

Abstract

In particle physics there exist two regions: the Standard Model which is fairly complete and the new physics sector which is completely unknown. In between and overlapping with both of these is neutrino physics. Neutrinos exist within the Standard Model but are not explained by it due to the discovery of neutrino oscillations. In this talk I will discuss where we stand with neutrino oscillations, where we might go with them, and how we might learn about the nature of neutrinos.

Overview of Neutrino Oscillations in the Three Flavor Paradigm

Peter B. Denton

Neutrinos from Home

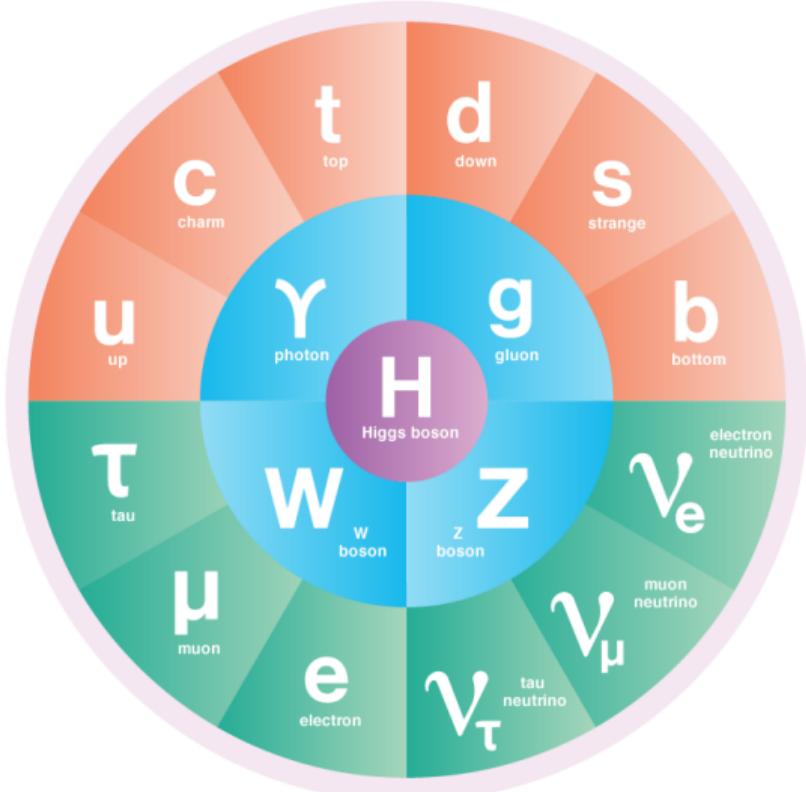
December 2-5, 2025



Brookhaven™
National Laboratory

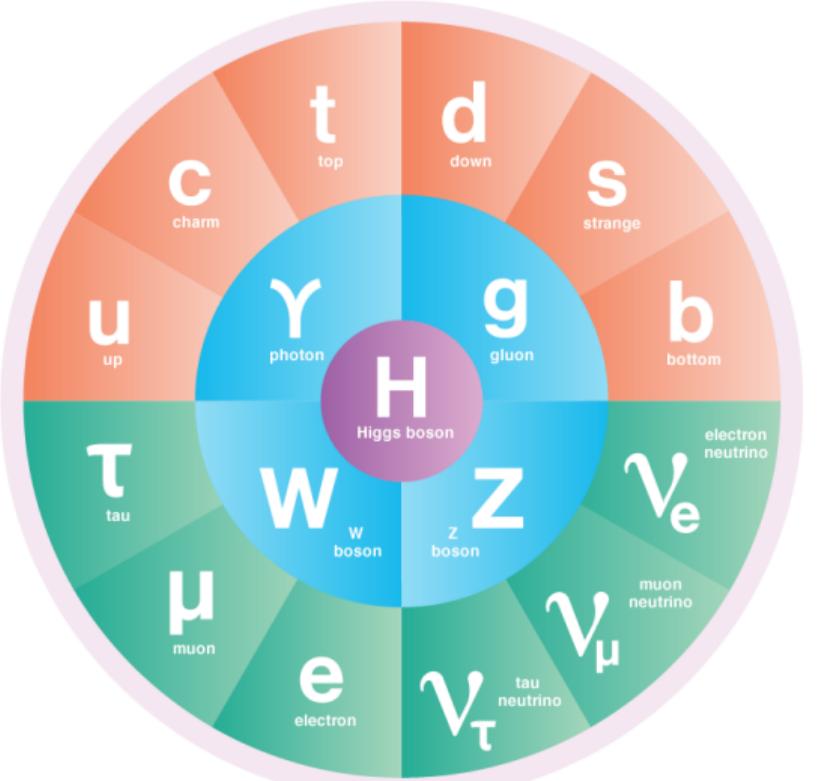


Particle physics



Symmetry Magazine

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2501.08374

Discovering/understanding particles:

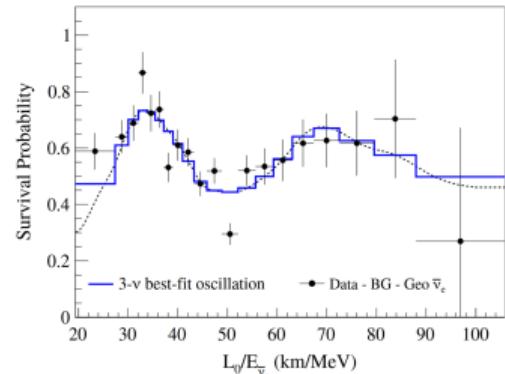
- ▶ Photon: easy
- ▶ Charged leptons: easy
- ▶ Light quarks: easy experimentally, harder theoretically
- ▶ Heavy quarks: hard experimentally, easy theoretically
- ▶ W & Z: hard experimentally, easy theoretically
- ▶ Gluons: easy experimentally, harder theoretically
- ▶ Higgs boson: hard experimentally, easy theoretically
- ▶ Neutrinos: hard experimentally, hard theoretically

Basic oscillation concepts

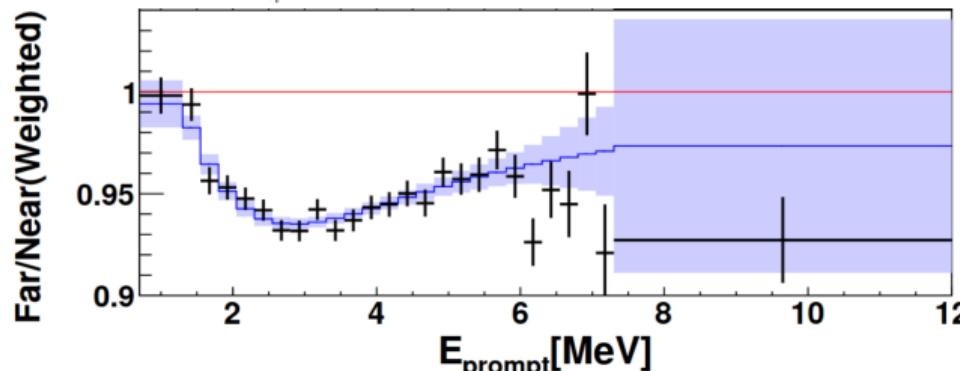
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2. Neutrino oscillate \Rightarrow must mix & masses must be different

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KamLAND [1303.4667](#)



Daya Bay [1809.02261](#)

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$$|\nu_\alpha\rangle = \sum_{i=1}^3 U_{\alpha i}^* |\nu_i\rangle \quad \alpha \in \{e, \mu, \tau\}$$

U is a unitary 3×3 matrix which has four degrees of freedom

Unitarity \Rightarrow 9 dofs, rephasing $\Rightarrow 9 - 5 = 4$

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The physical observable is the probability: $P(\nu_\alpha \rightarrow \nu_\beta; L, E)$

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Discrete symmetries:

$$T : L \rightarrow -L, \quad CP : \nu \leftrightarrow \bar{\nu} \Leftrightarrow U_{\alpha i} \rightarrow U_{\alpha i}^* \Leftrightarrow E \rightarrow -E$$

Assume CPT is conserved: $P(\nu_\alpha \rightarrow \nu_\beta) = P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha)$

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Assume that E and direction don't change during propagation

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- ▶ Fully decohered probabilities are easy!

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sum_{i=1}^3 P_{\alpha i} P_{ii} P_{i\beta} = \sum_{i=1}^3 |U_{\alpha i}|^2 |U_{\beta i}|^2$$

Everything is at the probability level not the amplitude level

This is the same expression as oscillation averaged probabilities

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Exercise: start from slide 5 and derive these expressions

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Can easily confirm unitarity:

$$\sum_{\beta} P(\nu_\alpha \rightarrow \nu_\beta) = 1$$

Three flavor

Three angles, three Δm^2 (two are close), one complex phase

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It is less easy to show that:

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\alpha) = & 1 - 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2 \left(\frac{\Delta m_{21}^2 L}{4E} \right) \\ & - 4|U_{\alpha 1}|^2|U_{\alpha 3}|^2 \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right) \\ & - 4|U_{\alpha 2}|^2|U_{\alpha 3}|^2 \sin^2 \left(\frac{\Delta m_{32}^2 L}{4E} \right) \end{aligned}$$

Many different ways to write these probabilities

Three flavor: appearance

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) = & -4\Re[U_{\alpha 1} U_{\beta 1}^* U_{\alpha 2}^* U_{\beta 2}] \sin^2 \left(\frac{\Delta m_{21}^2 L}{4E} \right) \\ & - 4\Re[U_{\alpha 1} U_{\beta 1}^* U_{\alpha 3}^* U_{\beta 3}] \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right) \\ & - 4\Re[U_{\alpha 2} U_{\beta 2}^* U_{\alpha 3}^* U_{\beta 3}] \sin^2 \left(\frac{\Delta m_{32}^2 L}{4E} \right) \\ + 8\Im[U_{\alpha 1} U_{\beta 1}^* U_{\alpha 2}^* U_{\beta 2}] & \sin \left(\frac{\Delta m_{21}^2 L}{4E} \right) \sin \left(\frac{\Delta m_{31}^2 L}{4E} \right) \sin \left(\frac{\Delta m_{32}^2 L}{4E} \right) \end{aligned}$$

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Final coefficient:

$$8\Im[U_{\alpha 1} U_{\beta 1}^* U_{\alpha 2}^* U_{\beta 2}] \equiv 8J = 8s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23} \sin \delta$$

This is the same for all appearance channels (up to sign)

C. Jarlskog [PRL 55 \(1985\)](#)

$$s_{ij} = \sin \theta_{ij}, c_{ij} = \cos \theta_{ij}$$

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5. This follows from CPT. CP: $\delta \rightarrow -\delta$ and T is $L \rightarrow -L$

Matter effect causes apparent CPT violation

Matter effect: constant

Call Schrödinger equation's eigenvalues m_i^2 and eigenvectors U_i .

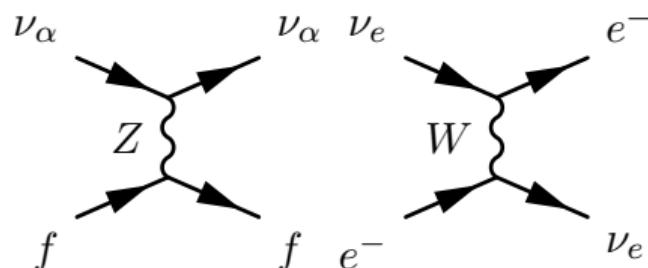
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In matter ν 's propagate in a new basis
that depends on $a \propto N_e E_\nu$.



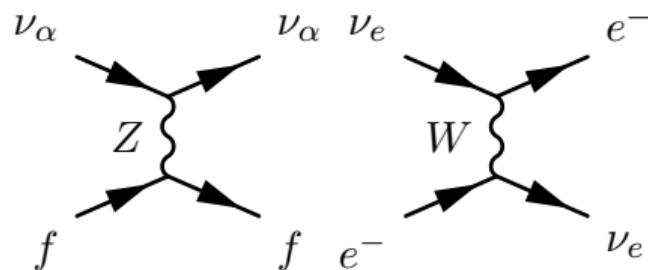
L. Wolfenstein PRD 17 (1978)

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Eigenvalues: $m_i^2 \rightarrow \widehat{m_i^2}(a)$

Eigenvectors are given by $\theta_{ij} \rightarrow \widehat{\theta}_{ij}(a)$ \Leftarrow Unitarity

Hamiltonian dynamics

$$H_{\text{flav}} = \frac{1}{2E} \left[U \begin{pmatrix} 0 & & \\ & \Delta m_{21}^2 & \\ & & \Delta m_{31}^2 \end{pmatrix} U^\dagger + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \right]$$

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$$U = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}$$

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For more on parameterizations see: [PBD](#), R. Pestes [2006.09384](#)

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Find eigenvalues and eigenvectors:

$$H_{\text{flav}} = \frac{1}{2E} \widehat{U} \begin{pmatrix} 0 & & \\ & \widehat{\Delta m^2}_{21} & \\ & & \widehat{\Delta m^2}_{31} \end{pmatrix} \widehat{U}^\dagger$$

H. Zaglauer, K. Schwarzer [Z.Phys. C40 \(1988\) 273](#)
 K. Kimura, A. Takamura, H. Yokomakura [hep-ph/0205295](#)
 PBD, S. Parke, X. Zhang [1907.02534](#)

Matter effect: varying

Solar neutrinos in an adiabatically changing matter potential
Solution = MSW effect

S. Mikheev, A. Smirnov [Nuovo Cim. C9 \(1986\) 17-26](#)

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$$P_{ee} = P_{e2}^\odot P_{22}^{\text{vac}} P_{2e}^{\text{det}} \approx 1 \times 1 \times |U_{e2}|^2 \approx \sin^2 \theta_{12}$$

Bonus question: do we see more solar neutrinos at day or night?

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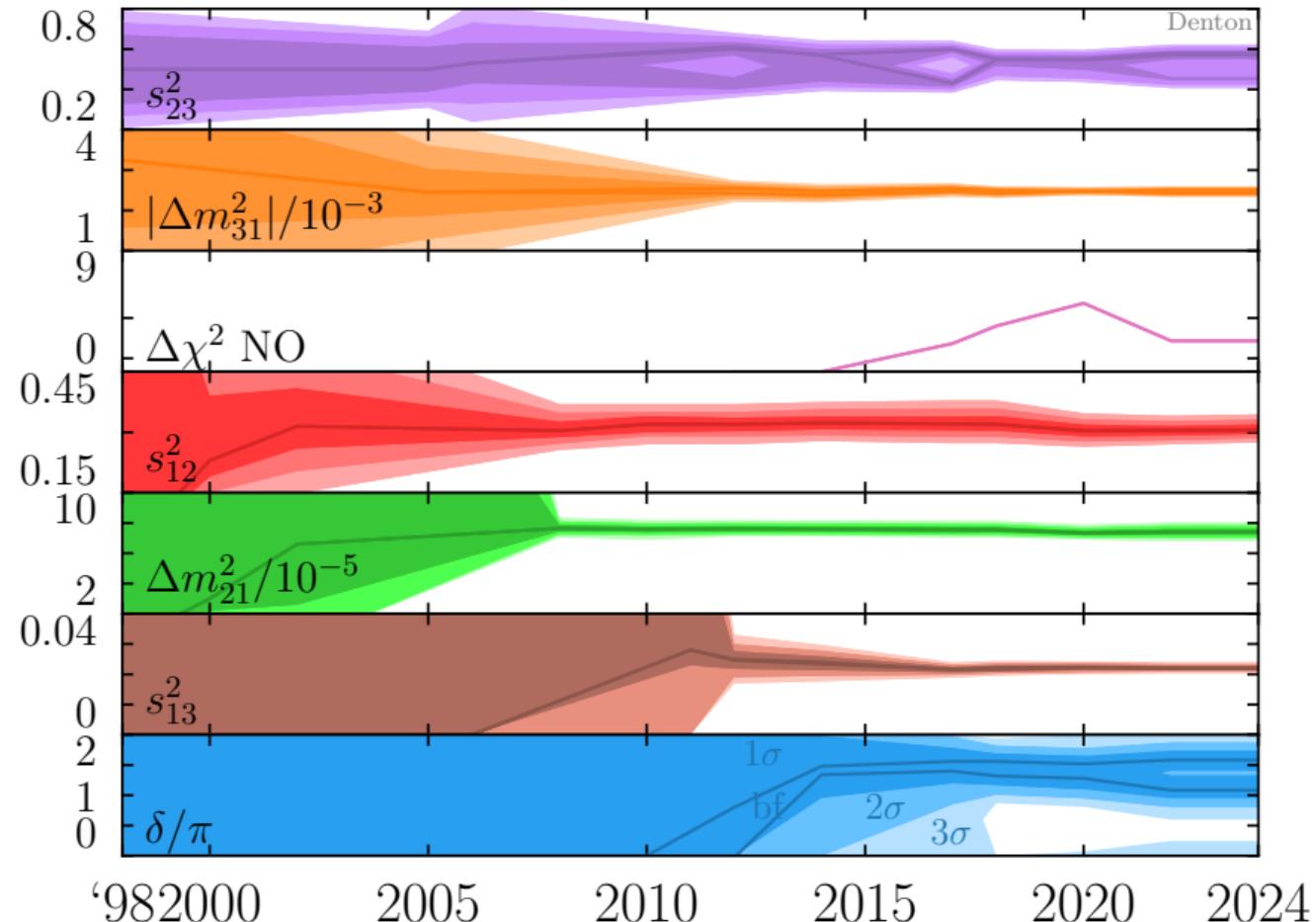
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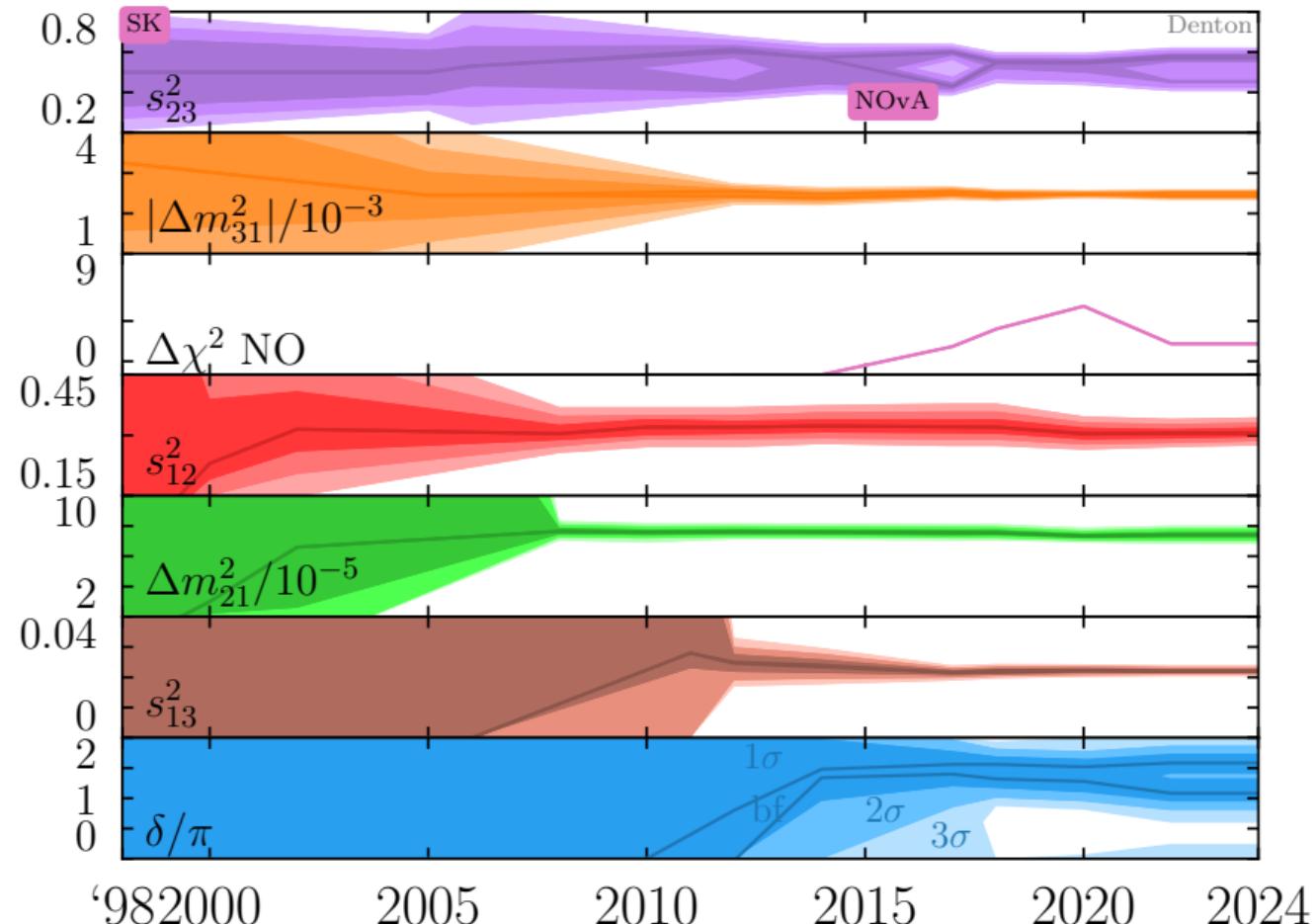
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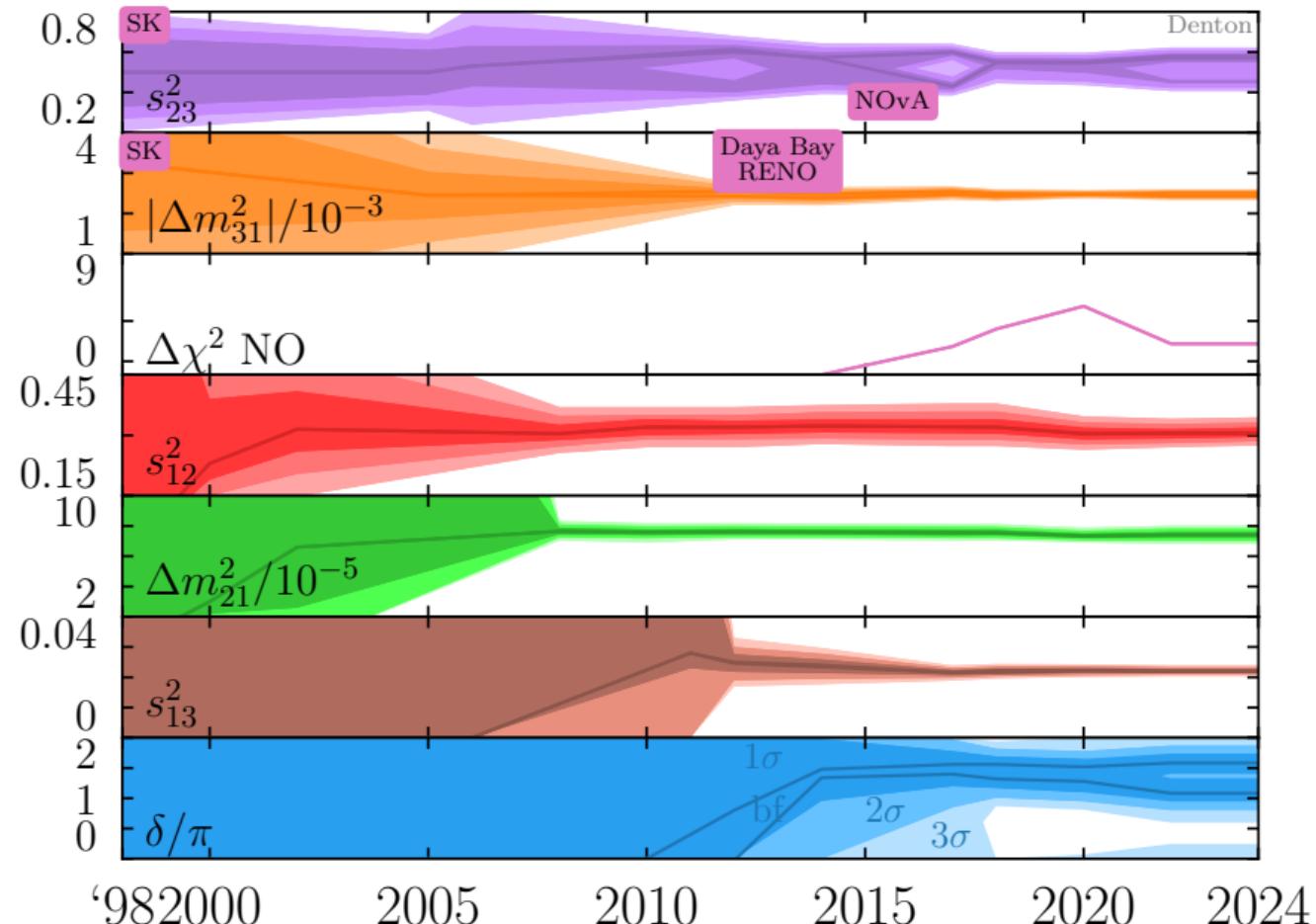
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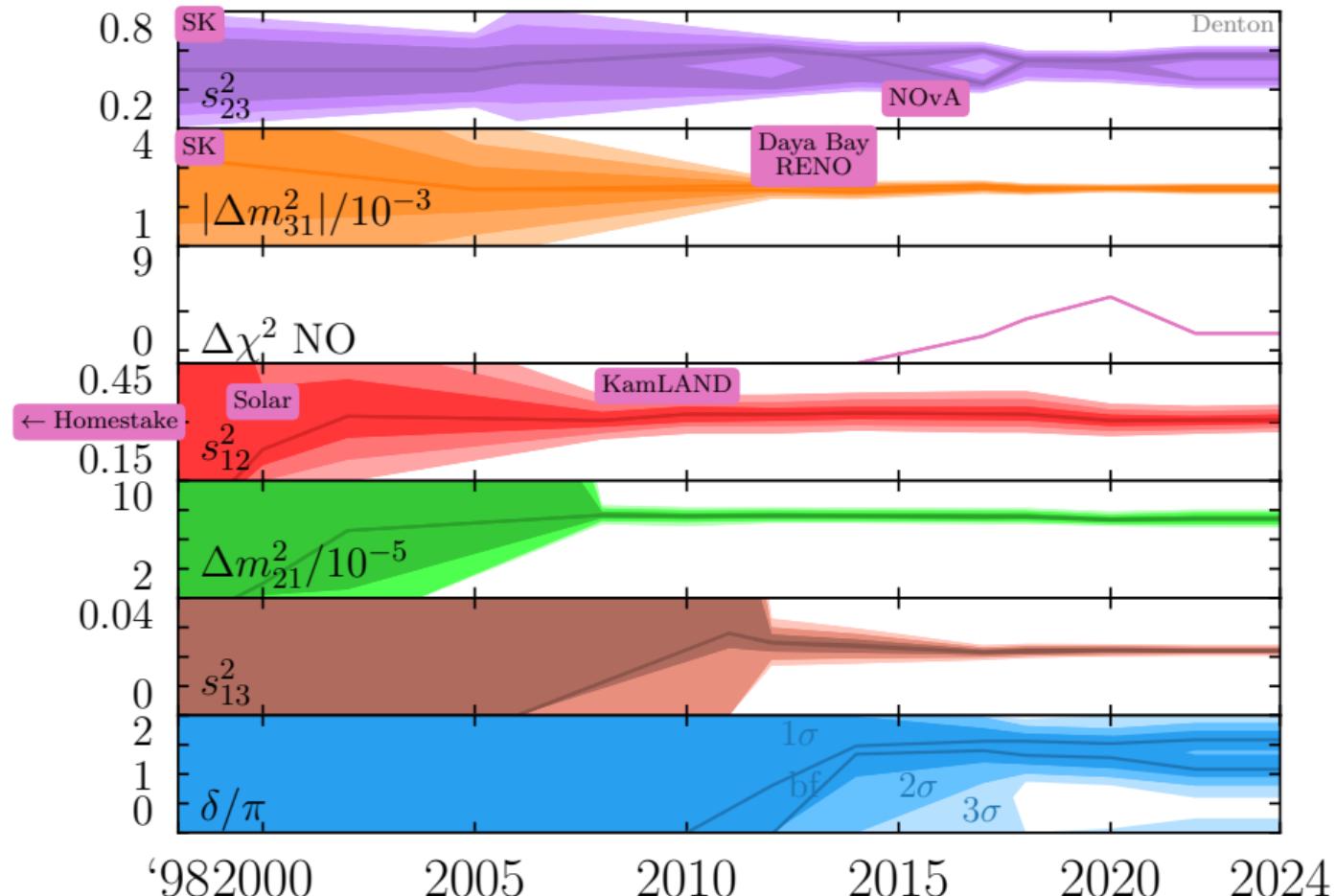
Neutrinos in SNe experience MSW effect too,
but they also experience neutrino-neutrino interactions

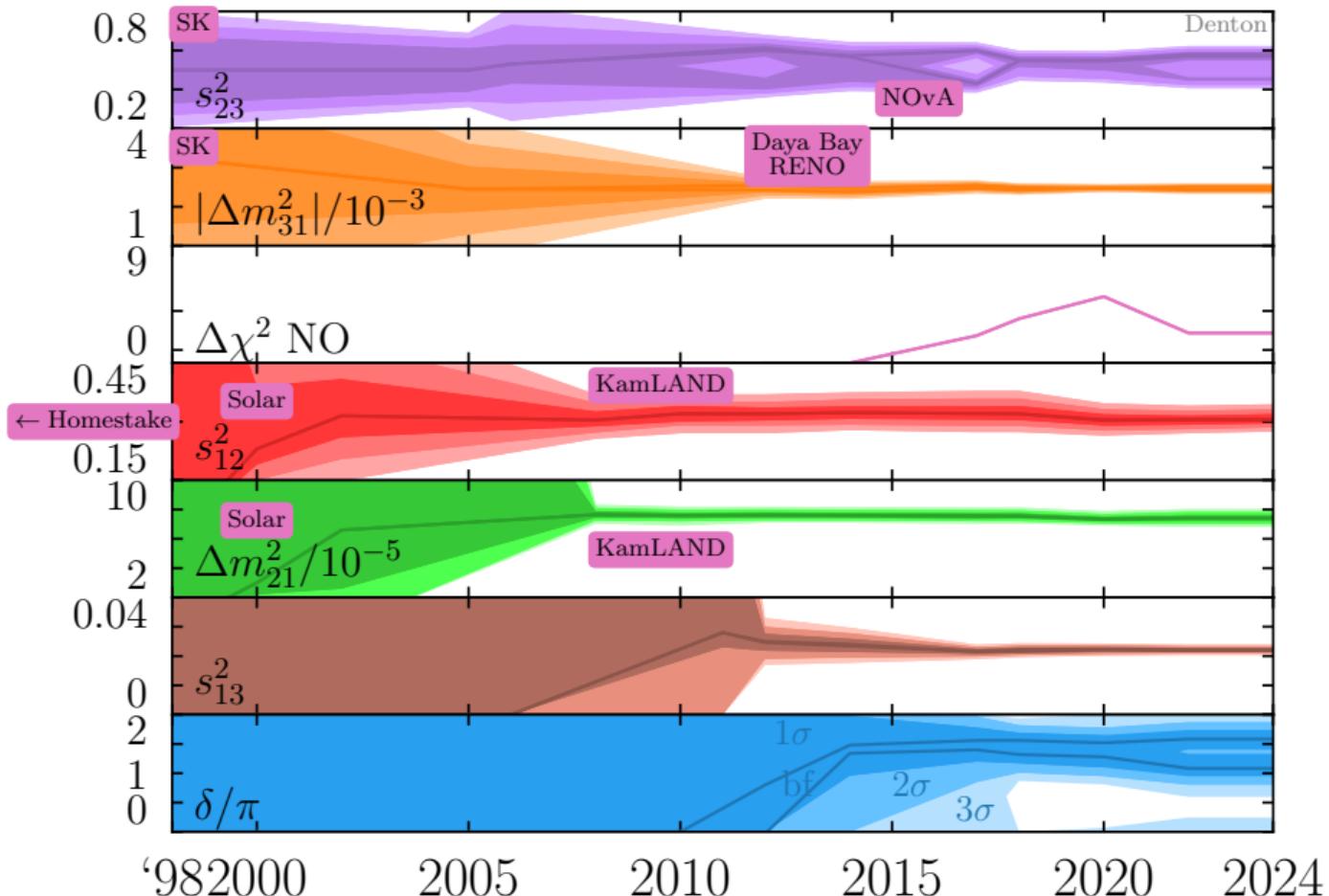
Propagation in SNe is much more involved

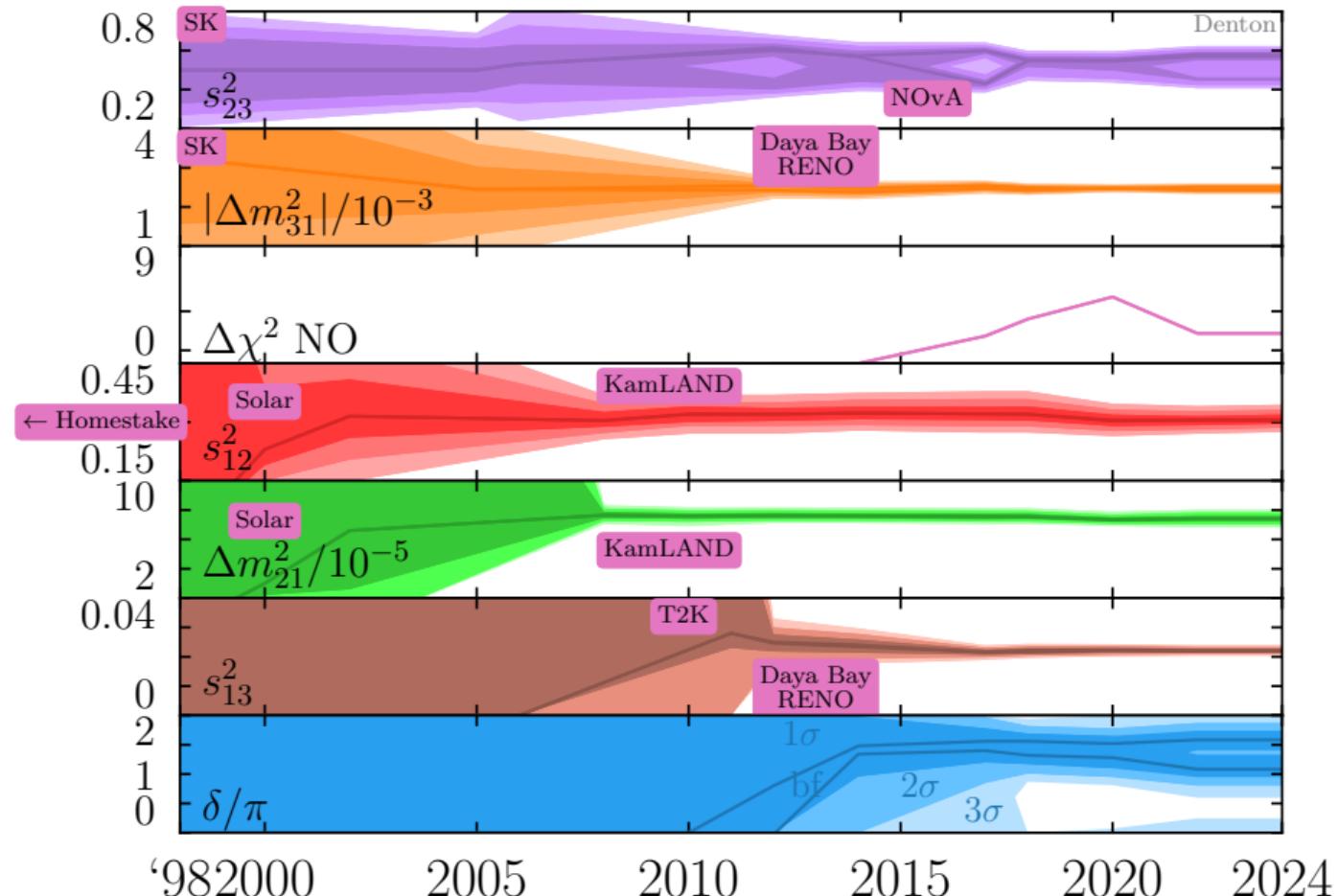


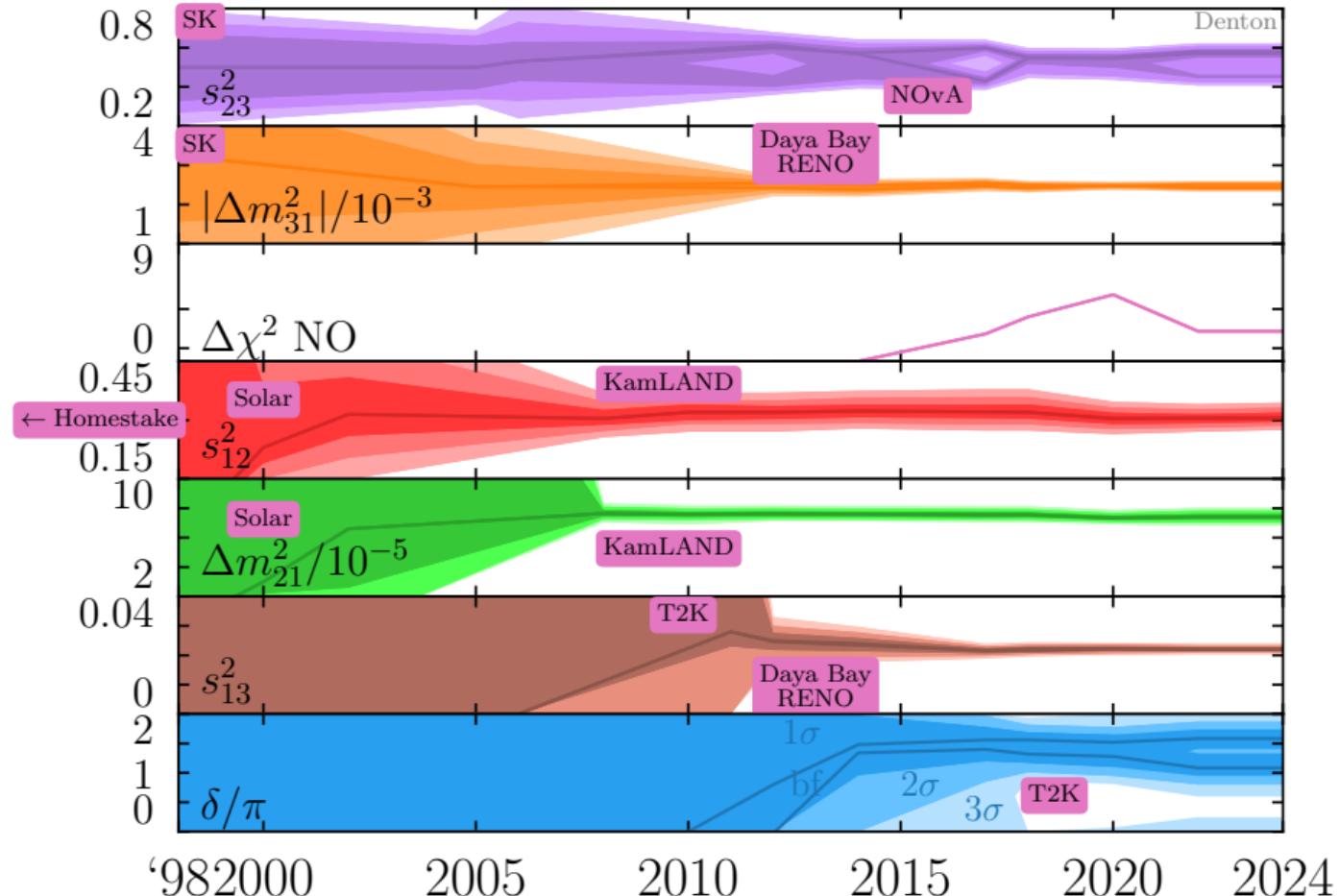




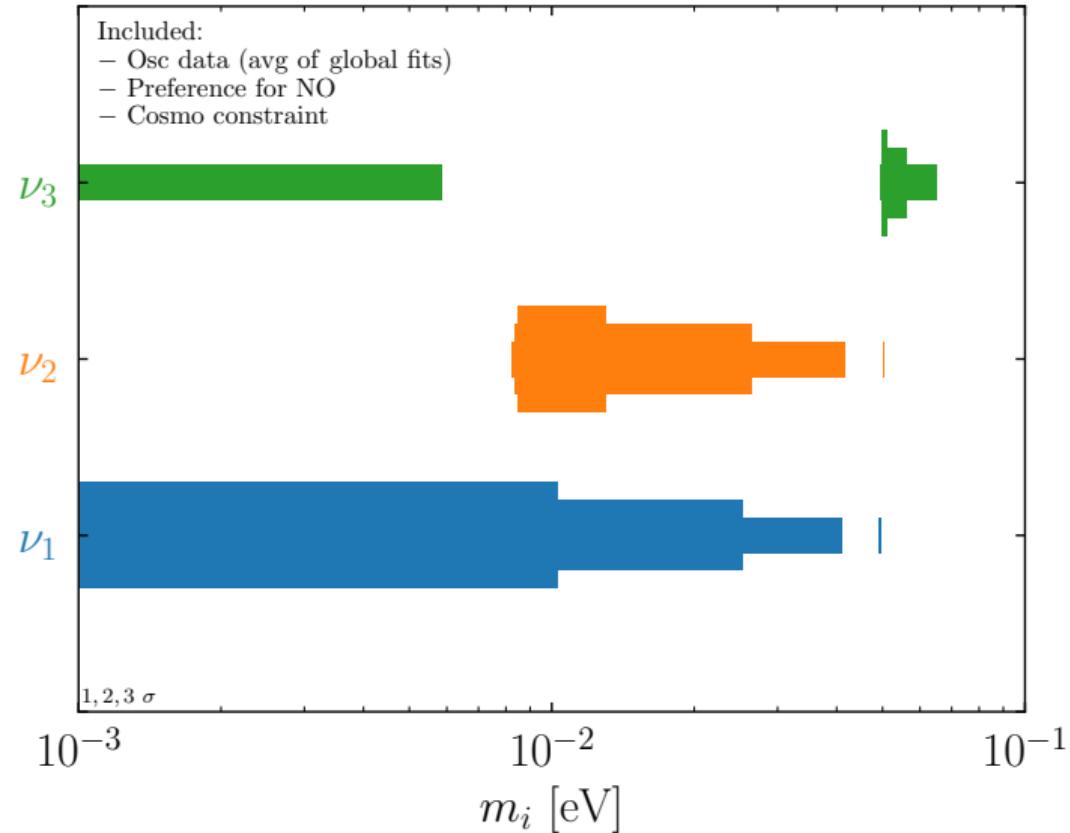








Absolute masses



Four known unknown in particle physics: all neutrinos

Atmospheric mass ordering

θ_{23} octant

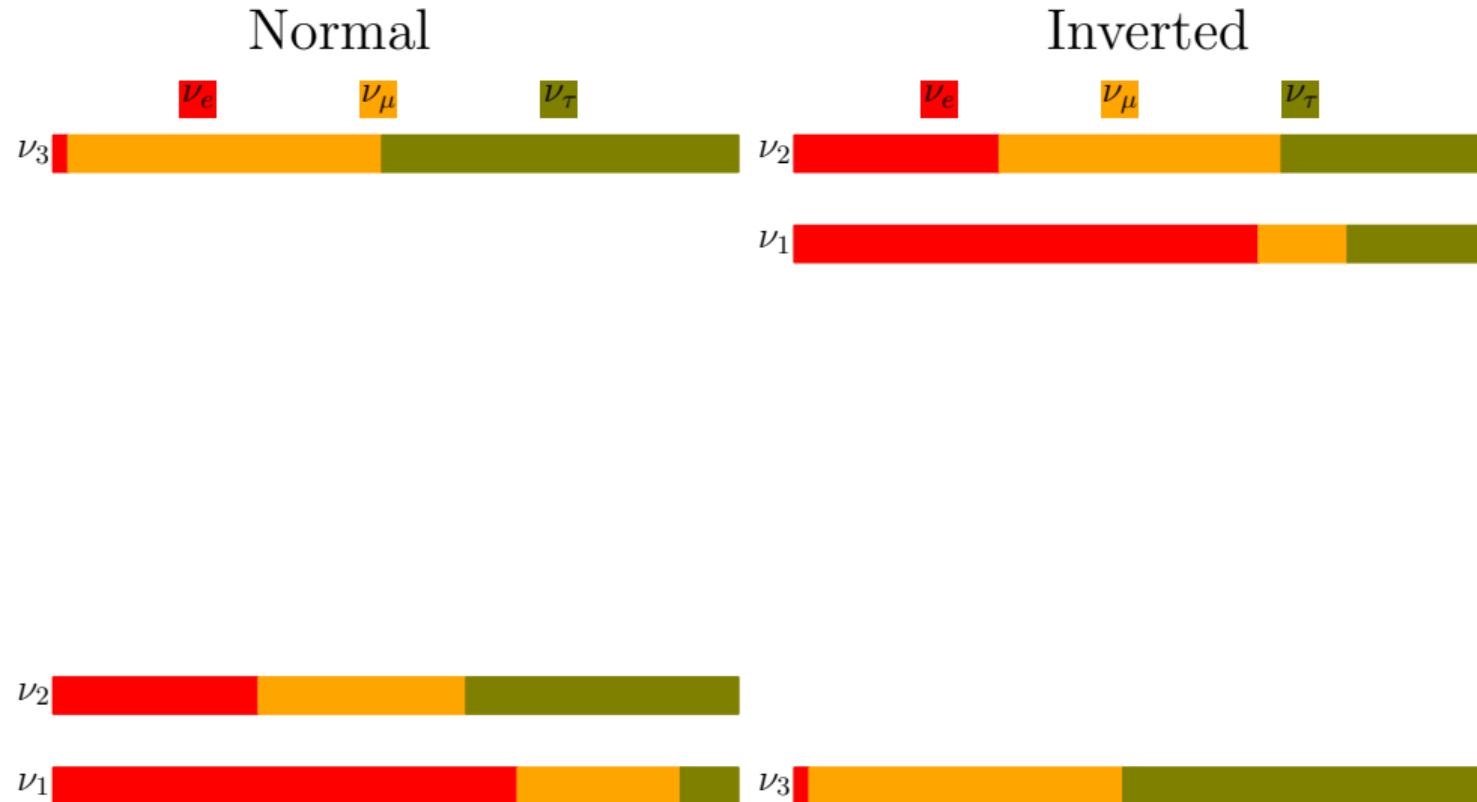
Complex phase

Absolute mass scale

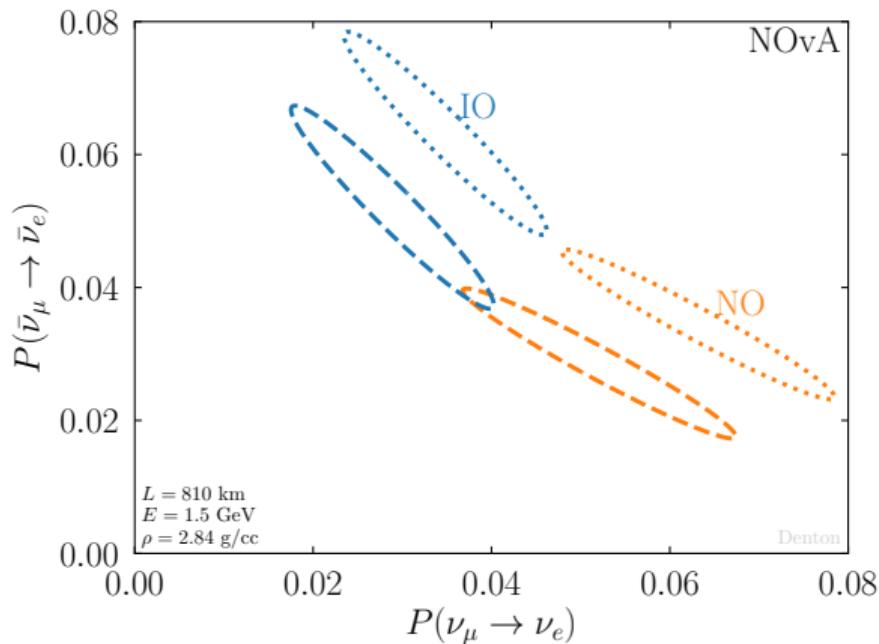
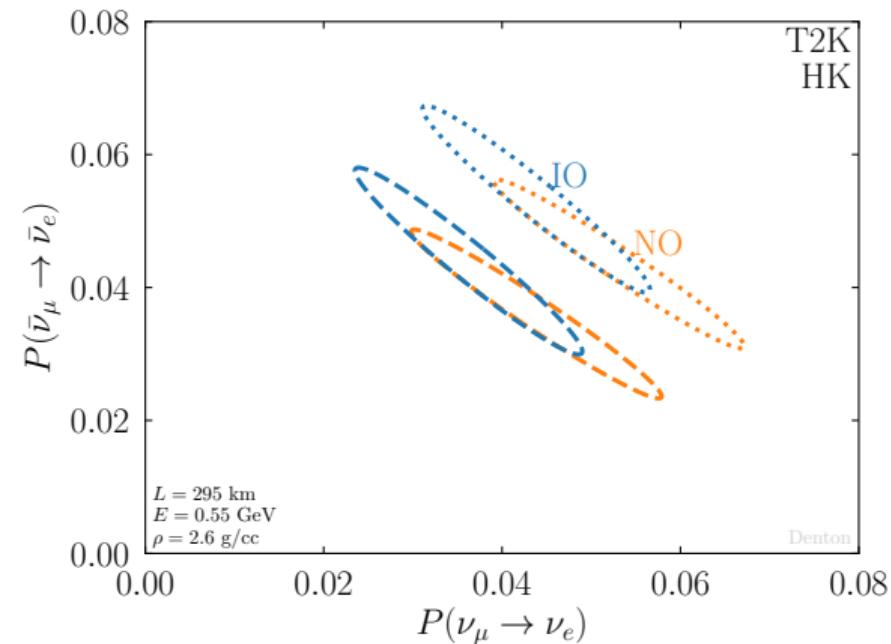
Cosmology, scattering, $0\nu\beta\beta$, ...

Atmospheric mass ordering

Mass ordering: what is it?



Mass ordering: what is it really?



Three other ways to determine mass ordering:
All require the matter effect

Mass ordering current status: oscillations

1. NOvA and T2K both prefer **NO** over **IO**
2. NOvA+T2K prefers **IO** over **NO**
3. SK still prefers **NO** over **IO** – statistics complicated
4. NOvA+T2K+SK still prefers **NO** over **IO**
5. + Daya Bay & RENO \Rightarrow slight preference **NO**
6. = no significant preference either way; with SK $\sim 2\sigma$

PBD, J. Gehrlein, R. Pestes [2008.01110](#)

K. Kelly, et al. [2007.08526](#)

I. Esteban, et al. [2007.14792](#)

F. Capozzi, et al. [2107.00532](#)

P. de Salas, et al. [2006.11237](#)

I. Esteban, et al. [2410.05380](#)

Mass ordering current status: all

From oscillations:

Normal : $m_1 + m_2 + m_3 > 60$ meV Inverted : $m_1 + m_2 + m_3 > 100$ meV

Cosmology: $m_1 + m_2 + m_3 < 90$ meV at 95% CL

E. Valentino, S. Gariazzo, O. Mena [2106.15267](#)
→ 20 meV precision with DESI, EUCLID, ...

Pushing to very low (negative?) masses!?

N. Craig, et al. [2405.00836](#)
Many caveats: T. Bertólez-Martínez, et al. [2411.14524](#)

See also KATRIN [2406.13516](#)

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PRIORS?

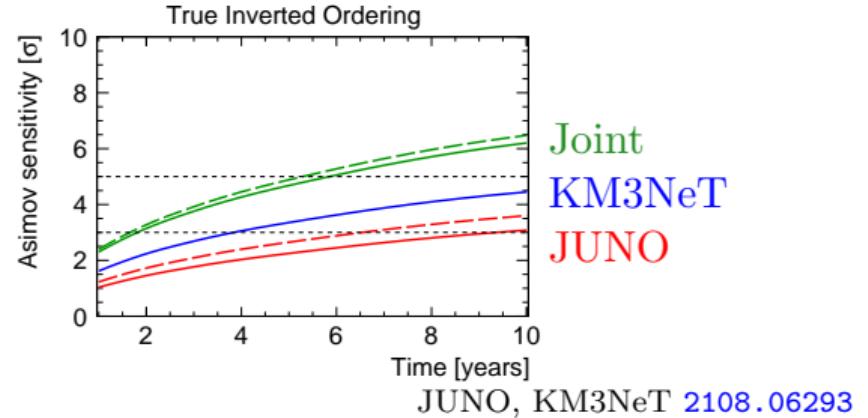
Some claim “decisive” Bayesian evidence for normal

R. Jimenez, et al. [2203.14247](#)

More general prior assumptions ⇒ no significant information from cosmology

S. Gariazzo, et al. [1801.04946](#)
S. Gariazzo, et al. [2205.02195](#)

Mass ordering: future sensitivities

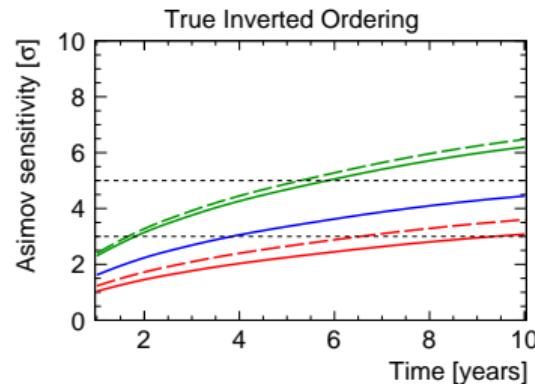
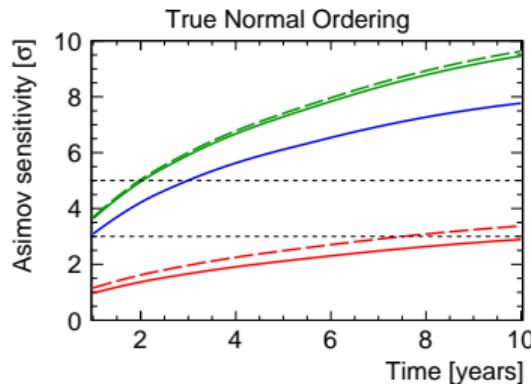


JUNO, KM3NeT [2108.06293](#)

JUNO, IceCube [1911.06745](#)

Note: if lower octant, KM3NeT is less sensitive

Mass ordering: future sensitivities



Joint
KM3NeT
JUNO

JUNO, KM3NeT [2108.06293](#)
JUNO, IceCube [1911.06745](#)

Note: if lower octant, KM3NeT is less sensitive

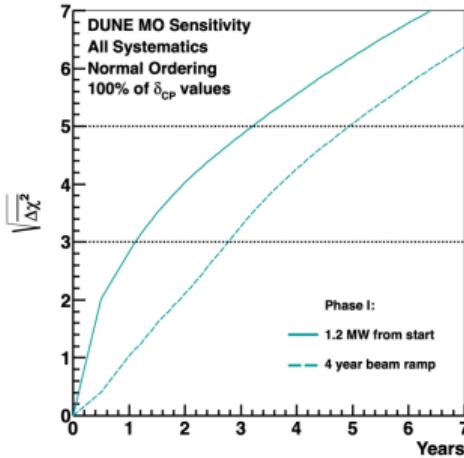
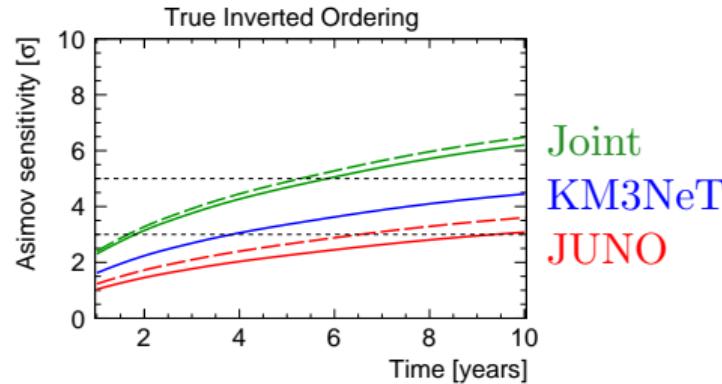
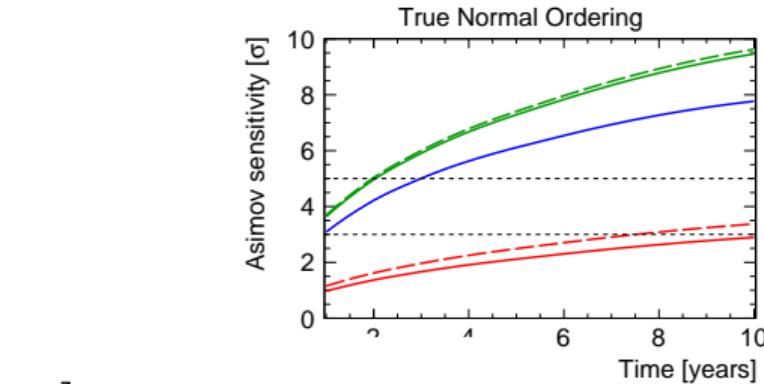
$$\Delta m_{ee}^2 = c_{12}^2 \Delta m_{31}^2 + s_{12}^2 \Delta m_{32}^2$$

$$\Delta m_{\mu\mu}^2 = s_{12}^2 \Delta m_{31}^2 + c_{12}^2 \Delta m_{32}^2 + \mathcal{O}(s_{13} \Delta m_{21}^2)$$

Differ by $\pm \sim 1.1\%$ in each mass ordering

H. Nunokawa, S. Parke, R. Funchal [hep-ph/0503283](#)

Mass ordering: future sensitivities



Matter effect \Rightarrow DUNE [2203.06100](#)

$$\Delta m_{ee}^2 = c_{12}^2 \Delta m_{31}^2 + s_{12}^2 \Delta m_{32}^2$$

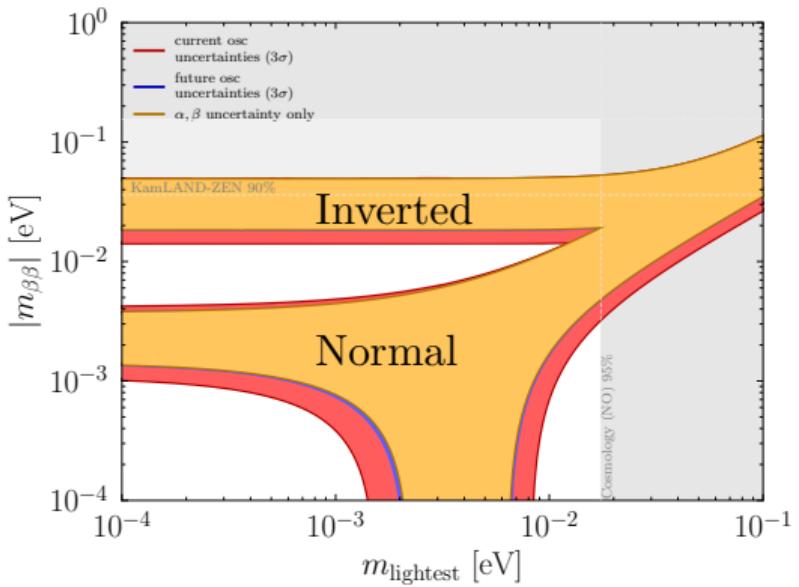
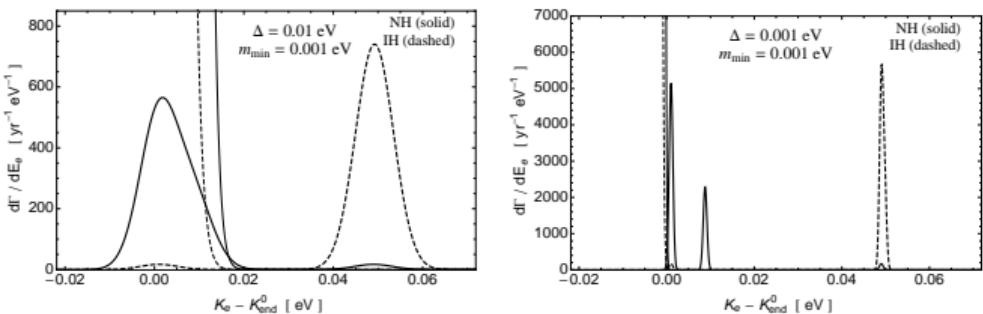
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Mass ordering: broad implications

- ▶ Affects cosmology
- ▶ Affects galactic SN signal
- ▶ Affects $0\nu\beta\beta$
- ▶ Affects flavor models
- ▶ Affects end point measurements
- ▶ Affects $C\nu B$



PBD, J. Gehrlein [2308.09737](#)

A. Long, C. Lunardini, E. Sabancilar [1405.7654](#)

Mass ordering: new physics degeneracies

In the presence of new physics such as NSI we have:

$$[\text{NO}] + [\epsilon = 0] \equiv [\text{IO}] + [\epsilon_{ee} = -2]$$

$$[\text{IO}] + [\epsilon = 0] \equiv [\text{NO}] + [\epsilon_{ee} = -2]$$

Equivalences hold even if all oscillation probabilities are *perfectly* measured

P. Bakhti, Y. Farzan [1403.0744](#)

P. Coloma, T. Schwetz [1604.05772](#)

P. Coloma, **PBD**, et al. [1701.04828](#)

PBD, S. Parke [2106.12436](#)

PBD, J. Gehrlein [2204.09060](#)



This is known as the **LMA-Dark** solution

Is the mass ordering robust?

Need **scattering** to break

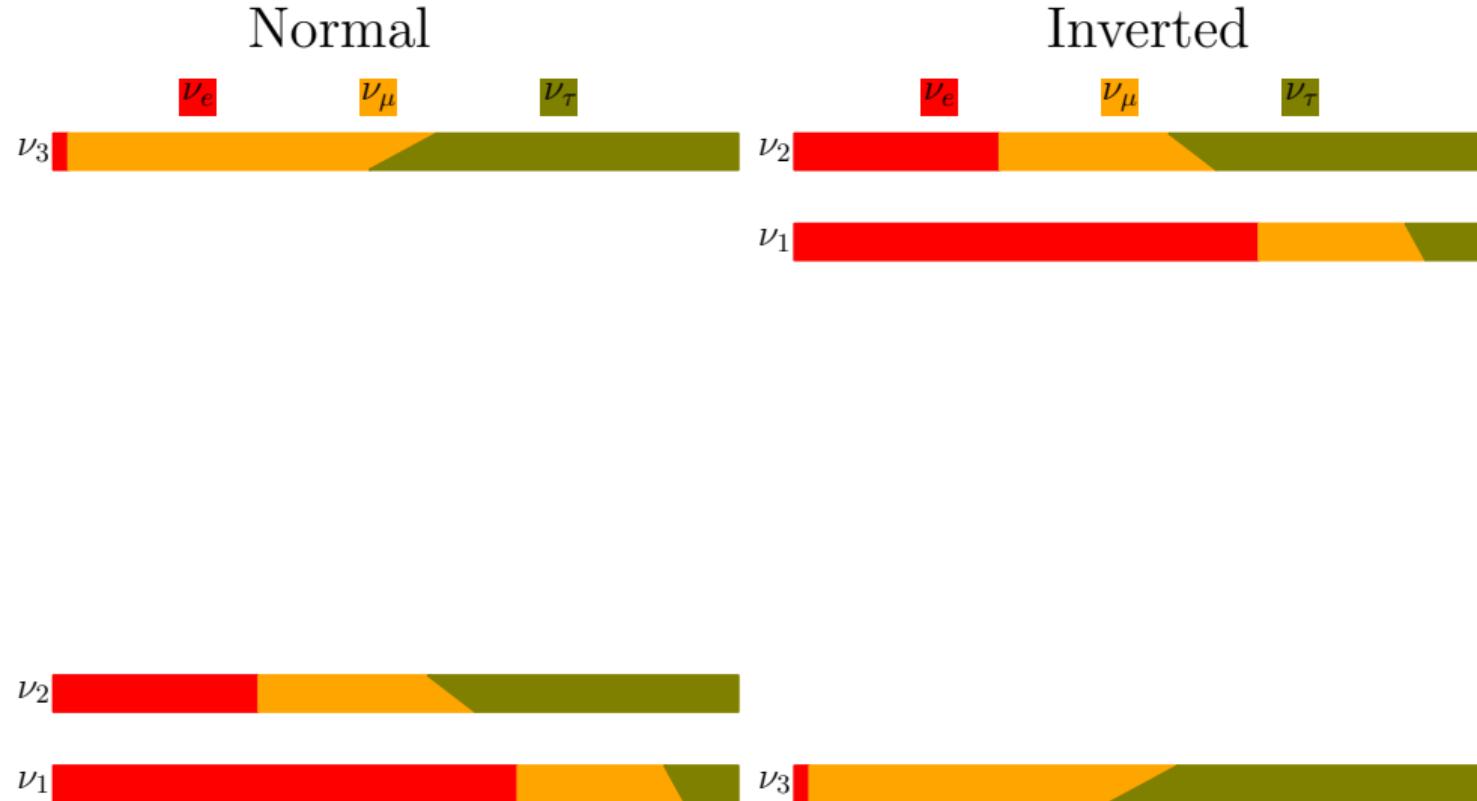


Can probe same NC $\epsilon = -2$ process in scattering, but...

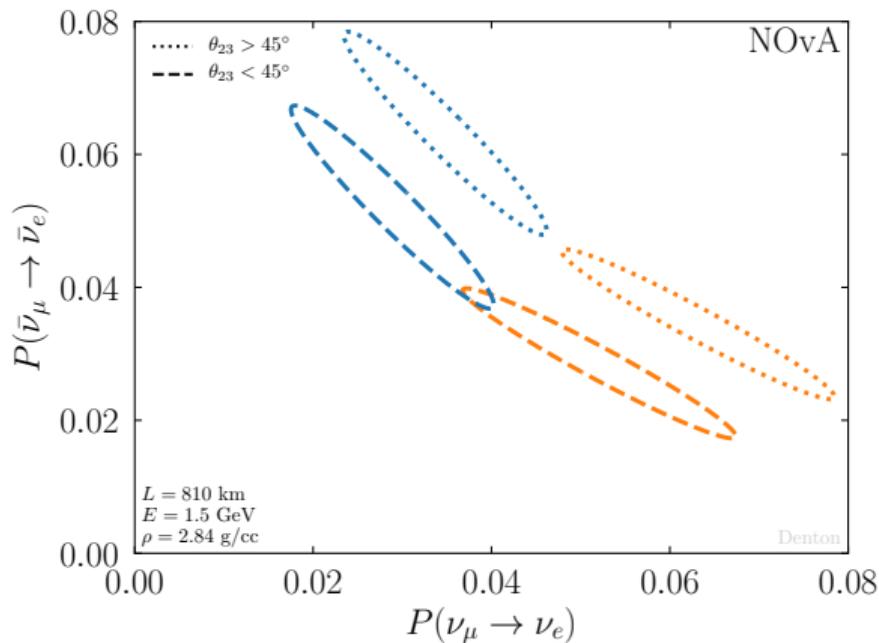
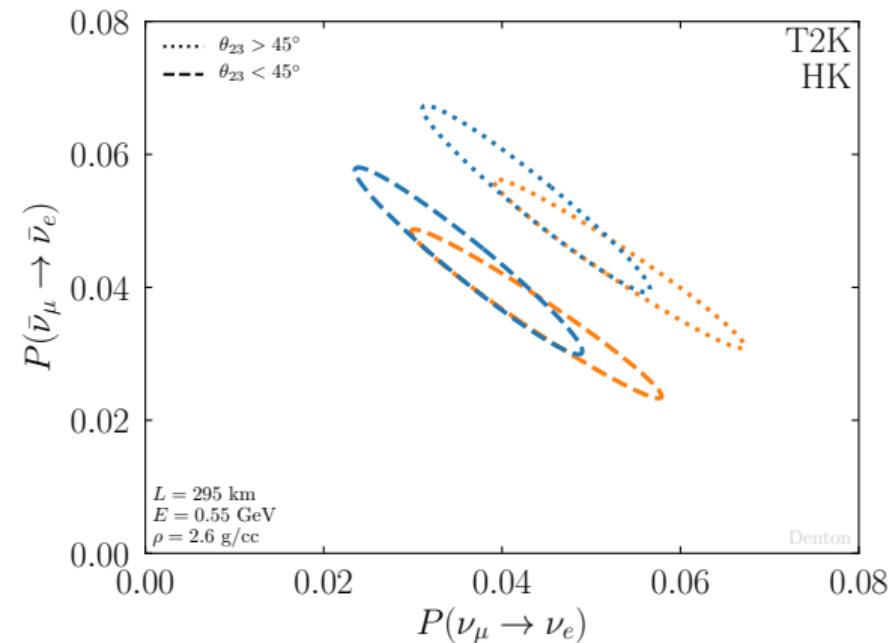
1. COHERENT for $M_{Z'} \gtrsim 50$ MeV and cosmology for $M_{Z'} \lesssim 5$ MeV
PBD, Y. Farzan, I. Shoemaker [1804.03660](#)
2. Dresden-II for ϵ_{ee} for any mediator mass
PBD, J. Gehrlein [2204.09060](#)
3. Can still evade with specific flavor structures
 $\epsilon_{\mu\mu} = \epsilon_{\tau\tau} = 2$ or certain u / d combinations
4. CCM & COHERENT can close all loopholes

θ_{23} octant

θ_{23} octant: what is it?

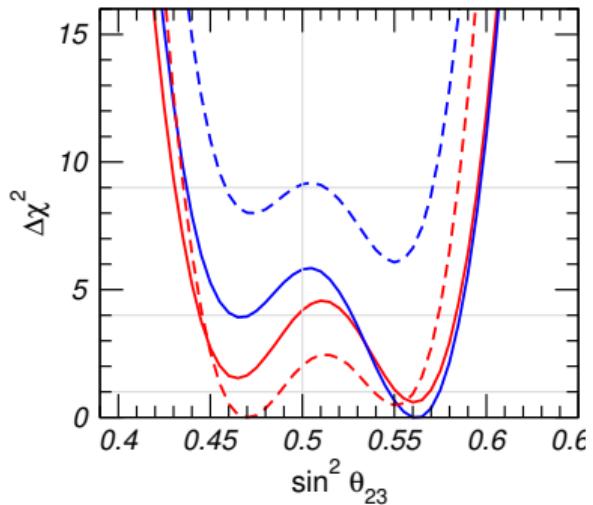


θ_{23} octant: what is it really?

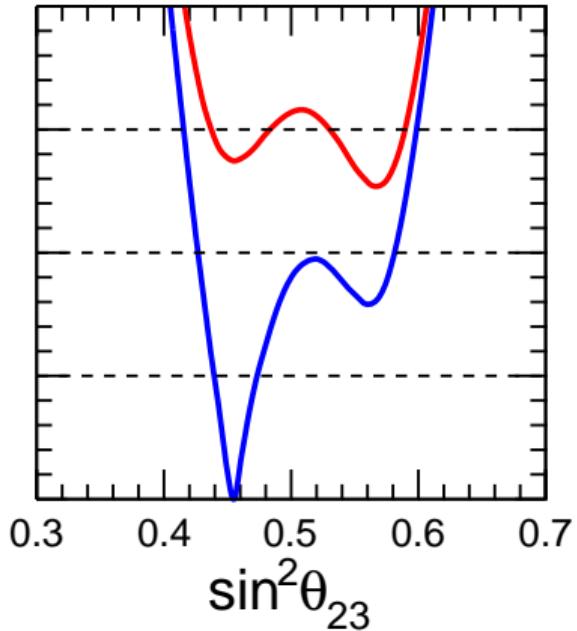


Lower octant more “normal” than upper octant

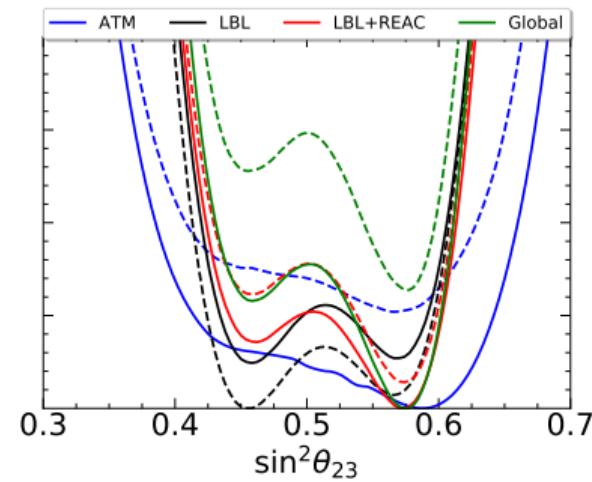
θ_{23} octant: current status



I. Esteban, et al. [2410.05380](#)



F. Capozzi, et al. [2107.00532](#)



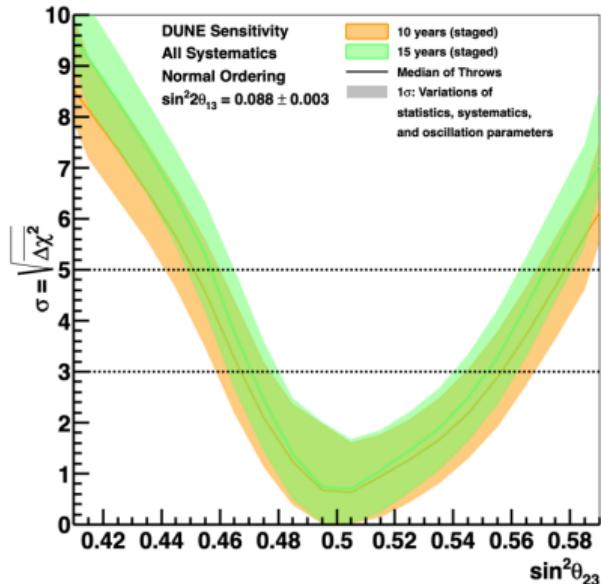
P. de Salas, et al. [2006.11237](#)

Upper/lower at $\sim 1\sigma$

Prefers **lower** at $\sim 1.5\sigma$

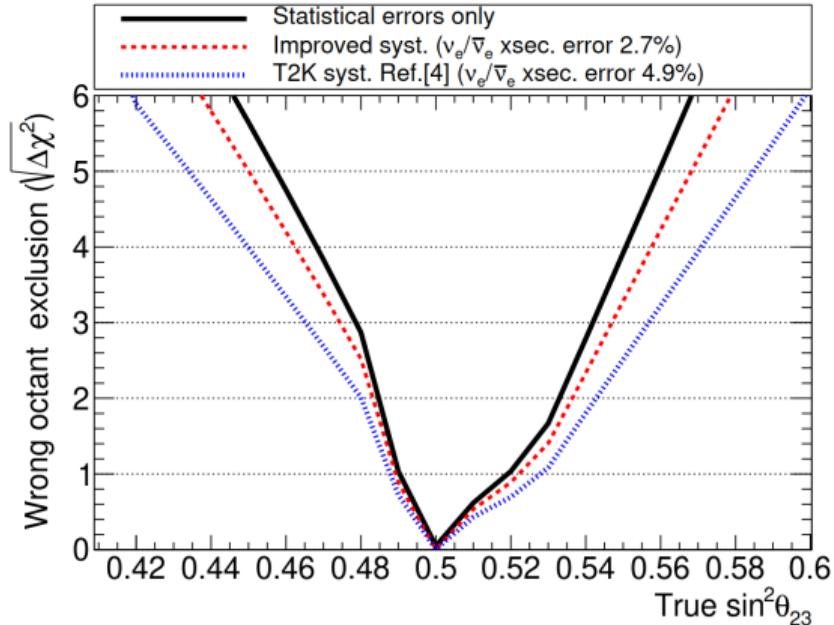
Prefers **upper** at $> 2\sigma$

θ_{23} octant: future sensitivities



$\sim 3 - 5\sigma$

DUNE 2002.03005



$\sim 3 - 5\sigma$

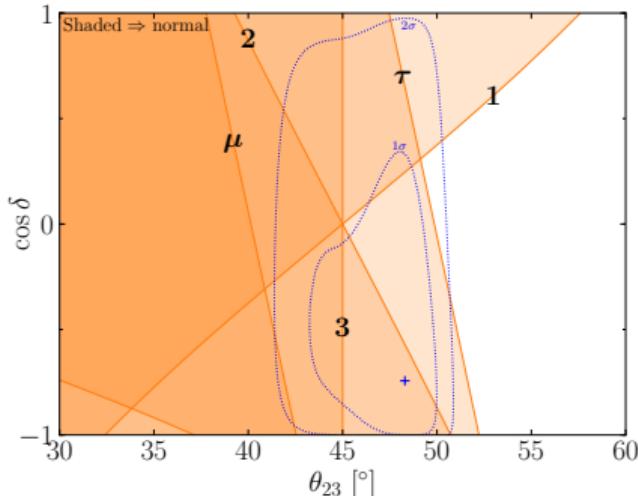
HK 2505.15019

θ_{23} : broader implications

μ - τ interchange/reflection symmetry

Normalcy

Is the heaviest neutrino mostly ν_τ ?
Is the lightest neutrino least ν_τ ?



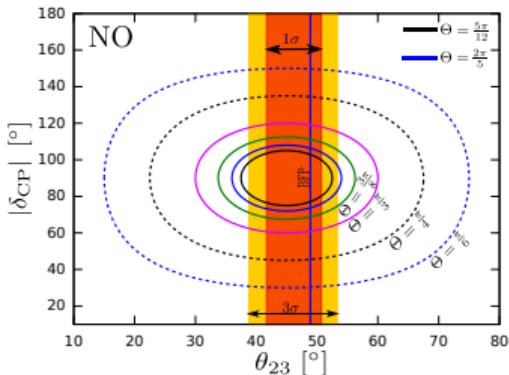
Quarks easily satisfy normalcy PBD 2003.04319

$$\nu_\mu \leftrightarrow \nu_\tau$$

$$M_\nu^* = X M_\nu X^T \quad X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$M_\nu \equiv U D_\nu U^\dagger$$

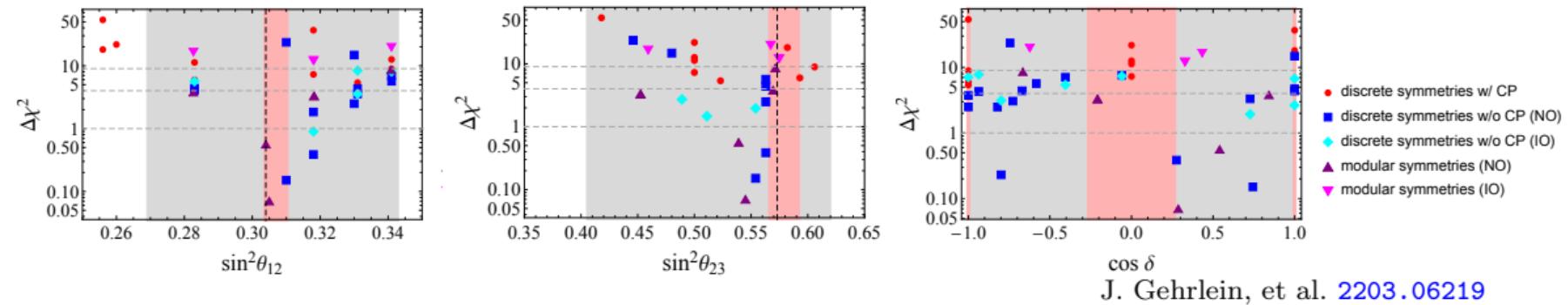
Predicts: $\theta_{23} = 45^\circ$, often $\theta_{13} = 0$



P. Chen, et al. 1512.01551

Parameter interplay

Models predict specific correlations among the parameters



Precision in all neutrino parameters is key!

Complex phase

δ and CP violation

$$J_{CP} = s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23} \sin \delta$$

C. Jarlskog [PRL 55, 1039 \(1985\)](#)



δ and CP violation

$$J_{CP} = s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23} \sin \delta$$

C. Jarlskog [PRL 55, 1039 \(1985\)](#)



1. Strong interaction: no observed EDM \Rightarrow CP (nearly) **conserved**

$$\frac{\bar{\theta}}{2\pi} < 10^{-11}$$

J. Pendlebury, et al. [1509.04411](#)

2. Quark mass matrix: non-zero but **small** CP violation

$$\frac{|J_{CKM}|}{J_{\max}} = 3 \times 10^{-4}$$

CKMfitter [1501.05013](#)

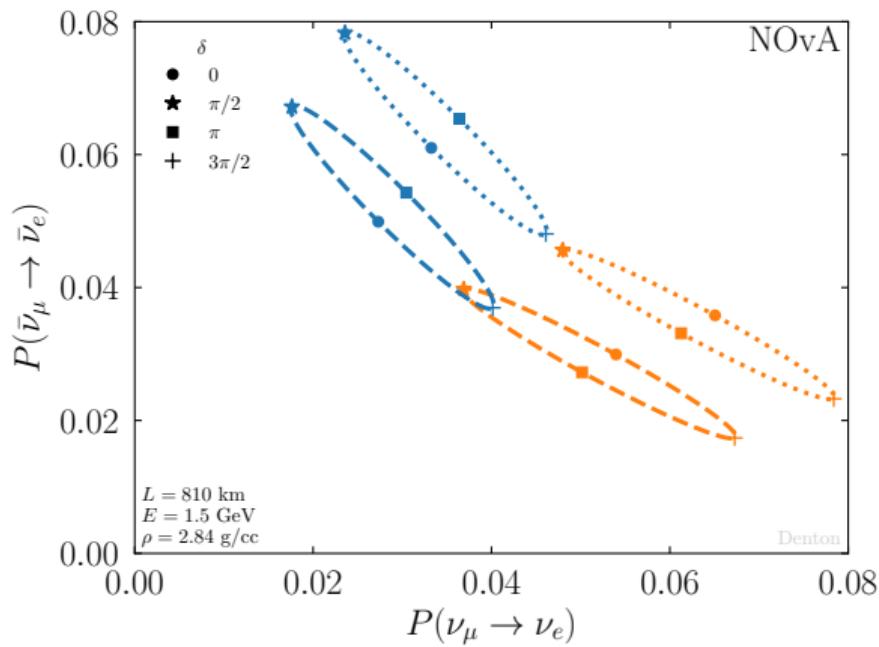
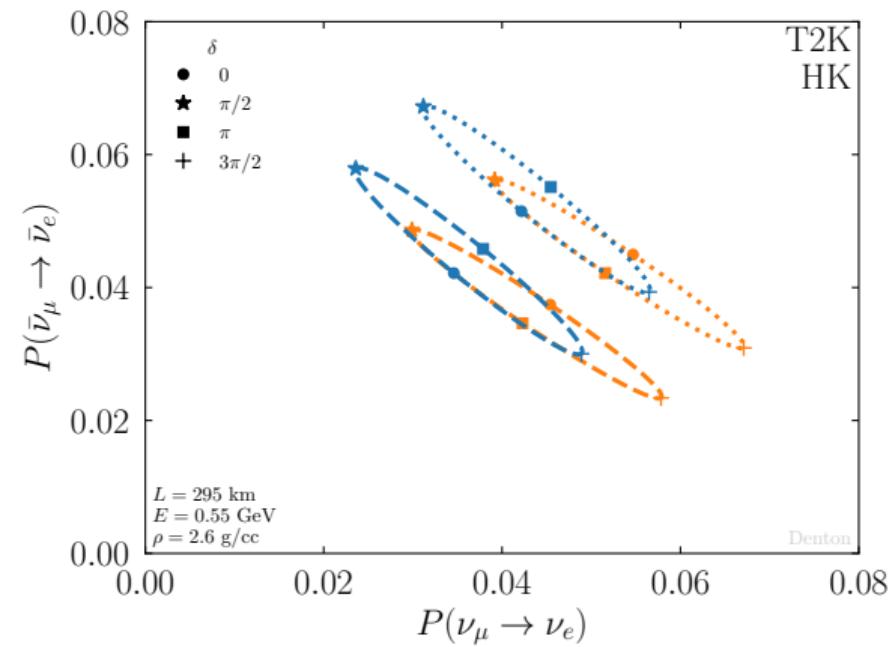
3. Lepton mass matrix: ?

$$\frac{|J_{PMNS}|}{J_{\max}} < 0.34$$

PBD, J. Gehrlein, R. Peses [2008.01110](#)

$$J_{\max} = \frac{1}{6\sqrt{3}} \approx 0.096$$

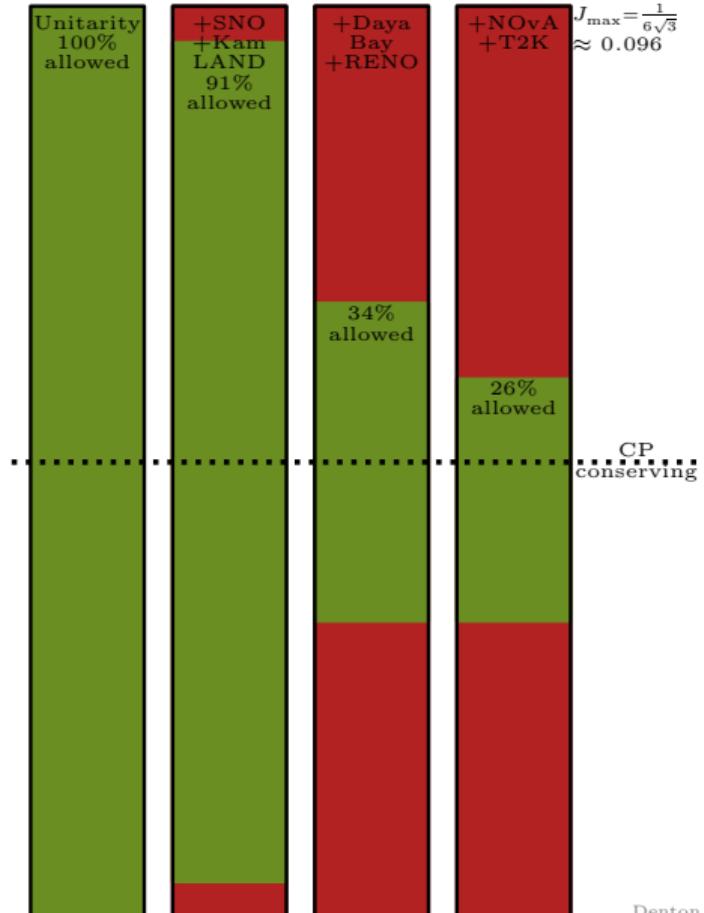
δ : what is it really?



δ, J : current status

Maximal CP violation is already ruled out:

1. $\theta_{12} \neq 45^\circ$ at $\sim 15\sigma$
2. $\theta_{13} \neq \tan^{-1} \frac{1}{\sqrt{2}} \approx 35^\circ$ at many (100) σ
3. $\theta_{23} = 45^\circ$ allowed at $\sim 1\sigma$
4. $|\sin \delta| = 1$ allowed



CP violation in oscillations

In vacuum at first maximum:

$$P_{\mu e} - \bar{P}_{\mu e} \approx 8\pi J \frac{\Delta m_{21}^2}{\Delta m_{32}^2}$$

$$J \equiv s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23} \sin \delta$$

C. Jarlskog [PRL 55, 1039 \(1985\)](#)

- ▶ Extracting δ from data requires every other oscillation parameter
- ▶ J requires only Δm_{21}^2 (up to matter effects)
- ▶ Instead of asymmetry, can be determined via triple sine dependence

Matter effects in triple sine term can be accounted for

$$\hat{J} \simeq \frac{J}{\sqrt{(c_{212} - c_{13}^2 a / \Delta m_{21}^2)^2 + s_{212}^2} \sqrt{(c_{213} - a / \Delta m_{ee}^2)^2 + s_{213}^2}}$$

[PBD](#), S. Parke [1902.07185](#)
[PBD](#), H. Minakata, S. Parke [1604.08167](#)

When δ and when J ?

If the goal is **CP violation** the Jarlskog invariant should be used

however

If the goal is **measuring the parameters** one must use δ

Given θ_{12} , θ_{13} , θ_{23} , and J , I can't determine the sign of $\cos \delta$ which is physical

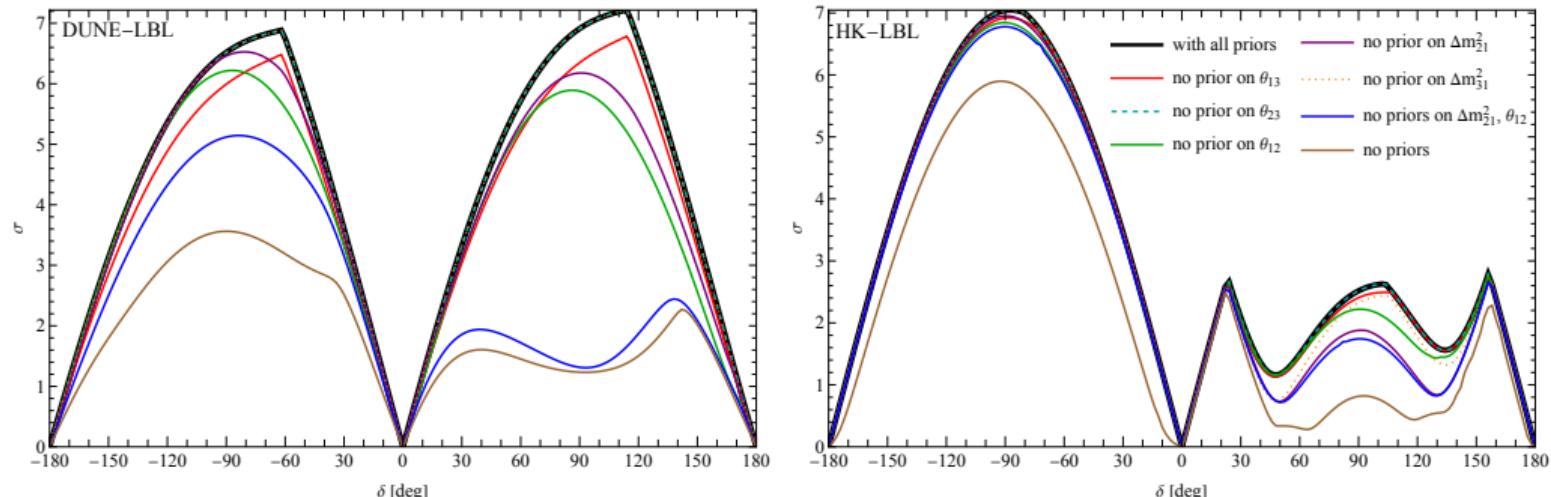
e.g. $P(\nu_\mu \rightarrow \nu_\mu)$ depends on $\cos \delta$
[PBD 2309.03262](#)

- ▶ T2K/HK are mostly sensitivity to $\sin \delta$; they should focus on J
T2K does this now!
- ▶ NOvA/DUNE has modest $\cos \delta$ sensitivity; both J and δ should be reported

δ : future sensitivities

DUNE and HK will make great measurements via appearance $\nu_\mu \rightarrow \nu_e$

$\nu + \bar{\nu}$ helps systematics but isn't strictly necessary

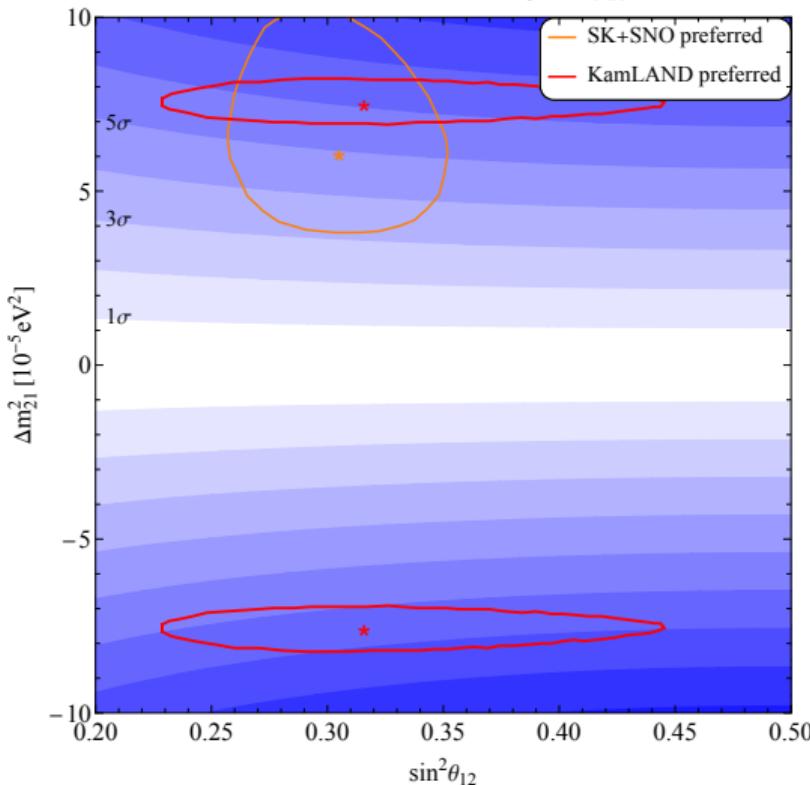


Need to know solar parameters to measure δ !

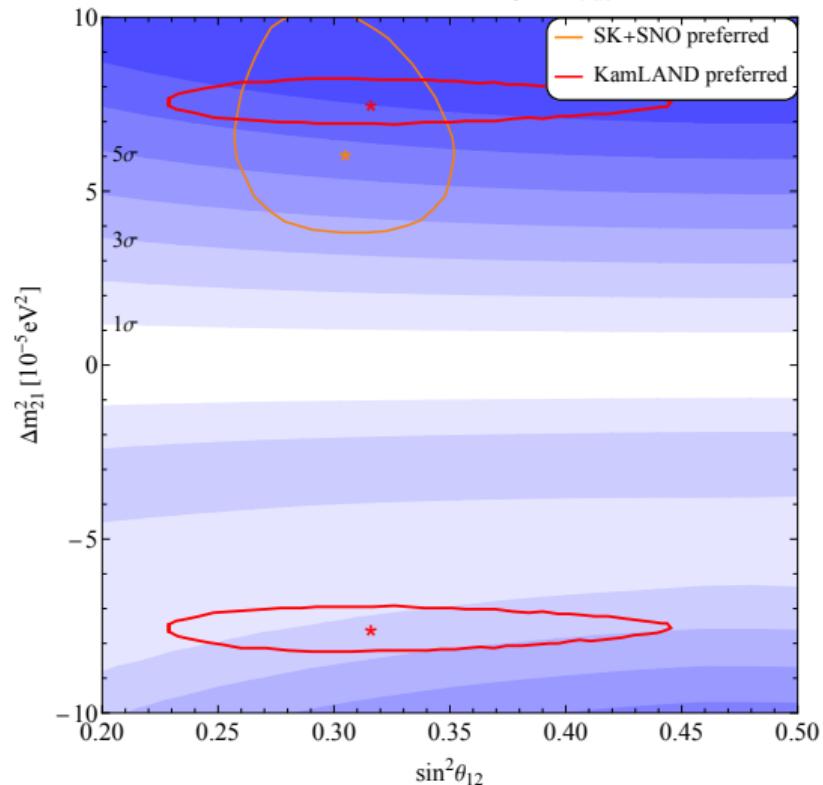
Current solar knowledge: okay
Future (JUNO): excellent
PBD, J. Gehrlein [2302.08513](#)

Impact of the true solar parameters on δ

DUNE-LBL CPV sensitivity at $\delta_{\text{true}} = -90^\circ$

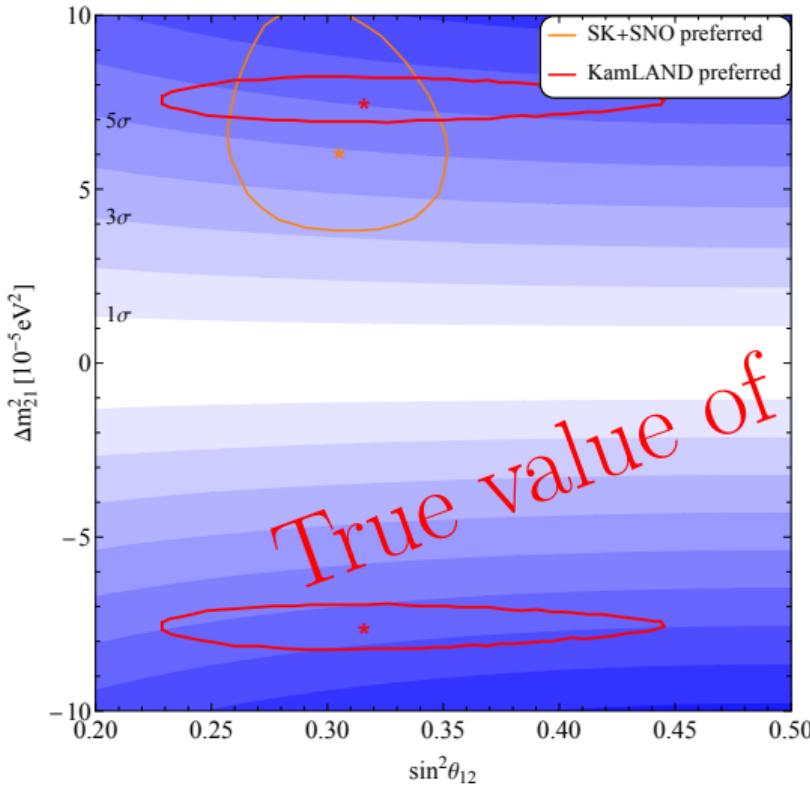


HK-LBL CPV sensitivity at $\delta_{\text{true}} = -90^\circ$

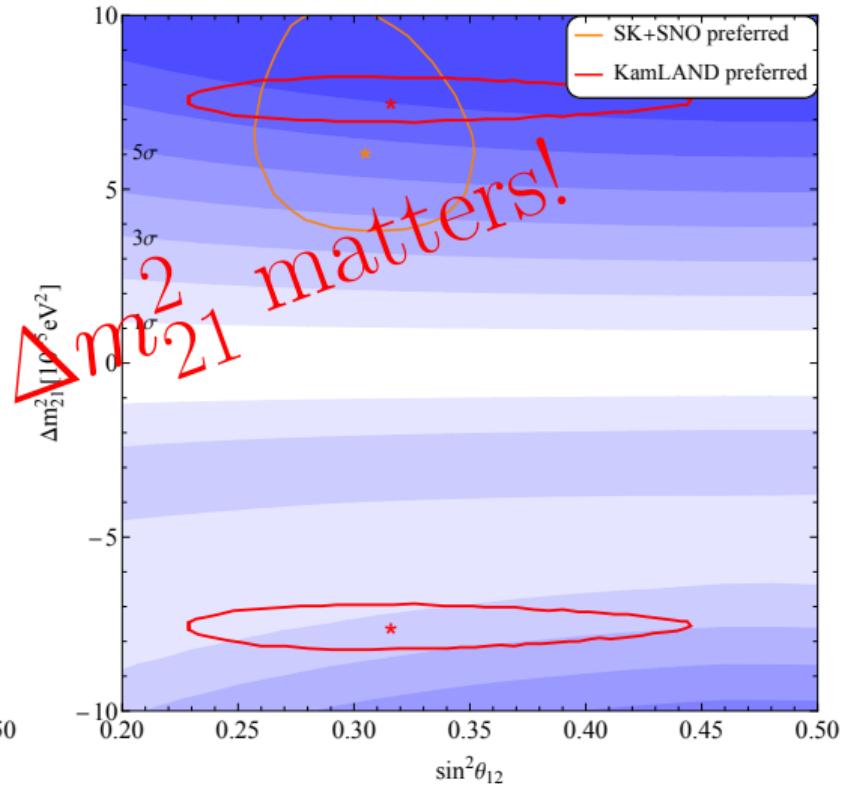


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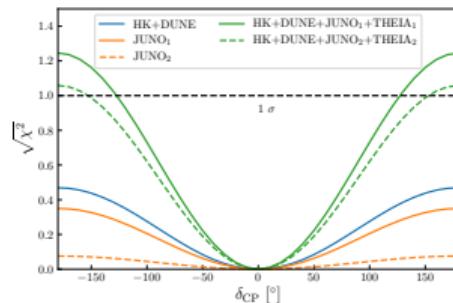
Non-standard CPV probes

1. Some information in solar due to loops in elastic scattering

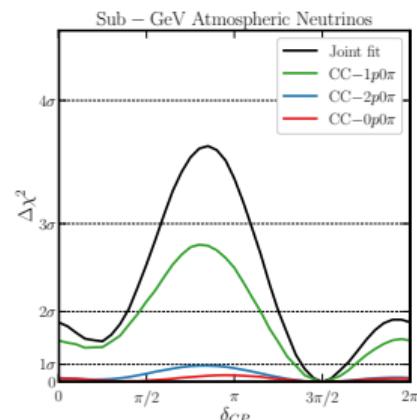
V. Brdar, X-J. Xu [2306.03160](#)
K. Kelly, et al. [2407.03174](#) requires 3k Borexinos

2. Sub-GeV \rightarrow sub-100 MeV atmospheric neutrinos

K. Kelly, et al. [1904.02751](#)
See also e.g. A. Suliga, J. Beacom [2306.11090](#)



Solar (no systematics)



Atmospheric neutrinos at DUNE

Non-standard CPV probes: disappearance

Possible to get at CPV with CPC processes

Disappearance probability:

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\alpha) &= 1 - 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2 \Delta_{21} \\ &\quad - 4|U_{\alpha 1}|^2|U_{\alpha 3}|^2 \sin^2 \Delta_{31} \\ &\quad - 4|U_{\alpha 2}|^2|U_{\alpha 3}|^2 \sin^2 \Delta_{32}, \end{aligned}$$

$$\Delta_{ij} = \Delta m_{ij}^2 L / 4E$$

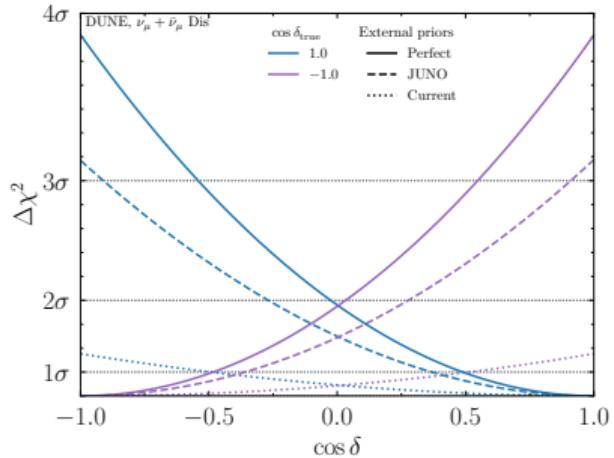
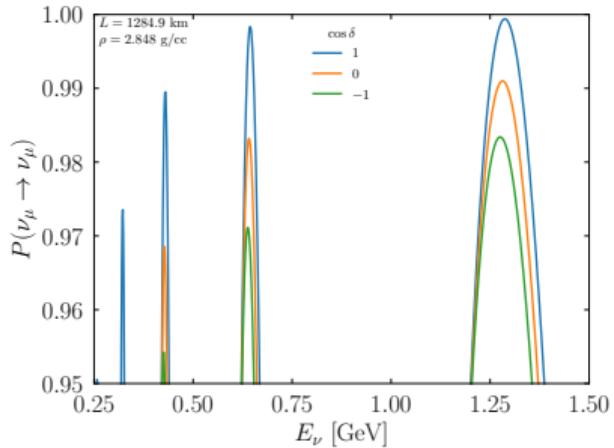
Can measure all three coeffs of each frequency \Rightarrow 2 dofs
 δ (and CPV) needs 4 dofs \Rightarrow two dis measurements

ν_e : Daya Bay and KamLAND/JUNO

ν_μ : precision at DUNE/HK

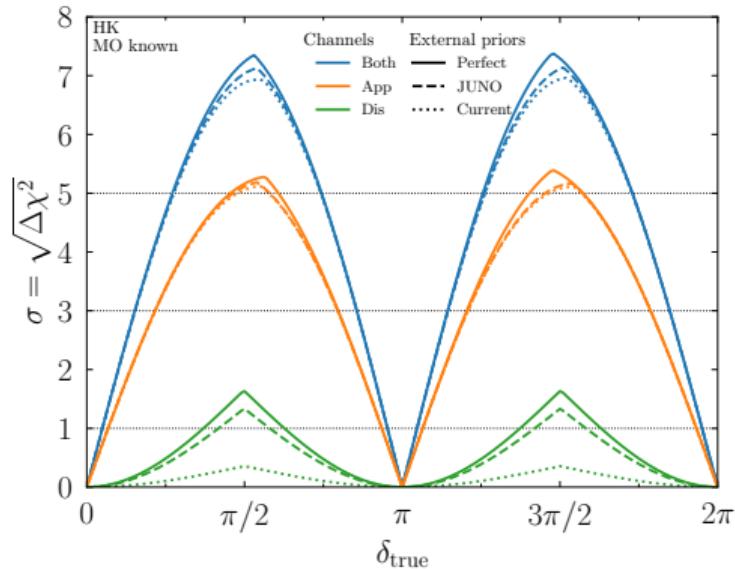
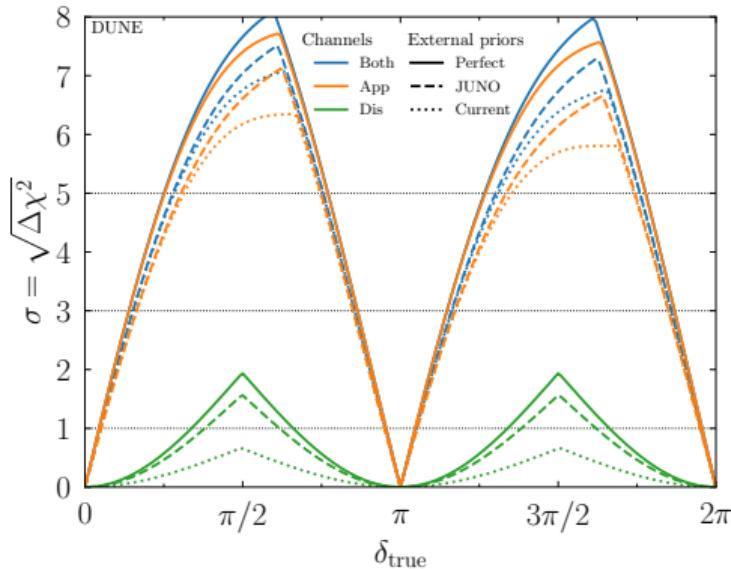
Important cross check

Different and cleaner systematics than appearance



CP violation discovery with disappearance

Need JUNO and DUNE or HK



PBD 2309.03262

New physics beyond standard three-flavor oscillations?

More new physics

Lots of neutrino anomalies and lots of ideas!

1. Gallium anomaly $\sim 5\sigma$

- ▶ No clear explanation PBD, H. Davoudiasl [2301.09651](#), V. Brdar, J. Gehrlein, J. Kopp [2303.05528](#), ...

2. ANITA and KM3NeT's curious high energy events, 3σ , 5σ , and beyond

- ▶ No clear explanation ANITA [1603.05218](#) KM3NeT Nature (2025)

3. LSND and MiniBooNE point to a ~ 1 eV sterile neutrino in appearance $\gtrsim 5\sigma$

- ▶ Tension with cosmology, ν_μ disappearance, MicroBooNE LSND [hep-ex/0104049](#)
- ▶ Many novel ideas such as heavier sterile that decays MiniBooNE [2006.16883](#)
- ▶ Still testing at MicroBooNE, ICARUS, and SBND A. Abdullahi, et al. [2308.02543](#)

4. IceCube's ν_e/ν_μ ratio is energy dependent $\sim 3\sigma$

- ▶ Sources can only do so much, maybe neutrino decay? PBD, I. Tamborra [1805.05950](#)
- ▶ A. Abdullahi, PBD [2005.07200](#)

5. NOvA and T2K seem to disagree on CPV $\sim 2\sigma$

- ▶ Could be vector NSI PBD, J. Gehrlein, R. Pestes [2008.01110](#), ...

6. Solar upturn? $\sim 2\sigma$

- ▶ Could be vector NSI J. Liao, D. Marfatia, K. Whisnant [1704.04711](#), ...
- ▶ Latest SuperK data indicates there may not be a problem Neutrinos from Home: December 2-5, 2025 45/46

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A. Abdullahi, et al. [2308.02543](#)
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More new physics

Lots of neutrino anomalies and lots of ideas!

1. Gallium anomaly $\sim 5\sigma$ BEST [2109.11482](#)
 - ▶ No clear explanation [PBD](#), H. Davoudiasl [2301.09651](#), V. Brdar, J. Gehrlein, J. Kopp [2303.05528](#), ...
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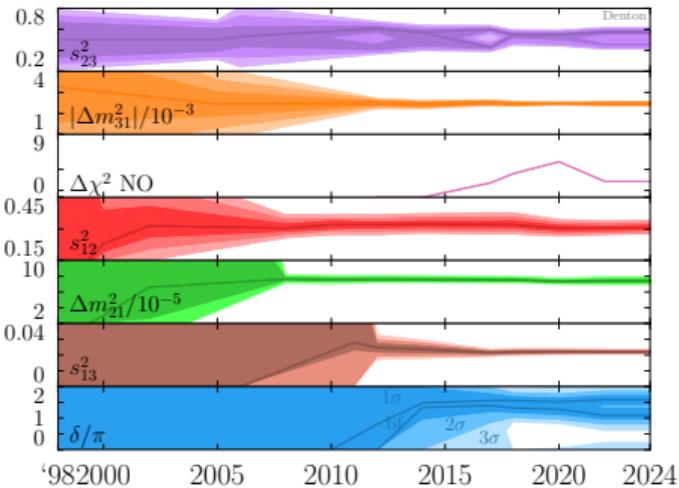
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Neutrino oscillation summary

- ▶ Four known unknowns in particle physics: all neutrinos
- ▶ Mass ordering will be measured (robustness?)
- ▶ θ_{23} octant is important for flavor models
- ▶ Multiple ways to determine CP violation: key cross check given systematics/BSM
- ▶ Rich new physics searches phenomenology!

Backups

References



SK [hep-ex/9807003](#)

M. Gonzalez-Garcia, et al. [hep-ph/0009350](#)

M. Maltoni, et al. [hep-ph/0207227](#)

SK [hep-ex/0501064](#)

SK [hep-ex/0604011](#)

T. Schwetz, M. Tortola, J. Valle [0808.2016](#)

M. Gonzalez-Garcia, M. Maltoni, J. Salvado [1001.4524](#)

T2K [1106.2822](#)

D. Forero, M. Tortola, J. Valle [1205.4018](#)

D. Forero, M. Tortola, J. Valle [1405.7540](#)

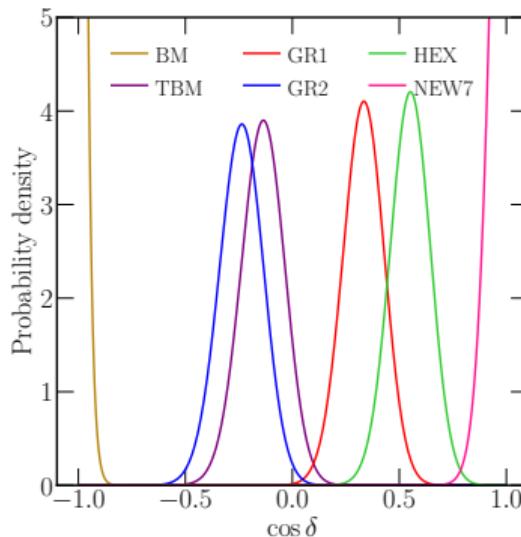
P. de Salas, et al. [1708.01186](#)

F. Capozzi et al. [2003.08511](#)

The importance of $\cos \delta$

- ▶ If only $\sin \delta$ is measured \Rightarrow sign degeneracy: $\cos \delta = \pm \sqrt{1 - \sin^2 \delta}$
- ▶ Most flavor models predict $\cos \delta$

J. Gehrlein, et al. [2203.06219](#)



L. Everett, et al. [1912.10139](#)

Neutrinos from Home: December 2-5, 2025 49/46

δ : what is it not?

$\delta \not\Rightarrow$ Baryogenesis

The amount of leptogenesis is a function of:

1. δ
2. the heavy mass scale
3. α, β (Majorana phases)
4. CP phases in the RH neutrinos
5. ...

C. Hagedorn, et al. [1711.02866](#)

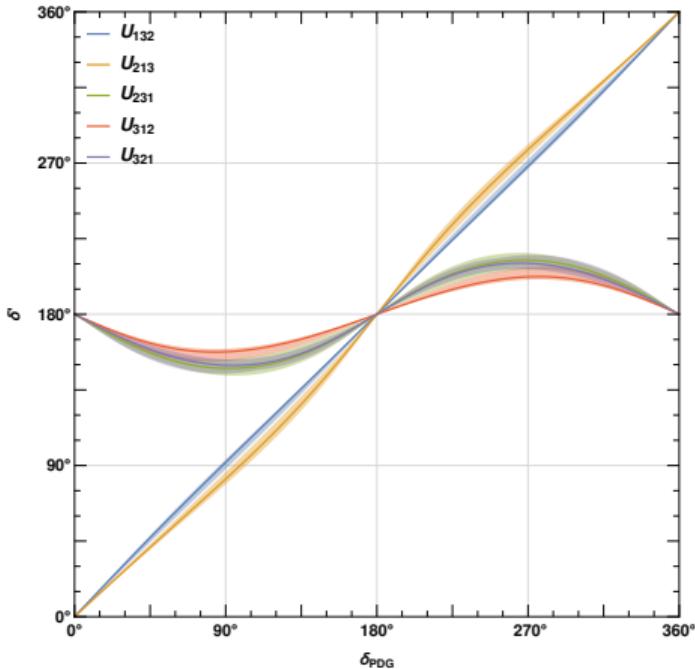
K. Moffat, et al. [1809.08251](#)

| | | |
|--------------------------------|-------------------|-----------------|
| Measuring $\delta = 0, \pi$ | $\not\Rightarrow$ | no leptogenesis |
| Measuring $\delta \neq 0, \pi$ | $\not\Rightarrow$ | leptogenesis |

Complex phase in different parameterizations

- ▶ Can relate the complex phase in one parameterization to that in another
- ▶ U_{132} and U_{213} similar to U_{123}
- ▶ δ constrained to $\sim [150^\circ, 210^\circ]$ in U_{231} , U_{312} , U_{321}
- ▶ Bands indicate 3σ uncertainty on θ_{12} , θ_{13} , θ_{23}
- ▶ “50% of possible values of δ ”
⇒ parameterization dependent

DUNE TDR II [2002.03005](#)



Quark mixing

From the PDG, V_{CKM} in the V_{123} parameterization is

$$\theta_{12} = 13.09^\circ \quad \theta_{13} = 0.2068^\circ \quad \theta_{23} = 2.323^\circ \quad \delta_{\text{PDG}} = 68.53^\circ$$

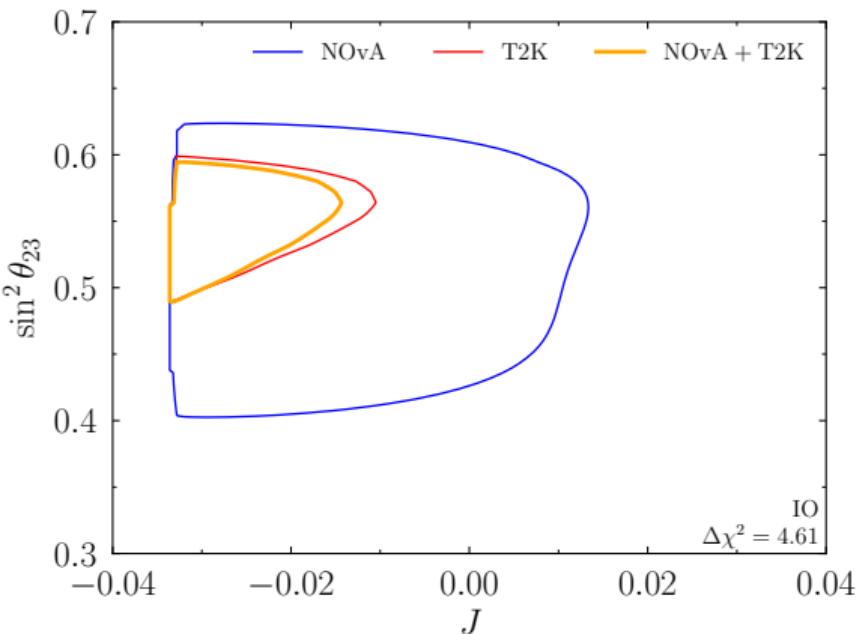
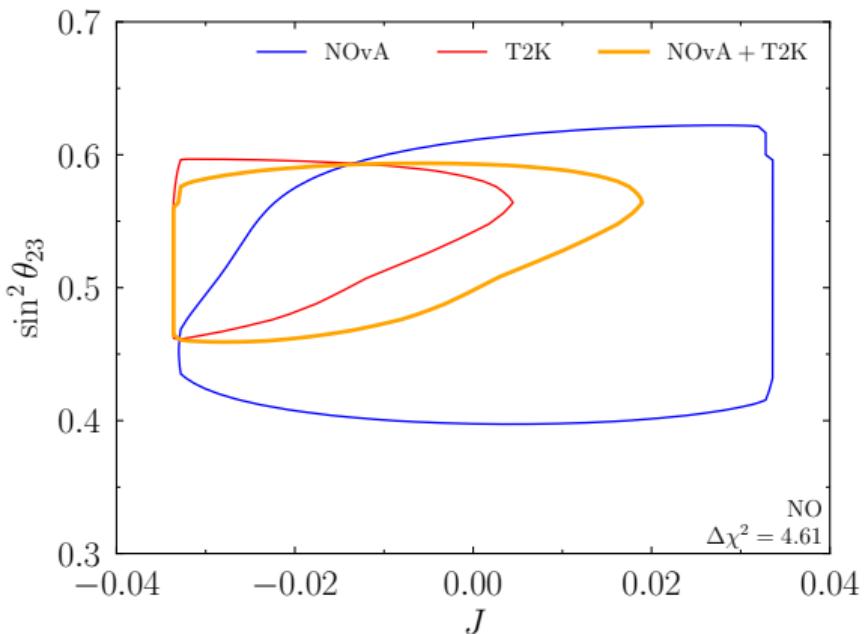
Looks like “large” CPV:

$$\sin \delta_{\text{PDG}} = 0.93 \sim 1$$

yet $J_{\text{CKM}}/J_{\text{max}} = 3 \times 10^{-4}$.

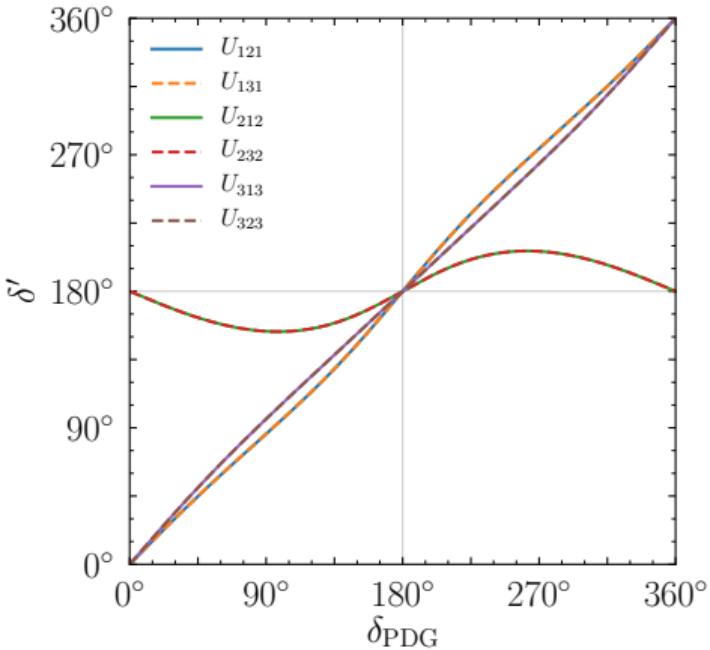
Switch to V_{212} parameterization, $\Rightarrow \delta' = 1^\circ$ and $\sin \delta' = 0.02$.

Standard oscillation parameters



Can see that the combination doesn't like the NO while it does like the IO
IO preferred over NO at $\Delta\chi^2 = 2.3$

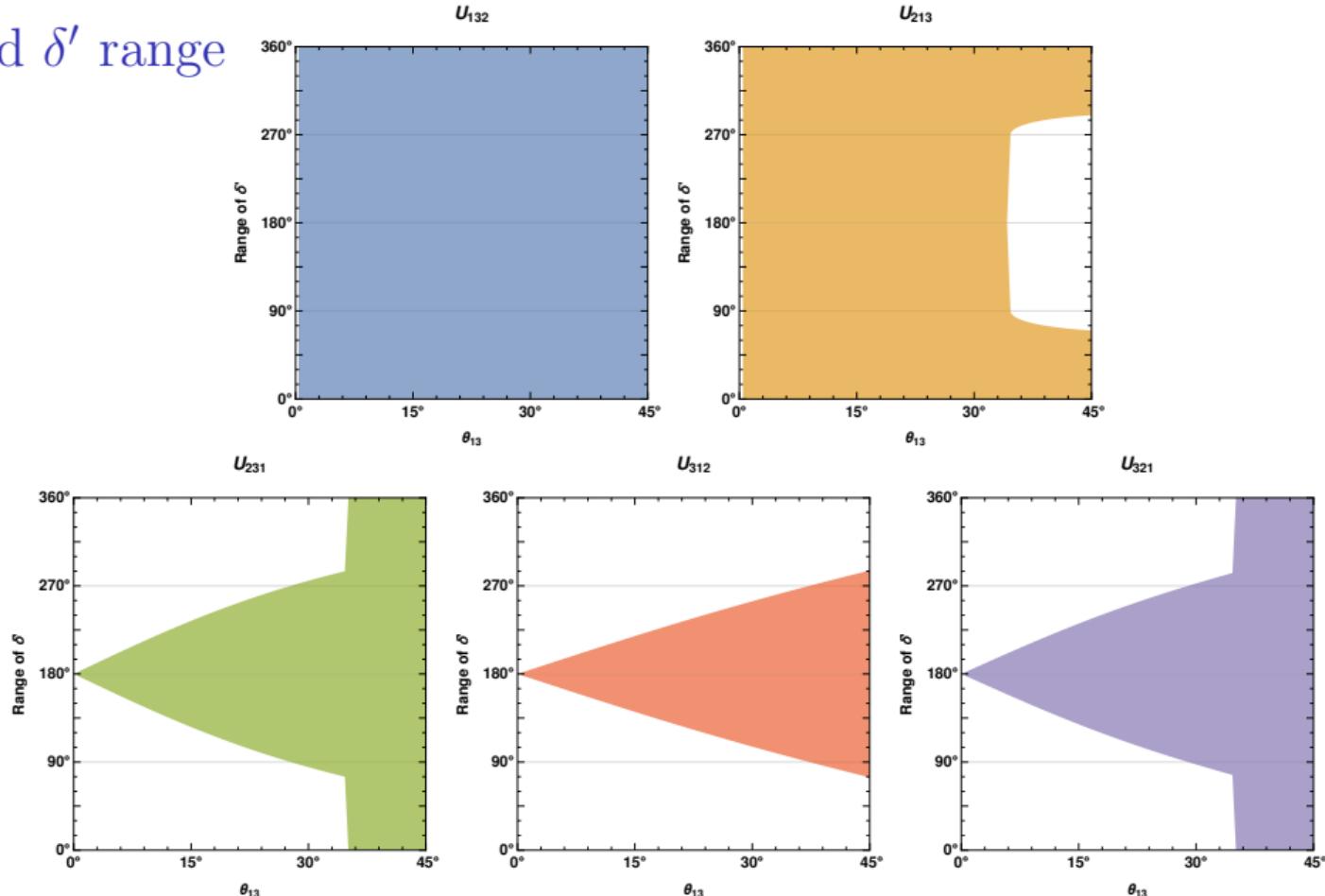
Repeated rotations



| | U_{121} | U_{131} | U_{212} | U_{232} | U_{313} | U_{323} |
|---------------|-----------|-----------|-----------|-----------|-----------|-----------|
| $ U_{e2} $ | ✓ | ✓ | ✓ | ✓ | ✗ | ✗ |
| $ U_{e3} $ | ✓ | ✓ | ✗ | ✗ | ✓ | ✓ |
| $ U_{\mu 3} $ | ✗ | ✗ | ✓ | ✓ | ✓ | ✓ |

Note that $e^{i\delta}$ must be on first or third rotation

Allowed δ' range



Many interesting new physics scenarios in oscillations

1. Sterile neutrinos
2. Non-standard neutrino interactions (NSI)
with any Lorentz structure: SPVAT
3. Non-standard neutrino self interactions
4. Neutrino decay
with visible or invisible final states
5. Unitarity violation
6. Many others: neutrino – dark matter interactions, environmental decoherence, and Lorentz invariance or CPT violation

Many interesting new physics scenarios in oscillations

1. Sterile neutrinos

PBD, Y. Farzan, I. Shoemaker [1811.01310](#)
PBD [2111.05793](#)
H. Davoudiasl, PBD [2301.09651](#)

2. Non-standard neutrino interactions (NSI)

with any Lorentz structure: SPVAT

PBD, Y. Farzan, I. Shoemaker [1804.03660](#)
P. Coloma, PBD, et al. [1701.04828](#)
PBD, J. Gehrlein, R. Pesters [2008.01110](#)
PBD, J. Gehrlein [2008.06062](#), [2204.09060](#)
PBD, A. Giannetti, D. Meloni [2210.00109](#), [2409.15411](#)

3. Non-standard neutrino self interactions

Barenboim, PBD, Oldengott [1903.02036](#)

4. Neutrino decay

with visible or invisible final states

PBD, I. Tamborra [1805.05950](#)
PBD, A. Abdullahi [2005.07200](#)

5. Unitarity violation

PBD [2109.14576](#)

PBD, J. Gehrlein [2109.14575](#)

6. Many others: neutrino – dark matter interactions, environmental decoherence, and Lorentz invariance or CPT violation

See e.g. PBD, J. Gehrlein, C.-F. Kong [2502.14027](#)

Shape-shifting sterile neutrinos

How to evade constraints?

Suppose:

1. Sterile neutrinos talk to dark matter
DM is ultralight boson
2. Dark matter talks to baryons

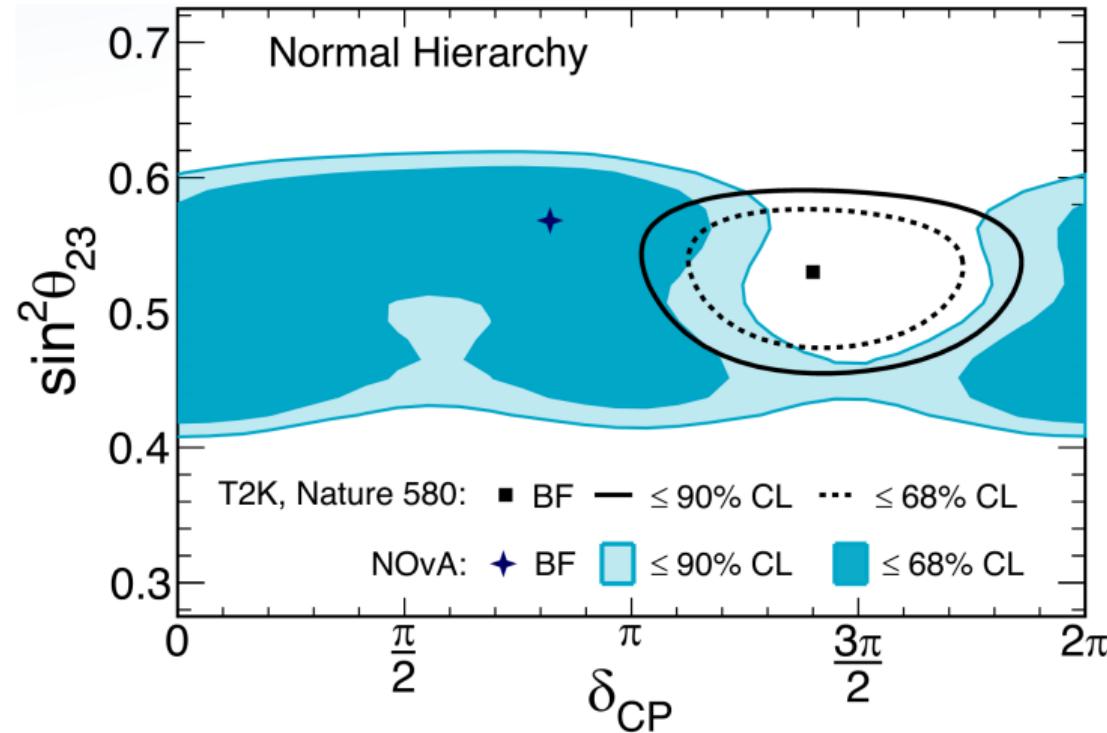
Then:

1. Sterile neutrinos aren't abundantly produced in the early universe
2. Mixing angle in the Sun is suppressed
3. Reactor constraints still exist

H. Davoudiasl,
[PBD 2301.09651](#)
[PBD 2301.11106](#)

CP violation at NOvA and T2K?

Excitement at the Neutrino conference!



A. Himmel for NOvA [10.5281/zenodo.3959581](https://zenodo.3959581)

NSI review

$$\mathcal{L}_{\text{NSI}} = -2\sqrt{2}G_F \sum_{\alpha,\beta,f,P} \epsilon_{\alpha\beta}^{f,P} (\bar{\nu}_\alpha \gamma^\mu \nu_\beta) (\bar{f} \gamma_\mu f)$$

Models with large NSIs consistent with CLFV:

Y. Farzan, I. Shoemaker [1512.09147](#) Y. Farzan, J. Heeck [1607.07616](#) D. Forero and W. Huang [1608.04719](#)
K. Babu, A. Friedland, P. Machado, I. Mocioiu [1705.01822](#) **PBD**, Y. Farzan, I. Shoemaker [1804.03660](#)
U. Dey, N. Nath, S. Sadhukhan [1804.05808](#) Y. Farzan [1912.09408](#) N. Bernal, Y. Farzan [2211.15686](#)
S. Abbaslu, Y. Farzan [2407.13834](#)

Affects oscillations via new matter effect

$$H = \frac{1}{2E} \left[UM^2 U^\dagger + a \begin{pmatrix} 1 + \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau} \end{pmatrix} \right]$$

Matter potential $a \propto G_F \rho E$

B. Dev, K. Babu, **PBD**, P. Machado, et al. [1907.00991](#)

Estimate size of effect: magnitude

$$|\epsilon_{e\beta}| \approx \frac{s_{12}c_{12}c_{23}\pi\Delta m_{21}^2}{2s_{23}w_\beta} \left| \frac{\sin \delta_{\text{T2K}} - \sin \delta_{\text{NOvA}}}{a_{\text{NOvA}} - a_{\text{T2K}}} \right| \approx \begin{cases} 0.22 & \text{for } \beta = \mu \\ 0.24 & \text{for } \beta = \tau \end{cases}$$

$a \propto \rho E$

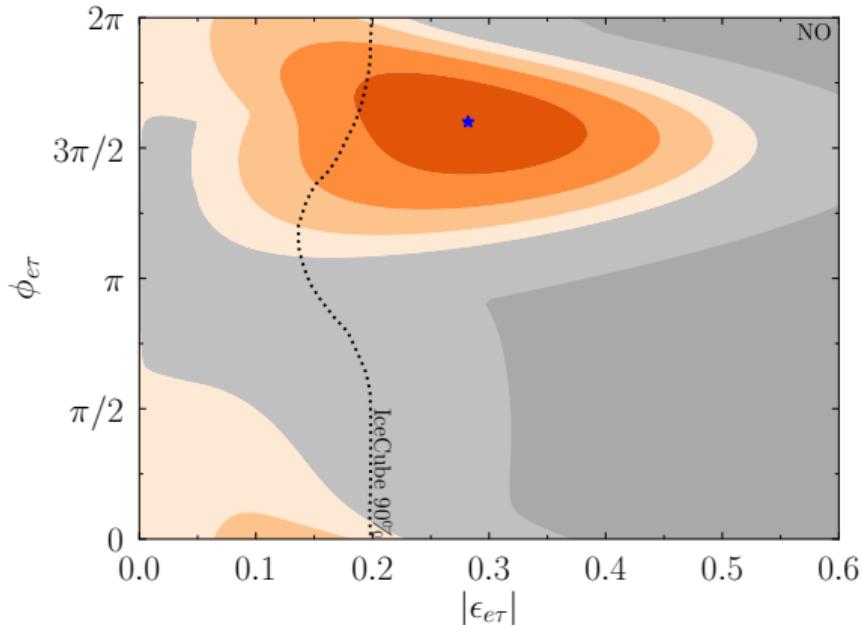
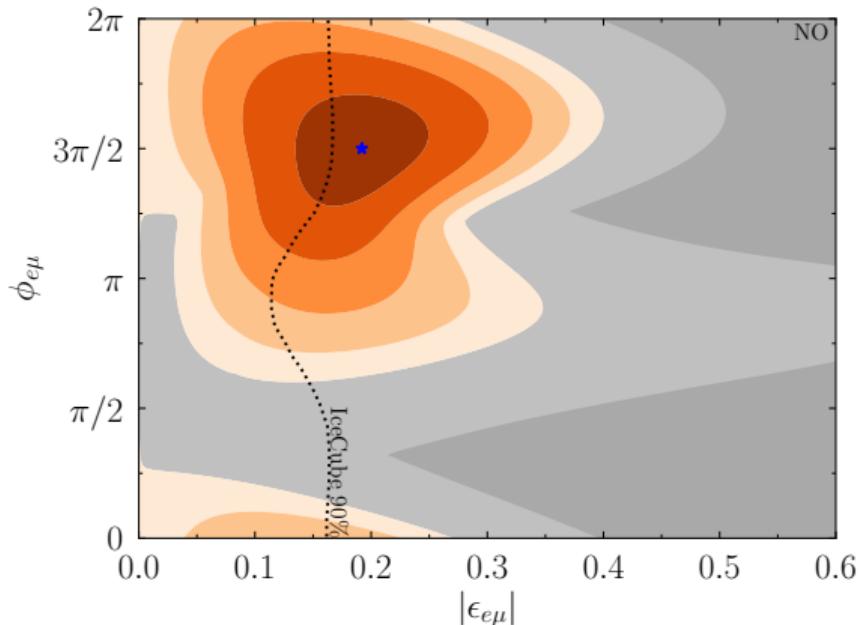
$w_\beta = s_{23}, c_{23}$ for $\beta = \mu, \tau$

Assumed upper octant $\theta_{23} > 45^\circ$

Consistency checks:

- ▶ $\sin \delta_{\text{NOvA}} = \sin \delta_{\text{T2K}} \Rightarrow |\epsilon| = 0$
- ▶ $\sin \delta_{\text{NOvA}} \neq \sin \delta_{\text{T2K}}$ and $a_{\text{NOvA}} = a_{\text{T2K}} \Rightarrow |\epsilon| \rightarrow \infty$
- ▶ Octant:
 1. LBL is governed by ν_3
 2. Upper octant $\Rightarrow \nu_3$ is more ν_μ
 3. More $\nu_\mu \Rightarrow$ need less new physics coupling to ν_μ to produce a given effect

NSI parameters



Orange is preferred over SM at integer values of $\Delta\chi^2$, dark gray is disfavored at 4.61

T. Ehrhardt, IceCube [PPNT \(2019\)](#)

$\epsilon_{\mu\tau}$, IO in backups

Other CP violating NSI constraints

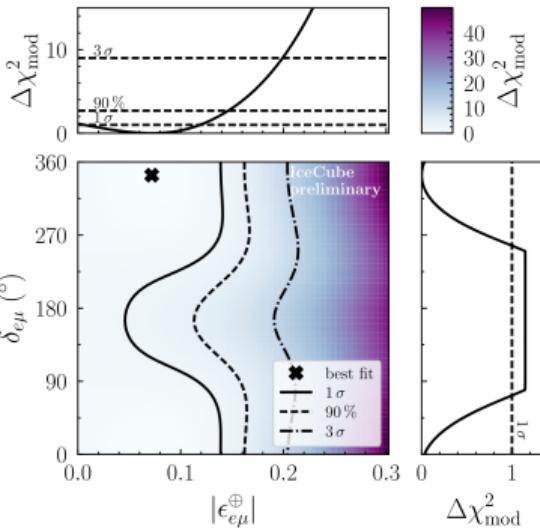
NSI effects grow with energy, density, and distance

Other CP violating NSI constraints

NSI effects grow with energy, density, and distance

Best probes:

- ▶ $\epsilon_{\mu\tau}$: atmospheric
- ▶ $\epsilon_{e\mu}, \epsilon_{e\tau}$: LBL appearance, atmospheric
- ▶ IceCube
 - ▶ Constraint is at LBL best fit with 3 yrs
10 yrs of data in the bank
 - ▶ Prefers non-zero $|\epsilon_{e\mu}|$ at $\sim 1\sigma$



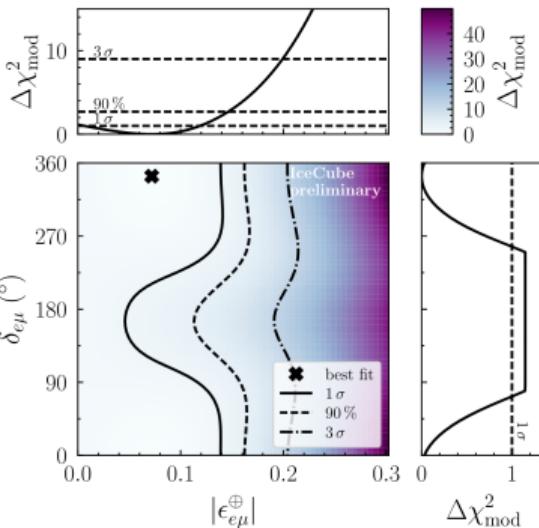
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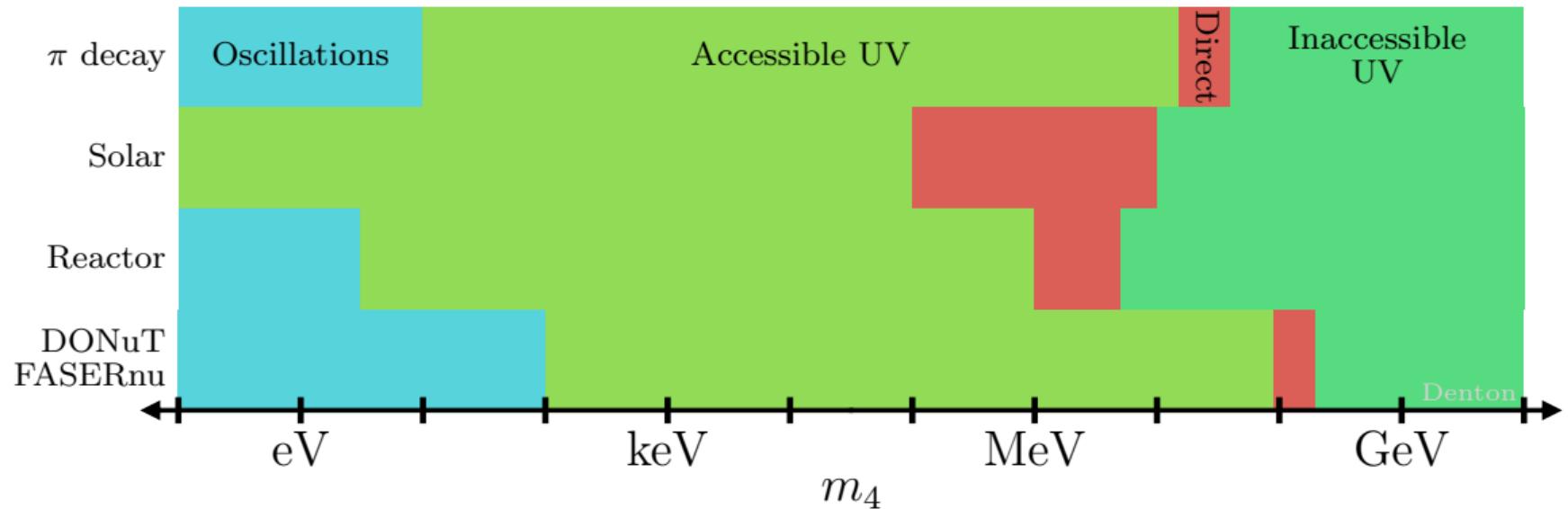
T. Ehrhardt, IceCube [PPNT \(2019\)](#)

- ▶ Super-K
 - ▶ Only consider real NSI
 - ▶ Comparable sensitivity as IceCube
- ▶ COHERENT
 - ▶ Only applies to NSI models with $M_{Z'} \gtrsim 10$ MeV
 - ▶ NSI u, d, e configuration matters
 - ▶ Comparable constraints

Super-K [1109.1889](#)

COHERENT [1708.01294](#)
PBD, Y. Farzan, I. Shoemaker [1804.03660](#)
PBD, J. Gehrlein [2008.06062](#)

Unitarity violation: a tale of two regimes



*Details depends on the specific experiment/channel

Unitarity violation: how to calculate

Kinematically **accessible** states

1. Unitary calculation of full $n \times n$ matrix
2. Oscillation averaged:

$$\sin^2 \frac{\Delta m_{41}^2 L}{4E} \rightarrow \frac{1}{2}$$

$$\sin \frac{\Delta m_{41}^2 L}{4E} \rightarrow 0$$

3. No matter effect:

$$H^{\text{mat}} = \text{diag}(V_{\text{CC}} + V_{\text{NC}}, V_{\text{NC}}, V_{\text{NC}}, 0, \dots)$$

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Kinematically **inaccessible** states

1. Nonunitary calculation of $m \times m$ matrix
 $m =$ number of kinematically accessible states
2. Rescale probability:

$$P_{\alpha\beta} = \frac{|\sum_{i=1}^{\text{acc}} U_{\alpha i}^* e^{i P_i L} U_{\beta i}|}{(\sum_{i=1}^{\text{acc}} U_{\alpha i}^* U_{\alpha i})(\sum_{i=1}^{\text{acc}} U_{\beta i}^* U_{\beta i})}$$

3. Cannot subtract multiples of $\mathbb{1}$
4. Rescale cross section/flux as appropriate
5. Rescale G_F in matter effect

Unitarity violation status from oscillations

3σ maximal deviations from unitarity

| Leptons | | |
|----------------|-------|--------|
| | Hu+ | Ellis+ |
| ν_e row | 0.003 | 0.05 |
| ν_μ row | 0.02 | 0.04 |
| ν_τ row | 0.2 | 0.82 |
| ν_1 col | 0.06 | 0.22 |
| ν_2 col | 0.09 | 0.27 |
| ν_3 col | 0.12 | 0.40 |

| Quarks | | |
|---------|---------|--------------------------|
| | u row | c row |
| d col | 0.0015 | $\sim 2.2\sigma$ tension |
| s col | - | 0.06 |
| b col | 0.005 | - |

Lepton constraints don't include anomalies

Care is required

S. Ellis, K. Kelly, S. Li [2008.01088](#)

Z. Hu, et al. [2008.09730](#)

S. Parke, M. Ross-Lonergan [1508.05095](#)

PDG

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| b col | 0.005 | - |

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Z. Hu, et al. [2008.09730](#)

S. Parke, M. Ross-Lonergan [1508.05095](#)

PDG

Vastly different mixing angle hierarchy

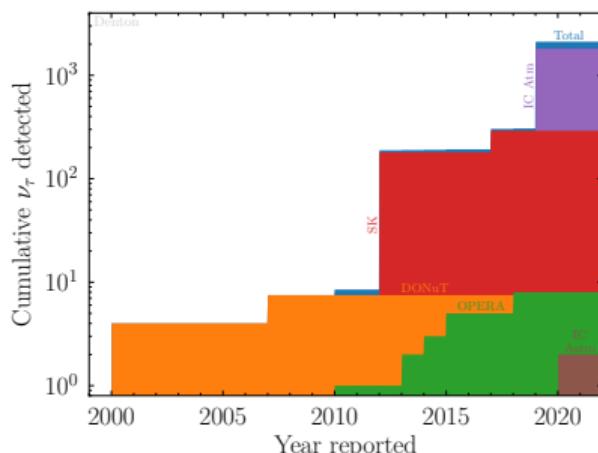


Like comparing apples and steak

Unitarity violation: tau row

Leptons: tau row is the weakest

1. Existing global analyses use OPERA and SNO
2. More data from atmospheric ν_τ appearance!



Also astrophysical ν_τ appearance; weak but distinct!

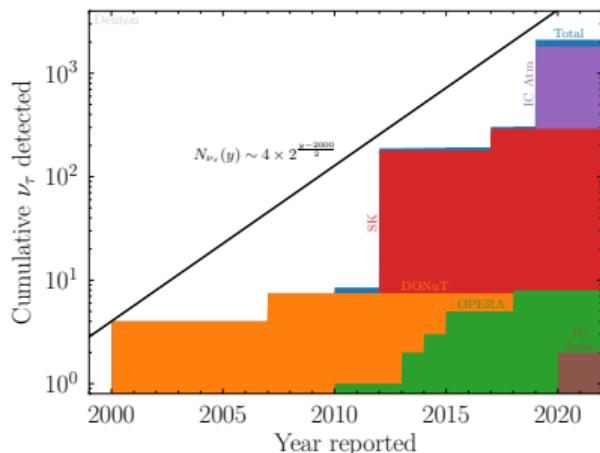
Atmospheric works because τ is in **direct** region

PBD, et al. [2203.05591](#)

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Atmospheric works because τ is in **direct** region

Tau neutrino data set doubles every two years!

PBD, et al. [2203.05591](#)

Unitarity violation

Consistency of the three-flavor oscillation picture?

and/or

Searches for unitarity violation?

Unitarity violation

Consistency of the three-flavor oscillation picture?

and/or

Searches for unitarity violation?

Not the same!

Lots of models to test standard three-flavor picture:
Sterile, unitarity violation, NSI, neutrino decay, decoherence, ...

Unitarity violation: what is it?

Our 3×3 matrix isn't unitary:

$$U_3 U_3^\dagger \neq \mathbb{1}$$

Addition of new flavor states $\nu_a, \nu_b, \nu_c, \dots$ and new mass states ν_4, ν_5, ν_6

$$U \rightarrow \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & \textcolor{red}{U_{e4}} & \cdots \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & \textcolor{red}{U_{\mu 4}} & \cdots \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & \textcolor{red}{U_{\tau 4}} & \cdots \\ \textcolor{red}{U_{a1}} & \textcolor{red}{U_{a2}} & \textcolor{red}{U_{a3}} & U_{a4} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Unitarity Violation \Rightarrow

New mass states not directly accessible by oscillations or decay

Thus check if U_3 is what it should be

Unitarity constraints

Unitary violation: the study of how $U_{3 \times 3}$ is not unitary independent of m_4, m_5, \dots
Constraints vary considerably in the literature:

$$1 - |U_{e1}|^2 - |U_{e2}|^2 - |U_{e3}|^2 < \begin{cases} 0.05 & \text{at } 2\sigma \\ 0.001 & \end{cases}$$

S. Parke, M. Ross-Lonergan [1508.05095](#)

Z. Hu, et al. [2008.09730](#)

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$$1 - |U_{e1}|^2 - |U_{e2}|^2 - |U_{e3}|^2 < \begin{cases} 0.05 & \text{at } 2\sigma \\ 0.001 & \end{cases}$$

All analyses *assume* unitarity

Throw out LSND, MiniBooNE, RAA, gallium, etc.

S. Parke, M. Ross-Lonergan [1508.05095](#)

Z. Hu, et al. [2008.09730](#)

Unitarity violation

- ▶ Could conceivably differentiate: 2 new states from 1, but not 3+ from 2
- ▶ Zero distance effect \Rightarrow near detector **with flux prediction**
 - E.g. RAA, Gallium
- ▶ Numerous parameterizations: α matrix, η matrix, submatrix & Cauchy-Schwartz
 - All apply to the inaccessible cases only
- ▶ There is an approximate correspondence to sterile and NSI

$$\alpha_{ee} \approx \frac{1}{2}(s_{14}^2 + s_{15}^2 + s_{16}^2) \approx -\epsilon_{ee}, \quad \dots$$

M. Blennow, et al. [1609.08637](#)

Applies one experiment at a time

- ▶ Additional EW precision information: W, Z, π , μ , τ decays

Care is required

S. Antusch, et al. [hep-ph/0607020](#)

S. Antusch, O. Fischer [1407.6607](#)

Unitarity violation: mass ranges for tau neutrinos

| experiment | (4,4) (m_4) | (5,3) (m_4) |
|-------------------------------------|--|---------------------------|
| atmospheric ν_μ disappearance | $\in [10 \text{ eV}, 15 \text{ MeV}]$ | $\gtrsim 40 \text{ MeV}$ |
| atmospheric ν_τ appearance | $\in [10 \text{ eV}, 15 \text{ MeV}]$ | $\gtrsim 40 \text{ MeV}$ |
| astrophysical ν_τ appearance | $\lesssim 15 \text{ MeV}$ | $\gtrsim 40 \text{ MeV}$ |
| solar ${}^8\text{B}$ | $\lesssim 5 \text{ MeV}$ | $\gtrsim 20 \text{ MeV}$ |
| DONuT/FASERnu | $\in [100 \text{ eV}, 90 \text{ MeV}]$ | $\gtrsim 200 \text{ MeV}$ |
| LBL ν_τ appearance (OPERA) | $\in [1 \text{ eV}, 15 \text{ MeV}]$ | $\gtrsim 40 \text{ MeV}$ |
| LBL ν_τ appearance (DUNE) | $\in [0.1 \text{ eV}, 15 \text{ MeV}]$ | $\gtrsim 40 \text{ MeV}$ |
| LBL ν_μ disappearance (DUNE) | $\in [0.1 \text{ eV}, 15 \text{ MeV}]$ | $\gtrsim 40 \text{ MeV}$ |
| CEvNS | $\in [10 \text{ eV}, 15 \text{ MeV}]$ | $\gtrsim 40 \text{ MeV}$ |

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