### **Towers of Hanoi**

Team Name:
Manager:
Recorder:
Presenter:
Analyst:
This is a Process Oriented Guided Inquiry Learning (POGIL) activity. You and your team will examine a working program. A series of questions will guide you through a cycle of exploration,

concept invention, and application. There is strong evidence that this is more effective (and less

By the time you are done with this activity, you and your team should be able to:

- understand and write simple recursive methods.
- think more critically.

boring) than a traditional lecture.

Your team's recorder is responsible for writing your team's answers to the numbered questions on this form.

# Playing the game

The *Towers of Hanoi* puzzle was invented in 1883 by Édouard Lucas. Our program is not about allowing humans to solve the puzzle but about generating solutions. To understand the puzzle itself, go to https://www.mathsisfun.com/games/towerofhanoi.html and play. First try to solve the puzzle with three disks, then four.

1. Is everyone done playing and ready to pay attention to the team?

You may need to go back and play the game again to answer some of the questions to come, but you should do so *deliberately*, because your team's manager assigned one or more people to find something out, not merely because you got bored with the conversation or thought you could answer a question better on your own.

2. What limitations do the rules place on moving disks?

If the three towers are labeled A, B, and C, here is a solution for two disks:

- $A \rightarrow B$
- $A \rightarrow C$
- $B \rightarrow C$
- 3. What is a solution for three disks?
- 4. What is a solution for four disks?

Stop here and wait for the other teams. If your instructor has given you a way to indicate that you have reached this point, use it now. Once all teams are ready, there will be a short discussion involving the whole class. Your team's presenter should be prepared to present any of your team's previous answers to the class. This discussion is also a good time for your team (through your presenter) to ask any questions you have. If your team is done before other teams, discuss the following open-ended question:

5. Is there any pattern to the sequence of moves in a solution?

subproblem?

Hard-coded solutions				
Exa	Examine Hanoi.java. This program contains several solutions, which we will examine in turn.			
As	As written, the program prints out a solution for one disk by calling hanoilHardCoded.			
6.	. Modify main to call hanoi2HardCoded instead. You will need three arguments instead of two. Why is the third argument needed?			
7.	Does hanoi2HardCoded produce a correct solution for two disks?			
8.	. Does hanoi3HardCoded produce a correct solution for three disks?			
9.	What is accomplished by the first three lines in hanoi3HardCoded?			
10.	What is accomplished by the fourth line in hanoi3HardCoded?			
11.	What is accomplished by the last three lines in hanoi3HardCoded?			

12. What relevance does the largest disk have while the last three lines are solving this

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- 13. These methods are said to have solutions "hard coded" into them: the computer doesn't so much solve the puzzle as spit out a prerecorded answer. Would this be a good approach for producing, say, a solution for seven disks? Why or why not?

# **Calling simpler methods**

- 14. Modify main to call hanoilCallingSimplerMethods. Does it produce a correct solution for one disk?
- 15. Does hanoi2CallingSimplerMethods produce a correct solution for two disks?
- 16. Does hanoi3CallingSimplerMethods produce a correct solution for three disks?
- 17. How does hanoi3CallingSimplerMethods differ from hanoi3HardCoded?

With main set up to call hanoi3CallingSimplerMethods, place a breakpoint in hanoi1CallingSimplerMethods and run the program in the debugger.

18. Complete the table below summarizing the state of the call stack.

Method	Arguments
hanoi3CallingSimplerMethods	start = A, $spare = B$ , $end = C$
main	args = <array 0="" length="" of=""></array>

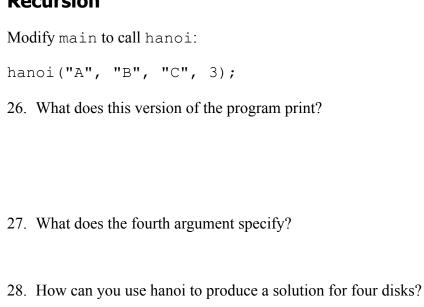
19. Does start have the same value throughout the call stack? If so, what is it? If not, why not?

20. Does spare have the same value throughout the call stack? If so, what is it? If not, why not?

21.	Does end have the same value throughout the call stack? If so, what is it? If not, why not?
22.	What is the same between hanoi3CallingSimplerMethods and hanoi2CallingSimplerMethods?
23.	What is different between hanoi3CallingSimplerMethods and hanoi2CallingSimplerMethods?
24.	Can you write hanoi4CallingSimplerMethods? Does it work? How do you know?

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- 25. Would this be a good approach for producing, say, a solution for seven disks? What about for 50 disks? Why or why not?

# **Recursion**



29. What about seven disks?

Now examine the code for the hanoi method.

30. How is hanoi similar to hanoi3CallingSimplerMethods?

	Method	Arguments	
33.	3. With main set to call hanoi using three disks, place a breakpoint on the line with the first println in hanoi and run the program in the debugger. Complete the table below summarizing the state of the call stack.		
Thi	s surprising action is called <i>recursion</i> .		
32.	hanoi3CallingSimplerMethods cal solve subproblems. What does hanoi call?	<b>led</b> hanoi2CallingSimplerMethods <b>to</b>	
31.	How is it different?		

Method	Arguments
hanoi	start = A, $spare = B$ , $end = C$ , $n = 3$
main	args = <array 0="" length="" of=""></array>
-	

- 34. hanoi calls itself to solve easier problems. In what sense are they easier?
- 35. For what argument values does hanoi *not* recursively call itself?

Comment out the first three lines of hanoi, as well as one of the closing curly braces, leaving only this:

```
hanoi(start, end, spare, n - 1);
StdOut.println(start + " -> " + end);
hanoi(spare, start, end, n - 1);
```

36. The lines you commented out make up the base case, the easiest problem. What happens when you run the program without the base case? Why?

37. Restoring those lines, replace n - 1 with n in the two recursive calls. What happens when you run the program? Why?

Stop here and wait for the other teams. If your instructor has given you a way to indicate that you have reached this point, use it now. Once all teams are ready, there will be a short discussion involving the whole class. Your team's presenter should be prepared to present any of your team's previous answers to the class. This discussion is also a good time for your team (through your presenter) to ask any questions you have. If your team is done before other teams, discuss the following open-ended question:

38. Did any members of your team previously believe it was impossible for a method to call itself? Why?

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