H_f = U_{23}(\theta_{23}, \delta) \tilde{H} U^\dagger(\theta_{23}, \delta)$$

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$$H_f = U_{23}(\theta_{23}, \delta) \tilde{H} U_{23}^\dagger(\theta_{23}, \delta)$$

$$\mathcal{A} = \prod_j \mathcal{A}_j = \prod_j e^{-iH_{f,j}L_j} = U_{23}(\theta_{23}, \delta) \left( \prod_j e^{-i\tilde{H}_j L_j} \right) U_{23}^\dagger(\theta_{23}, \delta)$$

$$U_{23}U_{23}^\dagger = \mathbb{1}$$

# Optimal Structure of the Probability for Atmospherics

3. Work in the tilde basis with  $\theta_{23}$  and  $\delta$  pulled out

► Can shift  $\delta$  from  $\theta_{13}$  to  $\theta_{23}$

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►  $U_{23}$  commutes with the matter potential

$$(2E)H_f = U_{23}(\theta_{23}, \delta) \left[ R_{13}R_{12} \begin{pmatrix} 0 & \Delta m_{23}^2 \\ & \Delta m_{31}^2 \end{pmatrix} R_{12}^T R_{13}^T + \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \right] U_{23}^\dagger(\theta_{23}, \delta)$$

$$H_f = U_{23}(\theta_{23}, \delta) \tilde{H} U_{23}^\dagger(\theta_{23}, \delta)$$

$$\mathcal{A} = \prod_j \mathcal{A}_j = \prod_j e^{-iH_{f,j}L_j} = U_{23}(\theta_{23}, \delta) \left( \prod_j e^{-i\tilde{H}_j L_j} \right) U_{23}^\dagger(\theta_{23}, \delta)$$

$\tilde{H}$  is real! And doesn't depend on  $\theta_{23}$  or  $\delta$

Reduces many unnecessary computations

$$U_{23}U_{23}^\dagger = 1$$

# Account for Matter in Atmospherics

- ▶ Calculate eigenvalues  $\lambda_i$  as in NuFast-LBL: exactly or approximately with Newton-Raphson corrections
- ▶ Further unitarity reductions:

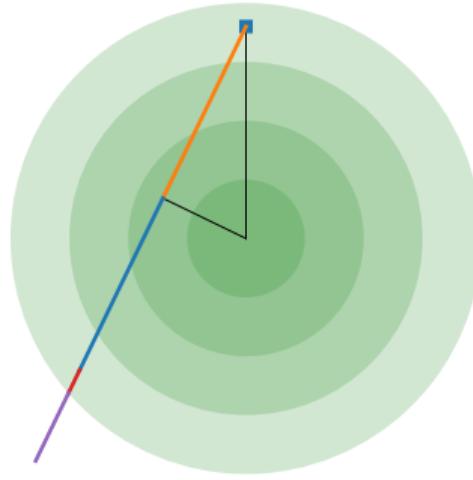
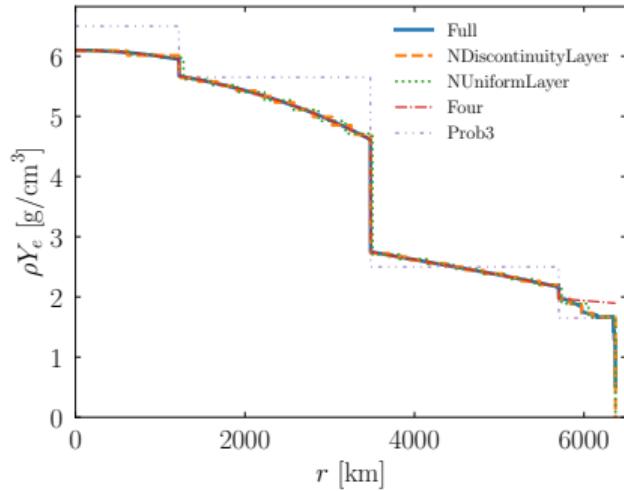
$$\mathcal{A}_{\alpha\beta} = \delta_{\alpha\beta} + V_{\alpha 2} V_{\beta 2}^* (e^{-i\Delta\lambda_{21}L/(2E)} - 1) + V_{\alpha 3} V_{\beta 3}^* (e^{-i\Delta\lambda_{31}L/(2E)} - 1)$$

- ▶ In the tilde basis, the  $\tilde{V}\tilde{V}^*$  terms are real
- ▶ Use adjugate matrices and advanced Eigenvector-Eigenvalue Identity

PBD, S. Parke, T. Tao, X. Zhang [1908.03795](#)  
A. Abdulahi, S. Parke [2212.12565](#)

$$\tilde{V}_{\alpha i} \tilde{V}_{\beta i} = \frac{\text{Adj}[\lambda_i \mathbb{1} - (2E)\tilde{H}]}{\prod_{k \neq i} (\lambda_i - \lambda_k)}$$

# Earth Trajectories

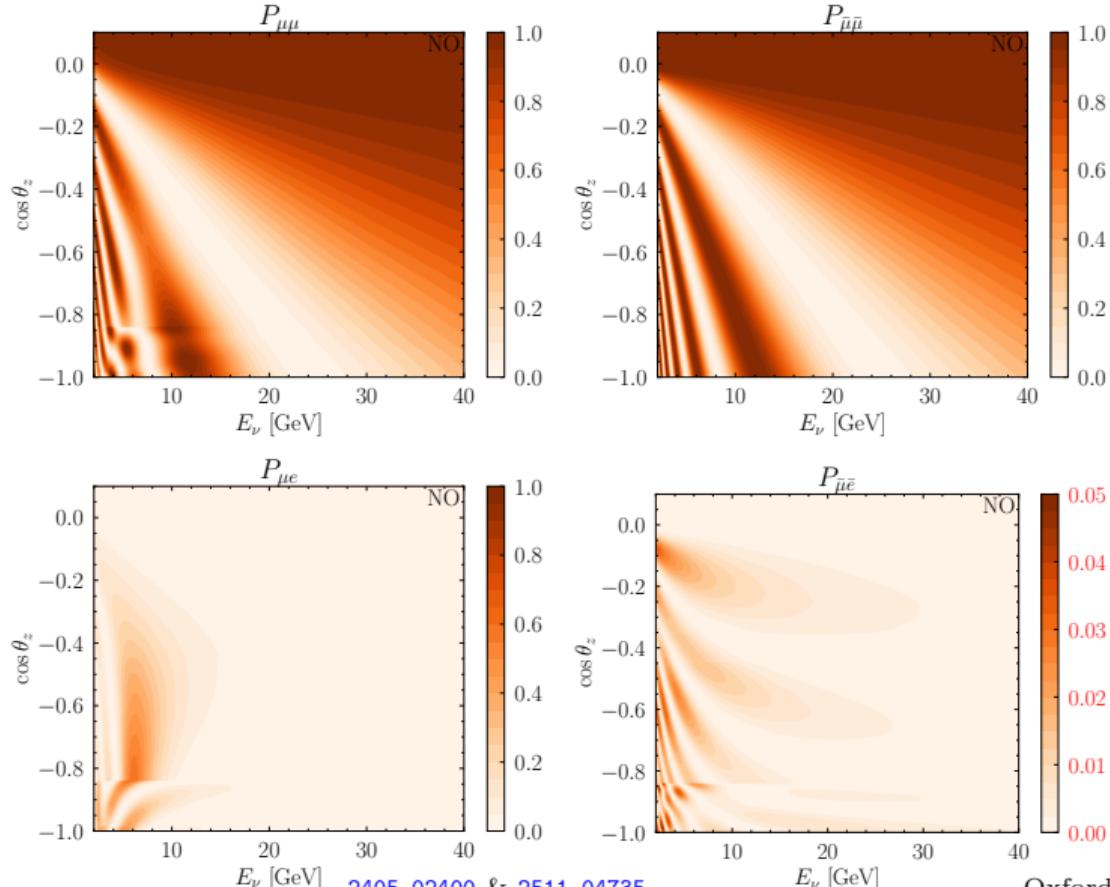


Trajectory goes from:

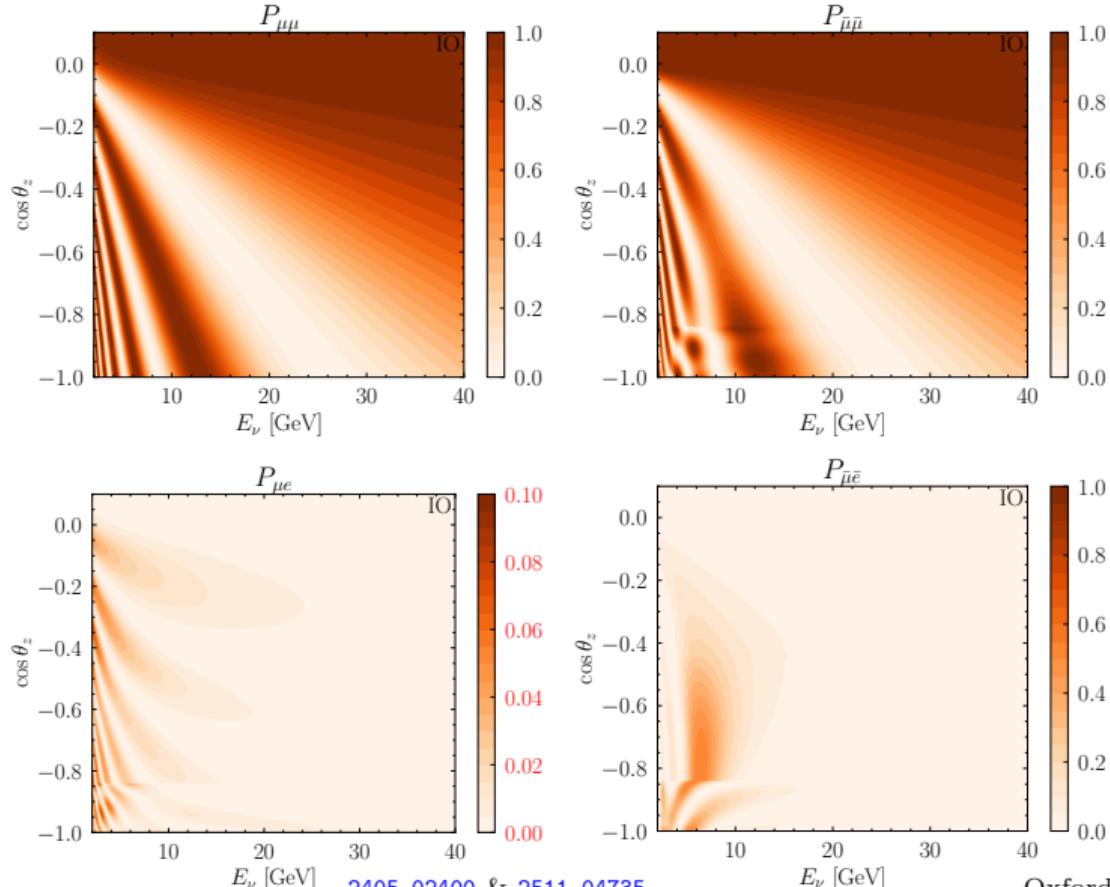
- ▶ Various Earth density profiles
- ▶ Varying or constant density in each shell

1. Production height to surface
2. Surface to detector depth
3. Detector depth to deepest point
4. Deepest point to detector depth:  
this is the transpose of #3

# Atmospheric Results: NO



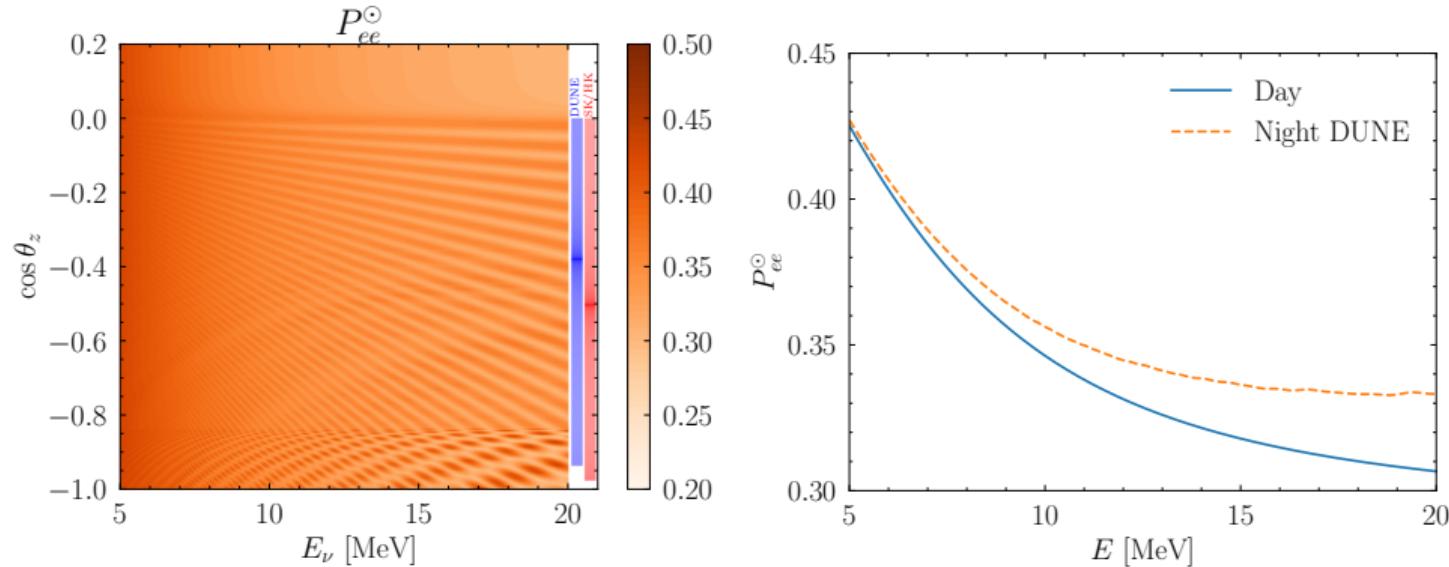
# Atmospheric Results: IO



# Nighttime Solar Neutrinos

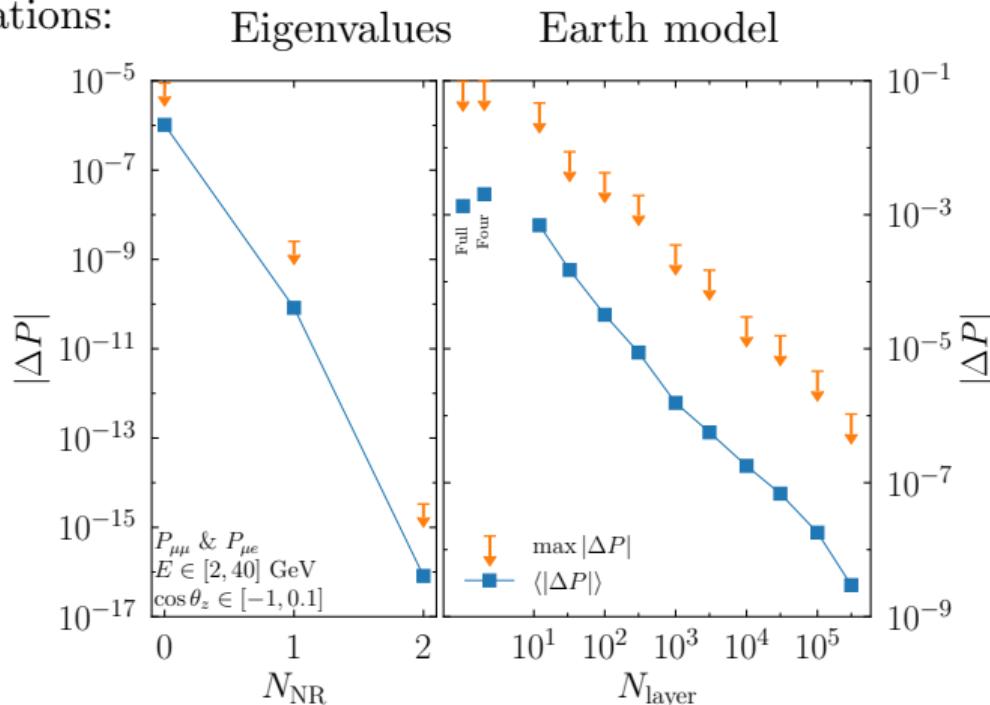
At night solar neutrinos experience partial regeneration:  
There are more  $\nu_e$ 's from the Sun at night than during the day!

SuperK has  $\sim 2\sigma$  evidence for this effect; DUNE and HK aim to measure it well



# NuFast-Earth Precision

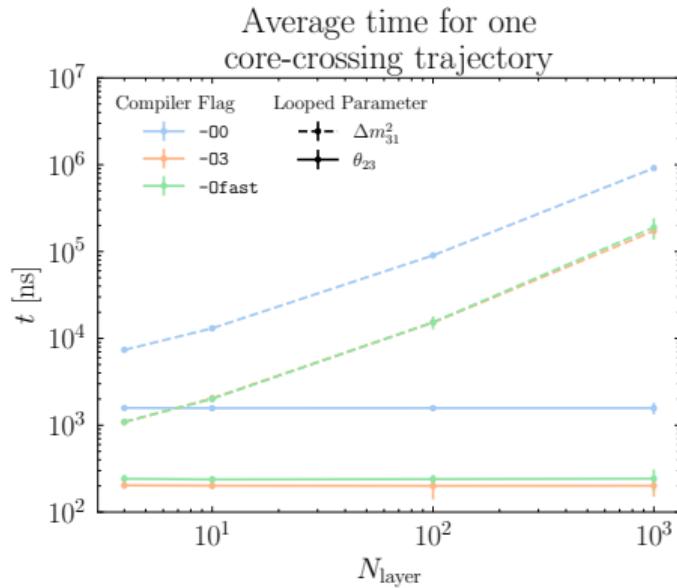
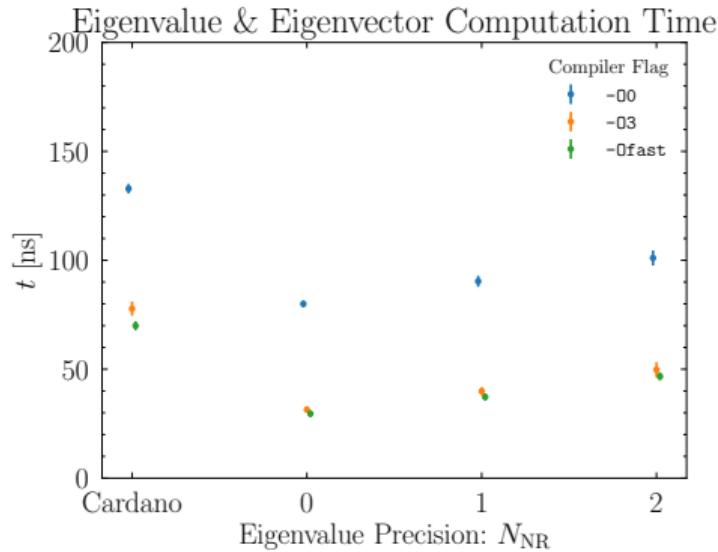
Two approximations:



“True” has exact eigenvalues and 1M layers

Calculate  $|\Delta P|$  across  $100 \times 100$  grid in energy and angle

# NuFast-Earth Speed



nuSQuIDS takes  $>1\text{M}$  ns per trajectory

C. Argüelles, J. Salvado, C. Weaver [2112.13804](https://arxiv.org/abs/2112.13804)

OscProb takes 40 000 ns per trajectory at 44 layers (compare to 6000 ns or 200 ns)

J. Coelho, R. Pestes, et al. [github.com/joaoabcoelho/OscProb](https://github.com/joaoabcoelho/OscProb)

Other codes are designed with different goals in mind

# NuFast-Earth Flowchart

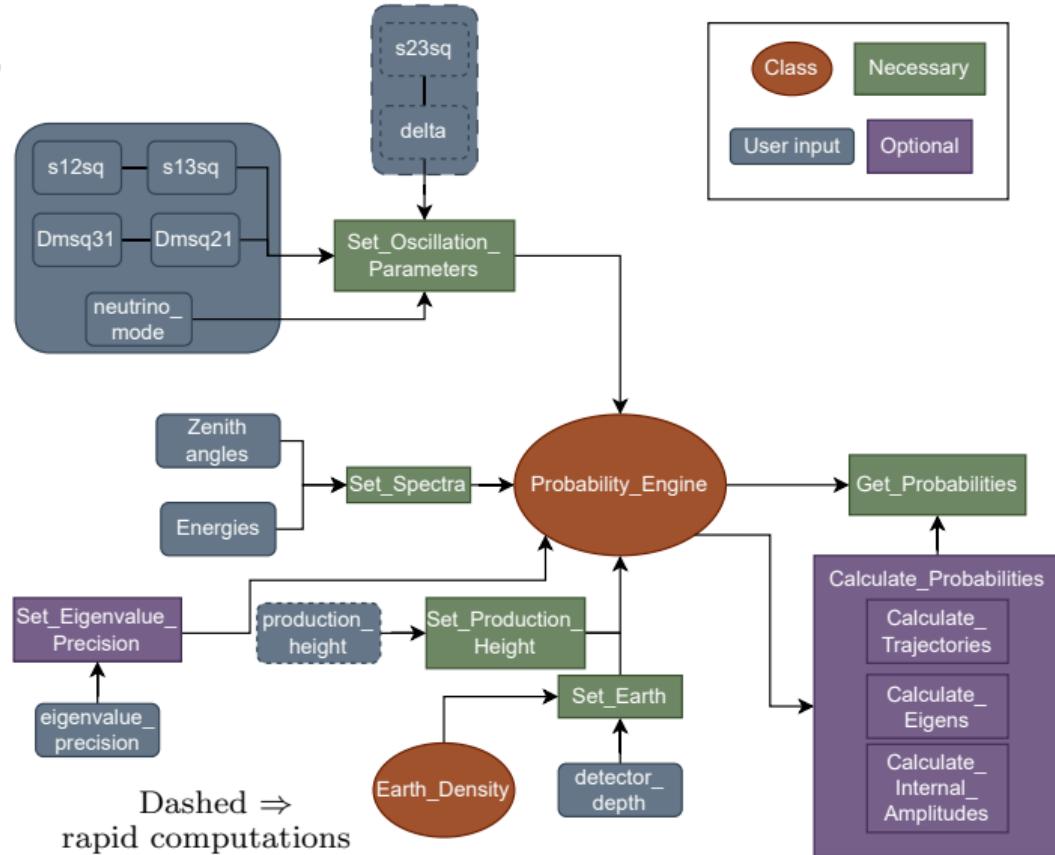
```
// Initialize the engine
Probability_Engine probability_engine;

// Set the oscillation parameters to nu-fit 6 best fit
// values:
probability_engine.Set_Oscillation_Parameters(0.307,
    0.02195, 0.561, 177 * M_PI / 180, 7.49e-5, 2.534e-3,
    true); // (s12sq, s13sq, s23sq, delta, Dmsq21, Dmsq31,
    neutrino_mode)

// Set energy and zenith angle arrays
std::vector<double> energies = {1, 2, 3, 4, 5}; // GeV
std::vector<double> coszs = {-1, -0.5, 0, 1}; // core-
// crossing to horizontal to down-going
probability_engine.Set_Spectra(energies, coszs);

// Create Earth model instance
PREM_NDiscontinuityLayer earth_density(2, 10, 10, 5); //
// 2 layers in the inner core, 10 layers in the outer
// core, 10 layers in the inner mantle, and 5 layers in
// the outer mantle
// Set Earth details
probability_engine.Set_Earth(2, &earth_density); // 
// detector depth in km
// Set production height (optional)
probability_engine.Set_Production_Height(10); // km,
// recalculations after changing this are fast

// Do the calculations
std::vector<std::vector<Matrix3r>> probabilities =
    probability_engine.Get_Probabilities();
```



# NuFast-Earth Recommendations

Many choices in how to use NuFast-Earth  
What is the best way for most cases?

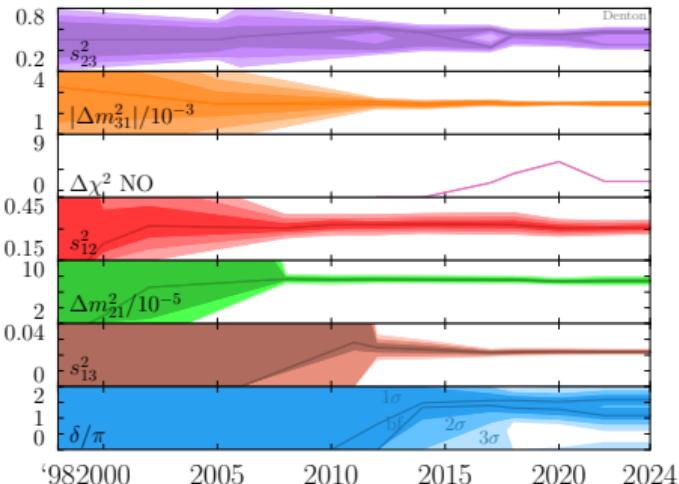
1. Eigenvalue precision: set the number of Newton-Raphson corrections to 1  
`probability_engine.Set_Eigenvalue_Precision(1);`
2. Earth model: 2, 10, 10, 5 layers in each of the four main zones  
`PREM_NDiscontinuityLayer earth_density(2, 10, 10, 5);`
3. Looping order: put  $\theta_{23}$ ,  $\delta$ , and production height on the innermost loops

## Discussion and Conclusions

- ▶ **Disappearance can discover CPV**
- ▶ Requires two good measurements: JUNO and DUNE/HK
- ▶ Can rule out some values of  $\delta$  at  $> 3\sigma$
- ▶ **LBL Experiments should break down  $\delta$  analyses into app vs. dis**
- ▶ Since systematics are different, provides a good cross check
  
- ▶ NuFast-LBL and NuFast-Earth provide **fast, precise, usable code**
- ▶ Leverages modern linear algebra and neutrino theory
- ▶ Orders of magnitude impact in computational cost
- ▶ LBL code implemented in pipelines for NOvA, T2K, DUNE, HK, & JUNO
- ▶ Atmospheric code being implemented now

# Backups

# References



SK [hep-ex/9807003](#)

M. Gonzalez-Garcia, et al. [hep-ph/0009350](#)

M. Maltoni, et al. [hep-ph/0207227](#)

SK [hep-ex/0501064](#)

SK [hep-ex/0604011](#)

T. Schwetz, M. Tortola, J. Valle [0808.2016](#)

M. Gonzalez-Garcia, M. Maltoni, J. Salvado [1001.4524](#)

T2K [1106.2822](#)

D. Forero, M. Tortola, J. Valle [1205.4018](#)

D. Forero, M. Tortola, J. Valle [1405.7540](#)

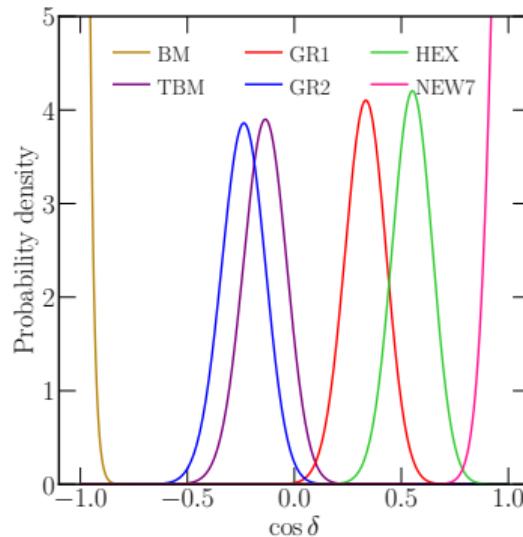
P. de Salas, et al. [1708.01186](#)

F. Capozzi et al. [2003.08511](#)

# The Importance of $\cos \delta$

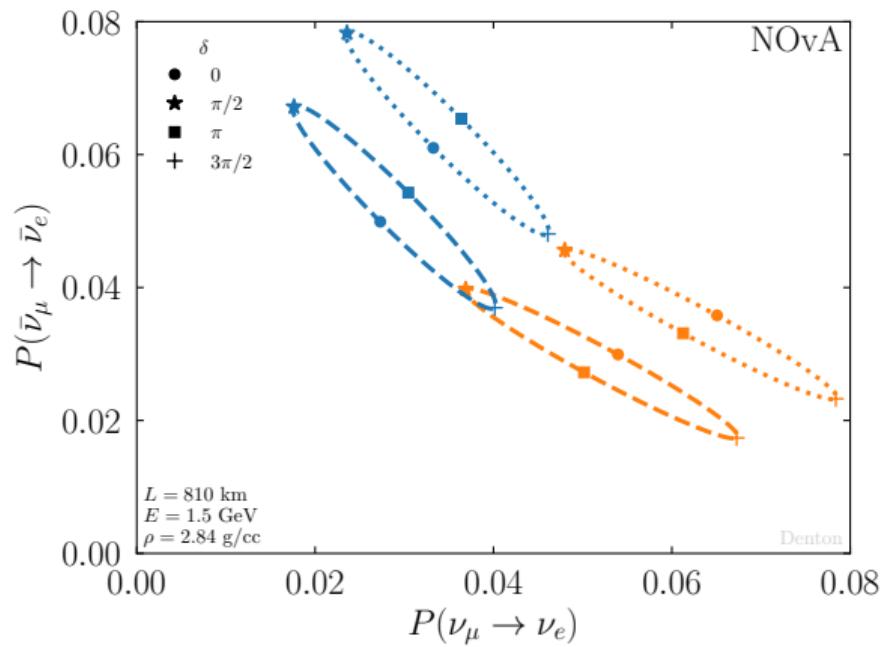
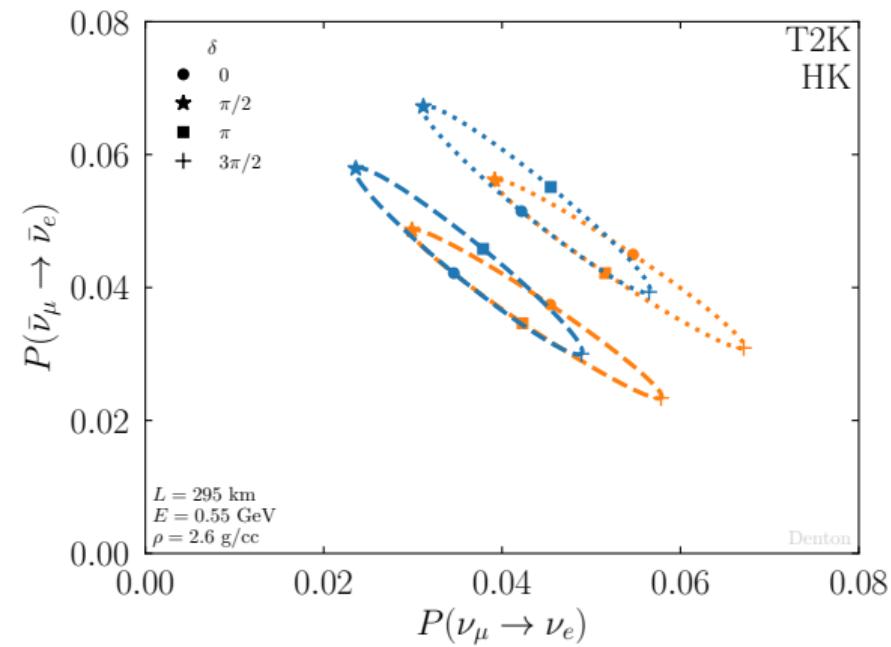
- ▶ If only  $\sin \delta$  is measured  $\Rightarrow$  sign degeneracy:  $\cos \delta = \pm \sqrt{1 - \sin^2 \delta}$
- ▶ Most flavor models predict  $\cos \delta$

J. Gehrlein, et al. [2203.06219](#)



L. Everett, et al. [1912.10139](#)

# $\delta$ : What is it Really?



# Appearance vs. Disappearance

Oscillation experiments can do  
appearance or disappearance experiments:

## Disappearance

K2K, MINOS, T2K, NO $\nu$ A  
KamLAND, Daya Bay, RENO, Double CHOOZ  
(Sort of) SNO, Borexino, SK-solar  
JUNO, DUNE, HK

## Appearance

T2K, NO $\nu$ A  
OPERA  
Atm  $\nu_\tau$  hints @ SK & IceCube  
DUNE, HK

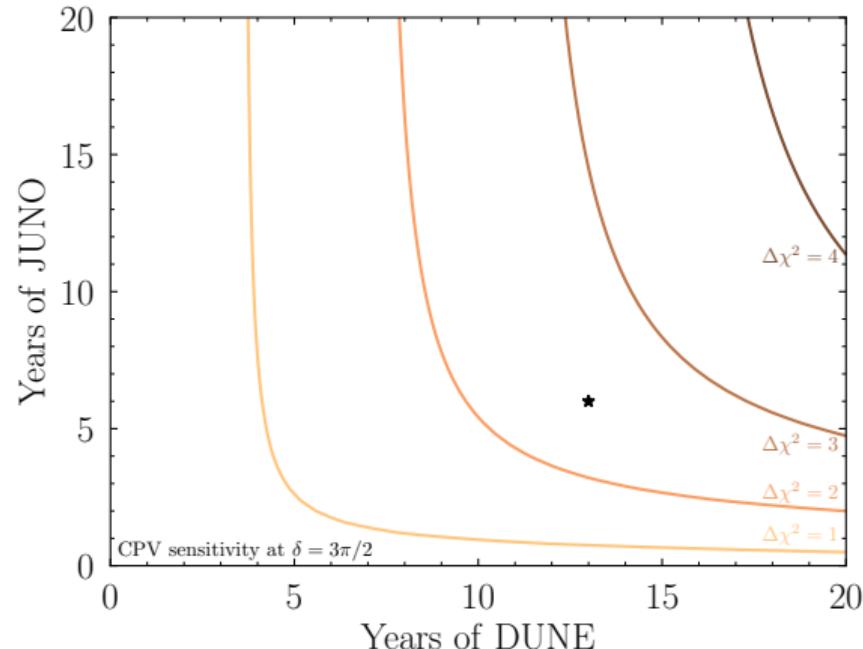
## Neither appearance nor disappearance

SK-atm, IceCube



# Varying Runtime/Power

Significance to disfavor  $|\cos \delta| = 1$  at  $\cos \delta = 0$



Improvement requires **both** experiments!

# Start at the End

What is needed for LBL experiments?

1. All 9 channels ( $\nu_\alpha \rightarrow \nu_\beta$ )
  - ▶ DUNE will certainly do  $\nu_\tau$  appearance  
See e.g. P. Machado, H. Schulz, J. Turner [2007.00015](#)
  - ▶  $\nu_\tau \rightarrow \nu_\beta$  channels are not needed, but come from free from unitarity
  - ▶ JUNO needs only  $\bar{\nu}_e \rightarrow \bar{\nu}_e$
2. Different energies, baselines, and densities
3.  $\nu$  and  $\bar{\nu}$
4. NO and IO
5. Oscillation parameters are mostly known
  - ▶ Don't need to consider e.g.  $\Delta m_{21}^2 > |\Delta m_{31}^2|$  or  $\theta_{23} \sim 10^\circ$

# All 9 Channels

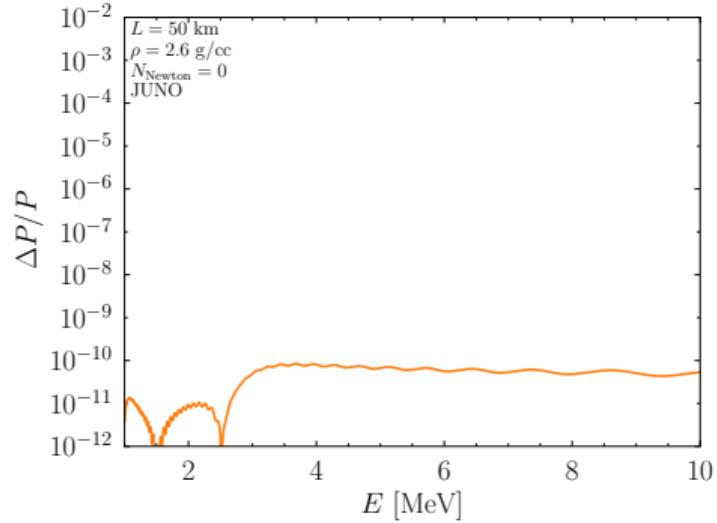
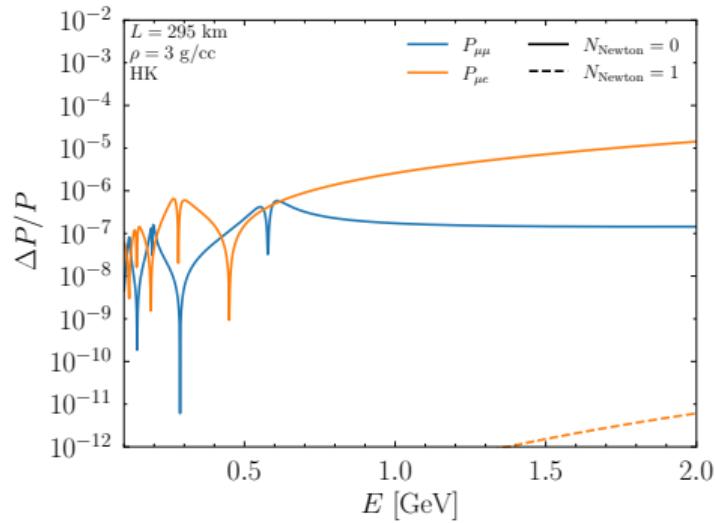
Given  $P_{ee}$ ,  $P_{\mu\mu}$ ,  $P_{\mu e}^{TC}$ , and  $P_{\mu e}^{TV}$ :

	$P_{\alpha e}$	$P_{\alpha \mu}$	$P_{\alpha \tau}$
$P_{e\beta}$	$P_{ee}$	$P_{\mu e}^{TC} - P_{\mu e}^{TV}$	$1 - P_{ee} - P_{\mu e}^{TC} + P_{\mu e}^{TV}$
$P_{\mu\beta}$	$P_{\mu e}^{TC} + P_{\mu e}^{TV}$	$P_{\mu\mu}$	$1 - P_{\mu\mu} - P_{\mu e}^{TC} - P_{\mu e}^{TV}$
$P_{\tau\beta}$	$1 - P_{ee} - P_{\mu e}^{TC} - P_{\mu e}^{TV}$	$1 - P_{\mu\mu} - P_{\mu e}^{TC} + P_{\mu e}^{TV}$	$-1 + P_{ee} + P_{\mu\mu} + 2P_{\mu e}^{TC}$

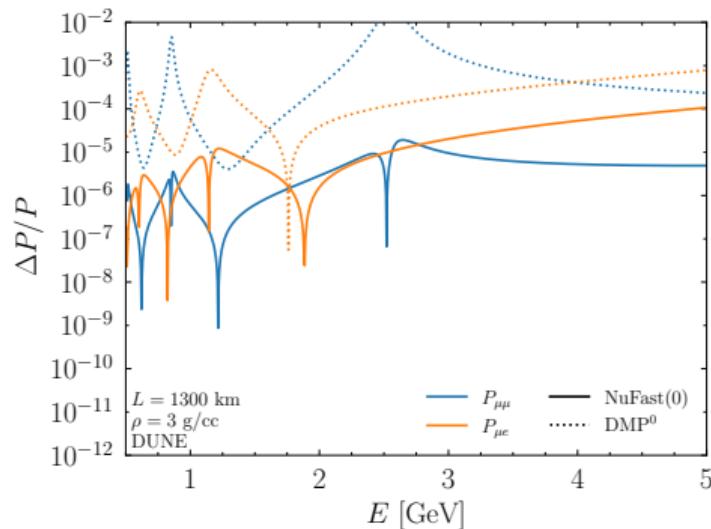
# Total LBL Approach

1. Inputs: 6 oscillation parameters, experimental details ( $L, E, \rho, Y_e$ )
2. Calculate  $\lambda_3$  approximately
  - ▶ Iteratively improve with Newton's method, if desired
3. Calculate the  $|V_{\alpha i}|^2$ 's with the Eigenvector-Eigenvalue Identity
4. Calculate the sines of the kinematic terms
5. Calculate the CP odd term with the NHS identity
6. Calculate key probabilities:  $P_{ee}$ ,  $P_{\mu\mu}$ , and  $P_{\mu e}$
7. Calculate remaining probabilities

# Precision

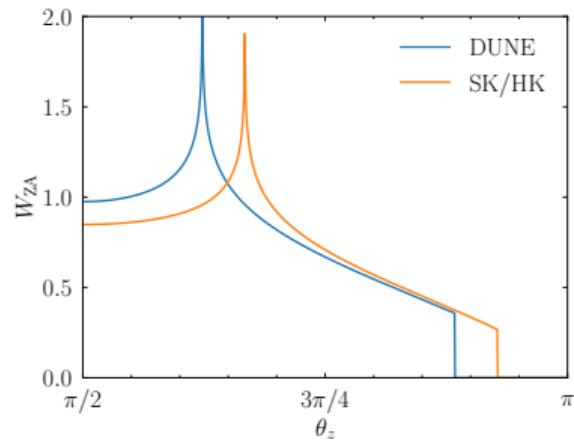
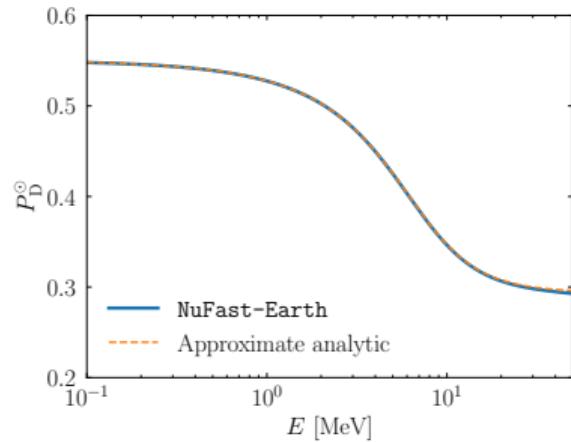


# Comparison to DMP



DMP: [PBD](#), H. Minakata, S. Parke [1604.08167](#)

# NuFast-Earth Solar Neutrino Validation



# NuFast-Earth LBL Validation

