Graphs

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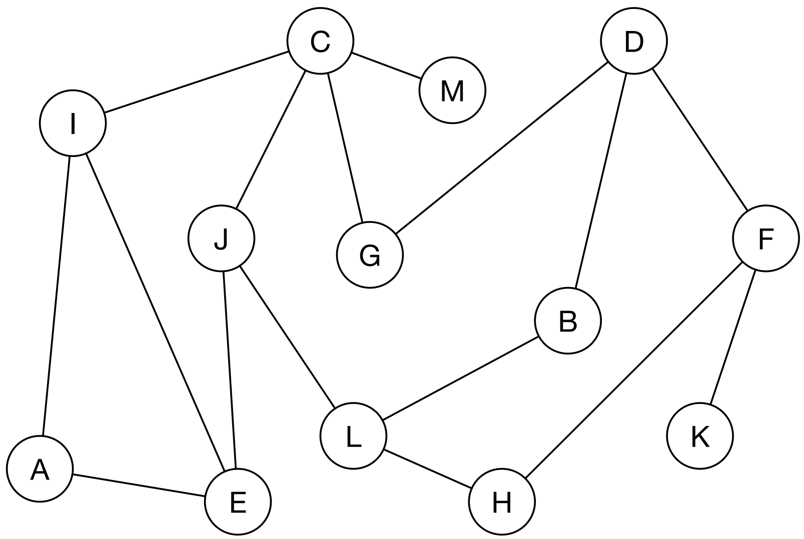
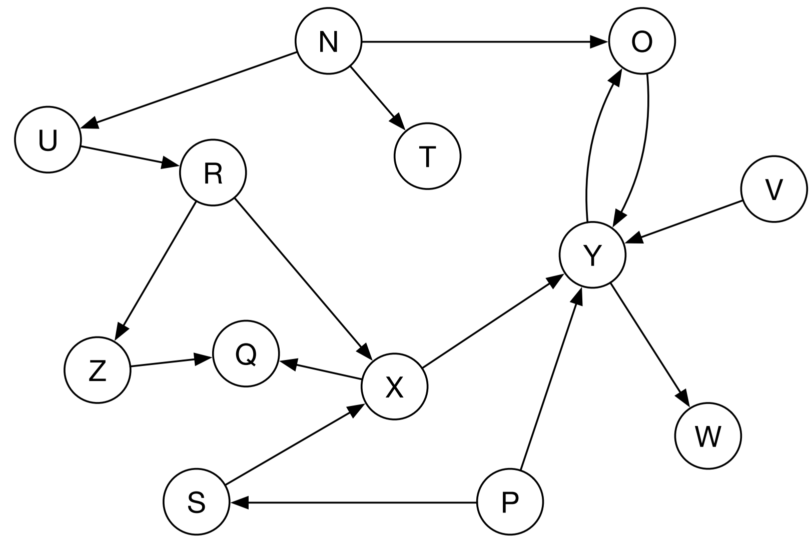
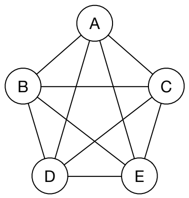
Terminology



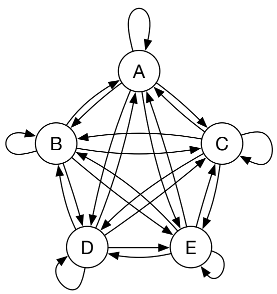


Circles are called *vertices* (singular: *vertex*) or *nodes*.
Lines or arrows are called *edges*.
A graph with one-way edges is *directed*.
The *neighbors* of a vertex are the vertices that can be reached in one step.

Size



An undirected graph with v vertices has at most $v\,(v - 1) / 2$ edges.



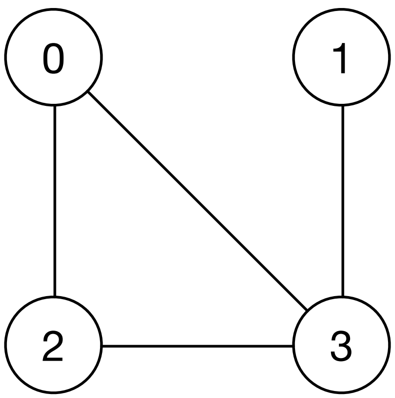
A directed graph with v vertices has at most v^2 edges (including self-loops).

In general, the number of edges $e \in O(v^2)$.

A graph can have zero edges!

Representations

Adjacency matrix



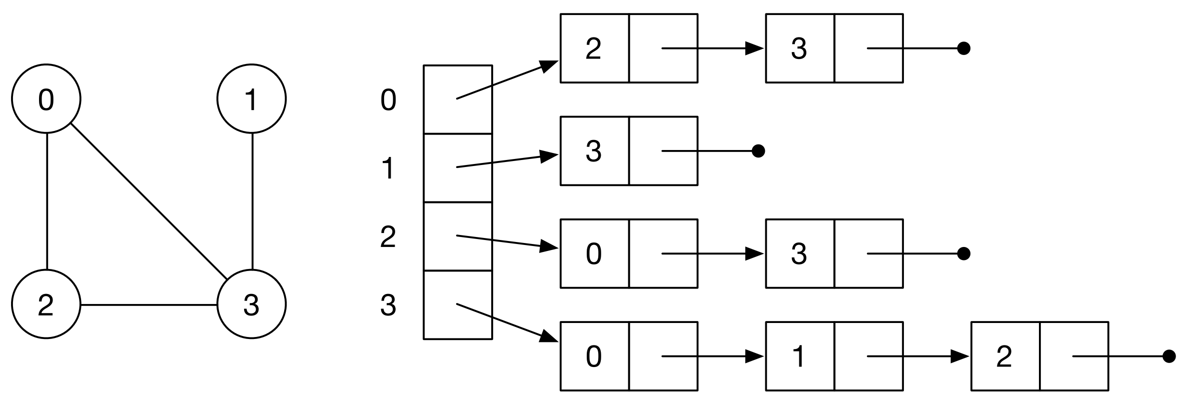
	0	1	2	3
0			x	x
1				x
2	x			x
3	x	x	x	

An entry at position *r*, *c* indicates an edge from vertex *r* to vertex *c*.
For an undirected graph, the matrix is always symmetric.

Space used: $\Theta(v^2)$
Time to see if an edge exists: $\Theta(1)$

Good for *dense* graphs (which have close to the maximum number of edges).

Adjacency lists



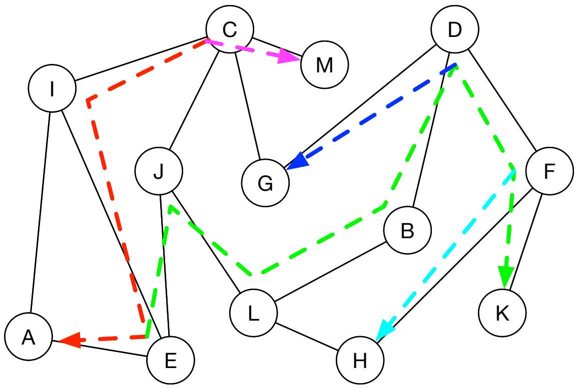
Row r is a linked list of the indices of vertex r's neighbors.

Space used: $\Theta(v + e)$
Time to see if an edge exists: $O(v)$

Good for *sparse* graphs (which have much less than the maximum number of edges).

Algorithms

Depth-first search



Must specify starting vertex.

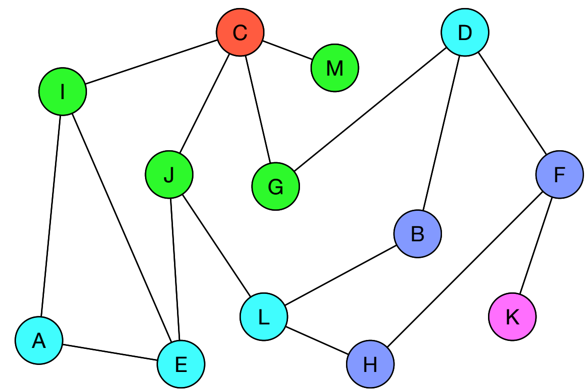
One of several orders starting at C: CIEAJLBDFKHGM.

Follow edges until you hit a dead end, then backtrack to the last fork.

Must keep track of visited vertices to prevent a loop.

Mark this vertex visited and add it to output
For each unvisited neighbor
    Search that neighbor

Breadth-first search



Must specify starting vertex.

One of several orders starting at C: CIJGMAELDBHFK.

Visit neighbors, then their neighbors, and so on.

Must keep track of visited nodes to prevent a loop.

Mark the start vertex visited and add it to a (previously empty) queue
While the queue is not empty:
    Dequeue a vertex *v* and add it to output
    For each unvisited neighbor of *v*:
        Mark that neighbor visited
        Enqueue that neighbor

This algorithm can be used to find shortest paths.

Review

Graphs are made of vertices and edges.

Some graphs are directed.

$e \in O(v^2)$

Graphs can be represented by adjacency matrices or adjacency lists.

Depth-first and breadth-first search are two of many useful graph algorithms.