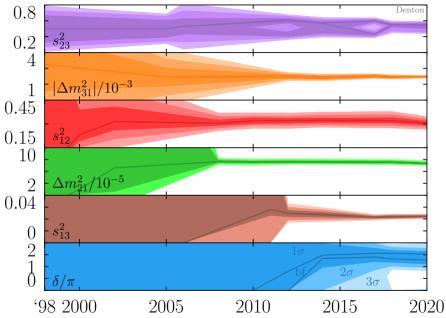
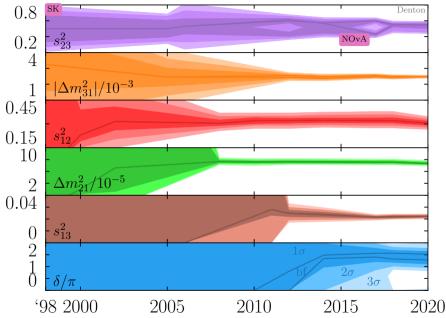
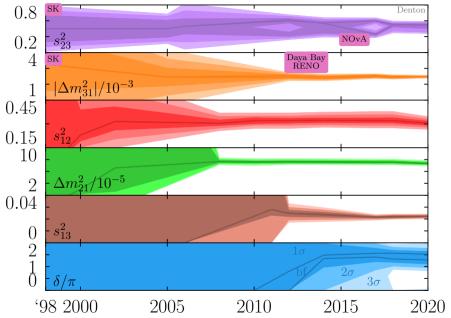
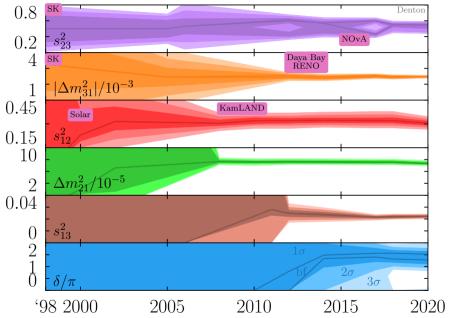
Flavor mixing, CP violation, and Unitarity

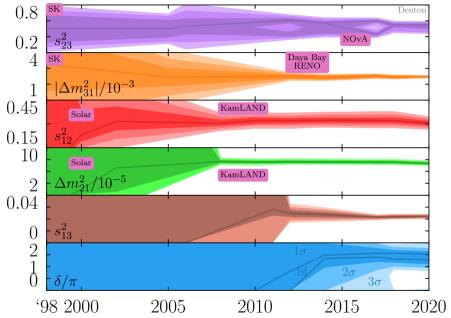


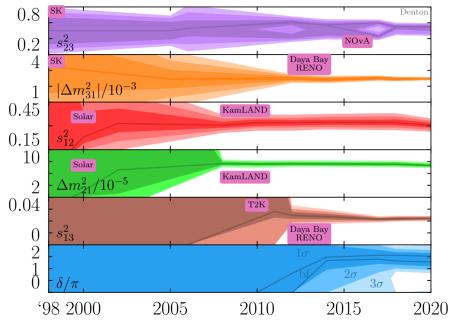


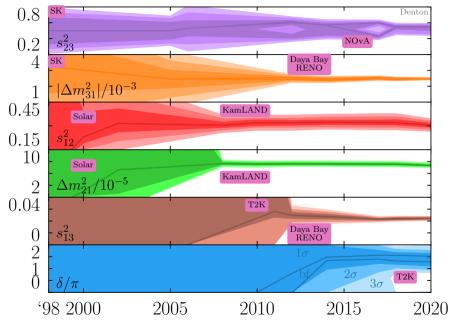












Four known unknown in particle physics: all neutrinos

Atmospheric mass ordering

 θ_{23} octant

Complex phase

Absolute mass scale

Four known unknown in particle physics: all neutrinos

Atmospheric mass ordering

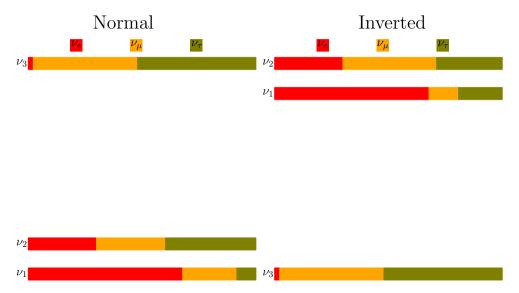
 θ_{23} octant

Complex phase

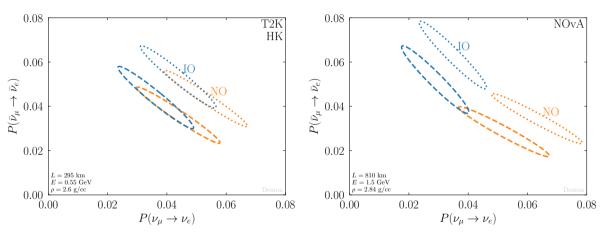
Absolute mass scale

Atmospheric mass ordering

Mass ordering: what is it?



Mass ordering: what is it really?



Mass ordering current status: oscillations

- 1. NOvA and T2K both prefer NO over IO
- 2. NOvA+T2K prefers IO over NO
- 3. SK still prefers NO over IO
- 4. NOvA+T2K+SK still prefers NO over IO
- 5. + Daya Bay & RENO \Rightarrow slight preference NO
- 6. $= 2.5 2.7\sigma$

K. Kelly, et al. 2007.08526

PBD, J. Gehrlein, R. Pestes 2008.01110

I. Esteban, et al. 2007.14792

F. Capozzi, et al. 2107.00532

P. de Salas, et al. 2006.11237

Mass ordering current status: all

Cosmology: $m_1 + m_2 + m_3 < 90 \text{ meV}$ at 95% CL

E. Valentino, S. Gariazzo, O. Mena 2106.15267

 \rightarrow 20 meV precision with DESI, EUCLID, ...

From oscillations:

Normal: $m_1 + m_2 + m_3 > 60 \text{ meV}$ Inverted: $m_1 + m_2 + m_3 > 100 \text{ meV}$

See also KATRIN 2105.08533

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See also KATRIN 2105,08533

PRIORS?

Some claim "decisive" Bayesian evidence for normal

R. Jimenez, et al. 2203.14247

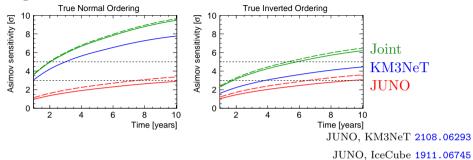
More general prior assumptions \Rightarrow no significant information from cosmology

S. Gariazzo, et al. 1801.04946

S. Gariazzo, et al. 2205.02195

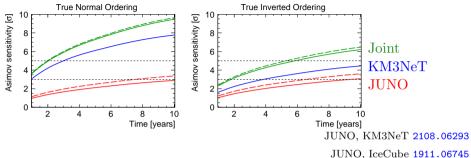
8/34

Mass ordering: future sensitivities



Note: if lower octant, KM3NeT is less sensitive

Mass ordering: future sensitivities



Note: if lower octant, KM3NeT is less sensitive

$$\Delta m_{ee}^2 = c_{12}^2 \Delta m_{31}^2 + s_{12}^2 \Delta m_{32}^2$$

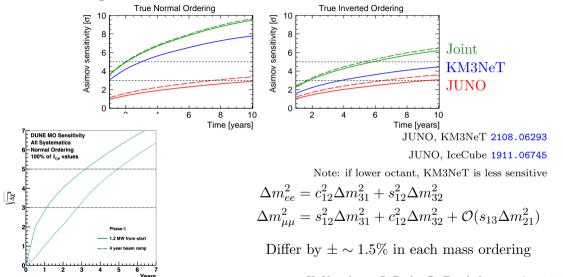
$$\Delta m_{\mu\mu}^2 = s_{12}^2 \Delta m_{31}^2 + c_{12}^2 \Delta m_{32}^2 + \mathcal{O}(s_{13} \Delta m_{21}^2)$$

Differ by $\pm \sim 1.5\%$ in each mass ordering

H. Nunokawa, S. Parke, R. Funchal hep-ph/0503283

Mass ordering: future sensitivities

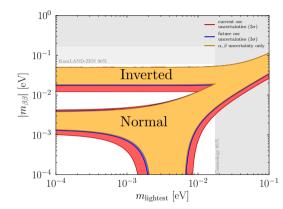
Matter effect \Rightarrow DUNE 2203.06100



H. Nunokawa, S. Parke, R. Funchal hep-ph/0503283

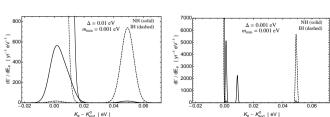
Mass ordering: broad implications

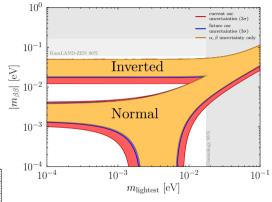
- ► Affects cosmology
- ightharpoonup Affects $0\nu\beta\beta$
- ► Affects end point measurements
- \triangleright Affects $C\nu B$



Mass ordering: broad implications

- ► Affects cosmology
- \triangleright Affects $0\nu\beta\beta$
- ► Affects end point measurements
- \triangleright Affects $C\nu B$





A. Long, C. Lunardini, E. Sabancilar 1405.7654

Mass ordering: new physics degeneracies

In the presence of new physics such as NSI¹ we have:

$$[NO] + [\epsilon = 0] \equiv [IO] + [\epsilon_{ee} = -2]$$

$$[IO] + [\epsilon = 0] \equiv [NO] + [\epsilon_{ee} = -2]$$

Equivalences hold even if all oscillation probabilities are perfectly measured

P. Bakhti, Y. Farzan 1403.0744

P. Coloma, T. Schwetz 1604.05772

PBD, S. Parke 2106.12436

PBD, J. Gehrlein 2204.09060





¹See Yasaman Farzan's talk on Friday, June 3!

Is the mass ordering robust?

Need **scattering** to break



Can probe same NC $\epsilon = -2$ process in scattering, but...

CHARM and NuTeV for $M_{Z'} \gtrsim 10 \text{ GeV}$

PBD, et al. 1701.04828

COHERENT for $M_{Z'} \gtrsim 50$ MeV and cosmology for $M_{Z'} \lesssim 5$ MeV

PBD, Y. Farzan, I. Shoemaker 1804.03660

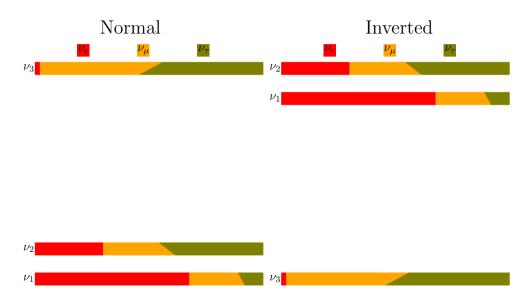
Dresden-II for any mediator mass

PBD, J. Gehrlein 2204.09060

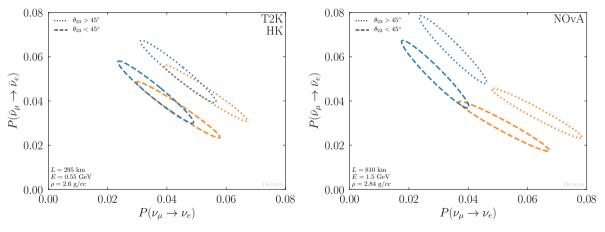
Can still evade with $\epsilon_{\mu\mu} = \epsilon_{\tau\tau} = 2$ or certain u / d combinations

θ_{23} octant

θ_{23} octant: what is it?

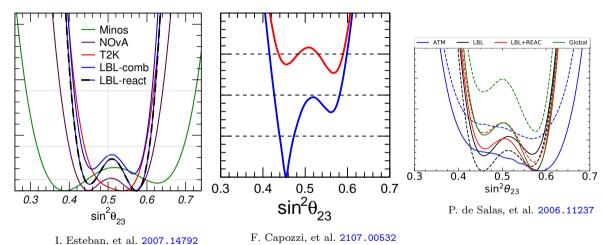


θ_{23} octant: what is it really?



Lower octant more "normal" than upper octant

θ_{23} octant: current status

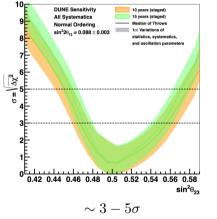


Prefers **upper** at $< 1\sigma$

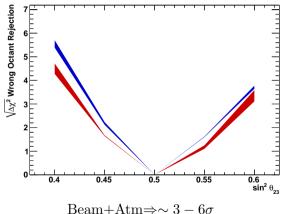
Prefers lower at $\sim 1.5\sigma$

Prefers **upper** at $> 2\sigma$

θ_{23} octant: future sensitivities



DUNE 2002.03005



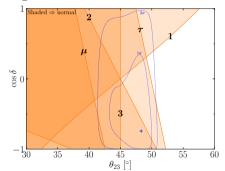
Beam+Atm $\Rightarrow \sim 3 - 6\sigma$

HK 1805.04163

θ_{23} : broader implications

Normalcy

Is the heaviest neutrino mostly ν_{τ} ? Is the lightest neutrino least ν_{τ} ?



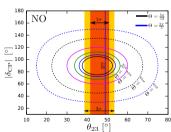
Quarks easily satisfy normalcy PBD 2003.04319

 $\mu\text{-}\tau$ interchange/reflection symmetry

$$M_{\nu}^{*} = X M_{\nu} X^{T} \qquad X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

 $M_{\nu} \equiv U D_{\nu} U^{\dagger}$

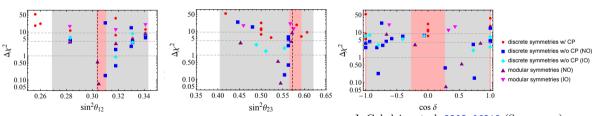
Predicts: $\theta_{23} = 45^{\circ}$, often $\theta_{13} = 0$



P. Chen, et al. 1512.01551

Parameter interplay

Models predict specific correlations among the parameters



Complex phase

δ and CP violation

 $J_{CP} = s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23}\sin\delta$

C. Jarlskog PRL 55, 1039 (1985)



δ and CP violation

$$J_{CP} = s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23}\sin\delta$$

C. Jarlskog PRL 55, 1039 (1985)



1. Strong interaction: no observed EDM \Rightarrow CP (nearly) conserved

$$\frac{\bar{\theta}}{2\pi} < 10^{-11}$$

J. Pendlebury, et al. 1509.04411

2. Quark mass matrix: non-zero but small CP violation

$$\frac{|J_{\rm CKM}|}{J_{\rm max}} = 3 \times 10^{-4}$$

CKMfitter **1501.05013**

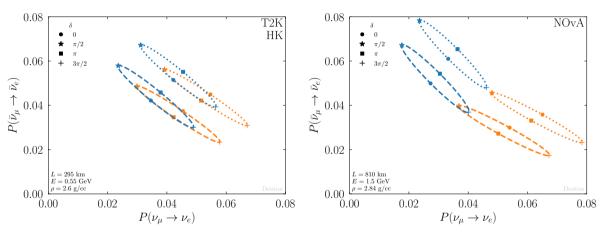
3. Lepton mass matrix: ?

$$\frac{|J_{\rm PMNS}|}{J_{\rm max}} < 0.34$$

PBD, J. Gehrlein, R. Pestes 2008.01110

 $J_{\rm max} = \frac{1}{6\sqrt{3}} \approx 0.096$

δ : what is it really?



 δ : what is it not?

$\delta \not\Rightarrow$ Baryogenesis

The amount of leptogenesis is a function of:

- 1. the heavy mass scale
- **2**. δ
- 3. α , β (Majorana phases)
- 4. CP phases in the RH neutrinos
- 5. ...

C. Hagedorn, et al. 1711.02866

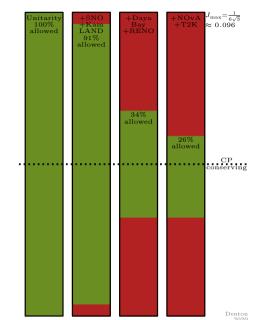
K. Moffat, et al. 1809.08251

$$\begin{array}{lll} \text{Measuring } \delta = 0, \pi & \not \Rightarrow & \text{no leptogenesis} \\ \text{Measuring } \delta \neq 0, \pi & \not \Rightarrow & \text{leptogenesis} \end{array}$$

δ , J: current status

Maximal CP violation is already ruled out:

- 1. $\theta_{12} \neq 45^{\circ}$ at $\sim 15\sigma$
- 2. $\theta_{13} \neq \tan^{-1} \frac{1}{\sqrt{2}} \approx 35^{\circ} \text{ at many (100) } \sigma$
- 3. $\theta_{23} = 45^{\circ}$ allowed at $\sim 1\sigma$
- 4. $|\sin \delta| = 1$ allowed



When δ and when J?

If the goal is **CP violation** the Jarlskog invariant should be used

however

If the goal is **measuring the parameters** one must use δ

Given θ_{12} , θ_{13} , θ_{23} , and J, I can't determine the sign of $\cos \delta$ which is physical e.g. $P(\nu_{\mu} \to \nu_{\mu})$ depends on $\cos \delta$ a tiny bit

- ▶ T2K/HK are mostly sensitivity to $\sin \delta$; they should focus on J
- T2K does this now!
- ▶ NOvA/DUNE has modest $\cos \delta$ sensitivity; both J and δ should be reported

Unitarity violation

Consistency of the three-flavor oscillation picture? and/or

Searches for unitarity violation?

Unitarity violation

Consistency of the three-flavor oscillation picture?

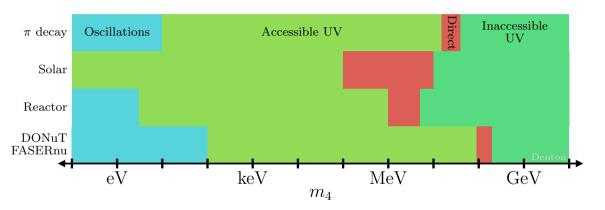
and/or

Searches for unitarity violation?

Not the same!

Lots of models to test standard three-flavor picture: Sterile, unitarity violation, NSI, neutrino decay, decoherence, . . .

Unitarity violation: a tale of two regimes



*Details depends on the specific experiment/channel

Unitarity violation: what is it?

Our 3×3 matrix isn't unitary:

$$U_3U_3^{\dagger} \neq 1$$

Addition of new flavor states $\nu_a, \nu_b, \nu_c, \dots$ and new mass states ν_4, ν_5, ν_6

$$U \to \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & \frac{U_{e4}}{U_{e4}} & \cdots \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} & \cdots \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} & \cdots \\ \frac{U_{a1}}{U_{a1}} & \frac{U_{a2}}{U_{a3}} & \frac{U_{a4}}{U_{a4}} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Unitarity Violation \Rightarrow

New mass states not directly accessible by oscillations or decay. Thus check if U_3 is what it should be

Unitarity violation: how to calculate

Kinematically accessible states

- 1. Unitary calculation of full $n \times n$ matrix
- 2. Oscillation averaged:

$$\sin^2 \frac{\Delta m_{41}^2 L}{4E} \to \frac{1}{2}$$
$$\sin \frac{\Delta m_{41}^2 L}{4E} \to 0$$

3. No matter effect:

$$H^{\text{mat}} = \text{diag}(V_{\text{CC}} + V_{\text{NC}}, V_{\text{NC}}, V_{\text{NC}}, 0, \dots)$$

Unitarity violation: how to calculate

Kinematically accessible states

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$$H^{\mathrm{mat}} = \mathrm{diag}(V_{\mathrm{CC}} + V_{\mathrm{NC}}, V_{\mathrm{NC}}, V_{\mathrm{NC}}, 0, \dots)$$

Kinematically **inaccessible** states

- 1. Nonunitary calculation of $m \times m$ matrix m = number of kinematically accessible states
- 2. Rescale probability:

$$P_{\alpha\beta} = \frac{|\sum_{i=1}^{\mathrm{acc}} U_{\alpha i}^* e^{iP_i L} U_{\beta i}|}{(\sum_{i=1}^{\mathrm{acc}} U_{\alpha i}^* U_{\alpha i})(\sum_{i=1}^{\mathrm{acc}} U_{\beta i}^* U_{\beta i})}$$

- 3. Cannot subtract multiples of 1
- 4. Rescale cross section/flux as appropriate
- 5. Rescale G_F in matter effect

Unitarity violation

- ► Could conceivably differentiate: 2 new states from 1, but not 3+ from 2
- ► Zero distance effect ⇒ near detector with flux prediction

E.g. RAA, Gallium

Numerous parameterizations: α matrix, η matrix, submatrix & Cauchy-Schwartz

All apply to the inaccessible cases only

▶ There is an approximate correspondence to sterile and NSI

$$\alpha_{ee} \approx \frac{1}{2} (s_{14}^2 + s_{15}^2 + s_{16}^2) \approx -\epsilon_{ee} , \dots$$

. .

Applies one experiment at a time

▶ Additional EW precision information: W, Z, π , μ , τ decays

Care is required

S. Antush, et al. hep-ph/0607020

M. Blennow, et al. 1609.08637

S. Antusch, O. Fischer 1407.6607

Unitarity violation status from oscillations

3σ maximal deviations from unitarity

${f Leptons}$			
	Hu+	Ellis+	
ν_e row	0.003	0.05	
ν_{μ} row	0.02	0.04	
ν_{τ} row	0.2	0.82	
$\nu_1 \text{ col}$	0.06	0.22	
ν_2 col	0.09	0.27	
ν_3 col	0.12	0.40	

Quarks				
u row	0.0015	$\sim 3\sigma$ tension		
c row	0.06			
t row	-			
$d \operatorname{col}$	0.005			
s col	0.06			
$b \operatorname{col}$	-			

Lepton constraints don't include anomalies

Care is required

S. Ellis, K. Kelly, S. Li 2008.01088

Z. Hu, et al. 2008.09730

S. Parke, M. Ross-Lonergan 1508.05095

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' '				

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t row	-			
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s col	0.06			
$b \operatorname{col}$	-			
	'			

Vastly different mixing angle hierarchy

$$\Rightarrow$$

Like comparing apples and hairstyles

Lepton constraints don't include anomalies

Care is required

S. Ellis, K. Kelly, S. Li 2008.01088

Z. Hu, et al. 2008.09730

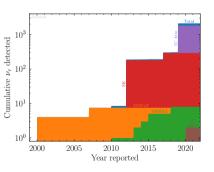
S. Parke, M. Ross-Lonergan 1508.05095

PDG

Unitarity violation: tau row

Leptons: tau row is the weakest

- 1. Existing global analyses use OPERA and SNO
- 2. More data from atmospheric ν_{τ} appearance!



PBD 2109.14576

Also astrophysical ν_{τ} appearance; weak but distinct! PBD, J. Gehrlein 2109.14575

Atmospheric works because τ is in direct region

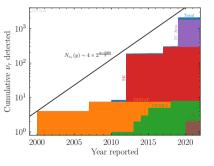
PBD, et al. 2203.05591 (whitepaper)

Peter B. Denton (BNL) 2109.14575 & 2109.14576 Neutrino 2022: June 1/2, 2022 32/34

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PBD 2109.14576

Also astrophysical ν_{τ} appearance; weak but distinct! PBD, J. Gehrlein 2109.14575

Atmospheric works because τ is in direct region

Tau neutrino data set doubles every two years!

PBD, et al. 2203.05591 (whitepaper)

Neutrino oscillation summary

- ► Four known unknowns in particle physics: all neutrinos
- ▶ Mass ordering will be measured

Robustness?

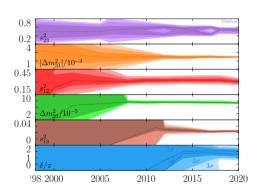
- \triangleright θ_{23} octant is important for flavor models
- \triangleright δ could shed light on CP violation
- ▶ Unitarity violation is phenomenologically very rich
- ▶ Lots of existing tau information to be utilized!

Precision is coming to neutrinos!



Backups

References



SK hep-ex/9807003

M. Gonzalez-Garcia, et al. hep-ph/0009350

M. Maltoni, et al. hep-ph/0207227

SK hep-ex/0501064

SK hep-ex/0604011

T. Schwetz, M. Tortola, J. Valle 0808.2016

M. Gonzalez-Garcia, M. Maltoni, J. Salvado 1001.4524

T2K 1106.2822

D. Forero, M. Tortola, J. Valle 1205.4018

D. Forero, M. Tortola, J. Valle 1405.7540

P. de Salas, et al. 1708.01186

F. Capozzi et al. 2003.08511

NSI review

$$\mathcal{L}_{\mathrm{NSI}} = -2\sqrt{2}G_F \sum_{\alpha,\beta,f,P} \epsilon_{\alpha\beta}^{f,P} (\bar{\nu}_{\alpha}\gamma^{\mu}\nu_{\beta})(\bar{f}\gamma_{\mu}f)$$

Models with large NSIs consistent with CLFV:

Y. Farzan, I. Shoemaker 1512.09147
 Y. Farzan, J. Heeck 1607.07616
 D. Forero and W. Huang 1608.04719
 K. Babu, A. Friedland, P. Machado, I. Mocioiu 1705.01822
 PBD, Y. Farzan, I. Shoemaker 1804.03660
 U. Dey, N. Nath, S. Sadhukhan 1804.05808
 Y. Farzan 1912.09408

Affects oscillations via new matter effect

$$H = \frac{1}{2E} \left[UM^2 U^{\dagger} + a \begin{pmatrix} 1 + \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau} \end{pmatrix} \right]$$

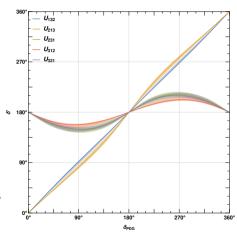
Matter potential $a \propto G_F \rho E$

B. Dev, K. Babu, PBD, P. Machado, et al. 1907.00991

Complex phase in different parameterizations

- ► Can relate the complex phase in one parameterization to that in another
- $ightharpoonup U_{132}$ and U_{213} similar to U_{123}
- δ constrained to $\sim [150^{\circ}, 210^{\circ}]$ in $U_{231}, U_{312}, U_{321}$
- ▶ Bands indicate 3σ uncertainty on θ_{12} , θ_{13} , θ_{23}
- ▶ "50% of possible values of δ "
 - ⇒ parameterization dependent

DUNE TDR II 2002.03005



Quark mixing

From the PDG, V_{CKM} in the V_{123} parameterization is

$$\theta_{12} = 13.09^{\circ}$$
 $\theta_{13} = 0.2068^{\circ}$ $\theta_{23} = 2.323^{\circ}$ $\delta_{PDG} = 68.53^{\circ}$

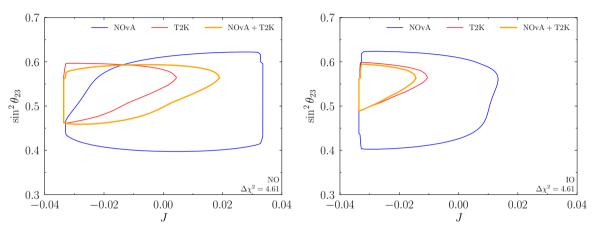
Looks like "large" CPV:

$$\sin \delta_{\rm PDG} = 0.93 \sim 1$$

yet $J_{CKM}/J_{max} = 3 \times 10^{-4}$.

Switch to V_{212} parameterization, $\Rightarrow \delta' = 1^{\circ}$ and $\sin \delta' = 0.02$.

Standard oscillation parameters



Can see that the combination doesn't like the NO while it does like the IO IO preferred over NO at $\Delta \chi^2 = 2.3$

CP violation in oscillations

In vacuum at first maximum:

$$P_{\mu e} - \bar{P}_{\mu e} \approx 8\pi J \frac{\Delta m_{21}^2}{\Delta m_{32}^2}$$

 $J \equiv s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23}\sin\delta$

C. Jarlskog PRL 55, 1039 (1985)

- \triangleright Extracting δ from data requires every other oscillation parameter
- ▶ J requires only Δm_{21}^2 (up to matter effects)

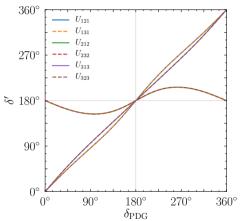
Matter effects are easily accounted for

$$\hat{J} \simeq \frac{J}{\sqrt{(c_{212} - c_{13}^2 a/\Delta m_{21}^2)^2 + s_{212}^2} \sqrt{(c_{213} - a/\Delta m_{ee}^2)^2 + s_{213}^2}}$$

PBD, S. Parke 1902.07185

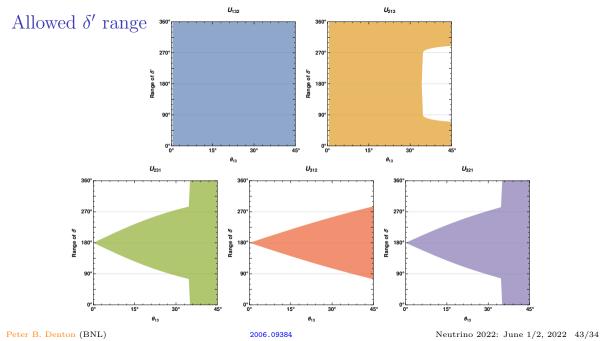
PBD, H. Minakata, S. Parke 1604.08167

Repeated rotations



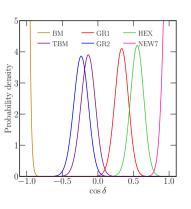
	U_{121}	U_{131}	U_{212}	U_{232}	U_{313}	U_{323}
$ U_{e2} $	1	1	1	1	X	X
$ U_{e3} $	1	1	X	X	1	1
$ U_{e2} $ $ U_{e3} $ $ U_{\mu 3} $	X	X	1	1	1	✓

Note that $e^{i\delta}$ must be on first or third rotation



The importance of $\cos \delta$

- ▶ If only $\sin \delta$ is measured \Rightarrow sign degeneracy: $\cos \delta = \pm \sqrt{1 \sin^2 \delta}$
- \blacktriangleright Most flavor models predict $\cos \delta$



J. Gehrlein, et al. 2203.06219

L. Everett, et al. 1912.10139

Unitarity violation: mass ranges for tau neutrinos

experiment	$(4,4) (m_4)$	$(5,3) (m_4)$
atmospheric ν_{μ} disappearance	$\in [10 \text{ eV}, 15 \text{ MeV}]$	$\gtrsim 40~{ m MeV}$
atmospheric ν_{τ} appearance	$\in [10 \text{ eV}, 15 \text{ MeV}]$	$\gtrsim 40~{ m MeV}$
astrophysical ν_{τ} appearance	$\lesssim 15~{ m MeV}$	$\gtrsim 40~{ m MeV}$
solar ⁸ B	$\lesssim 5~{ m MeV}$	$\gtrsim 20~{ m MeV}$
${ m DONuT/FASERnu}$	$\in [100 \text{ eV}, 90 \text{ MeV}]$	$\gtrsim 200~{ m MeV}$
LBL ν_{τ} appearance (OPERA)	$\in [1 \text{ eV}, 15 \text{ MeV}]$	$\gtrsim 40~{ m MeV}$
LBL ν_{τ} appearance (DUNE)	$\in [0.1 \text{ eV}, 15 \text{ MeV}]$	$\gtrsim 40~{ m MeV}$
LBL ν_{μ} disappearance (DUNE)	$\in [0.1 \text{ eV}, 15 \text{ MeV}]$	$\gtrsim 40~{ m MeV}$
CEvNS	$\in [10 \text{ eV}, 15 \text{ MeV}]$	$\gtrsim 40~{ m MeV}$

PBD, J. Gehrlein 2109.14575