

The best way to probe CP violation in the lepton sector is with long-baseline accelerator neutrino experiments in the appearance mode: the appearance of ν_e in predominantly ν_μ beams. Here we show that it is possible to discover CP violation with disappearance experiments only, by combining JUNO for electron neutrinos and DUNE or Hyper-Kamiokande for muon neutrinos. While the maximum sensitivity to discover CP is quite modest (1.6σ with 6 years of JUNO and 13 years of DUNE), some values of δ may be disfavored by $> 3\sigma$ depending on the true value of δ .

CP-Violation with Neutrino Disappearance

Peter B. Denton

University of Virginia

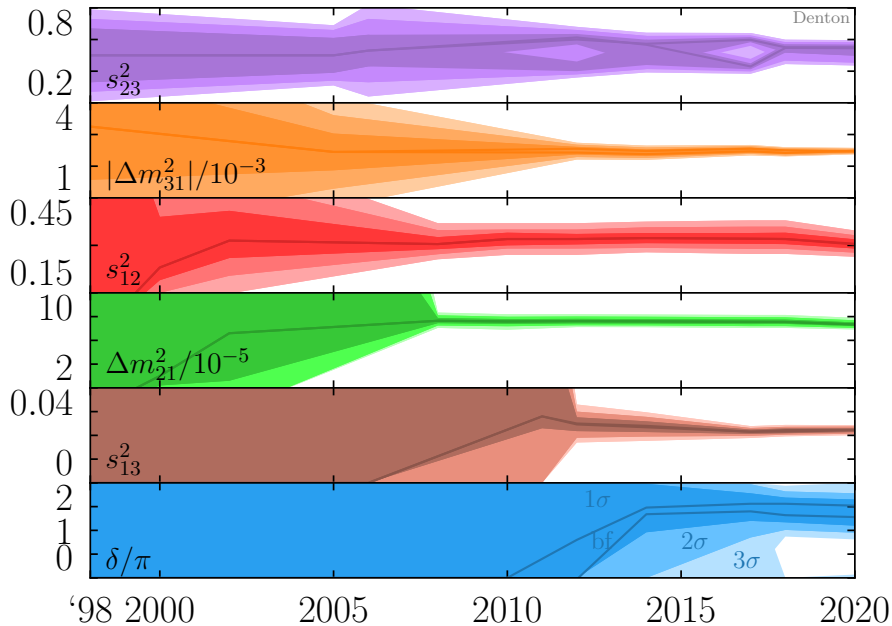
May 8, 2024

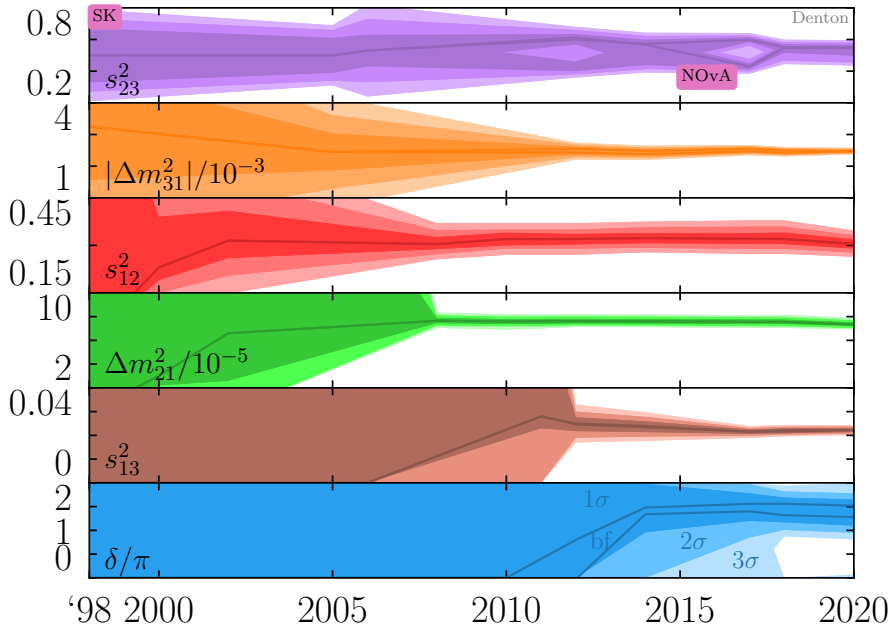
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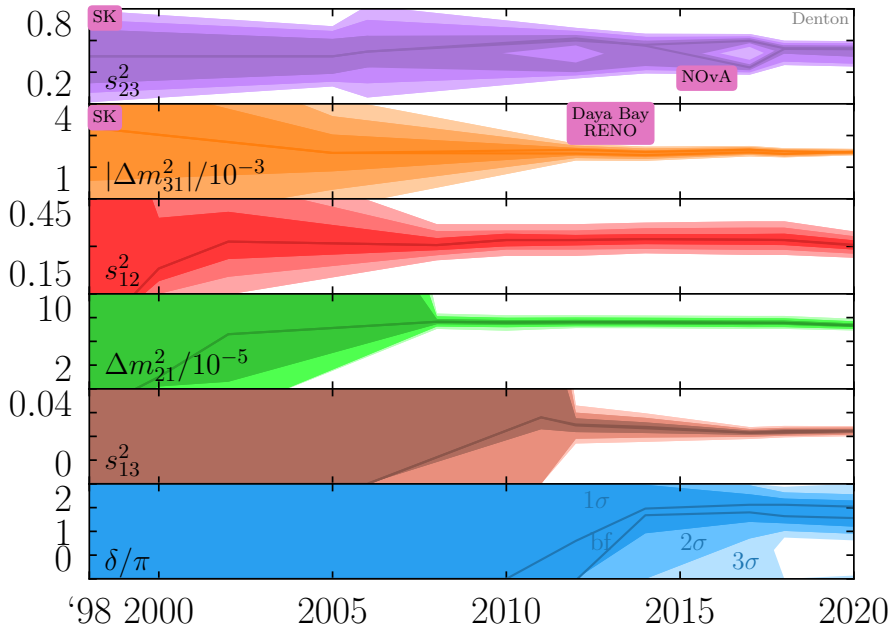


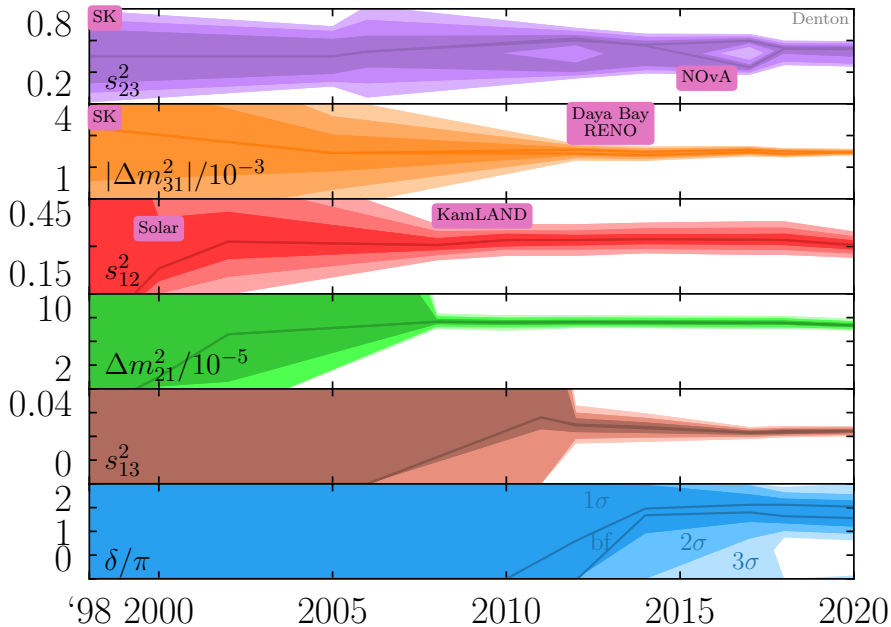
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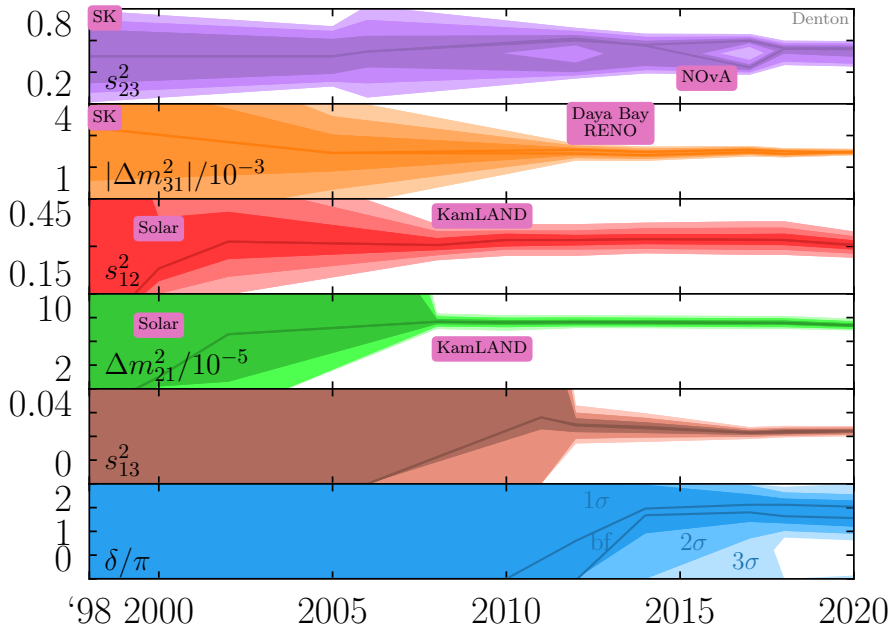


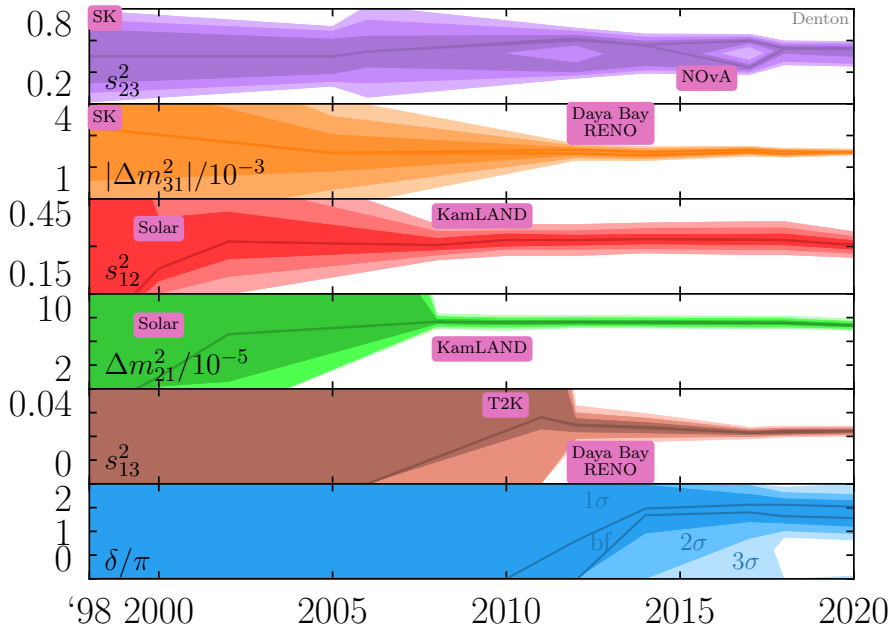


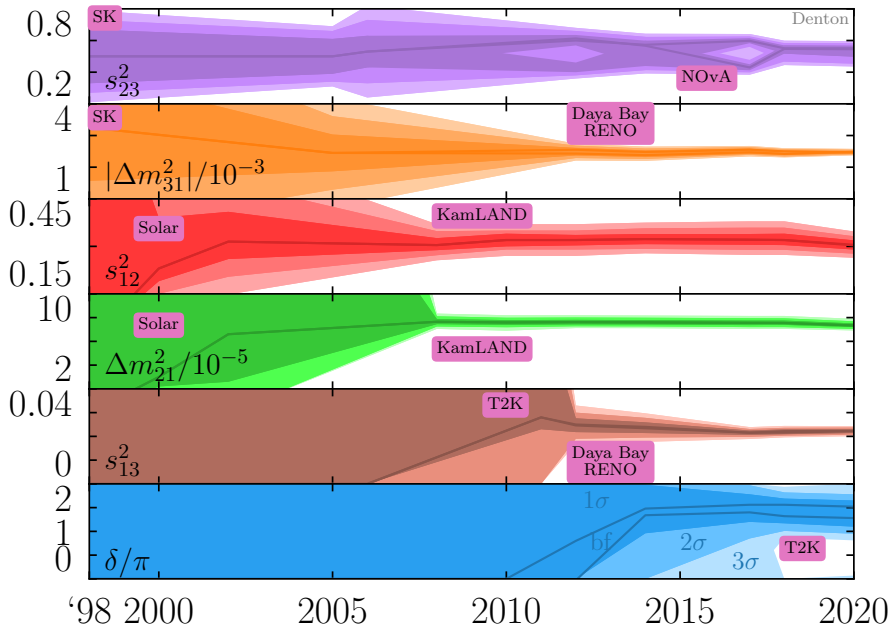












Four known unknown in particle physics: all neutrinos

Atmospheric mass ordering

θ_{23} octant

Complex phase

Absolute mass scale

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Outline

1. Why CPV is interesting
2. Other non-standard probes of CPV
3. Relationship between appearance, disappearance, CP, T, CPT
4. Three ways to see why there is CPV information in disappearance
 - 4.1 Parameter counting
 - 4.2 Direct analytic calculation
 - 4.3 Numerical test
5. Role of the matter effect
6. Sensitivities
7. New paper and code out last week! NuFast
8. Recommendation

Why is CPV interesting?

δ and CP violation

$$J_{CP} = s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23}\sin\delta$$

C. Jarlskog [PRL 55, 1039 \(1985\)](#)



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1. Strong interaction: no observed EDM \Rightarrow CP (nearly) **conserved**

$$\frac{\bar{\theta}}{2\pi} < 10^{-11}$$

J. Pendlebury, et al. [1509.04411](#)

2. Quark mass matrix: non-zero but **small** CP violation

$$\frac{|J_{CKM}|}{J_{\max}} = 3 \times 10^{-4}$$

CKMfitter [1501.05013](#)

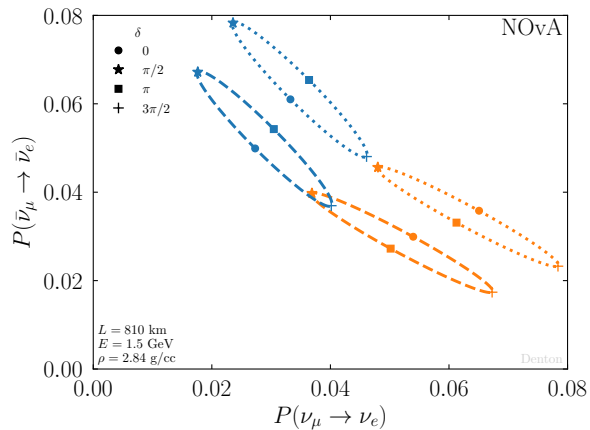
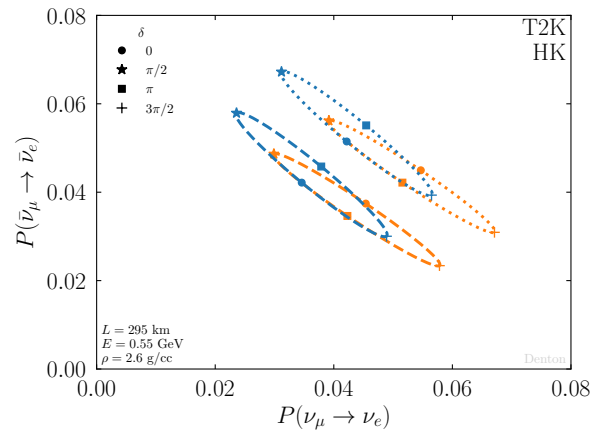
3. Lepton mass matrix: ?

$$\frac{|J_{PMNS}|}{J_{\max}} < 0.34$$

[PBD](#), J. Gehrlein, R. Pestes [2008.01110](#)

$$J_{\max} = \frac{1}{6\sqrt{3}} \approx 0.096$$

δ : what is it really?



δ : what is it not?

$\delta \not\Rightarrow$ Baryogenesis

The amount of leptogenesis is a function of:

1. the heavy mass scale
2. δ
3. α, β (Majorana phases)
4. CP phases in the RH neutrinos
5. ...

C. Hagedorn, et al. [1711.02866](#)

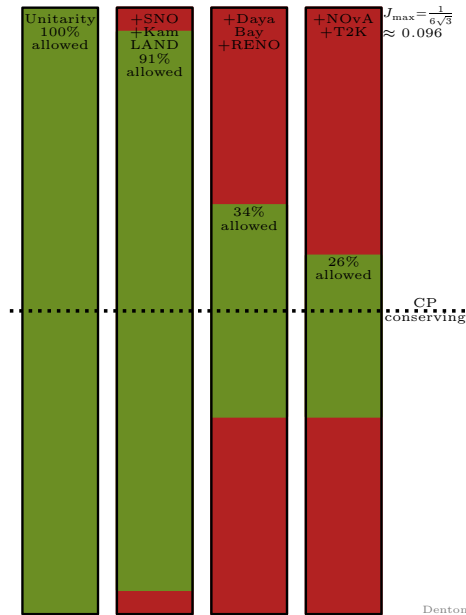
K. Moffat, et al. [1809.08251](#)

Measuring $\delta = 0, \pi$	$\not\Rightarrow$	no leptogenesis
Measuring $\delta \neq 0, \pi$	$\not\Rightarrow$	leptogenesis

δ, J : current status

Maximal CP violation is already ruled out:

1. $\theta_{12} \neq 45^\circ$ at $\sim 15\sigma$
2. $\theta_{13} \neq \tan^{-1} \frac{1}{\sqrt{2}} \approx 35^\circ$ at many (100) σ
3. $\theta_{23} = 45^\circ$ allowed at $\sim 1\sigma$
4. $|\sin \delta| = 1$ allowed



When δ and when J ?

If the goal is **CP violation** the Jarlskog invariant should be used

however

If the goal is **measuring the parameters** one must use δ

Given θ_{12} , θ_{13} , θ_{23} , and J , I can't determine the sign of $\cos \delta$ which is physical

e.g. $P(\nu_\mu \rightarrow \nu_\mu)$ depends on $\cos \delta$

Other non-standard CPV probes

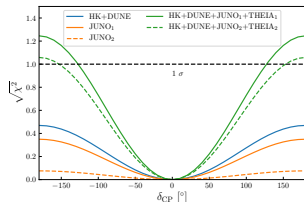
1. Some information in solar due to loops in elastic scattering

V. Brdar, X-J. Xu [2306.03160](#)

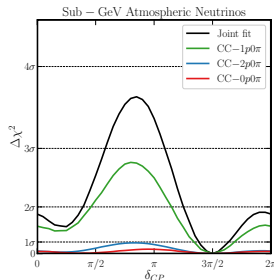
2. Sub-GeV atmospheric

K. Kelly, et al. [1904.02751](#)

See also e.g. A. Suliga, J. Beacom [2306.11090](#)



Solar (no systematics): $\sim 0.5\sigma$



Atmospherics at DUNE: $< 2\sigma$

Appearance, disappearance, and CP

Appearance vs. Disappearance

Oscillation experiments can do
appearance or disappearance experiments:

Disappearance

K2K, MINOS, T2K, $\text{NO}\nu\text{A}$

KamLAND, Daya Bay, RENO, Double CHOOZ

(Sort of) SNO, Borexino, SK-solar

JUNO, DUNE, HK

Neither appearance nor disappearance

SK-atm, IceCube

Appearance

T2K, $\text{NO}\nu\text{A}$

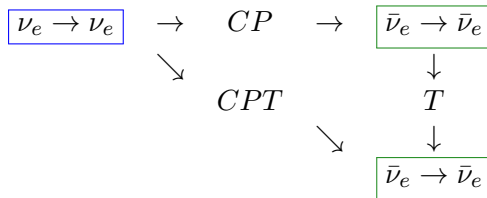
OPERA

Atm ν_τ hints @ SK & IceCube

DUNE, HK



CP, T: Disappearance

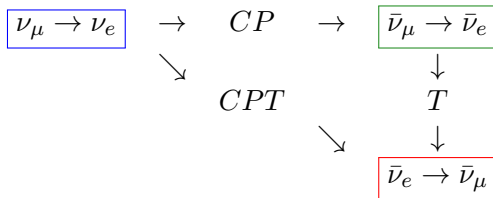


Disappearance measurements are even eigenstates of CP

$$CP[P(\nu_e \rightarrow \nu_e)] = P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \stackrel{CPT}{=} P(\nu_e \rightarrow \nu_e)$$

Assume that CPT is a good symmetry

CP, T: Appearance



Appearance measurements are not eigenstates of CP

Appearance and Disappearance, CP even and CP odd terms

Disappearance:

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\alpha) &= 1 - 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2 \Delta_{21} \\ &\quad - 4|U_{\alpha 1}|^2|U_{\alpha 3}|^2 \sin^2 \Delta_{31} \\ &\quad - 4|U_{\alpha 2}|^2|U_{\alpha 3}|^2 \sin^2 \Delta_{32} \\ &= P_{\alpha\alpha}^{CP+} \end{aligned}$$

$$\Delta_{ij} \equiv \Delta m_{ij}^2 L/4E$$

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Appearance:

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) &= -4\Re[U_{\alpha 1}U_{\beta 1}^*U_{\alpha 2}^*U_{\beta 2}] \sin^2 \Delta_{21} \\ &\quad - 4\Re[U_{\alpha 1}U_{\beta 1}^*U_{\alpha 3}^*U_{\beta 3}] \sin^2 \Delta_{31} \\ &\quad - 4\Re[U_{\alpha 3}U_{\beta 3}^*U_{\alpha 2}^*U_{\beta 2}] \sin^2 \Delta_{32} \\ &\quad \pm 8J_{CP} \sin \Delta_{21} \sin \Delta_{31} \sin \Delta_{32} \\ &= P_{\alpha\beta}^{CP+} + P_{\alpha\beta}^{CP-} \end{aligned}$$

$$\Delta_{ij} \equiv \Delta m_{ij}^2 L/4E$$

Sign depends on α, β

Conventional Wisdom

1. Appearance is sensitive to CPV

[True]

Conventional Wisdom

- | | |
|---|---------|
| 1. Appearance is sensitive to CPV | [True] |
| 2. Disappearance has no CPV sensitivity | [False] |

Conventional Wisdom

1. Appearance is sensitive to CPV [True]
2. Disappearance has no CPV sensitivity [False]
3. Any δ dependence in disappearance is in ν_μ not ν_e [Confusing/False]

$$\begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Correct Statements

- ▶ Appearance is the best way to measure δ and CPV
... given known oscillation parameters, systematics, and realistic experiments
 - ▶ Probes mostly $\sin \delta$ not $\cos \delta$
 - ▶ Don't need both ν and $\bar{\nu}$ (but systematics)
- ▶ **Disappearance can measure δ**
 - ▶ CPV can be discovered with only disappearance measurements
 - ▶ Probes mostly $\cos \delta$ not $\sin \delta$
 - ▶ Requires measurements of two flavors
 - ▶ “Works through unitarity” (as do nearly all oscillation measurements)

Parameter Counting

1. Four parameters in the PMNS matrix

Majorana phases are irrelevant in oscillations

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$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - 4 \sum_{i>j} C_{ij}^\alpha \sin^2 \Delta_{ij}$$

$$C_{ij}^\alpha = |U_{\alpha i}|^2 |U_{\alpha j}|^2$$

$$|U_{\alpha i}| = \left(\frac{C_{ij}^\alpha C_{ik}^\alpha}{C_{jk}^\alpha} \right)^{1/4}$$

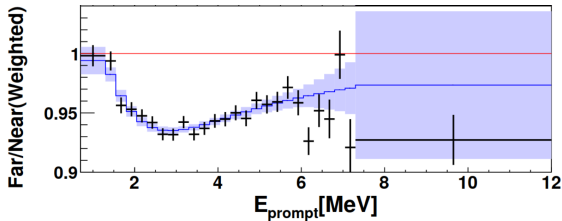
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Daya Bay [1809.02261](#)

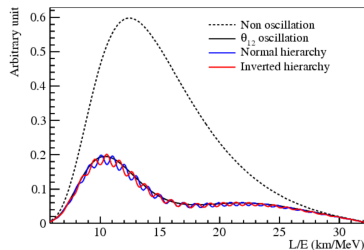
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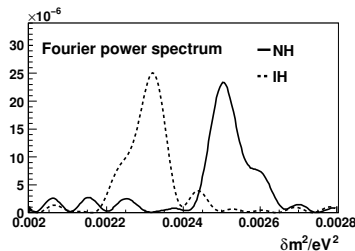
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JUNO [1507.05613](#)



L. Zhan, et al. [0807.3203](#)

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4. Given good measurements of the ν_e and ν_μ disappearance, 4 independent parameters will be measured

- ▶ Any row can be “simple” (e.g. $c_{12}c_{13}$, $s_{12}c_{13}$, ...) \Rightarrow no one row is ever enough
- ▶ That is, CPV is physical and cannot depend on parameterization

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6. If we determine $\cos \delta \neq \pm 1 \Rightarrow$ CP is violated!

Direct Analytic Calculation

Disappearance experiments measure various $|U_{\alpha i}|^2$ terms

Suppose 4 are measured: $|U_{e2}|^2$, $|U_{e3}|^2$, $|U_{\mu 2}|^2$, $|U_{\mu 3}|^2$

Actually this gives all 9 magnitudes by unitarity

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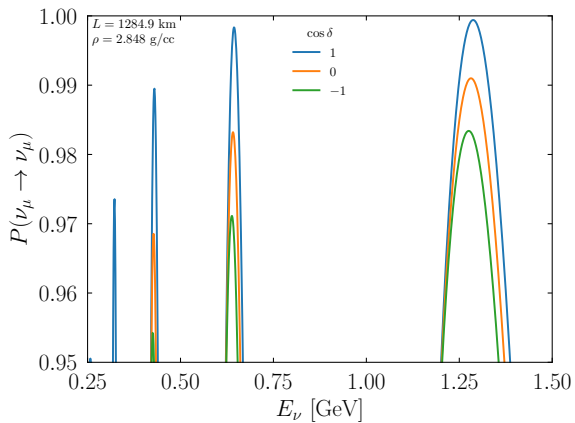
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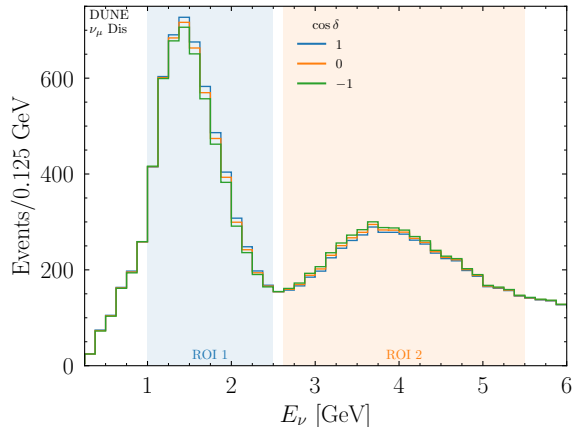
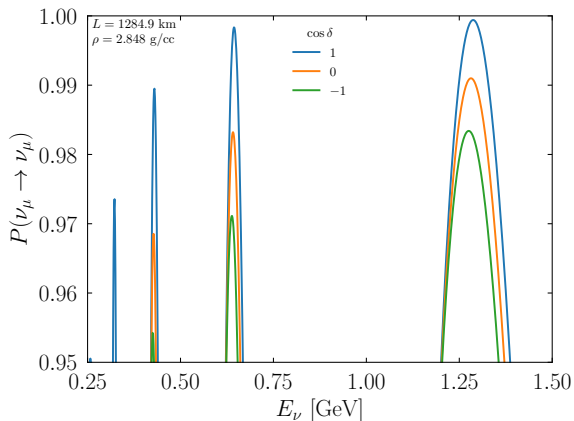
$$J_{CP}^2 = |U_{e2}|^2 |U_{\mu 2}|^2 |U_{e3}|^2 |U_{\mu 3}|^2 - \frac{1}{4} \left(1 - |U_{e2}|^2 - |U_{\mu 2}|^2 - |U_{e3}|^2 - |U_{\mu 3}|^2 + |U_{e2}|^2 |U_{\mu 3}|^2 + |U_{e3}|^2 |U_{\mu 2}|^2 \right)^2$$

Disappearance can tell us if CP is violated, but not if nature prefers ν 's or $\bar{\nu}$'s

Where is $|U_{\mu 2}|^2$?



Where is $|U_{\mu 2}|^2$?



	$\cos \delta$	ROI 1	ROI 2
6.5 yrs ν_μ rates	1	5506	5038
	0	5418	5115
	-1	5334	5193

Approximate size of $|U_{\mu 2}|^2$ signal: (21) sector

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PBD, J. Gehrlein [2302.08513](#)

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PBD, J. Gehrlein 2302.08513

- ▶ This term is

$$\approx -4c_{23}^2 (s_{12}^2 c_{12}^2 + s_{23} c_{23} s_{13} \sin 2\theta_{12} \cos 2\theta_{12} \cos \delta) \sin^2 \Delta_{21}$$

$$\approx -2 \quad (0.21 \quad + \quad 0.03 \cos \delta) \left(\frac{\pi}{33} \right)^2$$

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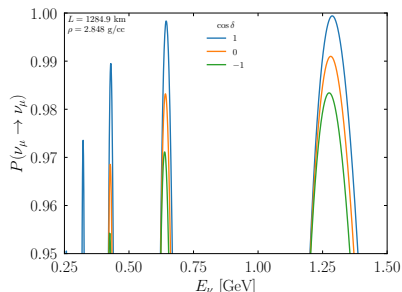
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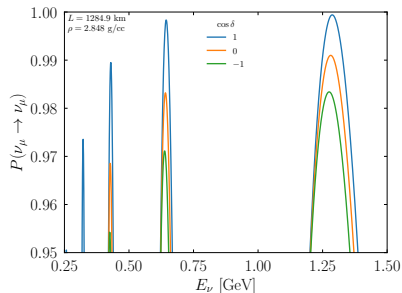
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Sign is wrong

Magnitude is ~ 16 too small

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Matter effects matter: (21) sector

- Let's start again at

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- ▶ Solar splitting modified by

$$\Delta m_{21}^2 \rightarrow \Delta m_{21}^2 \mathcal{S}_{\odot}$$
$$\mathcal{S}_{\odot} \approx \sqrt{(\cos 2\theta_{12} - c_{13}^2 a / \Delta m_{21}^2)^2 + \sin^2 2\theta_{12}} \approx 3.4$$

at $E = 1.3$ GeV
PBD, S. Parke [1902.07185](#)

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- ▶ Mixing angle is modified

$$\cos 2\theta_{12} = 0.37 \rightarrow \frac{\cos 2\theta_{12} - c_{13}^2 a / \Delta m_{21}^2}{\mathcal{S}_{\odot}} \approx -0.96 < 0$$

$$a \propto \rho E$$

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$$\approx -4c_{23}^2 (s_{12}^2 c_{12}^2 + s_{23} c_{23} s_{13} \sin 2\theta_{12} \cos 2\theta_{12} \cos \delta) \sin^2 \Delta_{21}$$

- ▶ Solar splitting modified by

$$\Delta m_{21}^2 \rightarrow \Delta m_{21}^2 \mathcal{S}_{\odot}$$
$$\mathcal{S}_{\odot} \approx \sqrt{(\cos 2\theta_{12} - c_{13}^2 a / \Delta m_{21}^2)^2 + \sin^2 2\theta_{12}} \approx 3.4$$

at $E = 1.3$ GeV
PBD, S. Parke [1902.07185](#)

- ▶ Mixing angle is modified

$$\cos 2\theta_{12} = 0.37 \rightarrow \frac{\cos 2\theta_{12} - c_{13}^2 a / \Delta m_{21}^2}{\mathcal{S}_{\odot}} \approx -0.96 < 0$$

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- ▶ This gets us **half** of the effect, and the correct sign

Matter effects matter: (32) sector

► $\frac{\Delta m_{\mu\mu}^2 L}{4E}$ in matter at the maximum is $\sim \pi$

H. Nunokawa, S. Parke, R. Funchal [hep-ph/0503283](#)
[PBD](#), S. Parke [2401.10326](#)

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H. Nunokawa, S. Parke, R. Funchal [hep-ph/0503283](#)
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- ▶ Adds in another $\approx 0.004\cos\delta$ effect
- ▶ Total is $\approx 0.008\cos\delta$ which agrees with exact calculation

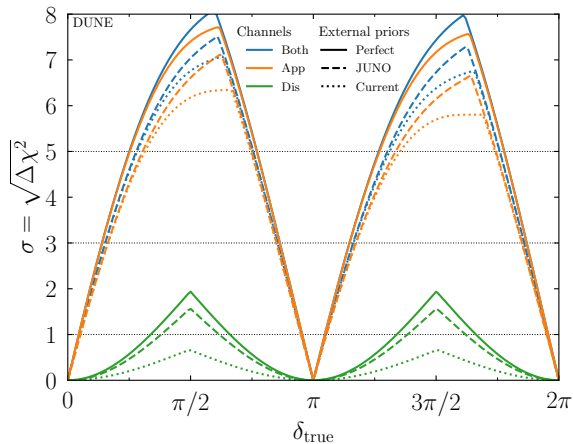
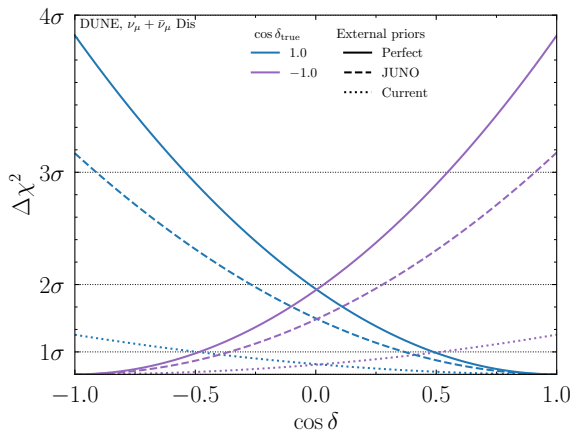
Numerical Studies

Inputs are *only*:

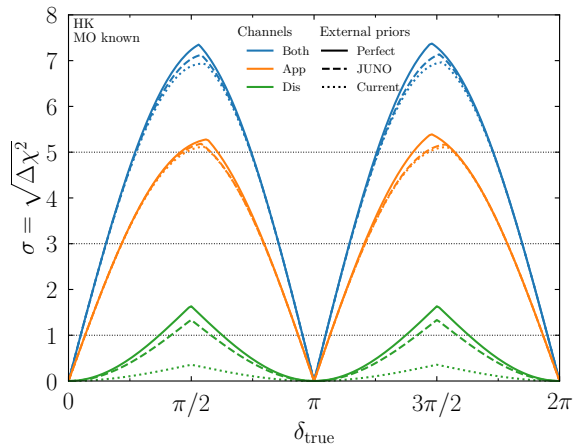
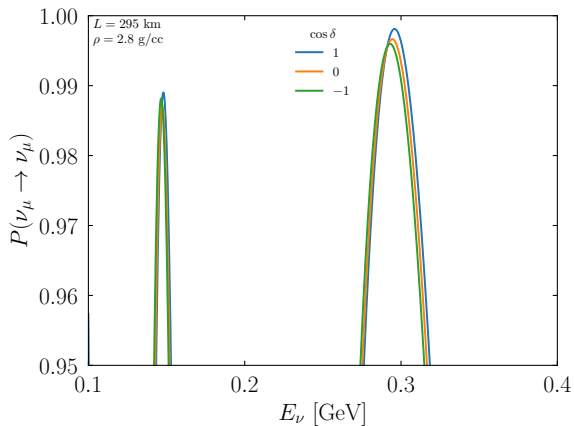
- ▶ Daya Bay data for θ_{13} 1809.02261
- ▶ KamLAND data for θ_{12} and Δm_{21}^2 1303.4667
- ▶ JUNO 6 yrs precision sensitivity on θ_{12} , Δm_{21}^2 , Δm_{31}^2 2204.13249
- ▶ DUNE 6.5+6.5 yrs disappearance channels sensitivity only 2103.04797

Also looked at varying JUNO's and DUNE's runtime, and at HK

JUNO and DUNE disappearance Sensitivities



JUNO and HK disappearance Sensitivities



NuFast: Fast Neutrino Oscillation Probabilities

Quick advertisement:

- ▶ “nearly 35 million core-hours were consumed by NOvA in the 54-hour period”
[FNAL Article](#)
- ▶ Most of the time spent was computing probabilities

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[FNAL Article](#)
- ▶ Most of the time spent was computing probabilities
- ▶ With S. Parke, presented a new algorithm, optimized for LBL:
 - ▶ NOvA, T2K, DUNE, HK, and JUNO
- ▶ Leverages past results

[PBD](#), H. Minakata, S. Parke [1604.08167](#)
[PBD](#), S. Parke, X. Zhang [1907.02534](#)
A. Abdulahi, S. Parke [2212.12565](#)

- ▶ Including the Eigenvector-Eigenvalue Identity:

$$|\hat{U}_{\alpha i}|^2 = \frac{\prod_{j=1}^{n-1} (\lambda_i - \xi_{\alpha}^j)}{\prod_{j=1, j \neq i}^n (\lambda_i - \lambda_j)}$$

[PBD](#), S. Parke, T. Tao, X. Zhang [1908.03795](#)

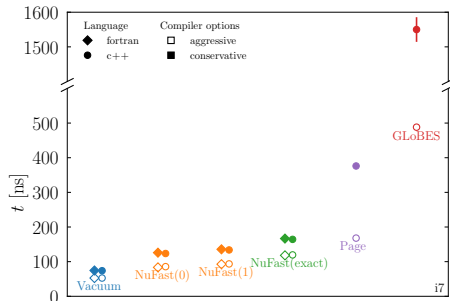
NuFast: Fast Neutrino Oscillation Probabilities

- Code is designed to be fast:

- c++
- f90

```
// NuFast.cpp
// Calculate the probabilities:
Probability_Matter_LBL(s12sq, s13sq, s23sq, delta,
    Dmsq21, Dmsq31, L, E, rho, Ye, N_Newton,
    &probs_returned);
// Print out the probabilities to terminal
for (int alpha = 0; alpha < 3; alpha++)
{
    for (int beta = 0; beta < 3; beta++)
    {
        printf("%d %d %g\n", alpha, beta,
            probs_returned[alpha][beta]);
    } // beta, 3
} // alpha, 3
```

- Faster than existing in the literature
- Far more precise than HK, DUNE, or JUNO will need
- Publicly available: github.com/PeterDenton/NuFast
- Atmospheric and nighttime solar in the works!



Discussion and Conclusions

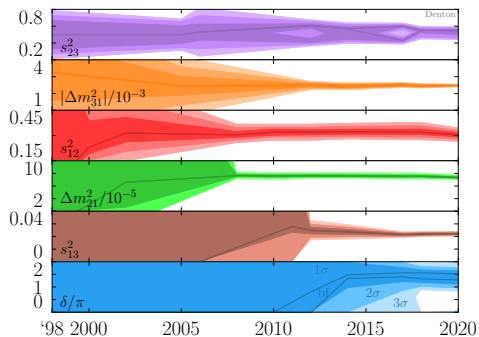
- ▶ **Disappearance can discover CPV**
- ▶ Requires two good measurements: JUNO and DUNE/HK
- ▶ Can rule out some values of δ at $> 3\sigma$
- ▶ Works in vacuum or matter; matter slightly minimizes HK's effect
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- ▶ Analyses already exist but...

Discussion and Conclusions

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- ▶ Analyses already exist but...
- ▶ **LBL Experiments should break down δ analyses into app vs. dis**
- ▶ Since systematics are different, provides a good cross check

Backups

References



SK [hep-ex/9807003](#)

M. Gonzalez-Garcia, et al. [hep-ph/0009350](#)

M. Maltoni, et al. [hep-ph/0207227](#)

SK [hep-ex/0501064](#)

SK [hep-ex/0604011](#)

T. Schwetz, M. Tortola, J. Valle [0808.2016](#)

M. Gonzalez-Garcia, M. Maltoni, J. Salvado [1001.4524](#)

T2K [1106.2822](#)

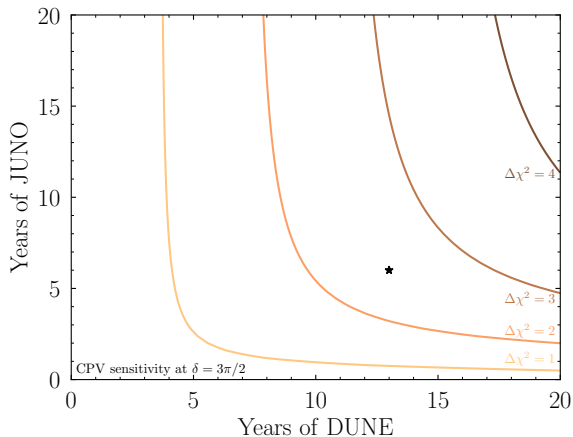
D. Forero, M. Tortola, J. Valle [1205.4018](#)

D. Forero, M. Tortola, J. Valle [1405.7540](#)

P. de Salas, et al. [1708.01186](#)

F. Capozzi et al. [2003.08511](#)

Varying Runtime/Power

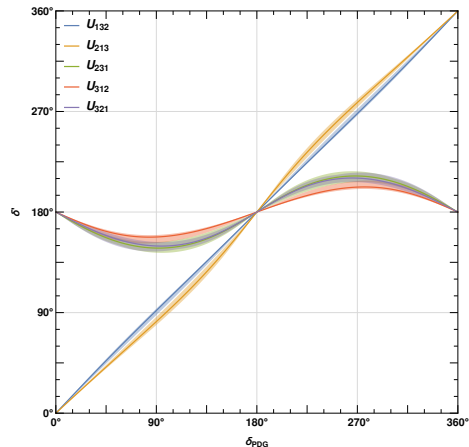


Improvement requires **both** experiments!

Complex phase in different parameterizations

- ▶ Can relate the complex phase in one parameterization to that in another
- ▶ U_{132} and U_{213} similar to U_{123}
- ▶ δ constrained to $\sim [150^\circ, 210^\circ]$ in U_{231} , U_{312} , U_{321}
- ▶ Bands indicate 3σ uncertainty on θ_{12} , θ_{13} , θ_{23}
- ▶ “50% of possible values of δ ”
 \Rightarrow parameterization dependent

DUNE TDR II [2002.03005](#)



Quark mixing

From the PDG, V_{CKM} in the V_{123} parameterization is

$$\theta_{12} = 13.09^\circ \quad \theta_{13} = 0.2068^\circ \quad \theta_{23} = 2.323^\circ \quad \delta_{\text{PDG}} = 68.53^\circ$$

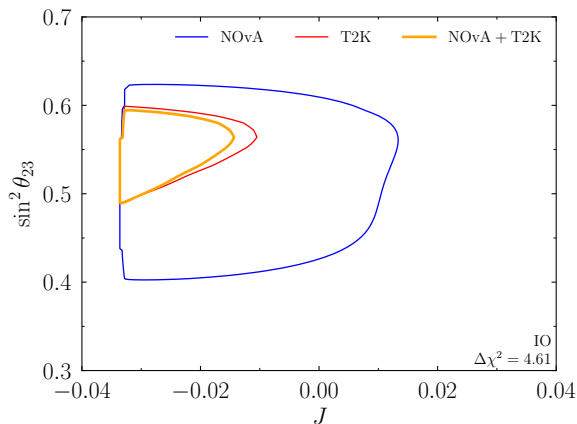
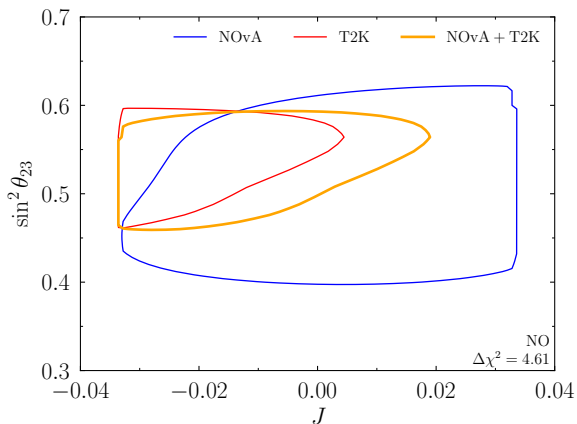
Looks like “large” CPV:

$$\sin \delta_{\text{PDG}} = 0.93 \sim 1$$

yet $J_{\text{CKM}}/J_{\text{max}} = 3 \times 10^{-4}$.

Switch to V_{212} parameterization, $\Rightarrow \delta' = 1^\circ$ and $\sin \delta' = 0.02$.

Standard oscillation parameters



Can see that the combination doesn't like the NO while it does like the IO
IO preferred over NO at $\Delta\chi^2 = 2.3$

CP violation in oscillations

In vacuum at first maximum:

$$P_{\mu e} - \bar{P}_{\mu e} \approx 8\pi J \frac{\Delta m_{21}^2}{\Delta m_{32}^2}$$

$$J \equiv s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23} \sin \delta$$

C. Jarlskog [PRL 55, 1039 \(1985\)](#)

- ▶ Extracting δ from data requires every other oscillation parameter
- ▶ J requires only Δm_{21}^2 (up to matter effects)

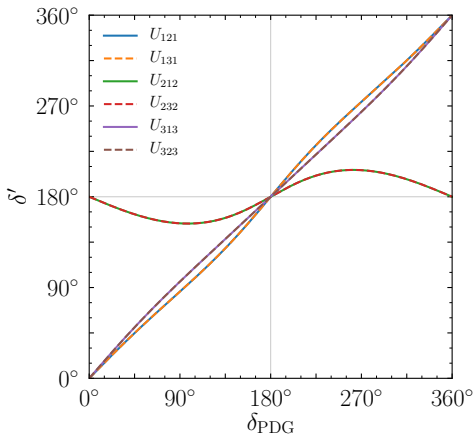
Matter effects are easily accounted for

$$\hat{J} \simeq \frac{J}{\sqrt{(c_{212} - c_{13}^2 a / \Delta m_{21}^2)^2 + s_{212}^2} \sqrt{(c_{213} - a / \Delta m_{ee}^2)^2 + s_{213}^2}}$$

[PBD](#), S. Parke [1902.07185](#)

[PBD](#), H. Minakata, S. Parke [1604.08167](#)

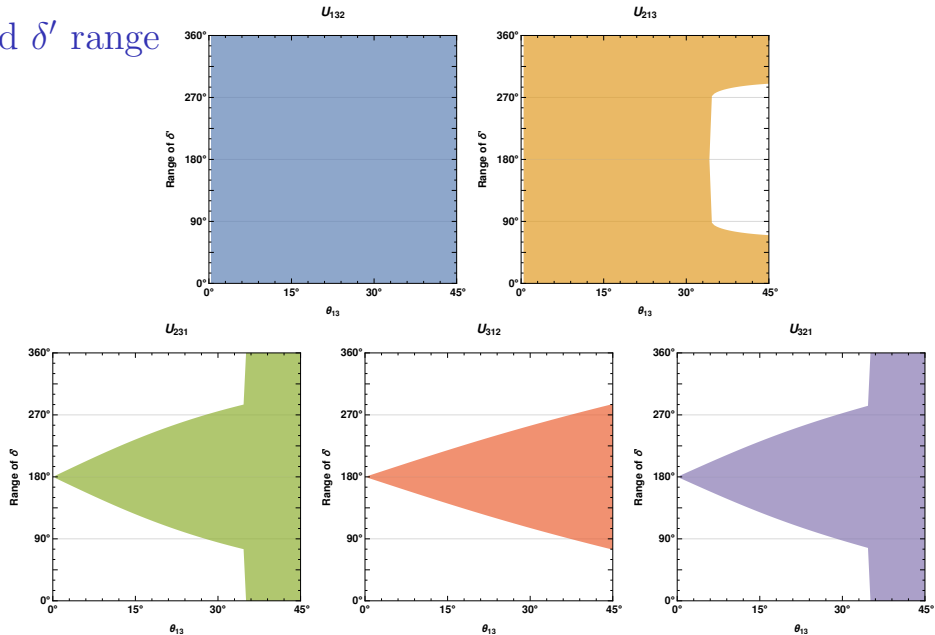
Repeated rotations



	U_{121}	U_{131}	U_{212}	U_{232}	U_{313}	U_{323}
$ U_{e2} $	✓	✓	✓	✓	✗	✗
$ U_{e3} $	✓	✓	✗	✗	✓	✓
$ U_{\mu 3} $	✗	✗	✓	✓	✓	✓

Note that $e^{i\delta}$ must be on first or third rotation

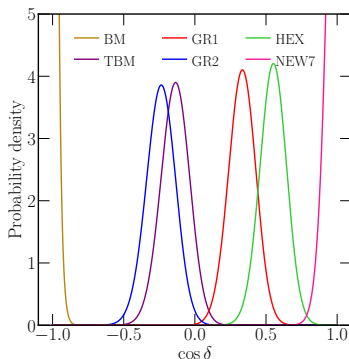
Allowed δ' range



The importance of $\cos \delta$

- ▶ If only $\sin \delta$ is measured \Rightarrow sign degeneracy: $\cos \delta = \pm \sqrt{1 - \sin^2 \delta}$
- ▶ Most flavor models predict $\cos \delta$

J. Gehrlein, et al. [2203.06219](#)



L. Everett, et al. [1912.10139](#)