

The best way to probe CP violation in the lepton sector is with long-baseline accelerator neutrino experiments in the appearance mode. I will show that it is possible to discover CP violation with disappearance experiments only, by combining JUNO for electron neutrinos and DUNE or Hyper-Kamiokande for muon neutrinos. While the maximum sensitivity to discover CP is quite modest, some values of δ may be disfavored by $> 3\sigma$ depending on the true value of δ .

Neutrino oscillation experiments will be entering the precision era in the next decade. Correctly estimating the confidence intervals from data for the oscillation parameters requires very large Monte Carlo data sets involving calculating the oscillation probabilities in matter many, many times. In this talk, I will leverage past work to present a new, fast, precise technique for calculating neutrino oscillation probabilities in matter optimized for long-baseline neutrino oscillations called NuFast. I will also present some preliminary results for atmospheric and solar neutrinos.

CP-Violation with Neutrino Disappearance and NuFast

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November 5, 2025

2309.03262 PRL

2405.02400 and 2511.XXXXX with S. Parke
github.com/PeterDenton/NuFast-LBL



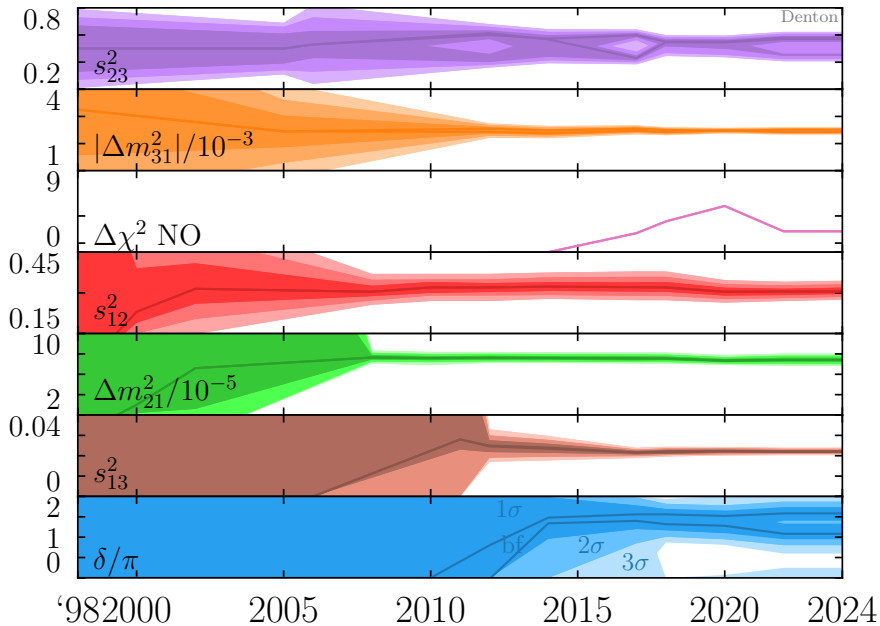
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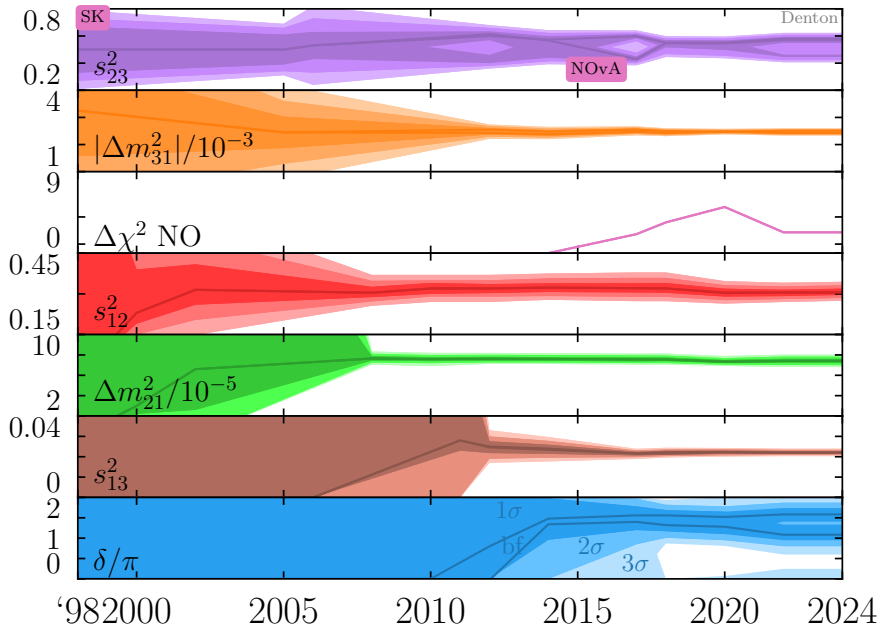


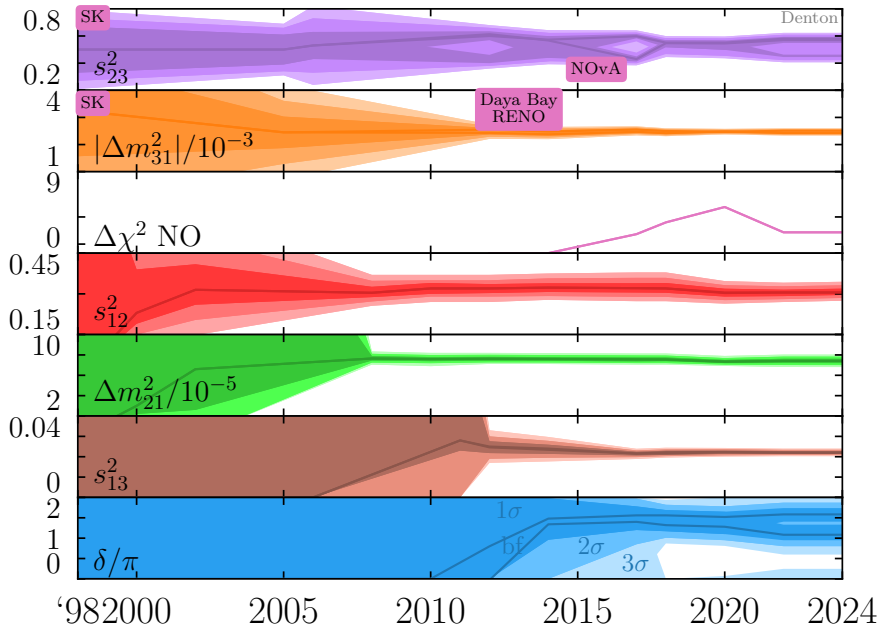
The BSM

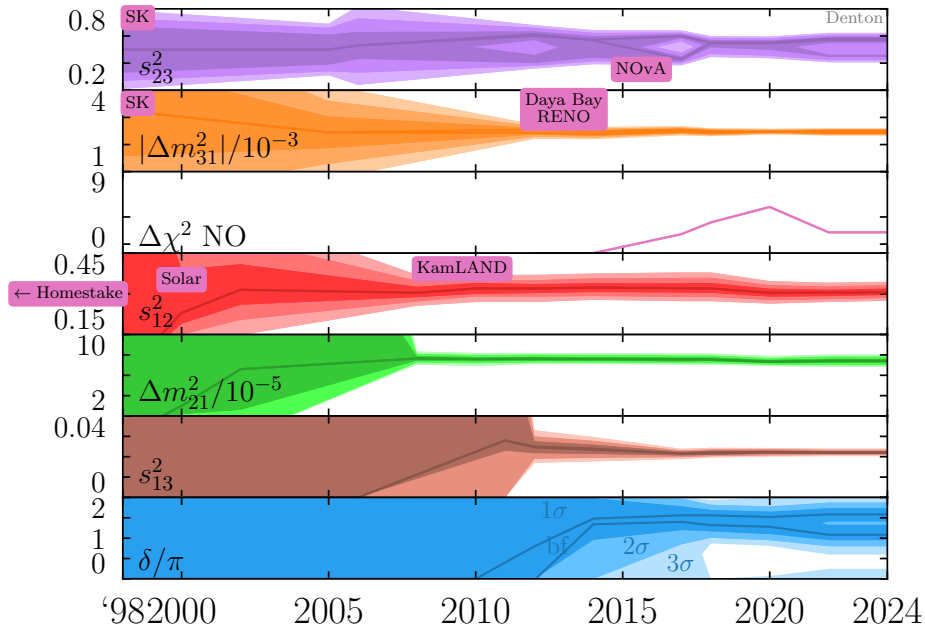
Physicists

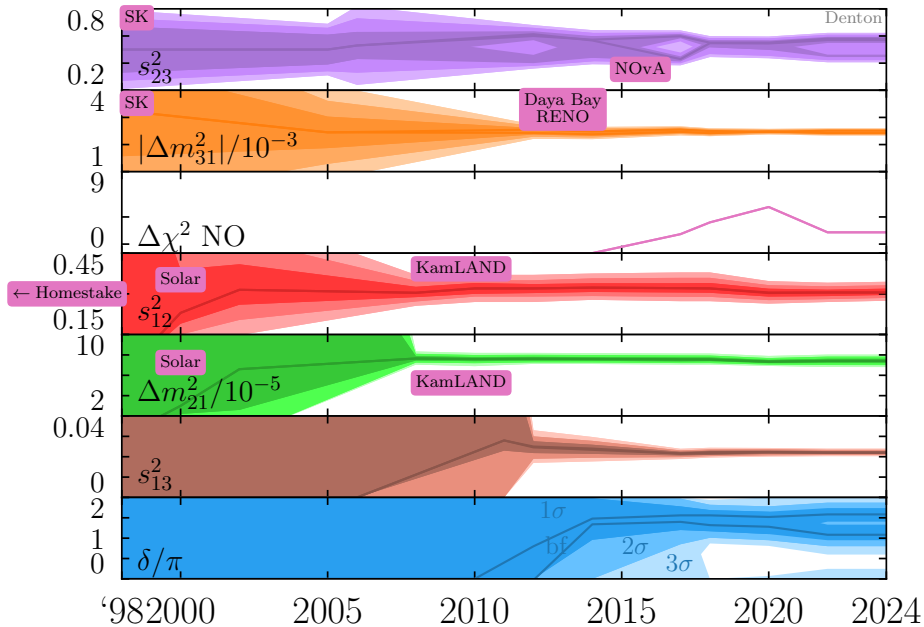


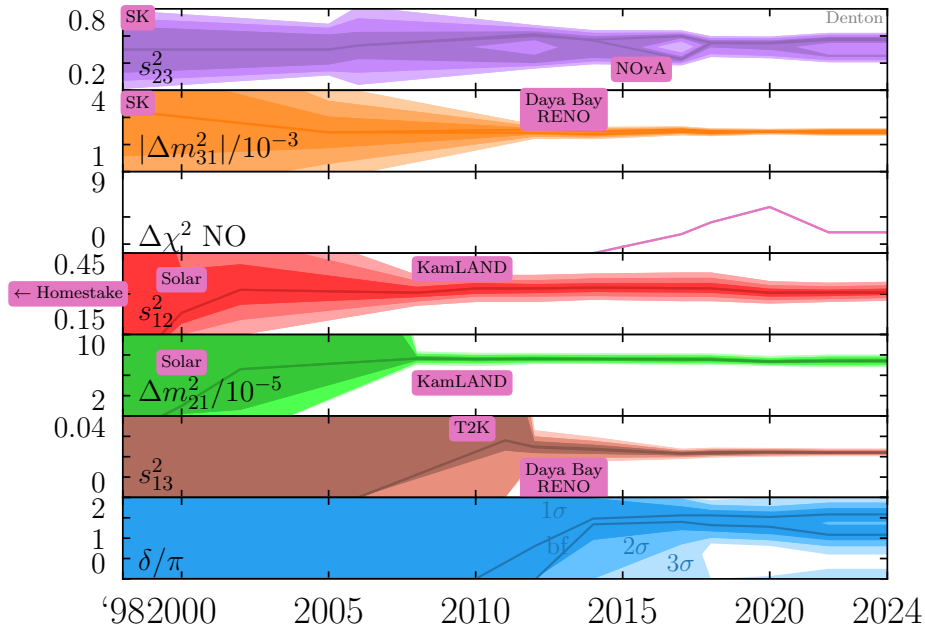


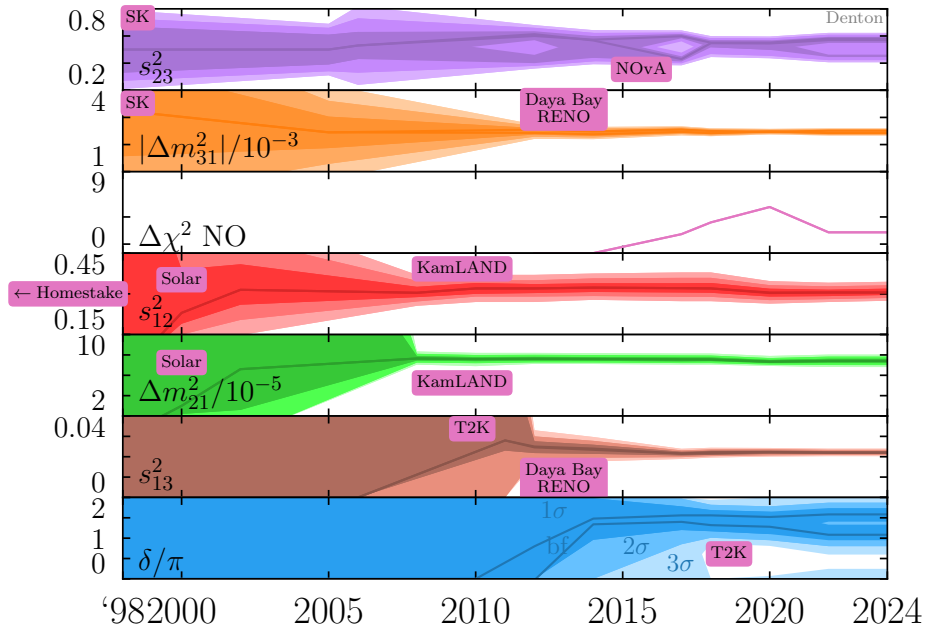












Atmospheric mass ordering

θ_{23} octant

Complex phase

Absolute mass scale

Atmospheric mass ordering

θ_{23} octant

Complex phase

Absolute mass scale

Outline

1. Why CPV is interesting
2. Other non-standard probes of CPV
3. Relationship between appearance, disappearance, CP, T, CPT
4. Three ways to see why there is CPV information in disappearance
 - 4.1 Parameter counting
 - 4.2 Direct analytic calculation
 - 4.3 Numerical test
5. Role of the matter effect
6. Sensitivities
7. NuFast-LBL
8. NuFast-Earth

Why is CPV interesting?

δ and CP Violation

$$J_{CP} = s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23}\sin\delta$$

C. Jarlskog [PRL 55, 1039 \(1985\)](#)



δ and CP Violation

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1. Strong interaction: no observed EDM \Rightarrow CP (nearly) **conserved**

$$\frac{\bar{\theta}}{2\pi} < 10^{-11}$$

J. Pendlebury, et al. [1509.04411](#)

2. Quark mass matrix: non-zero but **small** CP violation

$$\frac{|J_{CKM}|}{J_{\max}} = 3 \times 10^{-4}$$

CKMfitter [1501.05013](#)

3. Lepton mass matrix: ?

$$\frac{|J_{PMNS}|}{J_{\max}} < 0.34$$

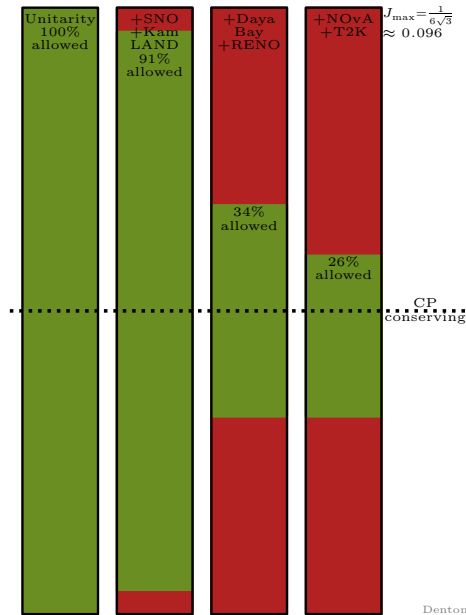
[PBD](#), J. Gehrlein, R. Pestes [2008.01110](#)

$$J_{\max} = \frac{1}{6\sqrt{3}} \approx 0.096$$

δ, J : Current Status

Maximal CP violation is already ruled out:

1. $\theta_{12} \neq 45^\circ$ at $\sim 15\sigma$
2. $\theta_{13} \neq \tan^{-1} \frac{1}{\sqrt{2}} \approx 35^\circ$ at many (100) σ
3. $\theta_{23} = 45^\circ$ allowed at $\sim 1\sigma$
4. $|\sin \delta| = 1$ allowed



When δ and When J ?

If the goal is **CP violation** the Jarlskog invariant should be used

however

If the goal is **measuring the parameters** one must use δ

Given θ_{12} , θ_{13} , θ_{23} , and J , I can't determine the sign of $\cos \delta$ which is physical

e.g. $P(\nu_\mu \rightarrow \nu_\mu)$ depends on $\cos \delta$

Other Non-standard CPV Probes

1. Some information in solar due to loops in elastic scattering

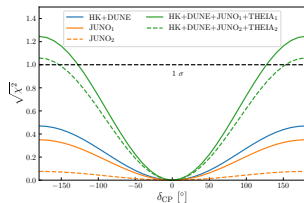
V. Brdar, X-J. Xu [2306.03160](#)

K. Kelly, et al. [2407.03174](#) requires 3k Borexinos

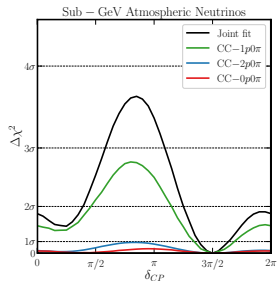
2. Sub-GeV \rightarrow sub-100 MeV atmospheric

K. Kelly, et al. [1904.02751](#)

See also e.g. A. Suliga, J. Beacom [2306.11090](#)



Solar (no systematics)



Atmospherics at DUNE

Appearance vs. Disappearance

Oscillation experiments can do
appearance or disappearance experiments:

Disappearance

K2K, MINOS, T2K, NO ν A

KamLAND, Daya Bay, RENO, Double CHOOZ

(Sort of) SNO, Borexino, SK-solar

JUNO, DUNE, HK

Neither appearance nor disappearance

SK-atm, IceCube

Appearance

T2K, NO ν A

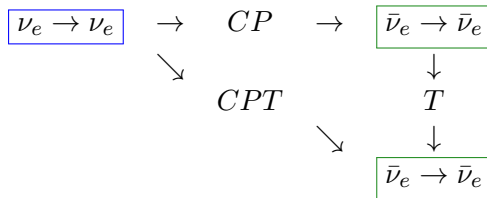
OPERA

Atm ν_τ hints @ SK & IceCube

DUNE, HK



CP, T: Disappearance

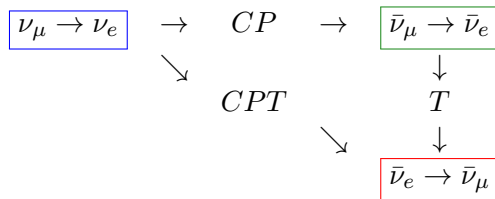


Disappearance measurements are even eigenstates of CP

$$CP[P(\nu_e \rightarrow \nu_e)] = P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \stackrel{CPT}{=} P(\nu_e \rightarrow \nu_e)$$

Assume that CPT is a good symmetry

CP, T: Appearance



Appearance measurements are not eigenstates of CP

Appearance, Disappearance, and CP

Appearance and Disappearance, CP Even and CP Odd Terms

Disappearance:

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\alpha) &= 1 - 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2 \Delta_{21} \\ &\quad - 4|U_{\alpha 1}|^2|U_{\alpha 3}|^2 \sin^2 \Delta_{31} \\ &\quad - 4|U_{\alpha 2}|^2|U_{\alpha 3}|^2 \sin^2 \Delta_{32} \\ &= P_{\alpha\alpha}^{CP+} \end{aligned}$$

$$\Delta_{ij} \equiv \Delta m_{ij}^2 L/4E$$

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Appearance:

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) &= -4\Re[U_{\alpha 1}U_{\beta 1}^*U_{\alpha 2}^*U_{\beta 2}] \sin^2 \Delta_{21} \\ &\quad - 4\Re[U_{\alpha 1}U_{\beta 1}^*U_{\alpha 3}^*U_{\beta 3}] \sin^2 \Delta_{31} \\ &\quad - 4\Re[U_{\alpha 3}U_{\beta 3}^*U_{\alpha 2}^*U_{\beta 2}] \sin^2 \Delta_{32} \\ &\quad \pm 8J_{CP} \sin \Delta_{21} \sin \Delta_{31} \sin \Delta_{32} \\ &= P_{\alpha\beta}^{CP+} + P_{\alpha\beta}^{CP-} \end{aligned}$$

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Sign depends on α, β

Conventional Wisdom

1. Appearance is sensitive to CPV

[True]

Conventional Wisdom

- | | |
|---|---------|
| 1. Appearance is sensitive to CPV | [True] |
| 2. Disappearance has no CPV sensitivity | [False] |

Conventional Wisdom

1. Appearance is sensitive to CPV [True]
2. Disappearance has no CPV sensitivity [False]
3. Any δ dependence in disappearance is in ν_μ not ν_e [Confusing/False]

$$\begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Correct Statements

- ▶ Appearance is the best way to measure δ and CPV
 - ... given known oscillation parameters, systematics, and realistic experiments
- ▶ Probes mostly $\sin \delta$ not $\cos \delta$
 - ▶ Don't need both ν and $\bar{\nu}$ (but systematics)
- ▶ **Disappearance can measure δ**
 - ▶ CPV can be discovered with only disappearance measurements
 - ▶ Probes mostly $\cos \delta$ not $\sin \delta$
 - ▶ Requires measurements of two flavors
 - ▶ “Works through unitarity” (as do nearly all oscillation measurements)

Parameter Counting

1. Four parameters in the PMNS matrix

Majorana phases are irrelevant in oscillations

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2. Disappearance experiments of one flavor can measure up to three amplitudes

- ▶ Electron neutrino row:
 - ▶ KamLAND measured one
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$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - 4 \sum_{i>j} C_{ij}^\alpha \sin^2 \Delta_{ij}$$

$$\Delta_{ij} \equiv \Delta m_{ij}^2 L/4E$$

$$C_{ij}^\alpha = |U_{\alpha i}|^2 |U_{\alpha j}|^2$$

$$|U_{\alpha i}| = \left(\frac{C_{ij}^\alpha C_{ik}^\alpha}{C_{jk}^\alpha} \right)^{1/4}$$

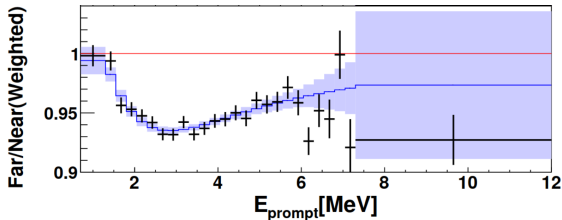
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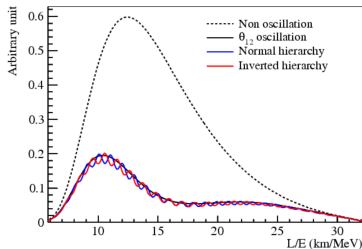
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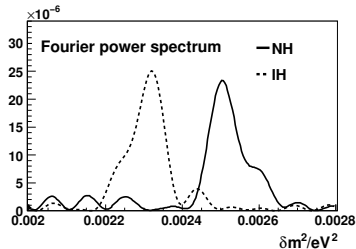
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JUNO [1507.05613](#)

[2309.03262](#)



L. Zhan, et al. [0807.3203](#)

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4. Given good measurements of the ν_e and ν_μ disappearance, 4 independent parameters will be measured

- ▶ Any row can be “simple” (e.g. $c_{12}c_{13}$, $s_{12}c_{13}$, ...) \Rightarrow no one row is ever enough
- ▶ That is, CPV is physical and cannot depend on parameterization

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6. If we determine $\cos \delta \neq \pm 1 \Rightarrow$ CP is violated!

Direct Analytic Calculation

Disappearance experiments measure various $|U_{\alpha i}|^2$ terms

Suppose 4 are measured: $|U_{e2}|^2$, $|U_{e3}|^2$, $|U_{\mu 2}|^2$, $|U_{\mu 3}|^2$

Actually this gives all 9 magnitudes by unitarity

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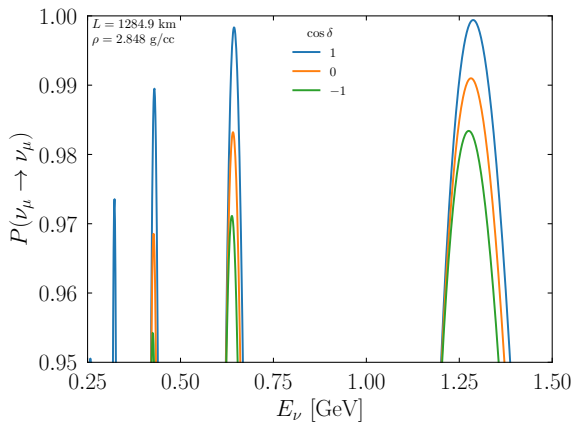
Actually this gives all 9 magnitudes by unitarity

$$J_{CP}^2 = |U_{e2}|^2 |U_{\mu 2}|^2 |U_{e3}|^2 |U_{\mu 3}|^2 \\ - \frac{1}{4} \left(1 - |U_{e2}|^2 - |U_{\mu 2}|^2 - |U_{e3}|^2 - |U_{\mu 3}|^2 + |U_{e2}|^2 |U_{\mu 3}|^2 + |U_{e3}|^2 |U_{\mu 2}|^2 \right)^2$$

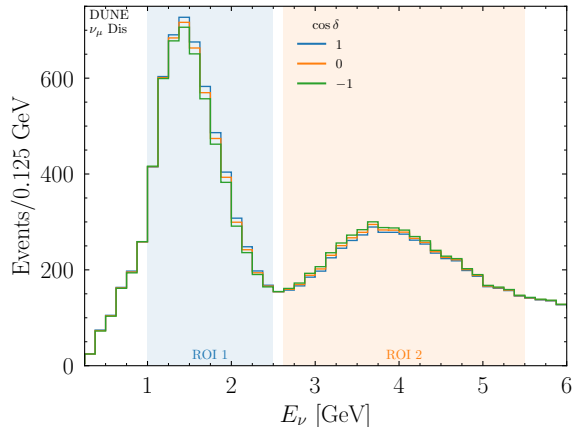
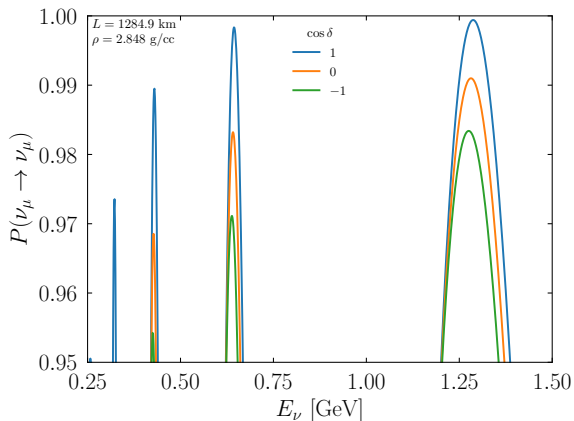
Can show that if any one $|U_{\alpha i}|^2 = 0 \Rightarrow J = 0$

Disappearance can tell us if CP is violated,
but not if Nature prefers ν 's or $\bar{\nu}$'s

Where is $|U_{\mu 2}|^2$?



Where is $|U_{\mu 2}|^2$?



	$\cos \delta$	ROI 1	ROI 2
6.5 yrs ν_μ rates	1	5506	5038
	0	5418	5115
	-1	5334	5193

Approximate Size of $|U_{\mu 2}|^2$ Signal: (21) Sector

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DUNE and HK can measure Δm_{21}^2 somewhat
PBD, J. Gehrlein [2302.08513](#)

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PBD, J. Gehrlein 2302.08513

- ▶ This term is

$$\begin{aligned}
 P_{\mu\mu} &\supset -4c_{23}^2 (s_{12}^2 c_{12}^2 + s_{23} c_{23} s_{13} \sin 2\theta_{12} \cos 2\theta_{12} \cos \delta) \sin^2 \Delta_{21} \\
 &\approx -2 \quad (0.21 + 0.03 \cos \delta) \left(\frac{\pi}{33}\right)^2 \\
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 \end{aligned}$$

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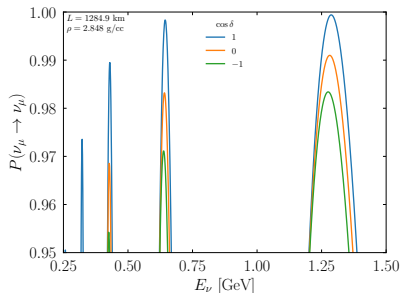
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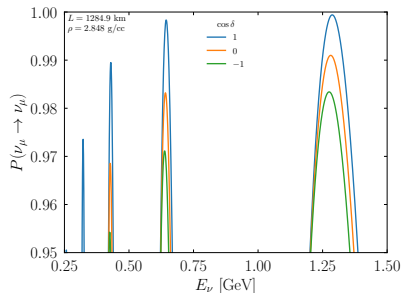
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Sign is wrong
Magnitude is ~ 16 too small

Matter Effects Matter: (21) Sector

- Let's start at

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- ▶ Solar splitting in matter modified by

$$\Delta m_{21}^2 \rightarrow \Delta m_{21}^2 \mathcal{S}_{\odot}$$

$$\mathcal{S}_{\odot} \approx \sqrt{(\cos 2\theta_{12} - c_{13}^2 a / \Delta m_{21}^2)^2 + \sin^2 2\theta_{12}} \approx 3.4$$

at $E = 1.3$ GeV
PBD, S. Parke 1902.07185

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- ▶ Mixing angle is modified

$$\cos 2\theta_{12} = 0.37 \rightarrow \frac{\cos 2\theta_{12} - c_{13}^2 a / \Delta m_{21}^2}{\mathcal{S}_{\odot}} \approx -0.96 < 0$$

$$a \propto \rho E$$

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at $E = 1.3$ GeV
PBD, S. Parke [1902.07185](#)

$$\cos 2\theta_{12} = 0.37 \rightarrow \frac{\cos 2\theta_{12} - c_{13}^2 a / \Delta m_{21}^2}{\mathcal{S}_{\odot}} \approx -0.96 < 0$$

- ▶ So the sign is swapped

$$a \propto \rho E$$

$$\sin 2\theta_{12} \cos 2\theta_{12} = 0.35 \rightarrow -0.26$$

Matter Effects Matter: (21) Sector

- ▶ Let's start at

$$P_{\mu\mu} \supset -4c_{23}^2 (s_{12}^2 c_{12}^2 + s_{23} c_{23} s_{13} \sin 2\theta_{12} \cos 2\theta_{12} \cos \delta) \sin^2 \Delta_{21}$$

- ▶ Solar splitting in matter modified by

$$\Delta m_{21}^2 \rightarrow \Delta m_{21}^2 \mathcal{S}_{\odot}$$

$$\mathcal{S}_{\odot} \approx \sqrt{(\cos 2\theta_{12} - c_{13}^2 a / \Delta m_{21}^2)^2 + \sin^2 2\theta_{12}} \approx 3.4$$

at $E = 1.3$ GeV
PBD, S. Parke 1902.07185

- ▶ Mixing angle is modified

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$$\sin 2\theta_{12} \cos 2\theta_{12} = 0.35 \rightarrow -0.26$$

- ▶ Also s_{13} increases in matter $\sim 15\%$: total effect is $0.004 \cos \delta$
- ▶ This gets us **half** of the effect, and the correct sign

Matter Effects Matter: (32) Sector

► $\frac{\Delta m_{\mu\mu}^2 L}{4E}$ in matter at the maximum is $\sim \pi$

H. Nunokawa, S. Parke, R. Funchal [hep-ph/0503283](#)
[PBD](#), S. Parke [2401.10326](#)

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H. Nunokawa, S. Parke, R. Funchal [hep-ph/0503283](#)
[PBD](#), S. Parke [2401.10326](#)

- ▶ The Δm_{32}^2 component is a bit off π at max

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H. Nunokawa, S. Parke, R. Funchal [hep-ph/0503283](#)
[PBD](#), S. Parke [2401.10326](#)

- ▶ The Δm_{32}^2 component is a bit off π at max

- ▶ Leading order in s_{13} :

$$\begin{aligned} &\approx -4s_{23}^2 (c_{12}^2 c_{23}^2 - 2s_{13}s_{12}c_{12}s_{23}c_{23}\cos\delta) \sin^2 \Delta_{32} \\ &\approx -2 \quad (0.0094 \quad -0.023 \cos\delta) 0.1 \quad (\text{matter}) \end{aligned}$$

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H. Nunokawa, S. Parke, R. Funchal [hep-ph/0503283](#)
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H. Nunokawa, S. Parke, R. Funchal [hep-ph/0503283](#)
PBD, S. Parke [2401.10326](#)

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- ▶ Adds in another $\approx 0.004\cos\delta$ effect
- ▶ Total is $\approx 0.008\cos\delta$ which agrees with numerical calculation

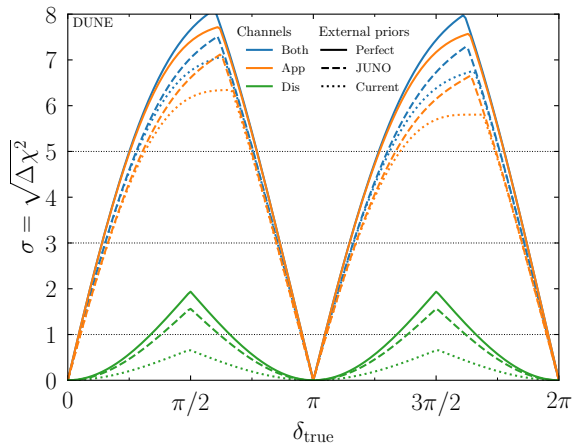
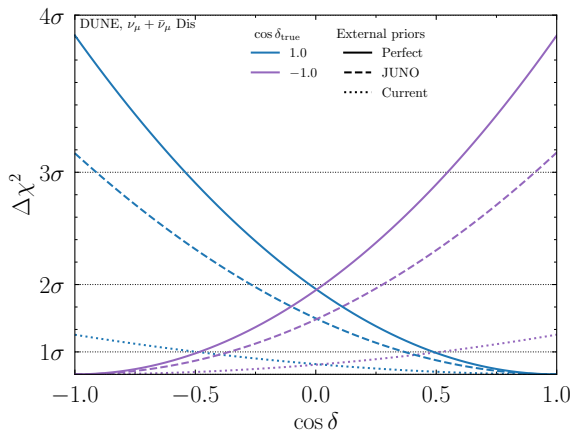
Numerical Studies

Inputs are *only*:

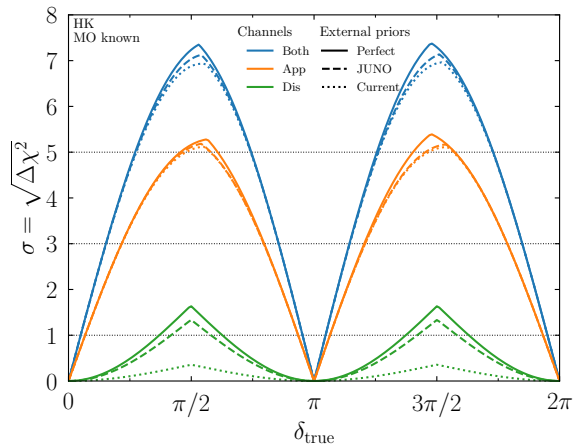
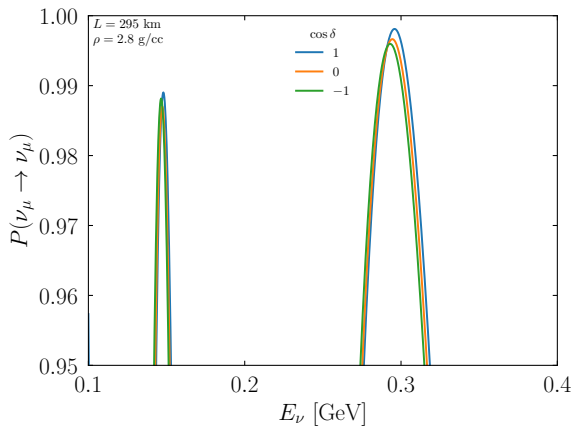
- ▶ Daya Bay data for θ_{13} 1809.02261
- ▶ KamLAND data for θ_{12} and Δm_{21}^2 1303.4667
- ▶ JUNO 6 yrs precision sensitivity on θ_{12} , Δm_{21}^2 , Δm_{31}^2 2204.13249
- ▶ DUNE 6.5+6.5 yrs disappearance channels sensitivity only 2103.04797

Also looked at varying JUNO's and DUNE's runtime, and at HK

JUNO and DUNE Disappearance Sensitivities

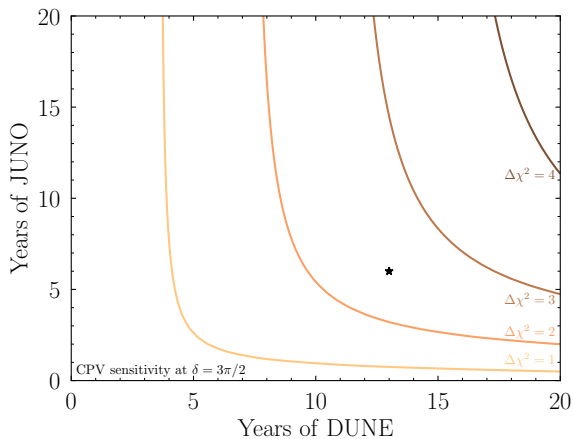


JUNO and HK Disappearance Sensitivities



Varying Runtime/Power

Significance to disfavor $|\cos \delta| = 1$ at $\cos \delta = 0$



Improvement requires **both** experiments!

NuFast-LBL

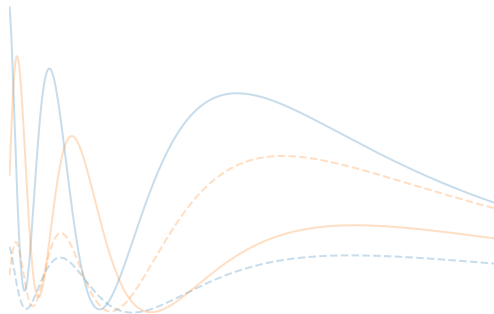
A fast code for long-baseline
neutrino oscillation probabilities in matter

2405.02400 with S. Parke
github.com/PeterDenton/NuFast-LBL

The Problem

$$\Delta m_{21}^2, \Delta m_{31}^2$$
$$s_{23}^2, s_{13}^2, s_{12}^2$$
$$\delta$$

\Rightarrow
in matter



Many Approaches

Solve the Schrödinger equation

$$i\frac{d}{dt}|\nu\rangle = H(t)|\nu\rangle$$

If $H(t) = H$ (constant density)

$$\mathcal{A}(\nu_\alpha \rightarrow \nu_\beta) = [e^{-iHL}]_{\beta\alpha} \quad P = |\mathcal{A}|^2$$

Exponential requires computing eigenvalues and eigenvectors of H

Many Approaches

Modify vacuum probabilities

- ▶ Get the eigenvalues by solving the cubic

Cardano 1545
V. Barger, et al. [PRD 22 \(1980\) 2718](#)

- ▶ Get the eigenvectors

H. Zaglauer, K. Schwarzer [Z.Phys. C40 \(1988\) 273](#)
K. Kimura, A. Takamura, H. Yokomakura [hep-ph/0205295](#)
[PBD](#), S. Parke, X. Zhang [1907.02534](#)
A. Abdulahi, S. Parke [2212.12565](#)

Fermilab computing experts bolster NOvA evidence, 1 million cores consumed

July 3, 2018 | Marcia Teckenbrock

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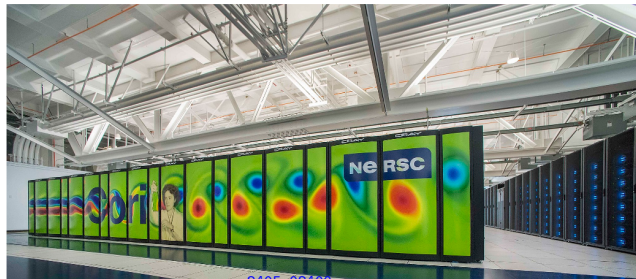
Array

How do you arrive at the physical laws of the universe when you're given experimental data on a renegade particle that interacts so rarely with matter, it can cruise through light-years of lead? You call on the power of advanced computing.

The NOvA neutrino experiment, in collaboration with the Department of Energy's Scientific Discovery through Advanced Computing (SciDAC-4) program and the HEPcloud program at DOE's Fermi National Accelerator Laboratory, was able to perform the largest-scale analysis ever to support the [recent evidence of antineutrino oscillation](#), a phenomenon that may hold clues to how our universe evolved.

Using Cori, the newest supercomputer at the [National Energy Research Scientific Computing Center \(NERSC\)](#), located at Lawrence Berkeley National Laboratory, NOvA used over 1 million computing cores, or CPUs, between May 14 and 15 and over a short timeframe one week later. This is the largest number of CPUs ever used concurrently over this duration — about 54 hours — for a single high-energy physics experiment. This unprecedented amount of computing enabled scientists to carry out some of the most complicated techniques used in neutrino physics, allowing them to dig deeper into the seldom seen interactions of neutrinos. This Cori allocation was more than 400 times the amount of Fermilab computing allocated to the NOvA experiment and 50 times the total computing capacity at Fermilab allocated for all of its rare-physics experiments. A continuation of the analysis was performed on NERSC's Cori and Edison supercomputers one week later. In total, [nearly 35 million core-hours were consumed by NOvA](#) in the 54-hour period. Executing the same analysis on a single desktop computer would take 4,000 years.

FNAL Newsroom



2405.02400

Monte-Carlo Estimates of Statistical Significances

Wilks' theorem is often wrong

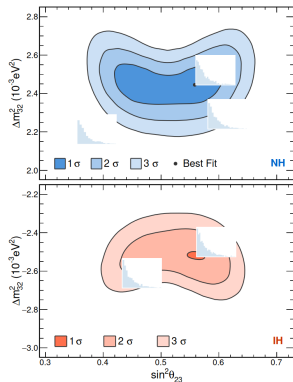
At each point in parameter space, simulate the experiment many times

“many” means $\gg 1/p$ for a desired p -value

This is sometimes called Feldman-Cousins

G. Feldman, R. Cousins [physics/9711021](#)

This isn't actually what was novel in the FC paper



Study found most of
the time was spent
computing probabilities

NOvA/T2K are $\sim 3\sigma$ experiments,
but DUNE/HK will be $\gtrsim 5\sigma$ experiments!

DUNE sensitivities require computing
the probabilities “a zillion times”

What is needed for LBL experiments?

1. All 9 channels ($\nu_\alpha \rightarrow \nu_\beta$)

- ▶ DUNE will certainly do ν_τ appearance

See e.g. P. Machado, H. Schulz, J. Turner [2007.00015](#)

- ▶ $\nu_\tau \rightarrow \nu_\beta$ channels are not needed, but come from free from unitarity
- ▶ JUNO needs only $\bar{\nu}_e \rightarrow \bar{\nu}_e$

2. Different energies, baselines, and densities

3. ν and $\bar{\nu}$

4. NO and IO

5. Oscillation parameters are mostly known

- ▶ Don't need to consider e.g. $\Delta m_{21}^2 > |\Delta m_{31}^2|$ or $\theta_{23} \sim 10^\circ$

How to Achieve Speed

1. Avoid costly operations

- ▶ `sin`, `cos` (and inverse functions) are very slow
- ▶ `sqrt` is quite slow, but not as bad as trigs
- ▶ Division is slower than multiplication ($0.2x$ may be faster than $x/5$)

2. Avoid complex numbers as possible

3. Reduce repeated calculations

- ▶ Compute $\frac{L}{4E}$ in the correct units once
- ▶ Compute each of the three $\sin \frac{\Delta m_{ij}^2 L}{4E}$ once

All of these are compiler dependent

Optimal Structure of the Probability

1. Amplitude requires four trig functions of kinematic variables $(\Delta m_{ij}^2 L/4E)$ ✗
2. Writing the probabilities out requires three trig functions ✓
3. Disappearance structure is straightforward:

$$P_{\alpha\alpha} = 1 - 4 \sum_{i>j} |V_{\alpha i}|^2 |V_{\alpha j}|^2 \sin^2 \frac{\Delta \lambda_{ij} L}{4E}$$

H in matter has eigenvalues λ_i and eigenvectors $V_{\alpha i}$

Optimal Structure of the Probability

4. Appearance structure:

T conserving:

$$P_{\mu e}^{TC} = 2 \sum_{i>j} (|V_{\tau k}|^2 - |V_{\mu i}|^2 |V_{ej}|^2 - |V_{\mu j}|^2 |V_{ei}|^2) \sin^2 \frac{\Delta\lambda_{ij}L}{4E}$$

Fun fact:

$$\begin{aligned} & 2\Re(V_{\alpha i} V_{\beta j}^* V_{\alpha j}^* V_{\beta i}) \\ &= |V_{\alpha k}|^2 |V_{\beta k}|^2 - |V_{\alpha i}|^2 |V_{\beta i}|^2 - |V_{\alpha j}|^2 |V_{\beta j}|^2 \\ &= |V_{\gamma k}|^2 - |V_{\alpha i}|^2 |V_{\beta j}|^2 - |V_{\alpha j}|^2 |V_{\beta i}|^2 \end{aligned}$$

T violating:

$$P_{\mu e}^{TV} = -8J \frac{\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2}{\Delta\lambda_{21} \Delta\lambda_{31} \Delta\lambda_{32}} \sin \frac{\Delta\lambda_{21}L}{4E} \sin \frac{\Delta\lambda_{31}L}{4E} \sin \frac{\Delta\lambda_{32}L}{4E}$$

Leverages NHS identity:

V. Naumov [IJMP 1992](#)

P. Harrison, W. Scott [hep-ph/9912435](#)

Account for Matter

1. Need the eigenvalues λ_i
2. For eigenvectors, naively need $\Re(V_{\alpha i} V_{\beta j}^* V_{\alpha j} V_{\beta i})$
3. Given our form, need only the $|V_{\alpha i}|^2$ and J
 - Don't need any phase information of the eigenvectors!

Leverages [PBD](#), S. Parke, X. Zhang [1907.02534](#)

4. Can compute the $|V_{\alpha i}|^2$ from the λ_i and submatrix eigenvalues (requires only a square root) using Eigenvector-Eigenvalue Identity

$$|V_{\alpha i}|^2 = \frac{\prod_{k=1}^{n-1} (\lambda_i - \xi_k^\alpha)}{\prod_{k=1; k \neq i}^n (\lambda_i - \lambda_k)}$$

See e.g. [PBD](#), S. Parke, T. Tao, X. Zhang [1908.03795](#)
Can actually avoid the $\sqrt{}$ in practice

Eigenvalues are Hard

The eigenvalues in matter λ_i depend on S :

$$S = \cos \left\{ \frac{1}{3} \cos^{-1} \left[\frac{2A^3 - 9AB + 27C}{2(A^2 - 3B)^{3/2}} \right] \right\}$$

where

$$A = \sum \lambda_i = \Delta m_{21}^2 + \Delta m_{31}^2 + a$$

$$B = \sum_{i>j} \lambda_i \lambda_j = \Delta m_{21}^2 \Delta m_{31}^2 + a[\Delta m_{21}^2(1 - |U_{e2}|^2) + \Delta m_{31}^2(1 - |U_{e3}|^2)]$$

$$C = \prod \lambda_i = a \Delta m_{21}^2 \Delta m_{31}^2 |U_{e1}|^2$$

Approximate Eigenvalues

1. Instead, approximate one eigenvalue
 - ▶ λ_3 is best because is never parametrically small and easy to approximate
2. From DMP:

$$\lambda_3 \approx \Delta m_{31}^2 + \frac{1}{2} \Delta m_{ee}^2 \left(x - 1 + \sqrt{(1-x)^2 + 4x s_{13}^2} \right)$$

$$x \equiv \frac{a}{\Delta m_{ee}^2} \quad \Delta m_{ee}^2 \equiv \Delta m_{31}^2 - s_{12}^2 \Delta m_{21}^2$$

H. Minakata, S. Parke [1505.01826](#)
[PBD](#), H. Minakata, S. Parke [1604.08167](#)
H. Nunokawa, S. Parke, R. Funchal [hep-ph/0503283](#)

3. Get other two eigenvalues by picking two of A , B , C conditions

Requires one more $\sqrt{}$

The Approximation

- ▶ This is the only approximation used in the entire approach
- ▶ In vacuum the approximation returns to the correct value

Many approximations in the literature are not correct in vacuum limit
See G. Barenboim, [PBD](#), S. Parke, C. Ternes [1902.00517](#)

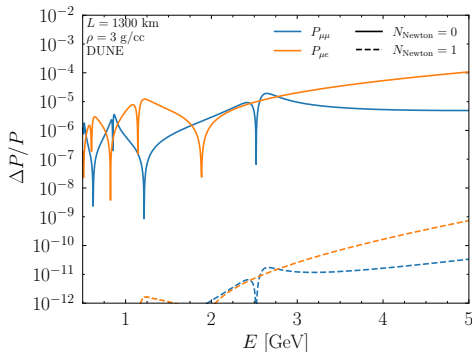
- ▶ In fact can iteratively improve λ_3 with rapid convergence via Newton's method:

$$\lambda_3 \rightarrow \lambda_3 - \frac{X(\lambda_3)}{X'(\lambda_3)}$$

$$X(\lambda) = \lambda^3 - A\lambda^2 + B\lambda - C = 0$$

- ▶ Precision improvement starts at 10^{-5} for the first step
 - ▶ The improvement is quadratic thereafter
- ▶ One line of code, just loop as many times as desired

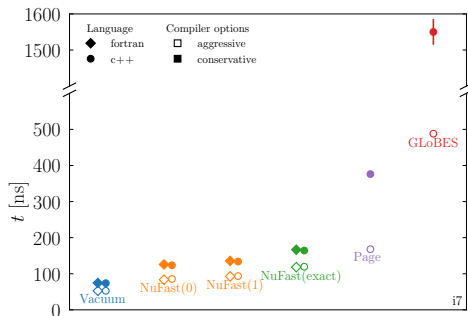
Is this approximation okay?
DUNE requires $\lesssim 1\%$ level precision



Slightly better for HK
 $\sim 10^{-10}$ for JUNO
 See backups

NuFast-LBL Speed

Is this algorithm fast?
How does it compare?



- ▶ “Conservative” = default, -O0
- ▶ “Aggressive” = -Ofast and -ffast-math
- ▶ Some variation expected due to architecture

See also J. Page [2309.06900](#)
and P. Huber, et al. [hep-ph/0701187](#)

NuFast-LBL: The Code

- ▶ Code is on github: github.com/PeterDenton/NuFast-LBL
- ▶ Implementations in c++ and f90
- ▶ Easy to use and there are comments!

```
// NuFast.cpp
// Calculate the probabilities:
Probability_Matter_LBL(s12sq, s13sq, s23sq, delta,
    Dmsq21, Dmsq31, L, E, rho, Ye, N_Newton,
    &probs_returned);
// Print out the probabilities to terminal
for (int alpha = 0; alpha < 3; alpha++)
{
    for (int beta = 0; beta < 3; beta++)
    {
        printf("%d %d %g\n", alpha, beta,
            probs_returned[alpha][beta]);
    } // beta, 3
} // alpha, 3
```

- ▶ $\bar{\nu}$: $E < 0$; IO: $\Delta m_{31}^2 < 0$
- ▶ Folder called **Benchmarks** to make the plots and speed tests in the paper
- ▶ Used in [NuOscillator](#) and [MaCh3](#)

NuFast-Earth

A fast code for neutrino oscillations through the Earth from the atmosphere, the Sun, or a supernova

2511.XXXXX with S. Parke

Extend to atmospheric neutrinos!

and solar and supernova neutrinos through the Earth

- ▶ NuFast-LBL assumes constant(ish) density profile
- ▶ The Earth has large discrete jumps between the four main regions
- ▶ Atmospheric and nighttime solar neutrinos depend on both energy and zenith angle: more computation required

Old and New

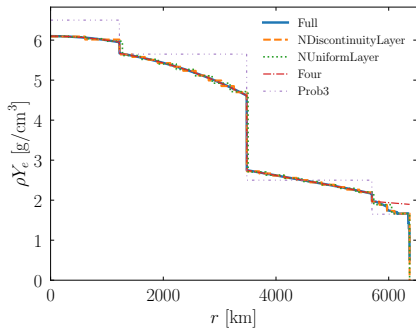
Can reuse from NuFast-LBL:

1. Eigenvalue computation
2. Eigenvalues to eigenvectors

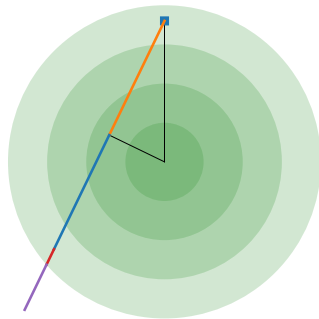
Newly developed for NuFast-Earth:

1. Work in basis without $U(\theta_{23}, \delta)$ rotation
2. Provide various models of the Earth
3. Break up trajectory into segments
4. Keep parts of the calculations for reuse

Earth Trajectories



- ▶ Various Earth density profiles
- ▶ Varying or constant density in each shell



Trajectory goes from:

1. Production height to surface
2. Surface to detector depth
3. Detector depth to deepest point
4. Deepest point to detector depth:
this is the transpose of #3

Procedure

Use the fact that the matter potential commutes with $U(\theta_{23}, \delta)$:

$$(2E)\tilde{H} = R_{13}R_{12}M^2R_{12}^TR_{13}^T + A$$

Compute amplitude for this Hamiltonian through each Earth layer

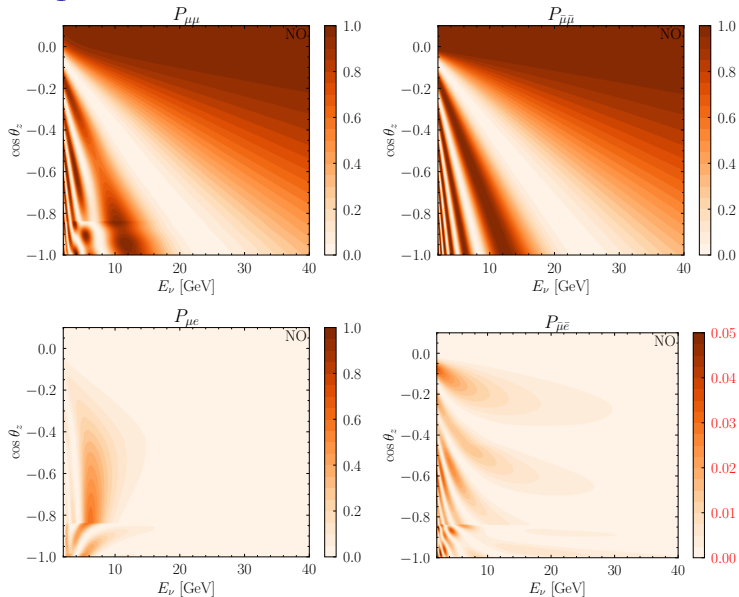
1. Makes the Hamiltonian real symmetric
2. Use adjugate approach to efficiently compute eigenvectors

PBD, S. Parke, T. Tao, X. Zhang [1908.03795](#)
A. Abdulahi, S. Parke [2212.12565](#)

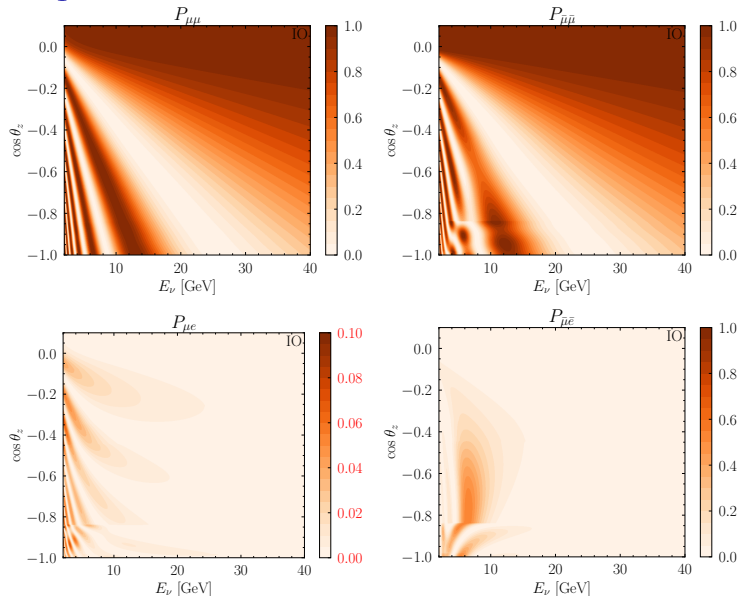
3. Can compute the amplitude through the whole Earth independent of θ_{23} and δ
4. Then add on the vacuum propagation through the atmosphere
5. Finally apply the $U(\theta_{23}, \delta)$ rotation and square for the probability

Changing θ_{23} , δ , or the production height does not require recomputing the amplitude through the Earth!

Atmospheric Results: NO



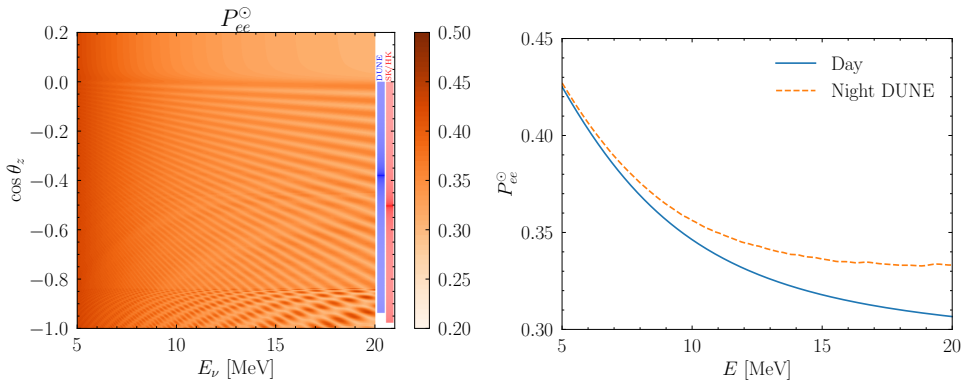
Atmospheric Results: IO



Nighttime Solar Neutrinos

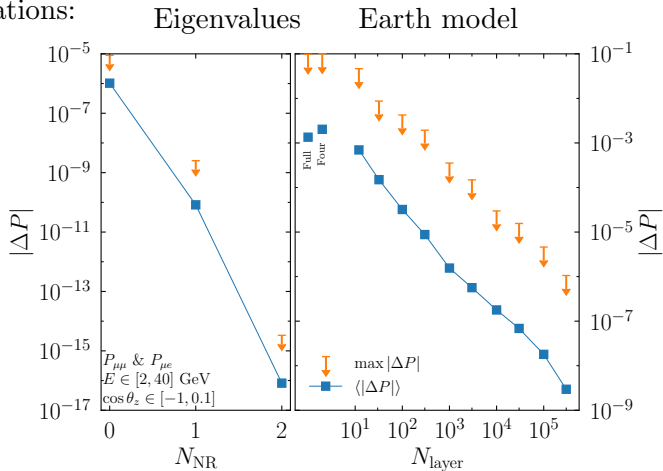
At night solar neutrinos experience partial regeneration:
There are more ν_e 's from the Sun at night than during the day!

SuperK has $\sim 2\sigma$ evidence for this effect; DUNE and HK aim to measure it well



NuFast-Earth Precision

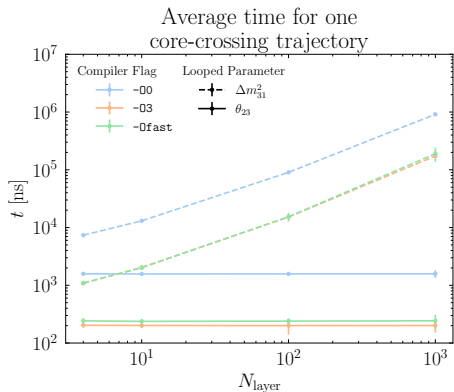
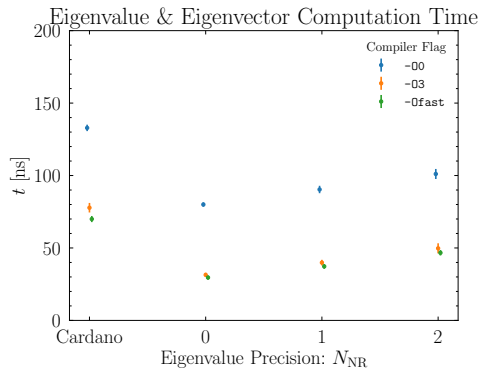
Two approximations:



“True” has exact eigenvalues and 1M layers

Calculate $|\Delta P|$ across 100×100 grid in energy and angle

NuFast-Earth Speed



nuSquIDS takes >1M ns per trajectory

C. Argüelles, J. Salvado, C. Weaver [2112.13804](#)

OscProb takes 40 000 ns per trajectory at 44 layers (compare to 6000 ns or 200 ns)

J. Coelho, R. Pestes, et al. github.com/joaoabcoelho/OscProb

Other codes are designed with different goals in mind

NuFast-Earth Flow

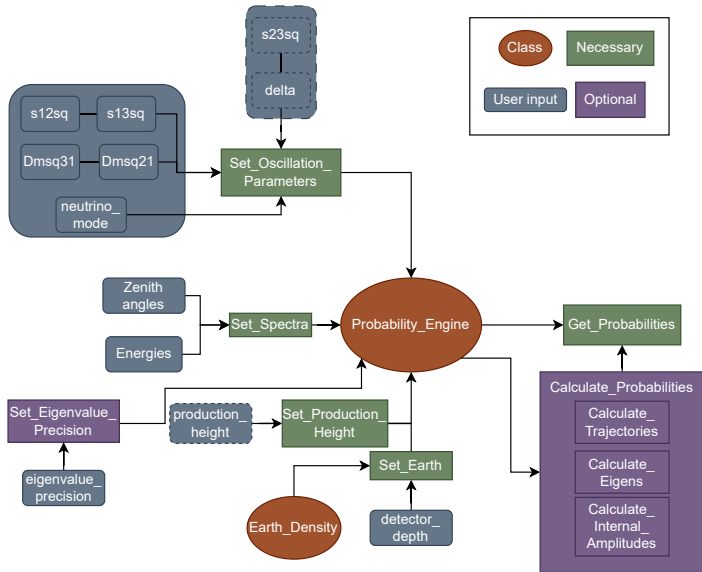
```
// Initialize the engine
Probability_Engine probability_engine;

// Set the oscillation parameters to nu-fit 6 best fit
// values:
probability_engine.Set_Oscillation_Parameters(0.307,
0.02195, 0.561, 177 * M_PI / 180, 7.49e-5, 2.534e-3,
true); //(s12sq, s13sq, s23sq, delta, Dmsq21, Dmsq31,
neutrino_mode)

// Set energy and zenith angle arrays
std::vector<double> energies = {1, 2, 3, 4, 5}; // GeV
std::vector<double> coszs = {-1, -0.5, 0, 1}; // core-
crossing to horizontal to down-going
probability_engine.Set_Spectra(energies, coszs);

// Create Earth model instance
PREM_NDiscontinuityLayer earth_density(2, 10, 10, 5); //
2 layers in the inner core, 10 layers in the outer
core, 10 layers in the inner mantle, and 5 layers in
the outer mantle
// Set Earth details
probability_engine.Set_Earth(2, &earth_density); //
detector depth in km
// Set production height (optional)
probability_engine.Set_Production_Height(10); // km,
recalculations after changing this are fast

// Do the calculations
std::vector<std::vector<Matrix3r>> probabilities =
probability_engine.Get_Probabilities();
```



NuFast-Earth Recommendations

Many choices in how to use NuFast-Earth
What is the best way for most cases?

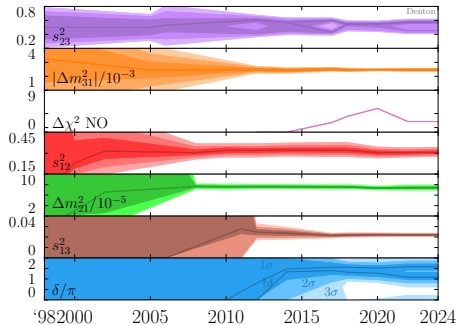
1. Eigenvalue precision: set the number of Newton-Raphson corrections to 1
`probability_engine.Set_Eigenvalue_Precision(1);`
2. Earth model: 2, 10, 10, 5 layers in each of the four main zones
`PREM_NDiscontinuityLayer earth.density(2, 10, 10, 5);`
3. Looping order: put θ_{23} , δ , and production height on the innermost loops

Discussion and Conclusions

- ▶ **Disappearance can discover CPV**
- ▶ Requires two good measurements: JUNO and DUNE/HK
- ▶ Can rule out some values of δ at $> 3\sigma$
- ▶ **LBL Experiments should break down δ analyses into app vs. dis**
- ▶ Since systematics are different, provides a good cross check
- ▶ NuFast-LBL and NuFast-Earth provide fast, precise, usable code
- ▶ Leverages modern linear algebra
- ▶ Uses precise approximations
- ▶ Huge impact in computational cost: minimize computations, save results
- ▶ LBL code implemented in pipelines for NOvA, T2K, DUNE, HK, & JUNO

Backups

References



SK [hep-ex/9807003](#)

M. Gonzalez-Garcia, et al. [hep-ph/0009350](#)

M. Maltoni, et al. [hep-ph/0207227](#)

SK [hep-ex/0501064](#)

SK [hep-ex/0604011](#)

T. Schwetz, M. Tortola, J. Valle [0808.2016](#)

M. Gonzalez-Garcia, M. Maltoni, J. Salvado [1001.4524](#)

T2K [1106.2822](#)

D. Forero, M. Tortola, J. Valle [1205.4018](#)

D. Forero, M. Tortola, J. Valle [1405.7540](#)

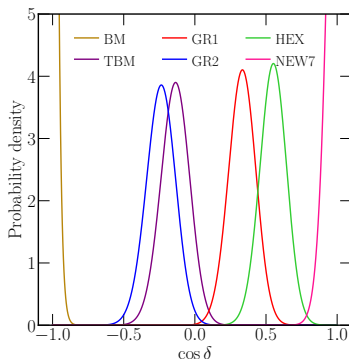
P. de Salas, et al. [1708.01186](#)

F. Capozzi et al. [2003.08511](#)

The Importance of $\cos \delta$

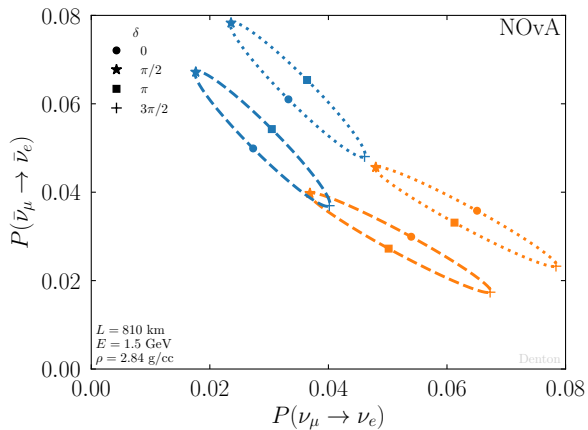
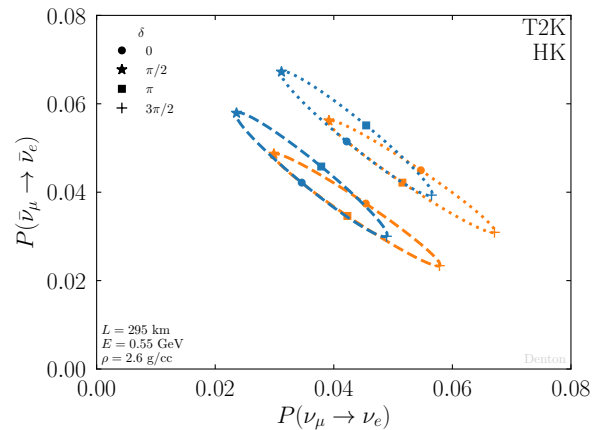
- ▶ If only $\sin \delta$ is measured \Rightarrow sign degeneracy: $\cos \delta = \pm \sqrt{1 - \sin^2 \delta}$
- ▶ Most flavor models predict $\cos \delta$

J. Gehrlein, et al. [2203.06219](#)



L. Everett, et al. [1912.10139](#)

δ : What is it Really?



All 9 Channels

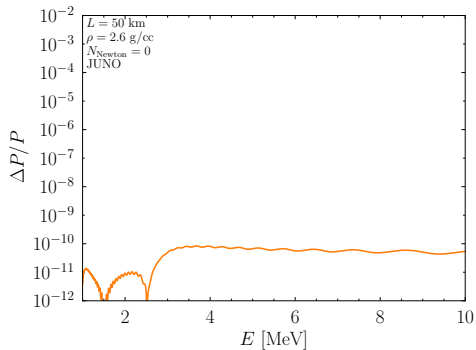
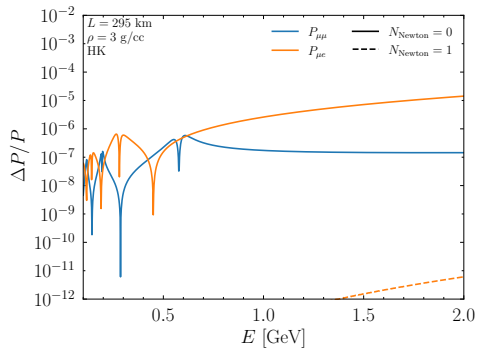
Given P_{ee} , $P_{\mu\mu}$, $P_{\mu e}^{TC}$, and $P_{\mu e}^{TV}$:

	$P_{\alpha e}$	$P_{\alpha\mu}$	$P_{\alpha\tau}$
$P_{e\beta}$	P_{ee}	$P_{\mu e}^{TC} - P_{\mu e}^{TV}$	$1 - P_{ee} - P_{\mu e}^{TC} + P_{\mu e}^{TV}$
$P_{\mu\beta}$	$P_{\mu e}^{TC} + P_{\mu e}^{TV}$	$P_{\mu\mu}$	$1 - P_{\mu\mu} - P_{\mu e}^{TC} - P_{\mu e}^{TV}$
$P_{\tau\beta}$	$1 - P_{ee} - P_{\mu e}^{TC} - P_{\mu e}^{TV}$	$1 - P_{\mu\mu} - P_{\mu e}^{TC} + P_{\mu e}^{TV}$	$-1 + P_{ee} + P_{\mu\mu} + 2P_{\mu e}^{TC}$

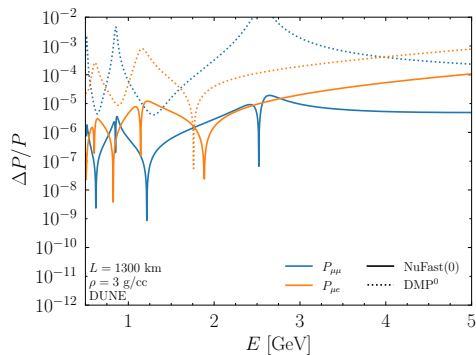
Total LBL Approach

1. Inputs: 6 oscillation parameters, experimental details (L , E , ρ , Y_e)
2. Calculate λ_3 approximately
 - ▶ Iteratively improve with Newton's method, if desired
3. Calculate the $|V_{\alpha i}|^2$'s with the Eigenvector-Eigenvalue Identity
4. Calculate the sines of the kinematic terms
5. Calculate the CP odd term with the NHS identity
6. Calculate key probabilities: P_{ee} , $P_{\mu\mu}$, and $P_{\mu e}$
7. Calculate remaining probabilities

Precision

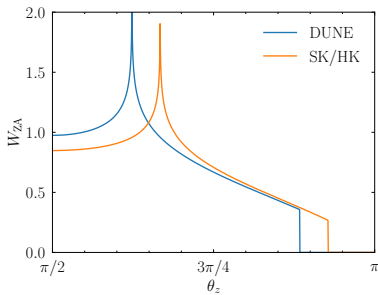
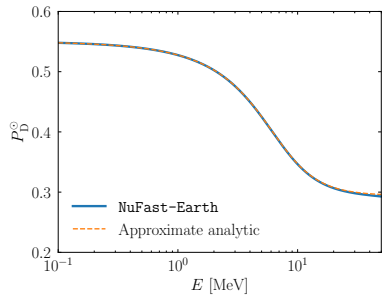


Comparison to DMP



DMP: [PBD](#), H. Minakata, S. Parke [1604.08167](#)

NuFast-Earth Solar Neutrino Validation



NuFast-Earth LBL Validation

