

Recursion and Critical Thinking

Overview

Recursion

Factorial

Fibonacci

Merge sort

Quicksort

General template

Base and recursive cases

How to think recursively

Iteration versus recursion

Critical Thinking

Recursion

Factorial

$$\begin{aligned} 5! &= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \\ &= 5 \cdot 4! \end{aligned}$$

```
public static int fact(int n) {  
    if (n == 1) {  
        return 1;  
    } else {  
        return n * fact(n - 1);  
    }  
}
```

Fibonacci

$$f(0) = 1$$

$$f(1) = 1$$

$$f(n) = f(n - 1) + f(n - 2)$$

```
public static int fibo(int n) {  
    if (n < 2) {  
        return 1;  
    } else {  
        return fibo(n - 1) + fibo(n - 2);  
    }  
}
```

Merge sort

If there are 0 or 1 things to sort

Do nothing

Otherwise

Divide the array in half

Recursively sort each half

Merge the sorted halves

Quicksort

If there are 0 or 1 things to sort

Do nothing

Otherwise

Divide the array into small and large numbers

Recursively sort each piece

General template

If the problem is simple enough

Solve it directly

Otherwise

Find some simpler subproblems

Solve them recursively

Use the solutions to solve the original problem

Base and recursive cases

You must have at least one base case.

Any recursive call must get closer to a base case.

If not, bottomless recursion leads to a stack overflow.

How to think recursively

What is a simple base case?

Often $n = 0$ or $n = 1$

You may need more than one base case

Test the base case(s) before writing the recursive case(s)

What problem is slightly easier than the one you're trying to solve?

Often $n - 1$, a portion of an array, or all but one of your data

Solve the hard problem *assuming you already have a solution for any easier problem.*

Iteration versus recursion

Recursion can do anything that iteration (loops) can do, and vice versa

Some algorithms are more naturally stated iteratively
(e.g., those with nested loops)

Some algorithms are more naturally stated recursively
(e.g., those with multiple recursive calls)

Critical thinking

Critical means “analytical”, not “disapproving”

What patterns and trends are there?

What similarities and differences between instances are there?

What assumptions are you making? Are they valid?

What are the implications and consequences?

Write down your assumptions, analysis, hypotheses, and conclusions to make your thoughts precise

Review

Recursion solves problems by combining solutions to easier subproblems.

There must be at least one base case and at least one recursive case.

Critical thinkers look for patterns and consider consequences.