Picture construction and animation with Expander2

fldit-www.cs.tu-dortmund.de/∼peter/Expander2/Exp2Pic.pdf

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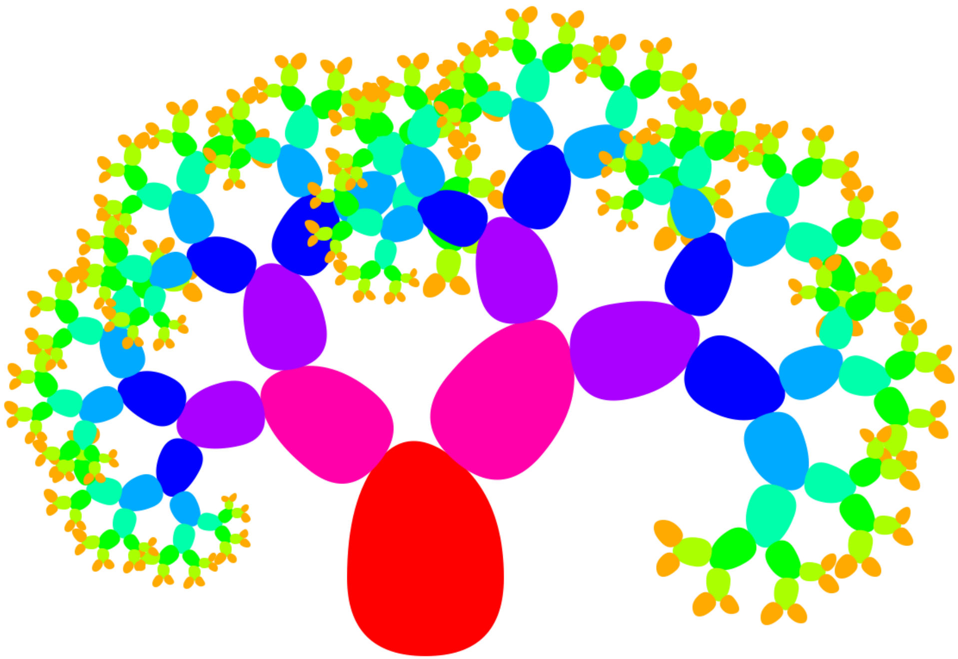
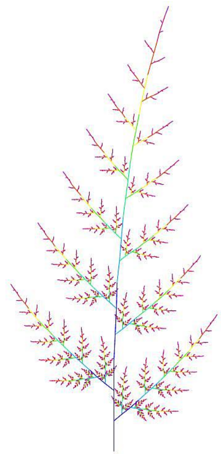
May 11, 2017



Contents

1. Expander2 5
2. Swinging types 8
3. Picture generation 10
4. The Haskell types 14
5. Turtle actions 15
6. From turtle actions to a picture 17
7. Recursive drawing 18
8. Picture scanner 19
9. Oscillable widgets 20
10. Putting it all together 23
11. Examples 26

Expander2

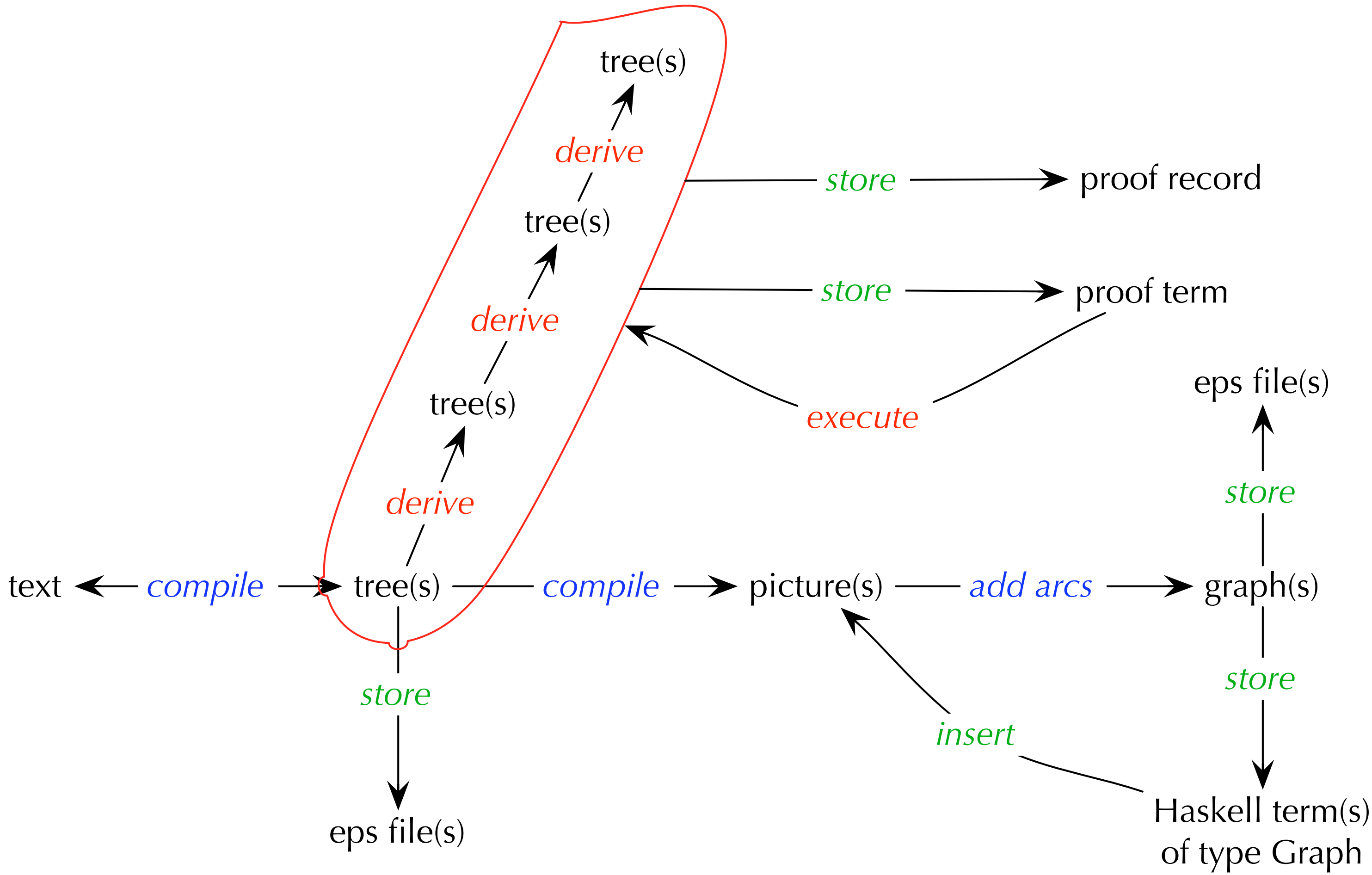


- Horn axioms ($p(t) \Leftarrow \varphi$) and co-Horn axioms ($p(t) \Rightarrow \varphi$) defining functions, relations or non-deterministic functions (transition systems),
- logical **top-down derivations** into *True* or another **solved formula** (constraint):
  - *prove* φ: $\varphi \quad \vdash \quad True$
  - *solve* φ: $\varphi \quad \vdash \quad$ solved formula
  - *refute* φ: $\neg\varphi \quad \vdash \quad True$
  - *verify* p: $p(x) \Rightarrow \varphi \quad \vdash \quad True$
  - *evaluate* p: $p(x) \Leftarrow \varphi \quad \vdash \quad True$
  - *evaluate* t: $t \equiv x \quad \vdash \quad c \equiv x$

- **rewrite sequences** that generate/manipulate/normalize functional terms or check Kripke models,
- rules at 3 levels of automation/interaction:
  - **Simplifications** are equivalence transformations that partially evaluate terms and formulas.
  - **Narrowing** and **rewriting** apply all or some **axioms** to **goals**, exhaustively or selectively, interactively or automatically, stepwise or iteratively.
  - **Induction**, *coinduction* and other proper **expansions** are applied interactively and stepwise. (Fixpoint) induction and coinduction apply **goals** (as hypotheses) to **axioms** and thus show the former by *solving* the latter.

Expander2

- The semantics is given by the **initial** resp. **final model** of the axioms.
- Relations and predicates are interpreted as the **least** or **greatest solutions** of their Horn resp. co-Horn axioms.
- These dualities admit the uniform treatment of constructor- and destructor-based data types, finite and infinite objects, positive and negative propositions.
- *3 representations of a formula/term:*
  text, **tree** (rooted graph) and **picture** (list of 2-dimensional widgets).
  All representations can be edited, moved and scaled.
  The pictorial ones can also be rotated and connected by arcs of different shapes.



Expander's representations of terms/formulas and derivations

Swinging types

- **Sums** $\coprod_{i\in I} t_i$ formalize/implement selection and case analysis.
  Products $\prod_{i\in I} t_i$ formalize/implement tupling and relationships.
- A recursively defined type T is created from

constructors		destructors
$c : \mathit{composed\ type} \to T$	or	$d : T \to \mathit{composed\ type}$
(initial) algebras		(final) coalgebras
context-free languages		transition systems

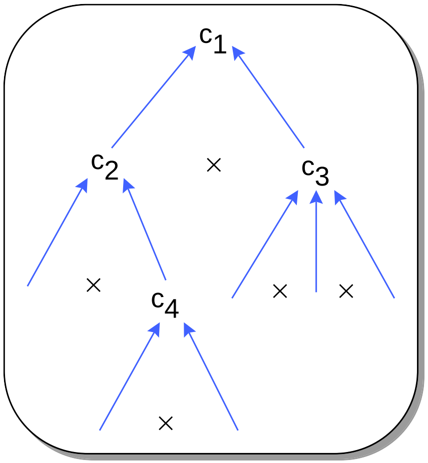
  Constructor-based types are called **visible**.
  Destructor-based types are called **hidden**.
- More constructors lead to **supertypes**. More destructors lead to **subtypes**.
- Functions $f : T \to \mathit{composed\ type}$ on a visible type T are defined by **recursion**.
  Functions $g : \mathit{composed\ type} \to T$ into a hidden type T are defined by **corecursion**.

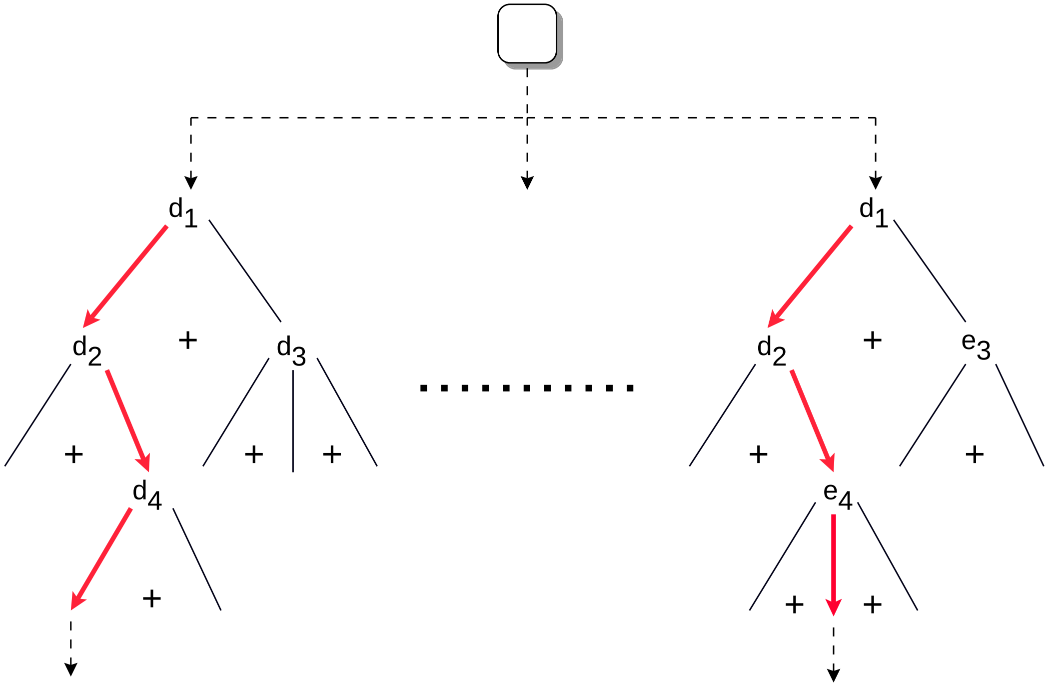
Swinging types

- **Horn clauses** $r(t) \Leftarrow \varphi$ define **least relations**
  (least solution of $r(t) \Leftarrow \varphi$ in r).
  Co-Horn clauses $r(t) \Rightarrow \varphi$ define **greatest relations**
  (greatest solution of $r(t) \Rightarrow \varphi$ in r).
  Properties of least relations are proved by **induction**.
  Properties of greatest relations are proved by **coinduction**.

- Least or greatest **congruences** $\equiv: t \times t$ and **quotients** $A/\equiv^A$
  formalize/implement (visible or hidden) abstraction.
  Least or greatest **invariants** $all : t$ and **substructures** $all^A \subseteq A$
  formalize/implement (visible or hidden) restriction.

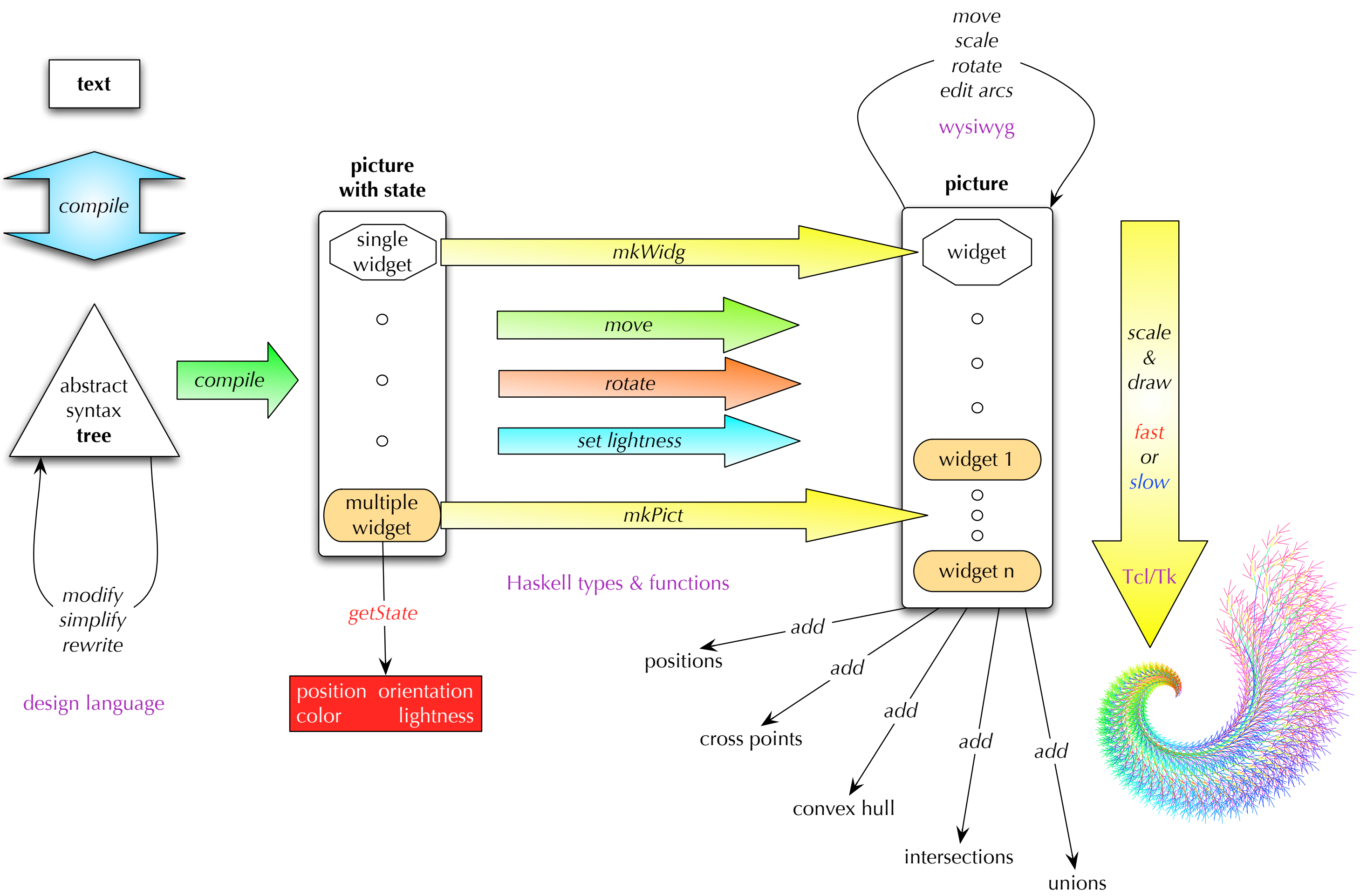
- The standard models are **initial/least** or **final/greatest** solutions of domain equations ($A \leq B$ iff $\exists\ A \to B$), constructed as **suprema/colimits** or **infima/limits** of **ascending** or **descending** chains.





An element of the initial model for constructors $c_i : s_{i,1} \times \ldots \times s_{i,n_i} \to s_i$ (left) versus an element of the final model for destructors $d_i : s_i \to s_{i,1} + \cdots + s_{i,n_i}$ (right).

Picture generation



- Several interpreters translate a **tree** into a **picture**:
  - *alignment, linear equations, matrices, matrix solution,*
  - *partition, polygons, polygon solution, rectangles*
- They create a widget in an **initial state**
  (position (0,0), orientation 0, color, lightness 0)
- Some operations on a list of pictures *ps*, the contents of a file *F*, a single widget *w* or a list of actions *acts*:
  - *color(c,ps), dark(ps), file(F), flipH/V(ps), gif(F), grow(ps), grow5/R(n,ps), hframe(ps), light(i,ps), matrix(w), meet(n,ps), odots(ps), outline(ps), place(w,points), rainbow(w,n,d,a), rainbow2(w,n), reverse(ps), rframe(ps), shelves(n,d,ps), shineB/W(w,d,a), shuffle(ps), split(ps), splitS(sc,ps), tabA/B(n,d,ps), tabAS/BS(n,d,sc,ps), turt(acts), turt(ps)*
  - **animators**: *fadeB/W(w), fast(w), flash(w), new, old, osciL/P/W(...), peaks(w,m), pulse(w), repeat(ps), rotate/C(w,a)*
  - Further operations on list *terms* are provided by the simplifier.

The Haskell types

```
type Picture = [Widget_]
type Arcs    = [[Int]]

data Widget_ = Arc Color ArcStyleType Point Float (Float,Float) |
               Arc0 State ArcStyleType Float Float |
```

Arc0, Path0 and Tree0 are abstract versions of Arc, Path resp. Tree which they are turned into before being displayed.

```
               Bunch Widget_ [Int] |
```

Bunch w ns represents widget w together with arcs leading from w to the widgets at positions ns.

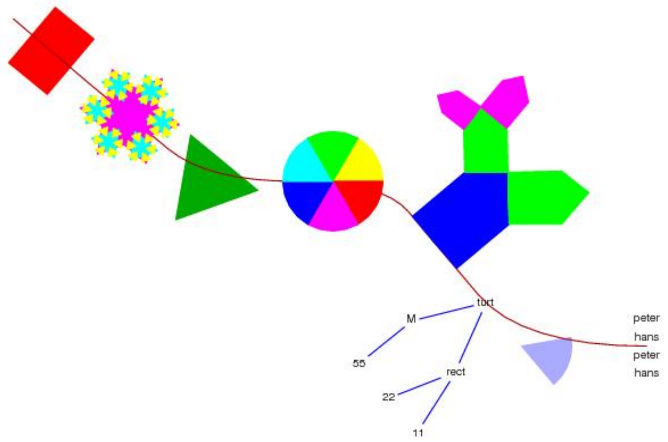
ms++ns is the list of direct successors of Bunch w ms ns.

```
               Circ State Float | CircA State Float | Dot Color Point |
```

CircA and RectA ignore the scale of enclosing turtles.

```
               Fast Widget_ | File_ String | Gif String Point |
```

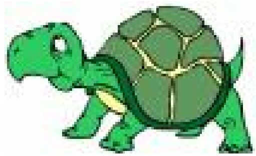
```
New | Old | Path Color Int [Point] |
Path0 State Int Point [Point] |
Poly State Int [Float] Float |
Rect State Float Float | RectA State Float Float |
Repeat Widget_ | Snow State Int Float |
Text_ State [String] |
Tree Color Color (Term TNode) |
Tree0 State String Color [Term TNode] |
Tria State Float |
Turtle State Float [TurtleAct] | White
```



The Haskell types

```
data TurtleAct = Move Float | MoveA Float | Jump Float | JumpA Float |
                  MoveA and JumpA ignore the scale of the enclosing turtle.
                 Turn Float | Open Color Int | Scale Float |
                 Close | Draw | Widg Widget_ | WidgB Widget_
                  Widg w ignores the orientation of the enclosing turtle,
                  WidgB w adds it to the orientation of w.
type State     = (Point,Float,Color,Int)
type TNode     = (String,Point)
type Point     = (Float,Float)
```

Turtle actions



```
drawAt ps = f [open] p0 ps
      where f acts p (q:ps) = f (acts++acts') q ps
                 where acts' = if p == q then [Widg w]
                               else [Turn a,Jump d,Turn (-a),Widg w]
                       (a,d) = (angle p q,distance p q)
            f acts _ _ _ = acts++[Close]
```

From turtle actions to a picture

```
mkPict (Turtle (p,a,c,i) sc acts) = g pict c' n ps
 where (pict,(_,_,c',n,_,ps):_) = foldl f ([],[(p,a,c,0,sc,[p])]) acts
       f (pict,(p,a,c,n,sc,ps):s) (Move d) =
                                  (pict,(q,a,c,n,sc,ps++[q]):s)
                                  where q = successor p a (d*sc)
       f (pict,(p,a,c,n,sc,ps):s) (Jump d) =
                                  (g pict c n ps,(q,a,c,n,sc,[q]):s)
                                  where q = successor p a (d*sc)
       f (pict,(p,a,c,n,sc,ps):s) (Turn b) =
                                  (pict,(p,a+b,c,n,sc,ps):s)
       f (pict,s@((p,a,c,m,sc,_):_)) (Open d n) =
                                  (pict,(p,a,d,n,sc,[p]):s)
       f (pict,s@((p,a,c,n,sc,ps):_)) (Scale sc') =
                                  (pict,(p,a,c,n,sc*sc',ps):s)
```

From turtle actions to a picture

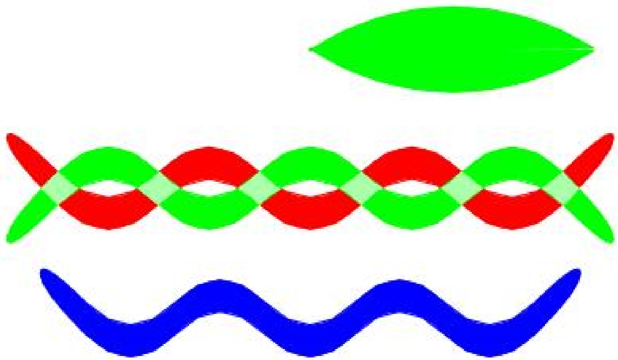
```
f (pict,(_,_,c,n,_,ps):s) Close =
                          (pict++[Path (mkLight i c) n ps],
                           s)

f (pict,(p,a,c,n,sc,ps):s) Draw =
                          (pict++[Path (mkLight i c) n ps],
                           (p,a,c,n,sc,[p]):s)

f (pict,s@((p,a,_,_,sc,_):_)) (Widg w) =
                          (pict++[scaleWidg sc (moveWidg p a w)],
                           s)
```

Recursive drawing

```
drawPict pict = action
                if fast || all isFast pict then mapM_ drawWidget pict
                else let scan = head scans
                     run <- scan.isRunning
                     if run then scan.addScan pict
                            else scan.startScan0 delay pict

drawWidget (Circ ((x,y),_,c,i) r) = action
                canv.oval (round2 (x-r,y-r)) (round2 (x+r,y+r))
                  [Outline (outColor c i), Fill (fillColor c i)]
                done
...
drawWidget w | isWidg w = drawWidget (mkWidg w)
             | isPict w = drawPict (mkPict w)
```

Picture scanner

```
struct Scanner = startScan0 :: Int -> Picture -> Action
                 ...

scanner :: TkEnv -> (Widget_ -> Action) -> Template Scanner
scanner tk act =
  template (as,run,running) := ([],undefined,False)
  in let startScan0 n bs = action as := bs; startScan n
         startScan n = action if running then run.stop
                                run0 <- tk.periodic n loop
                                run := run0; run.start; running := True
         loop = action case as of a:s -> if noRepeat a then as := s
                                           act a; if isFast a then loop
                                    _ -> stopScan
         addScan bs = action as := bs++as
         stopScan = action if running then run.stop; running := False
         isRunning = request return running
     in struct ..Scanner
```



Oscillable widgets

```
struct Oscillable = maxheight :: Float
                    actseq :: Float -> [TurtleAct]

oscillate obj = f h++g 1
                where f a = if a == 0 then [] else acts a++f (a-1)
                      g a = if a == h then [Fast (w h)]
                                      else acts a++g (a+1)
                      acts a = [Fast v,wait,Fast (delWidg v)]
                               where v = w a
                      w = turtle . obj.actseq
                      h = obj.maxheight
```

```
oleafF h c = struct maxheight = h
                    actseq a = leafF h a c

oplait n d c c' = struct maxheight = 85
                         actseq a = f a c++f (-a) c'
                                    where f = wave True 3 n d

owave n d c = struct maxheight = 85
                     actseq a = wave False 3 n d a c

leafF h d c = [open,Jump y,Turn 90,Jump (-x),Turn a,Widg w,Turn (-a),
               Jump (x+x),Turn (-a),Widg (flipWidg False w),Close]
              where p@(x,y) = ((h*h-d*d)/(d+d),h/2)
                    (dist,a) = (angle p0 p,distance p0 p)
                    w = Arc0 (p0,0,c,0) Chord dist (a+a)

wave b k n d a c = Open c k:
                   if b then right:Jump (-y/2):left:acts else acts
        where right = Turn 90; left = Turn (-90)
              acts = Jump (-fromInt n*x):right:Jump (-5):
                     left:border a++border (-a)++[Close]
```

```
border a = foldl1 (<++>) (take n (repeat (step a)))++
           [right,Move 10,right]
step a = [Turn a,Move d,Turn (-a-a),Move d,Turn a]
(x,y) = successor p0 a d
```

Putting it all together

```
scaleAndDraw = action
  mapM_ (.stopScan) scans
  scan <- scanner tk drawWidget
  scans := [scan]
  let pict = pictures!!curr
  sizes <- mkSizes font (stringsInPict pict)
  (ns,ws) <- getEnclosed pict
  let (pict1,(x1,y1,x2,y2)) = f pict 0
      f (w:pict) i = (w':pict',minmax4 (widgFrame sizes w') bds)
                     where w' = scaleWidg (sc i) w
                           (pict',bds) = f pict (i+1)
      f _ _        = ([],(0,0,0,0))
      sc i = if just rect && i `elem` ns then rscale else scale
```

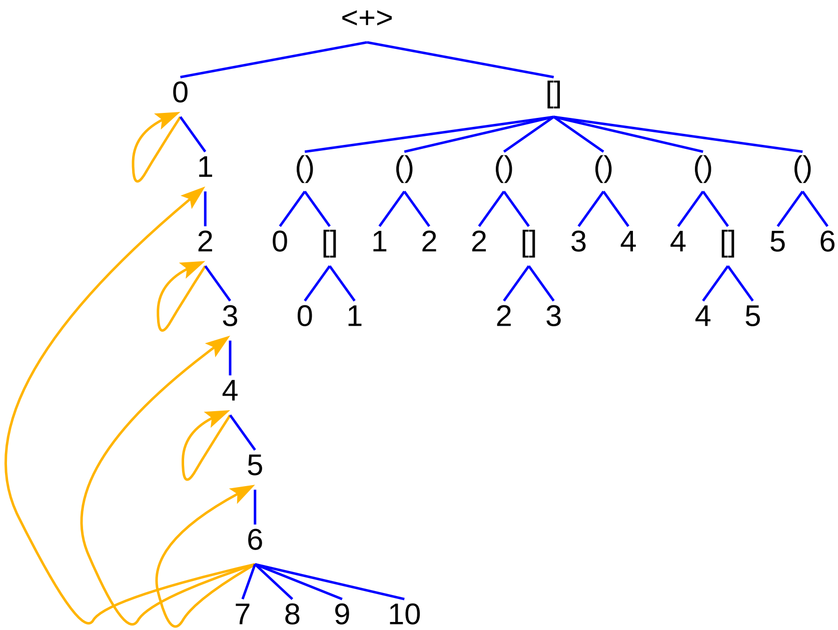
```
pict2 = map (transXY (5-x1) (5-y1)) pict1
pict3 = filter (not . isRedDot) pict2
compl = map (hullLines sizes) . minus1 pict3
anchor w = Dot c p where p = coords w
                         c = if any (interior p) (compl w)
                             then RGB 150 150 150 else black
(hull,rs) = convexPath sizes qs pict3
qs = if just rect then filter (`inRect`(get rect)) ps else ps
     where ps = map coords pict3
hullNos = zipWithIndices addNo rs
          where addNo i p = Text_ (p,0,dark red,0) [show i]
hulls = concatMap (hullPoints sizes) (removeSingles sizes pict3)
```

```
pictures := updList pictures curr
                    (zipWithIndices (scaleWidg (recip (sc pict2))))
widthX := max 100 (round (x2-x1+10))
widthY := max 100 (round (y2-y1+10))
canv.set [ScrollRegion (0,0) (widthX,widthY)]
if partBit then drawPict pict2
else case drawMore of
     0 -> drawPict pict2
     1 -> drawPict (pict3++map anchor pict3)
     2 -> drawPict (pict3++hull++hullNos)
     3 -> drawPict (pict3++markCross hulls)
     4 -> drawPict (pict3++meetHulls (showStrands True) hulls)
     5 -> drawPict (pict3++meetHulls (showStrands False) hulls)
     n -> drawPict (pict3++uniquePaths (meetHulls f hulls))
          where f = mergeStrands Path (n-6)
mapM_ drawArrow (getArcs sizes pict2 (edges!!curr))
if just rect then drawWidget (get rect)
```

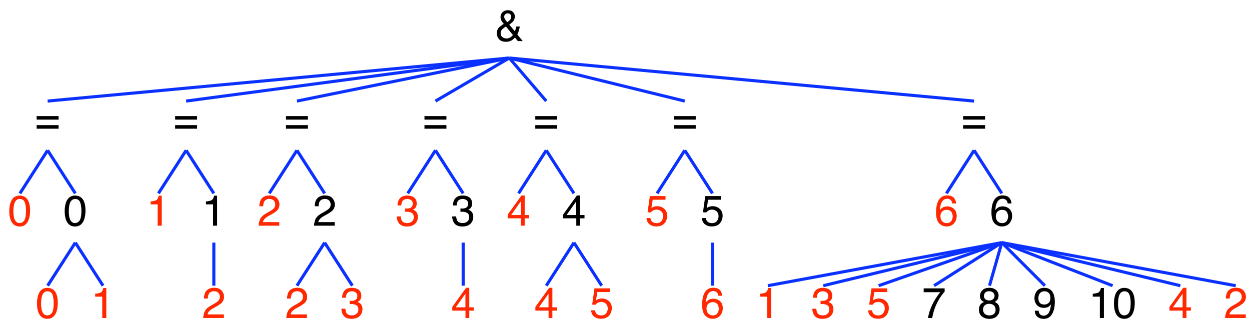
Examples

Example: NDA (Examples/TRANS0)

```
defuncts:   states
fovars:     n
axioms:     states = [0..10]                               &
            (n < 6 & n `mod` 2 = 0 ==> n -> [n,n+1])       &
            (n < 6 & n `mod` 2 =/= 0 ==> n -> n+1)         &
            6 -> [1,3,5,7..10]
```


<+>
0
[]
1
()
()
()
()
()
()
2
0
[]
1
2
2
[]
3
4
4
[]
5
6
3
0
1
2
3
4
5
4
5
6
7
8
9
10

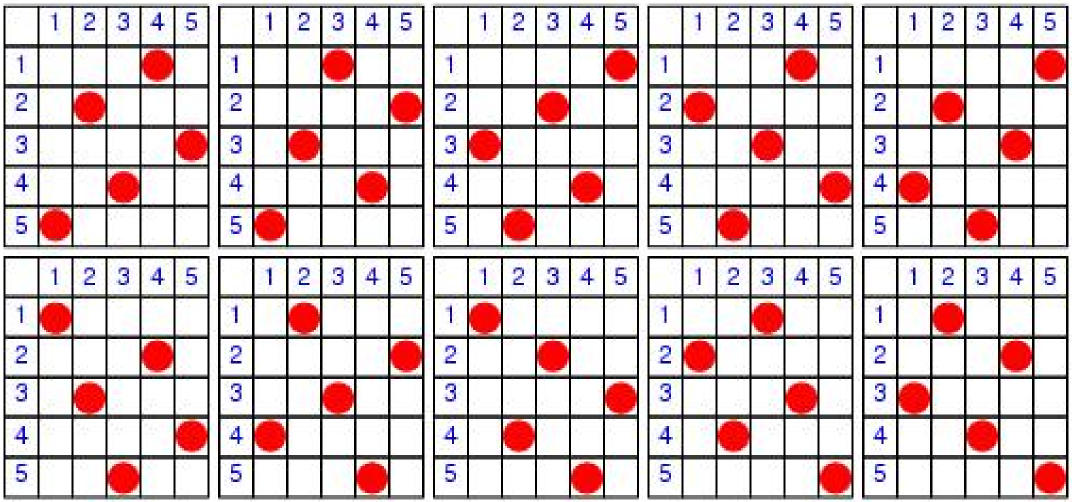
	0	1	2	3	4	5	6
0	●	●					
1			●				
2			●	●			
3					●		
4					●	●	
5							●



Example: Five queens (Examples/QUEENS)

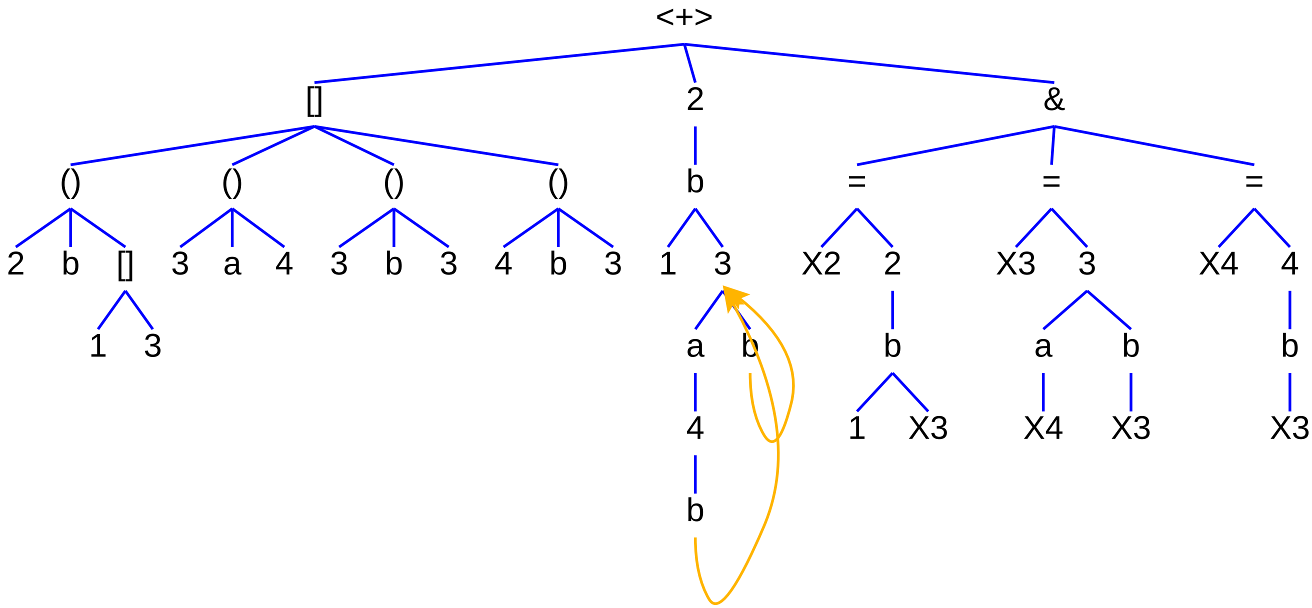
```
preds:      cmp loop queens
fovars:     n x y xs ys ps s s'

axioms:     (x =/= y+n & x =/= y-n ==> cmp(x)(y,n))                  &
            (xs `gives` x & zipAll(cmp(x))(ys)[1..length(ys)]
                 ==> (xs,ys) -> (xs-x,x:ys))                          &
            loop(xs,([],ys),zip(xs)(ys))                              &
            (s -> s' ==> (loop(xs,s,ps) <=== loop(xs,s',ps)))         &
            (xs = [1..n] ==> (queens(n,ps) <=== loop(xs,(xs,[]),ps)))
```

```
conjects:  queens(5,ps)
      ps = [(1,4),(2,2),(3,5),(4,3),(5,1)]
    | ps = [(1,3),(2,5),(3,2),(4,4),(5,1)]
    | ps = [(1,5),(2,3),(3,1),(4,4),(5,2)]
    | ps = [(1,4),(2,1),(3,3),(4,5),(5,2)]
    | ps = [(1,5),(2,2),(3,4),(4,1),(5,3)]
    | ps = [(1,1),(2,4),(3,2),(4,5),(5,3)]
    | ps = [(1,2),(2,5),(3,3),(4,1),(5,4)]
    | ps = [(1,1),(2,3),(3,5),(4,2),(5,4)]
    | ps = [(1,3),(2,1),(3,4),(4,2),(5,5)]
    | ps = [(1,2),(2,4),(3,1),(4,3),(5,5)]
```



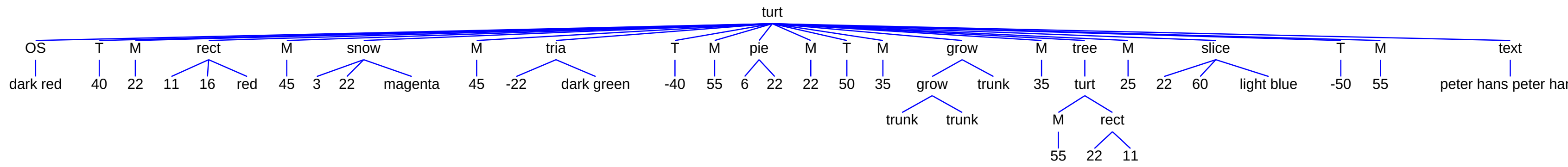
Example: Labelled transition relation (Examples/TRANS1)

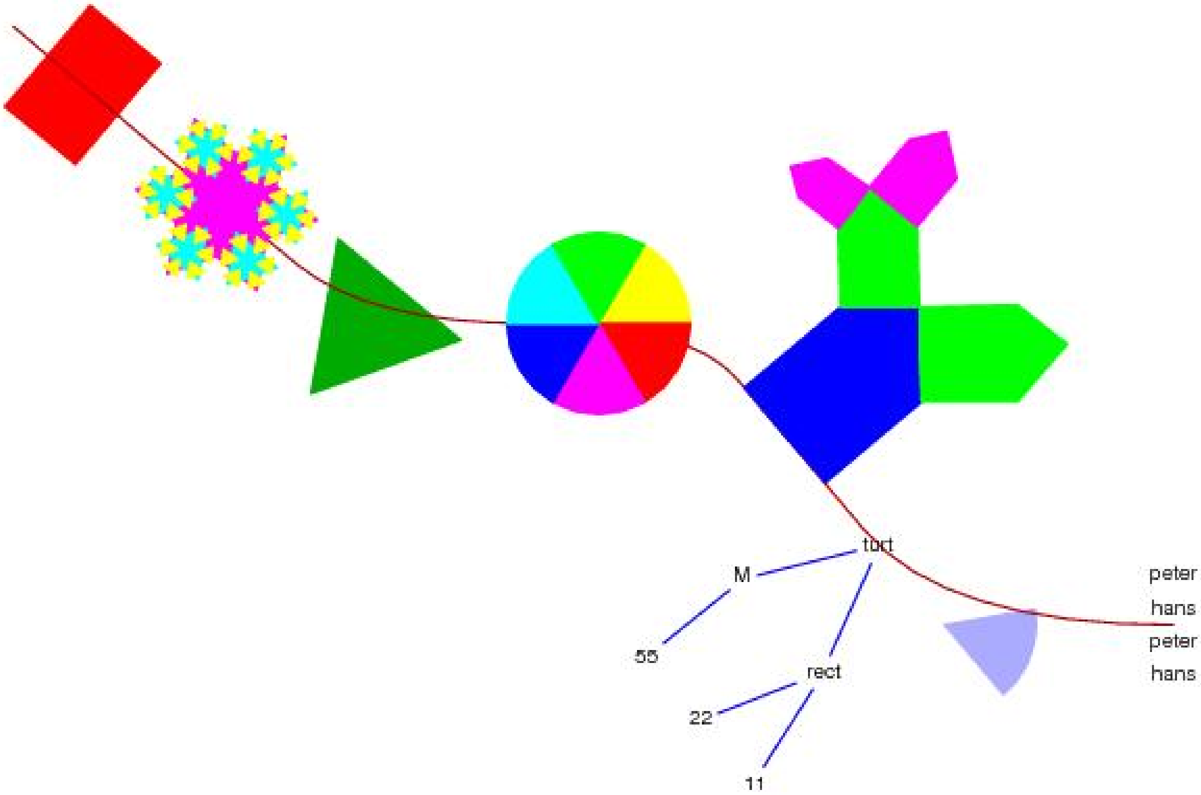
```
constructs: a b
defuncts:   states
axioms:     states = [1,2,3,4] & labels = [a,b] &
            (2,b) -> [1,3] & (3,b) -> 3 & (3,a) -> 4 & (4,b) -> 3
```



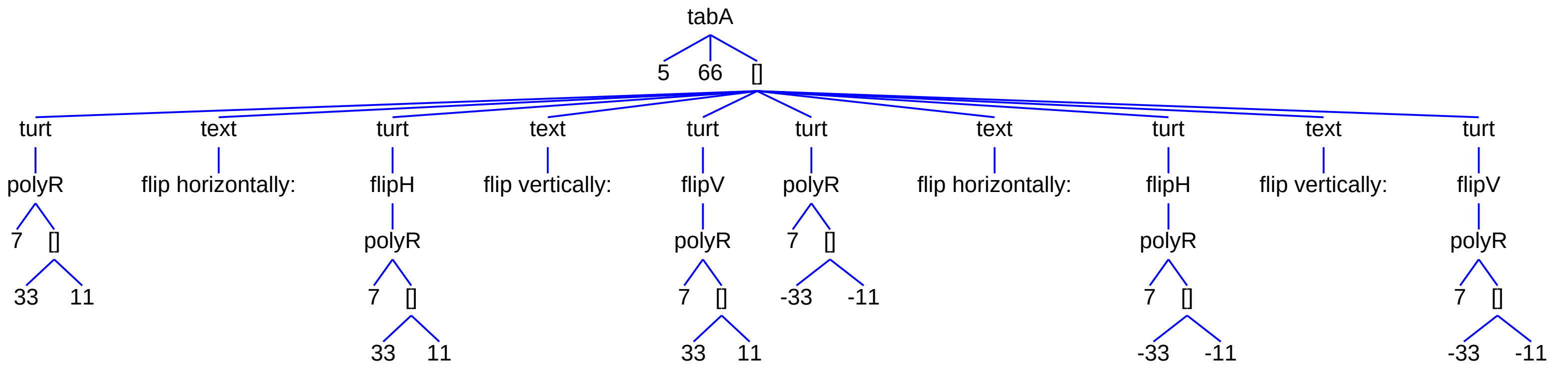
	a	b
2		1 3
3	4	3
4		3

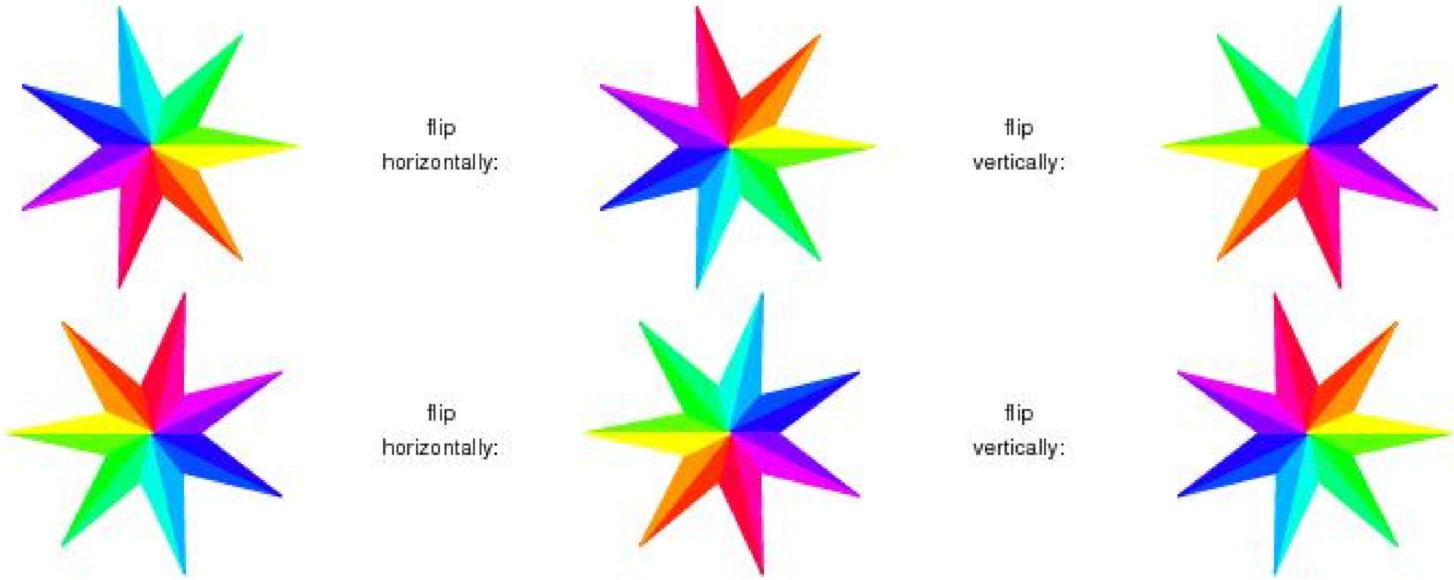
Example: Turtle (Examples/turt)



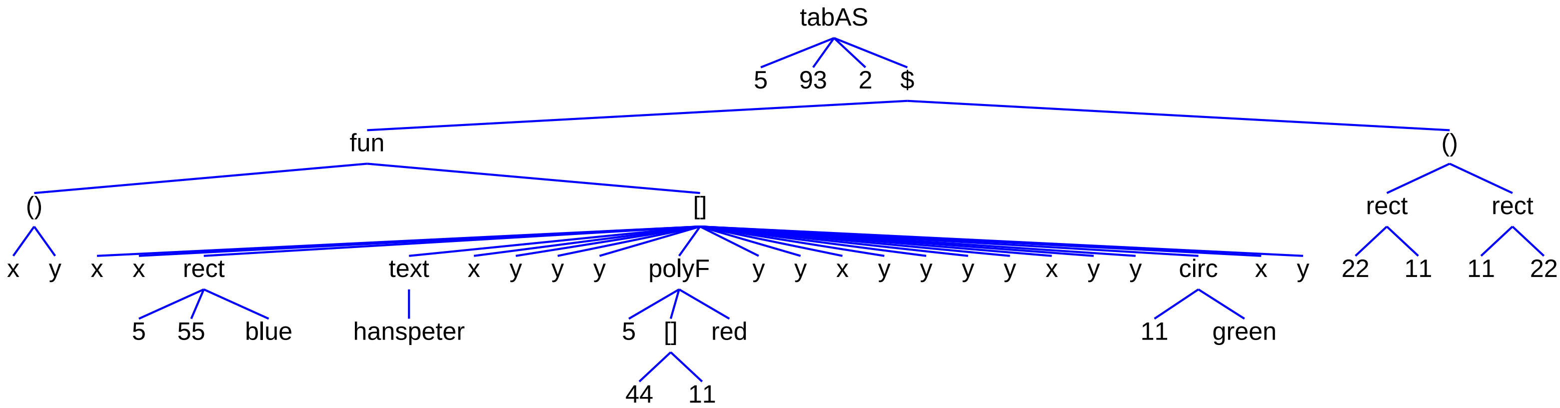


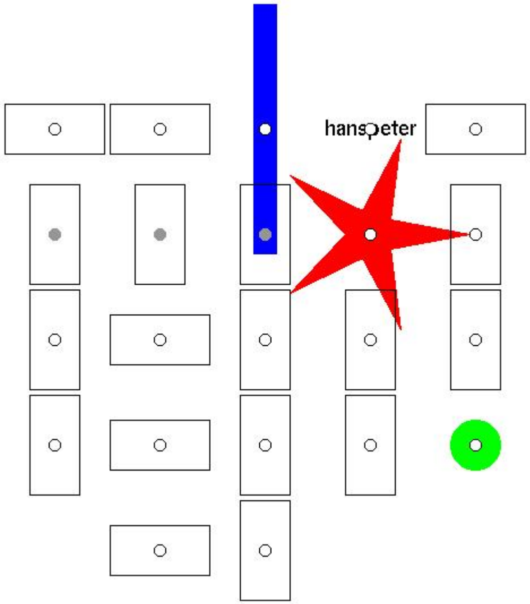
Example: Table (Examples/polytab)



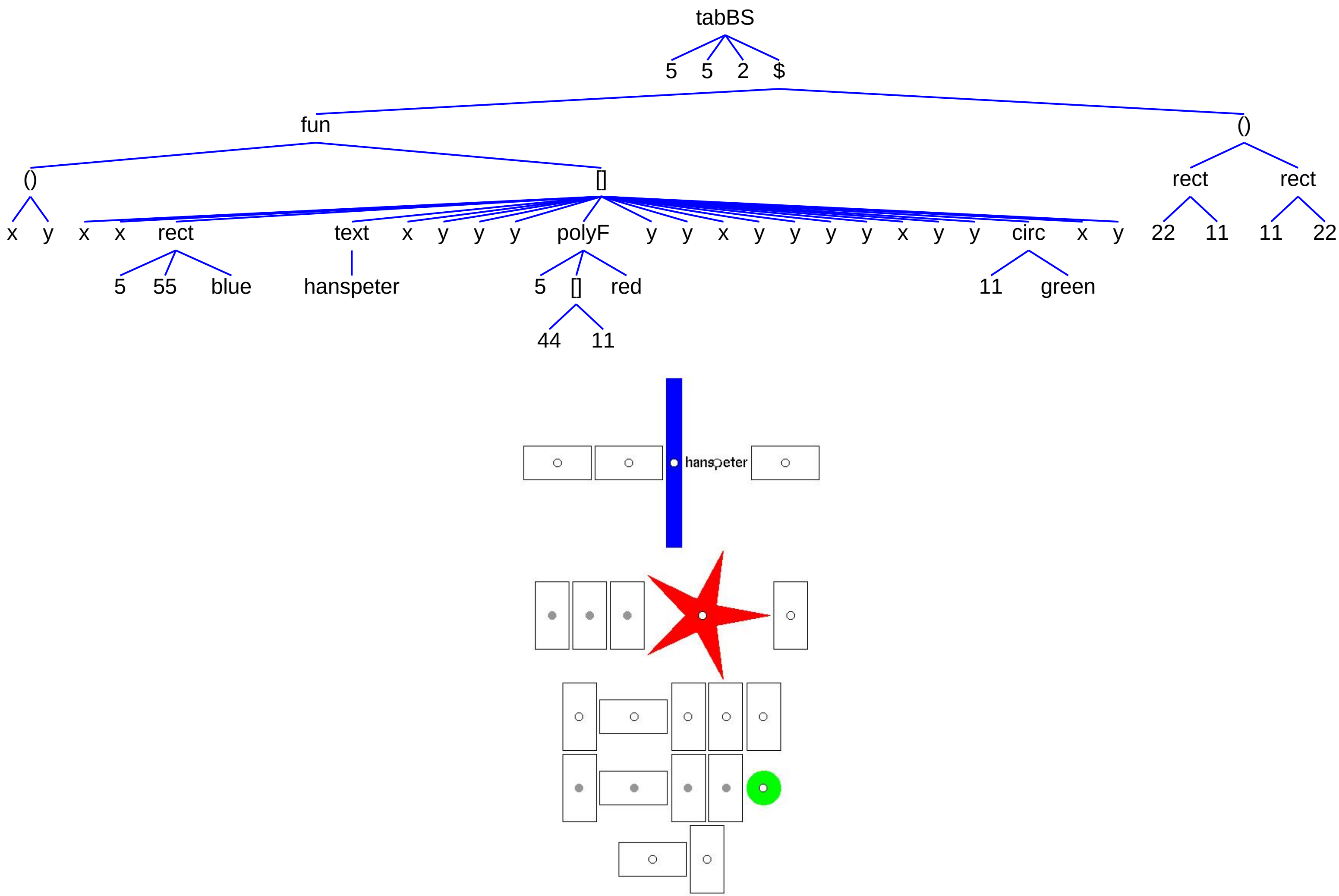


Example: Shelves (Examples/shelvesA)

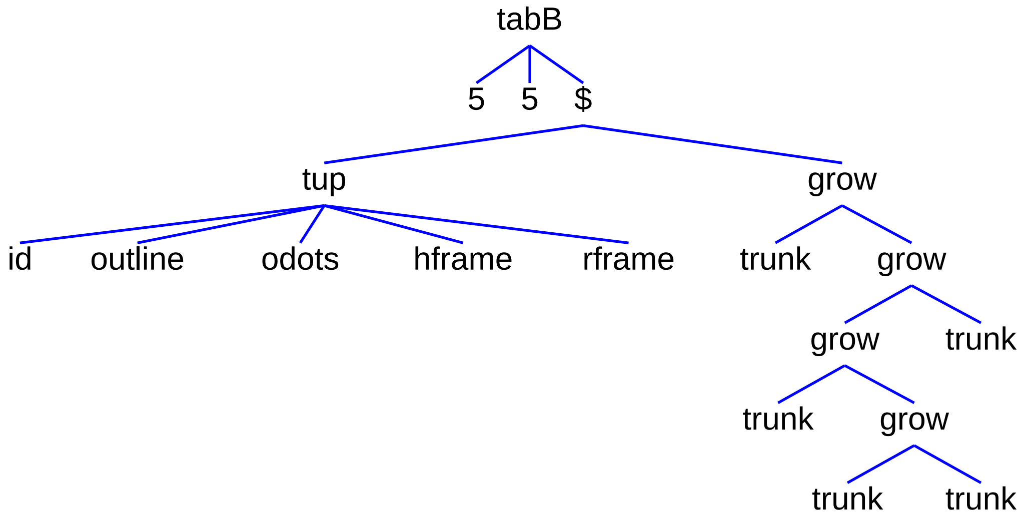


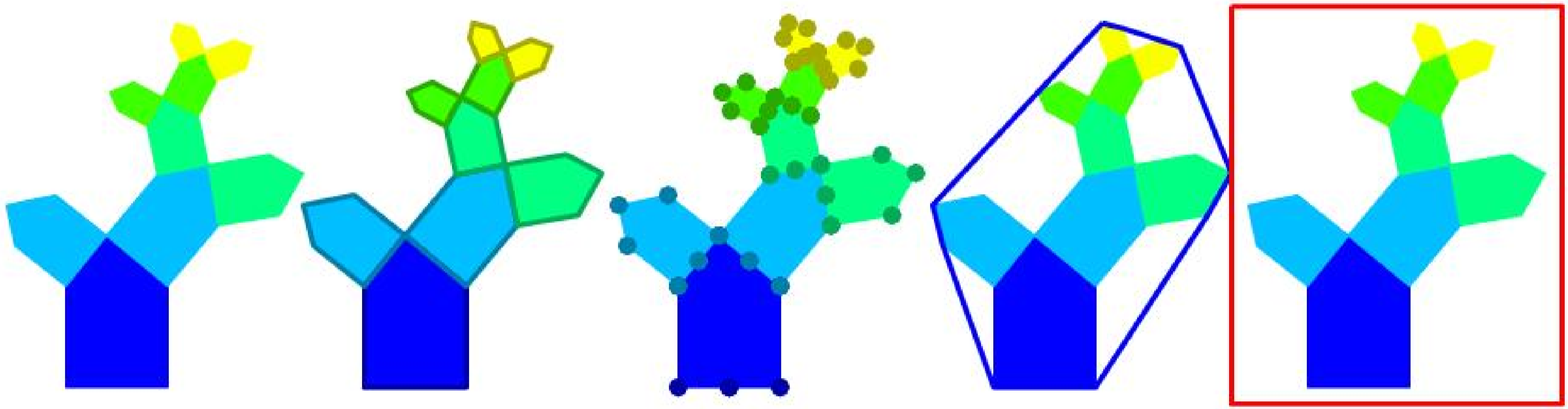


Example: Shelves (Examples/shelvesB)

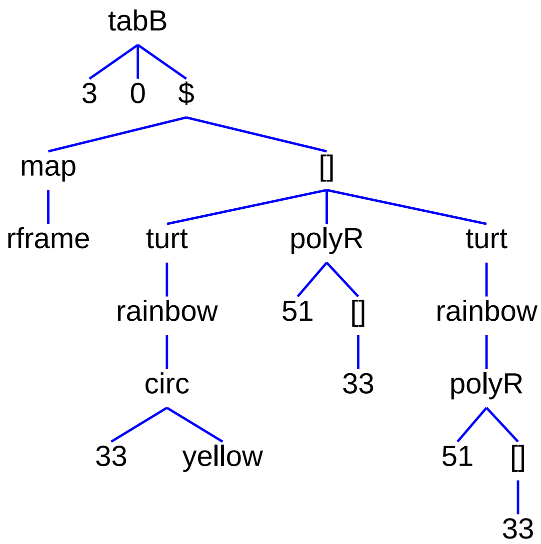


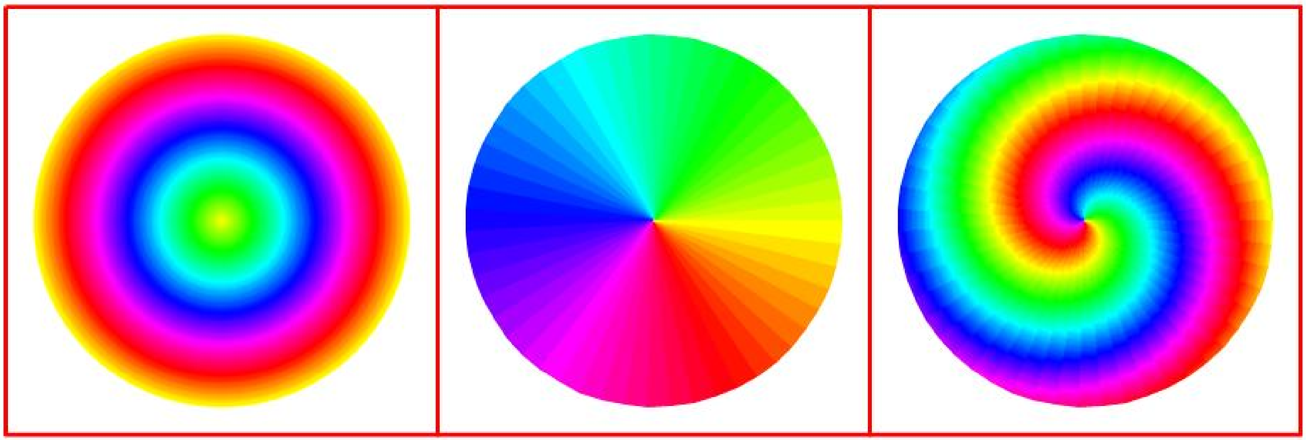
Example: Decorations (Examples/hull)



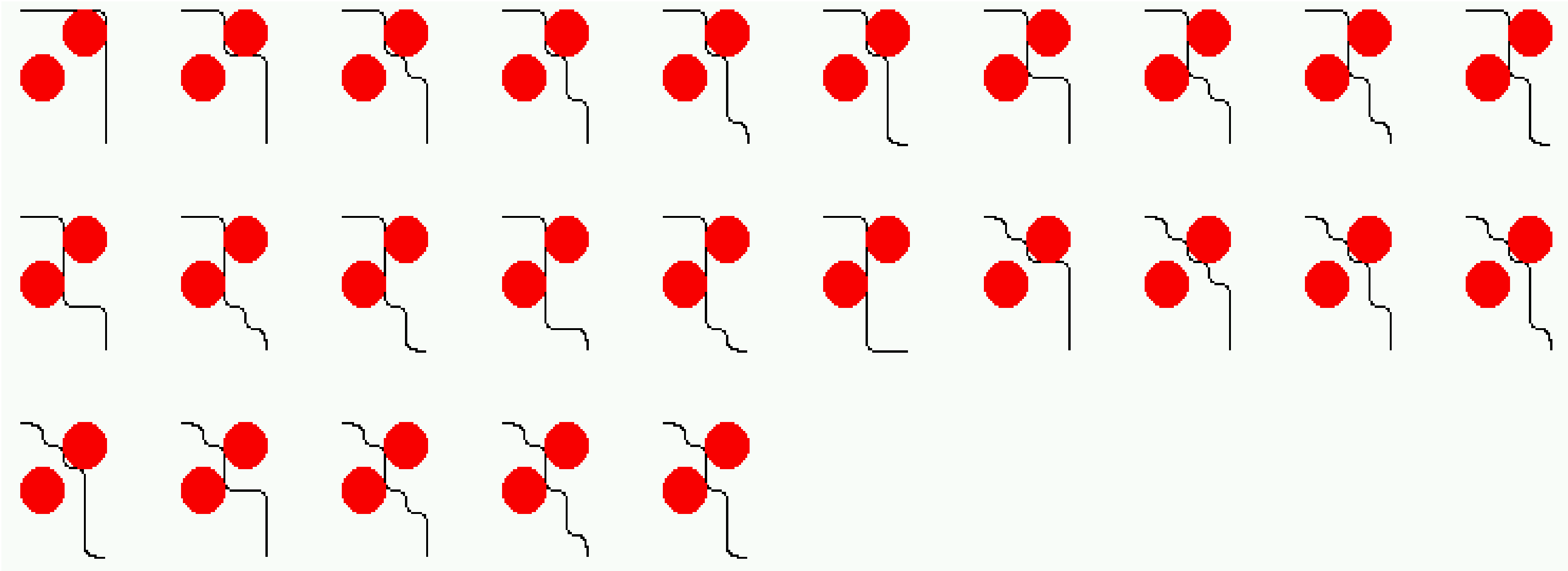


Example: Rainbows (Examples/raintabB)





Example: Pathfinder (Examples/ROBOT)

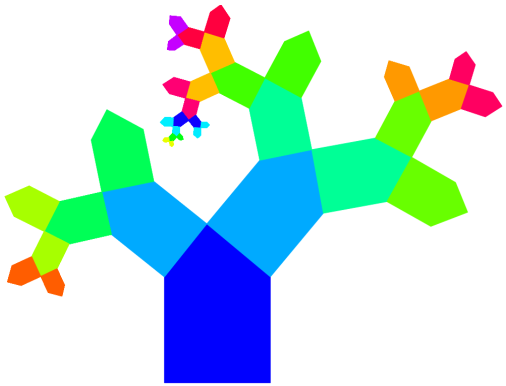


```
preds:      loop
constructs: turt path place circ red
defuncts:   cs
fovars:     x' y' p q ps pa
```

```
axioms:

cs = [(2,6),(6,2)]                                                &

((p = (x+2,y) | p = (x,y+2)) & p `NOTin` cs ==> (x,y) -> p)      &

loop((8,12),path(ps),path(ps++[(8,12)]))                          &
(p < (8,12) & p -> q
     ==> (loop(p,path(ps),pa) <=== loop(q,path(ps++[p]),pa)))    &

((x,y) < (x,y') <=== y < y')                         &
((x,y) < (x',y) <=== x < x')                         &
((x,y) < (x',y') <=== x < x' & y < y')

conjects:

Any pa: (loop((0,0),path[],pa) & turt(pa:place(circ(2,red),cs)) = z)
```

Example: Plan formation (Examples/ROBOTACTS)

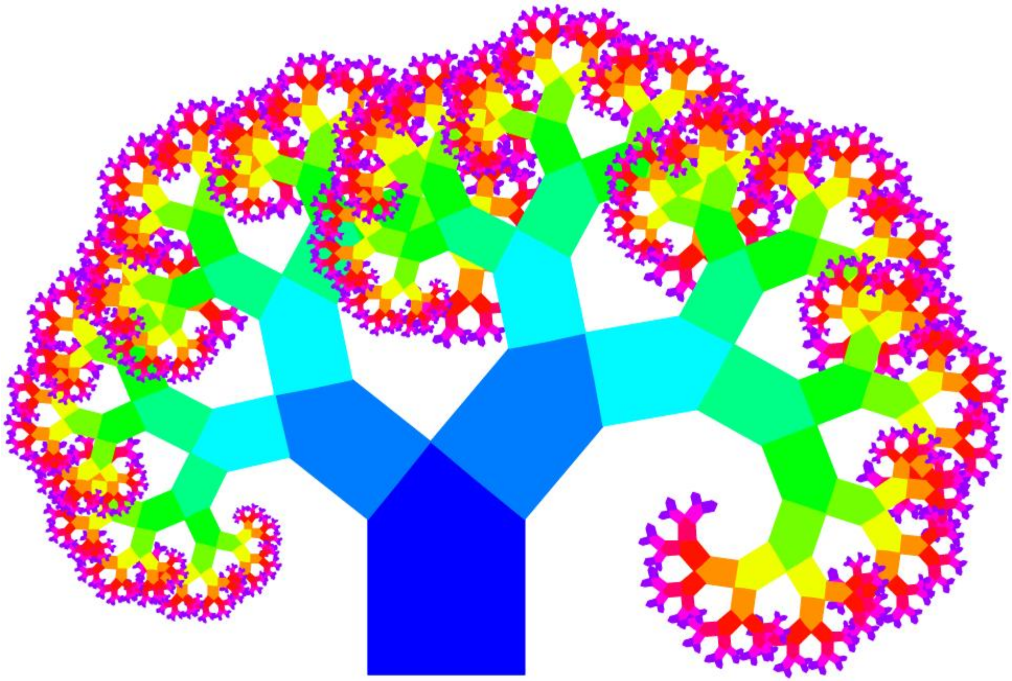
```
preds:      loop
constructs: turt pathS place circ red O C blue M R L
defuncts:   cs
fovars:     x' y' s s' act act' acts acts1 acts2

axioms:

(s = (x+2,y) & s `NOTin` cs ==> (x,y) -> (s,[M(2)]))          &
(s = (x,y+2) & s `NOTin` cs ==> (x,y) -> (s,[R,M(2),L]))      &

loop((8,12),acts,acts)                                         &
(s < (8,12) & s -> (s',acts)
   ==> (loop(s,acts1,acts2) <=== loop(s',acts1++acts,acts2)))  &

conjects:

Any acts: (loop((0,0),[],acts) &
           turt(O(blue):acts++[C,place(circ(2,red),cs)]) = z)
```



Example: Pythagorean trees (Examples/PYTREE)

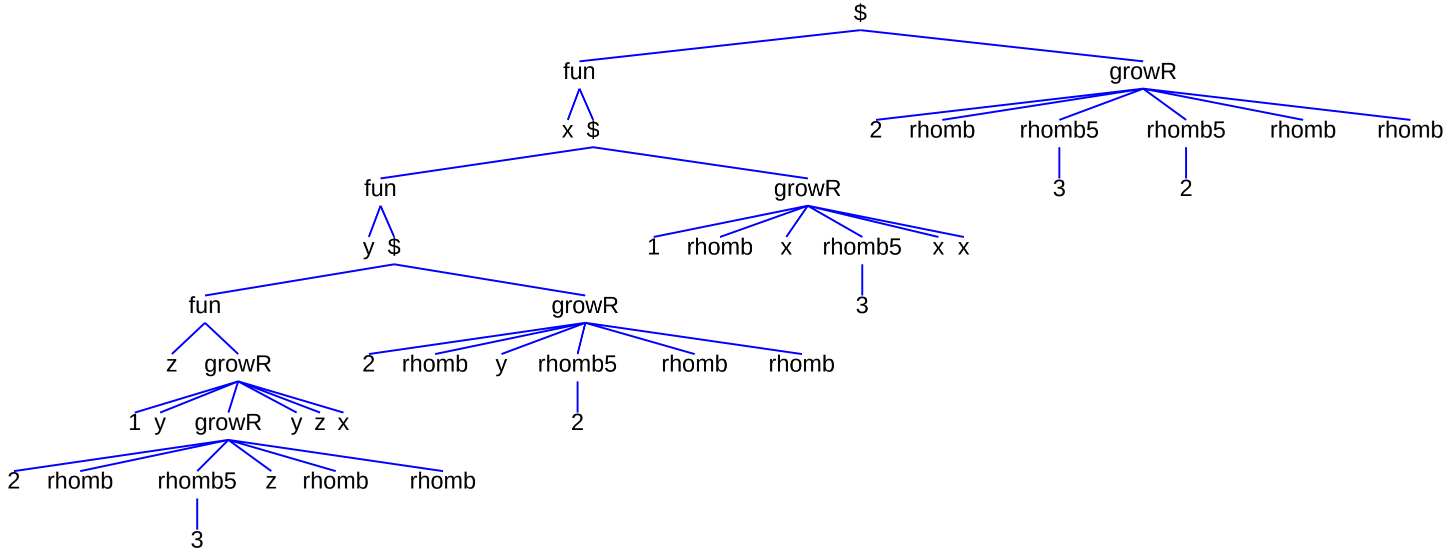
```
fovars:     x y
axioms:     trunk -> flipV(trunk)                    &
            trunk -> grow(trunk,trunk)               &
            flipV(flipV(x)) -> x
conjects:   trunk                                    <+>
            flipV(trunk)                             <+>
            grow(trunk,trunk)                        <+>
            grow(trunk,flipV(trunk))                 <+>
            pytree1                                  <+>
            pytree2                                  <+>
            file(pytree1code)
```

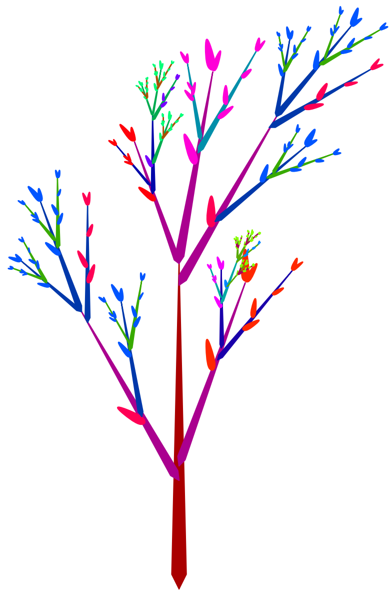
```
trunk c = path0 c 2 [p0,(-15,0),(-15,-30),(-3,-45),(15,-30),(15,0),p0]
grow c acts1 acts2 =
    Widg (trunk c):Turn 180:open:Jump 15:Turn 90:Jump 30:Turn 38.6598:
    Scale 0.640313:Jump 15:acts1++close2++open:Jump 3:Turn 90:Jump 45:
    Turn 129.806:Scale 0.781023:Jump 15:acts2++close2
fractal "pytree" n c = open:f n c++[Close]
    where f 0 c = []
          f n c = grow c acts acts where acts = f (n-1) (nextCol n c)
```

fractal "pytree" 12 blue ⇝ 

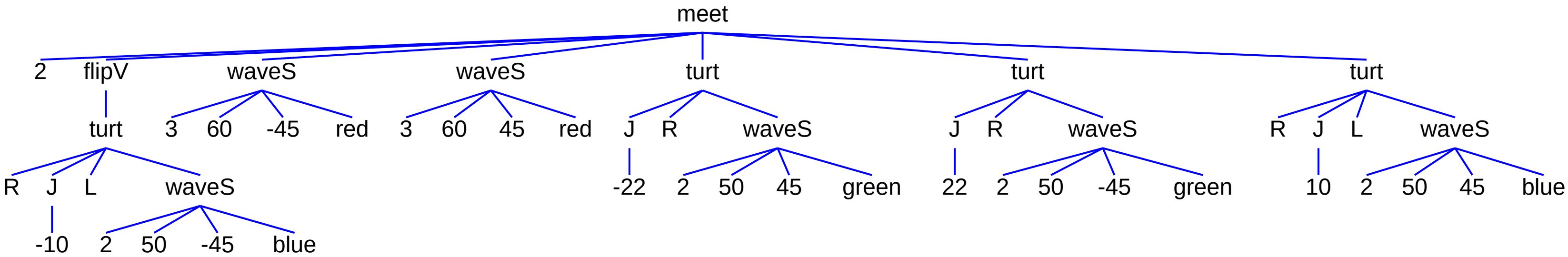
Example: Various trees (Examples/NICETREE)

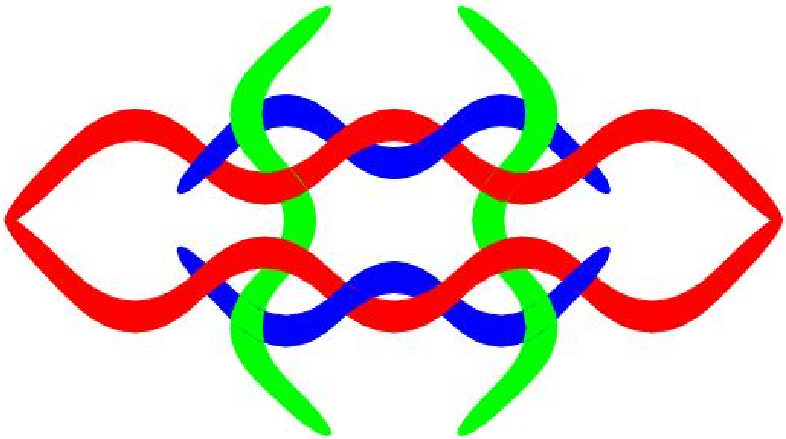
```
fovars:     n x
axioms:     rhomb -> leaf(1.5,20)                                  &
            rhomb -> leafF(15,6)                                   &
            rhomb -> turt(blosF(10,5,2,red),blosF(5,3,1,yellow))  &
            rhomb -> polyR(5,[9,3])                                &
            rhomb -> rhomb5(1)                                     &
            rhomb -> flipV(rhomb)                                  &
            rhomb -> grow5(1,rhomb,rhomb,rhomb,rhomb,rhomb)        &
            rhomb -> growR(1,rhomb,rhomb,rhomb,rhomb,rhomb)        &
            ...                                                    &
            flipV(flipV(x)) -> x
```


$
fun growR
x $ 2 rhomb rhomb5 rhomb5 rhomb rhomb
fun growR 3 2
y $ 1 rhomb x rhomb5 x x
fun growR 3
z growR 2 rhomb y rhomb5 rhomb rhomb
1 y growR y z x 2
2 rhomb rhomb5 z rhomb rhomb
3

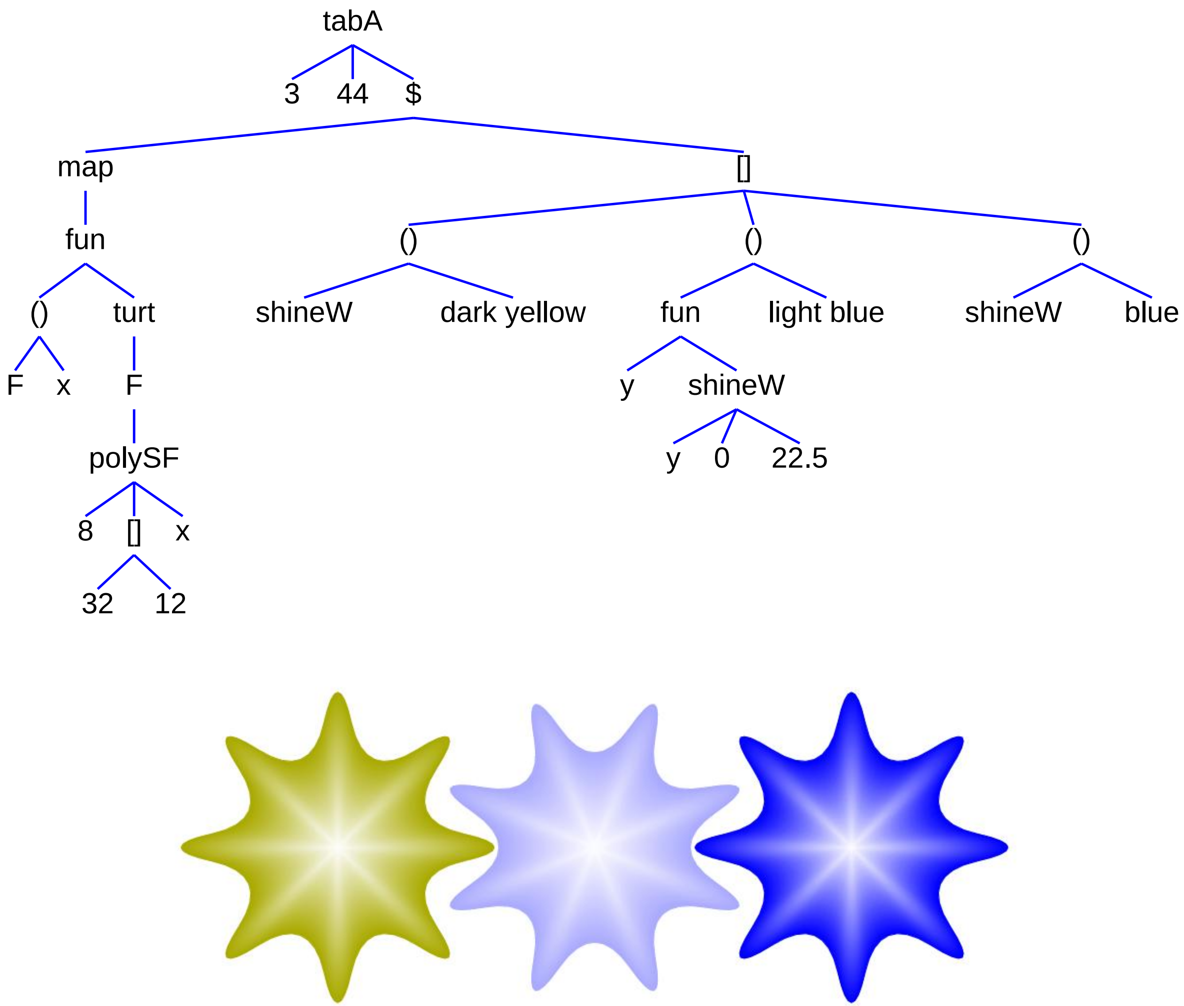


Example: Tattoo (Examples/tattoo)

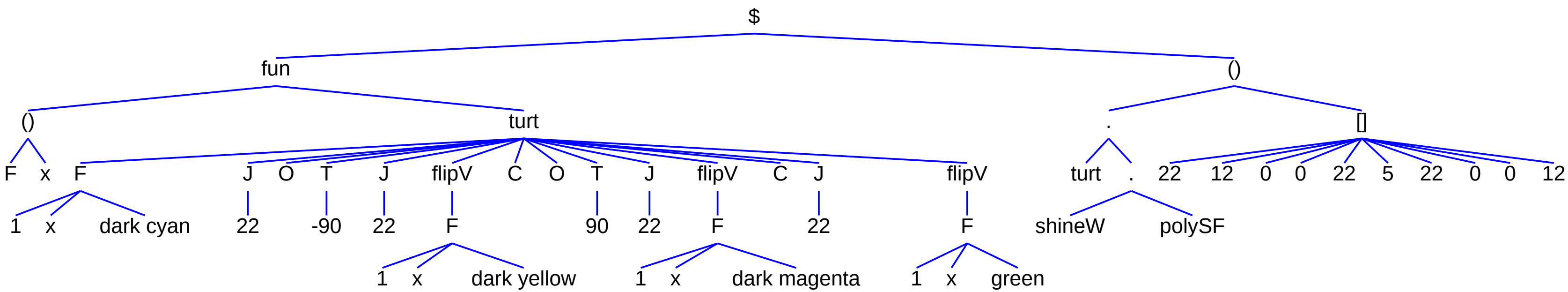


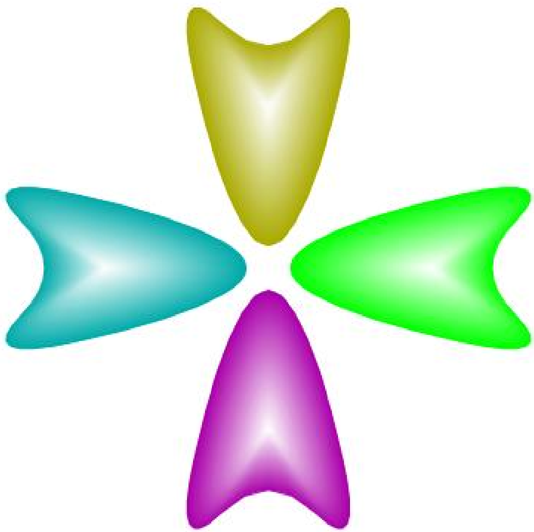


Example: Stars (Examples/shinepoly3)

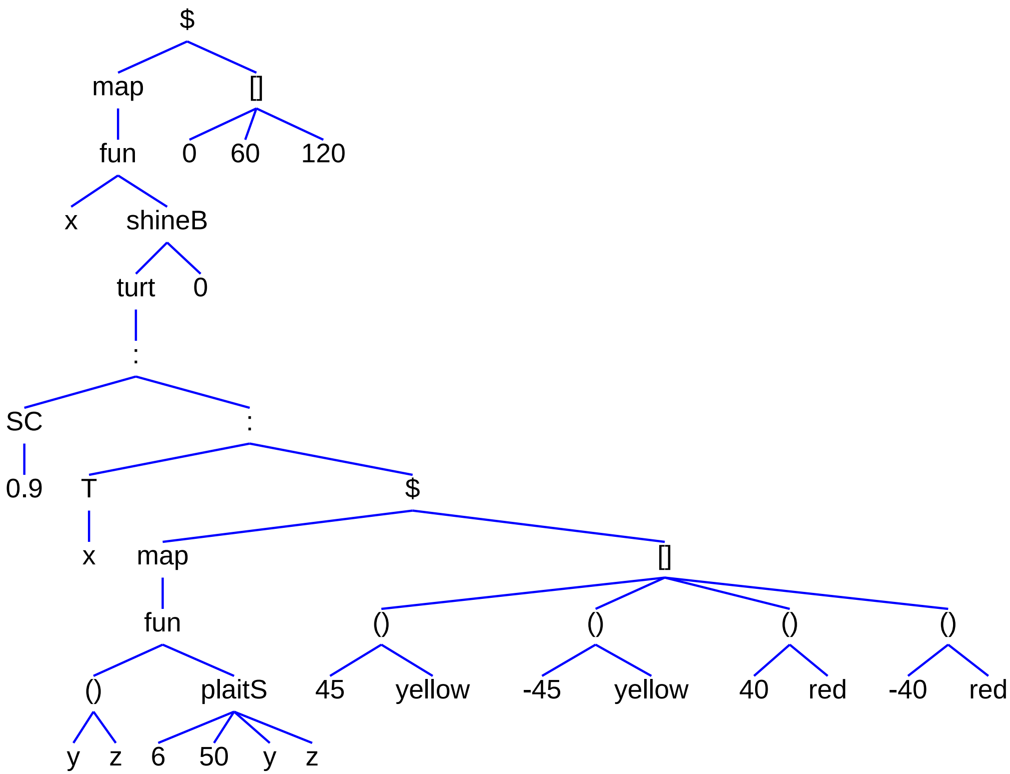


Example: Shuttles (Examples/shuttles2)

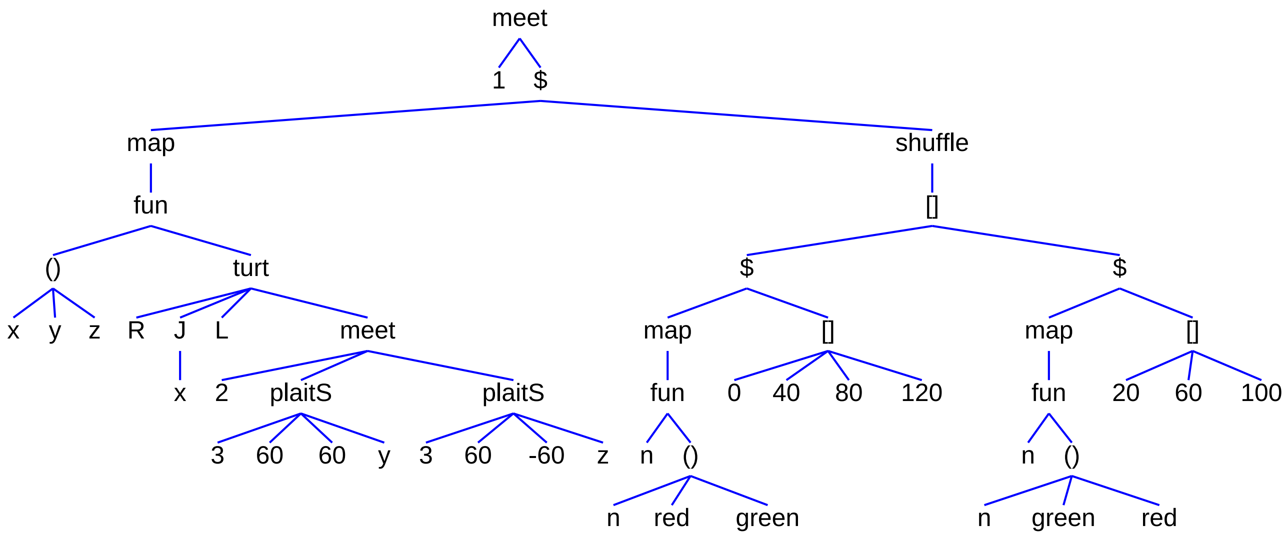


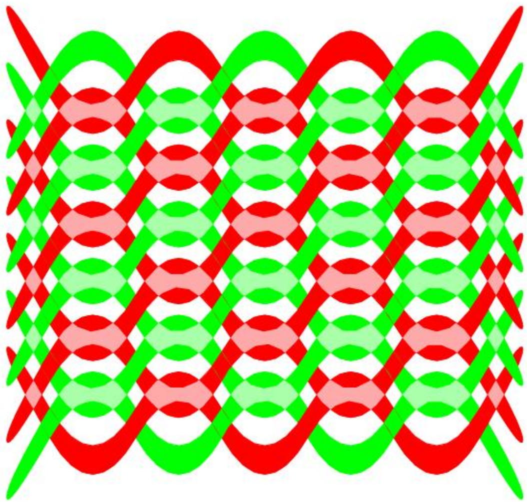


Example: Windmill (Examples/shineplait4)

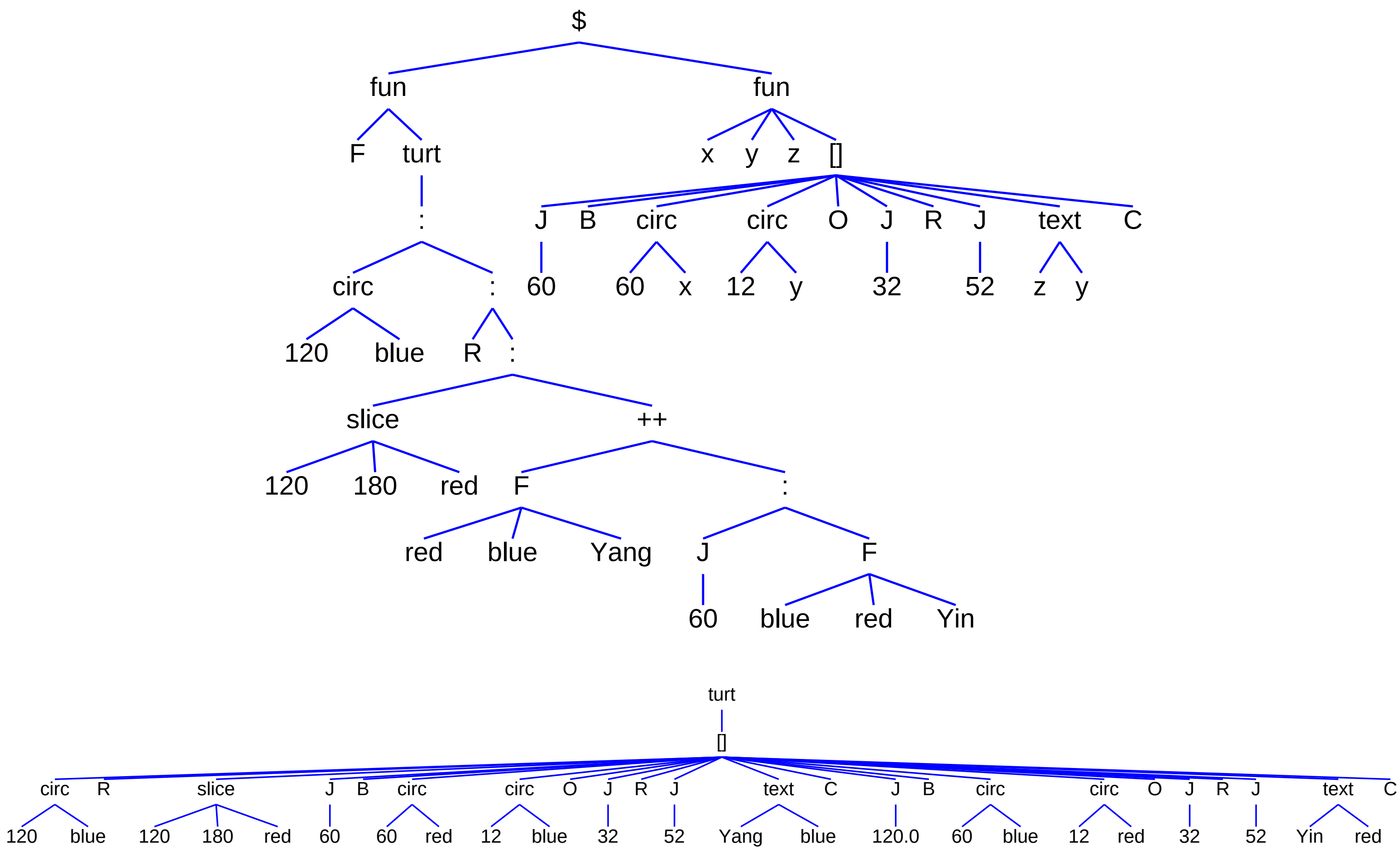


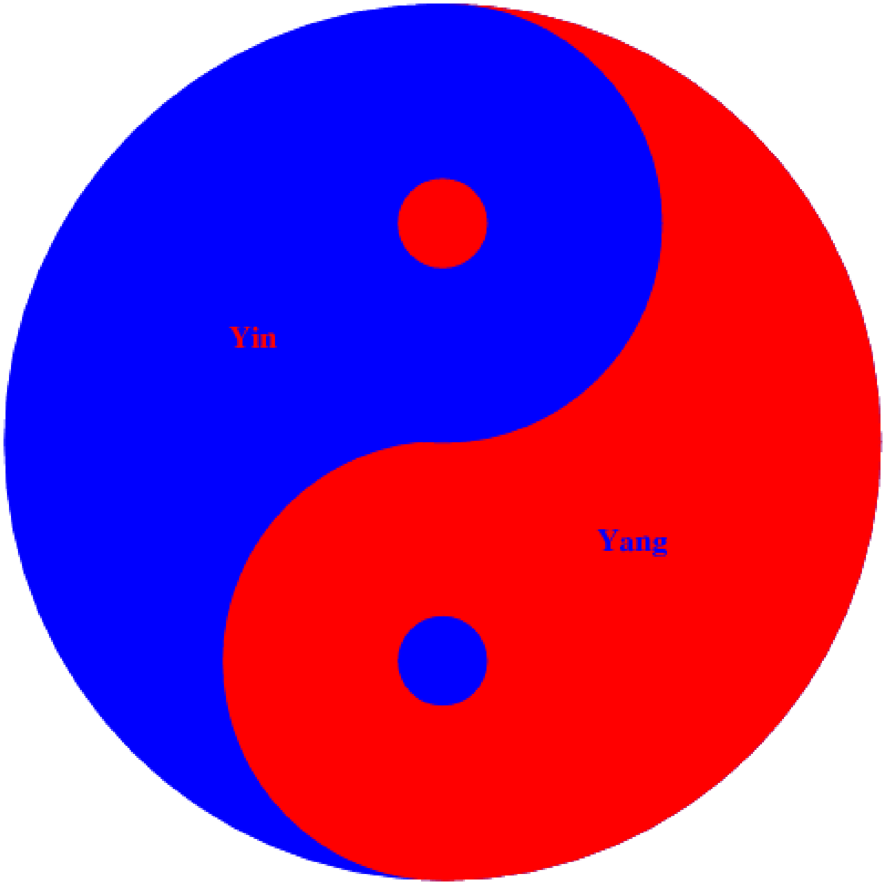
Example: Carpet (Examples/CARPET)



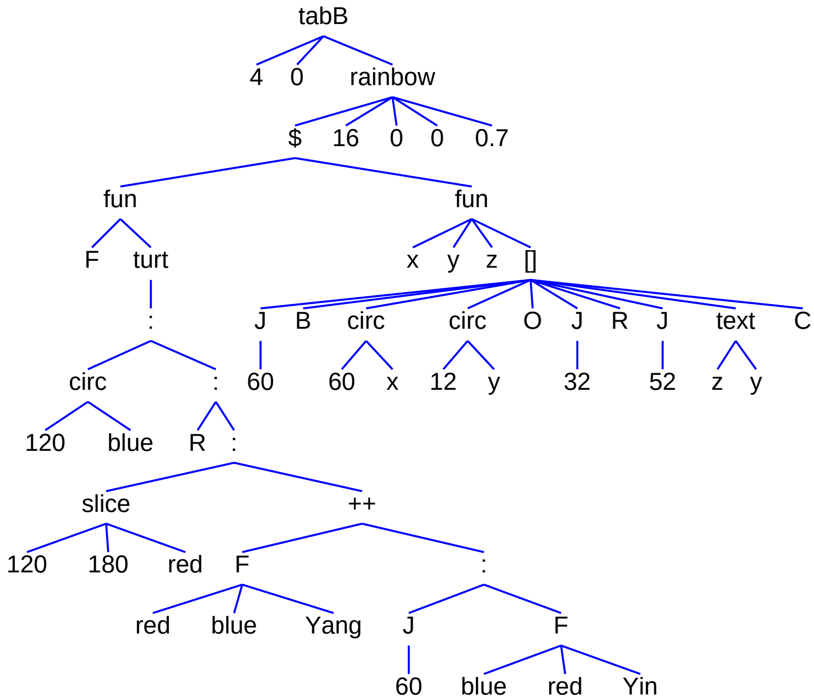


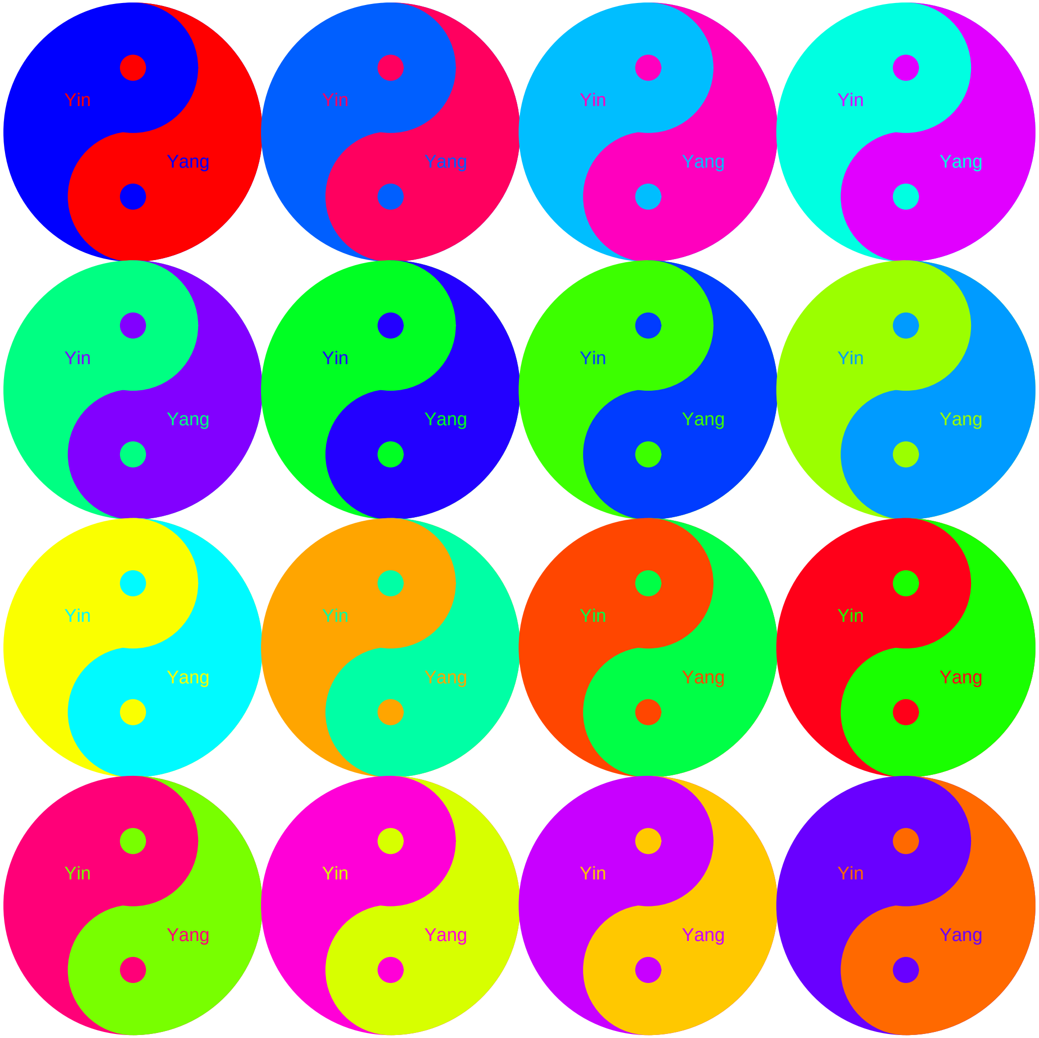
Example: Yin & Yang (Examples/yinyang)




Yin
Yang

Example: Yin & Yang (Examples/yinyangR)

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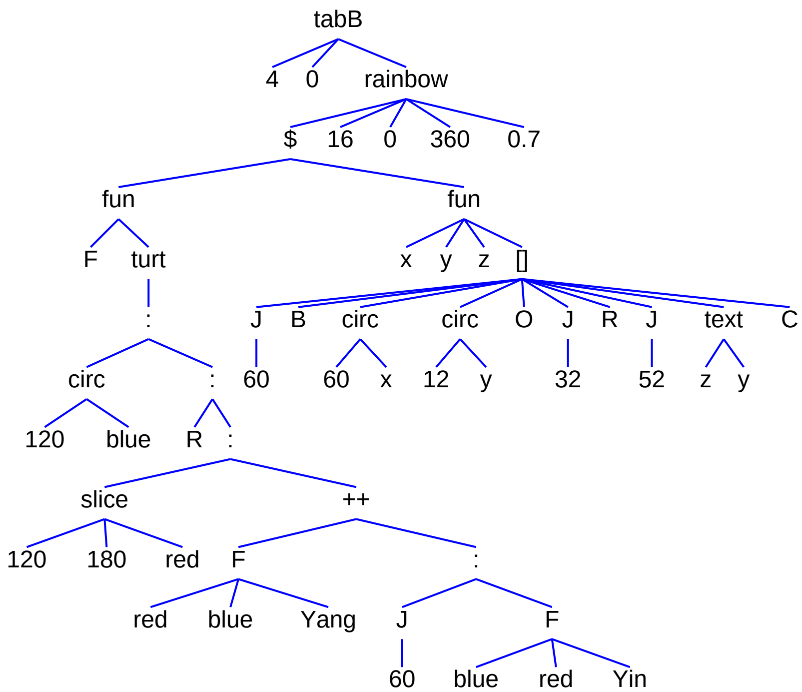
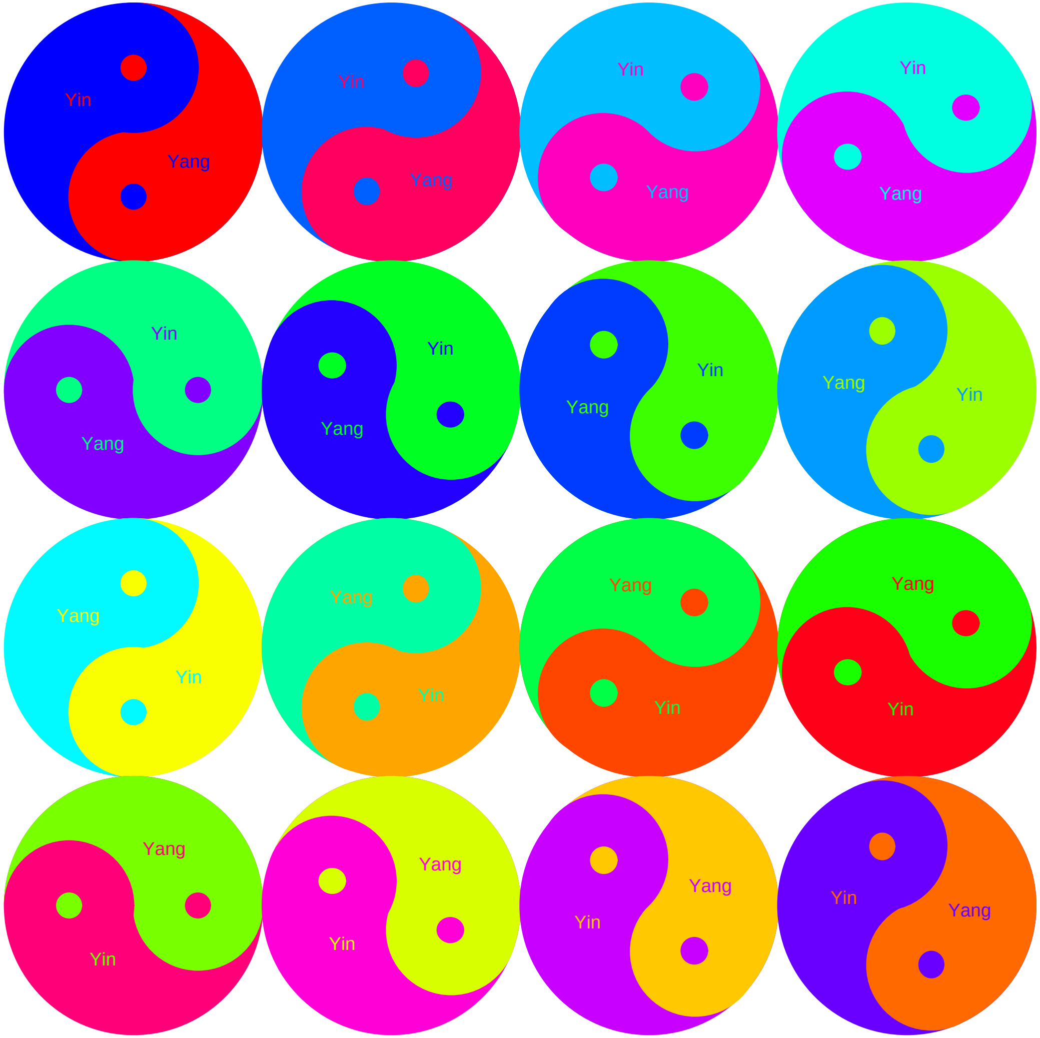
Example: Yin & Yang (Examples/yinyangRR)

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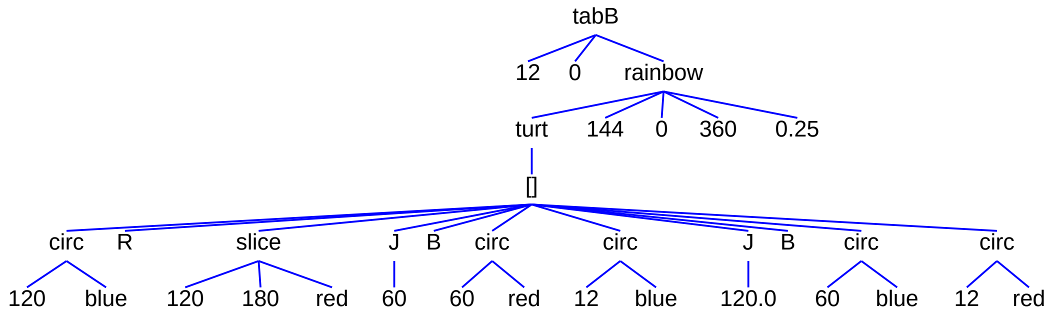
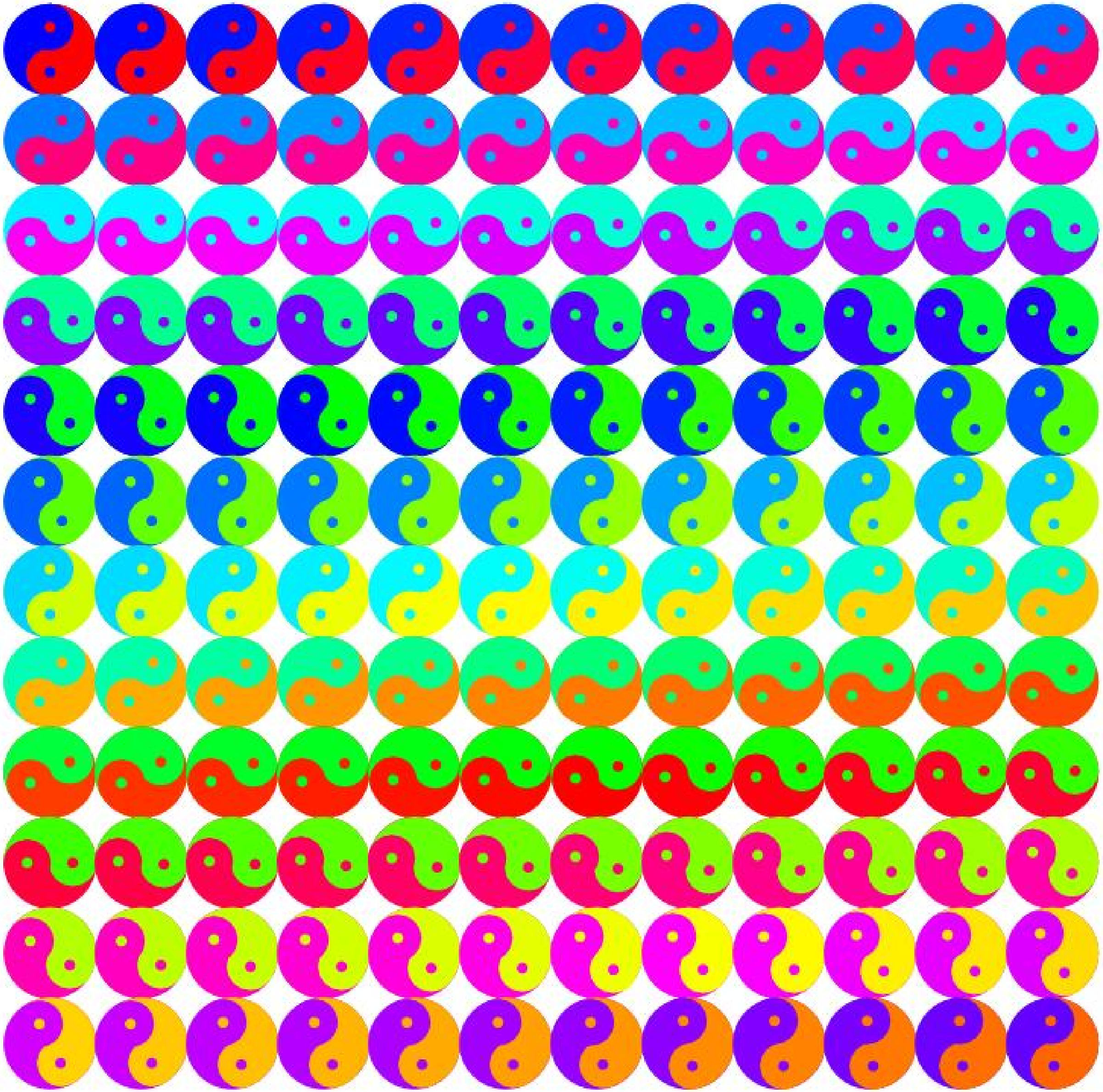
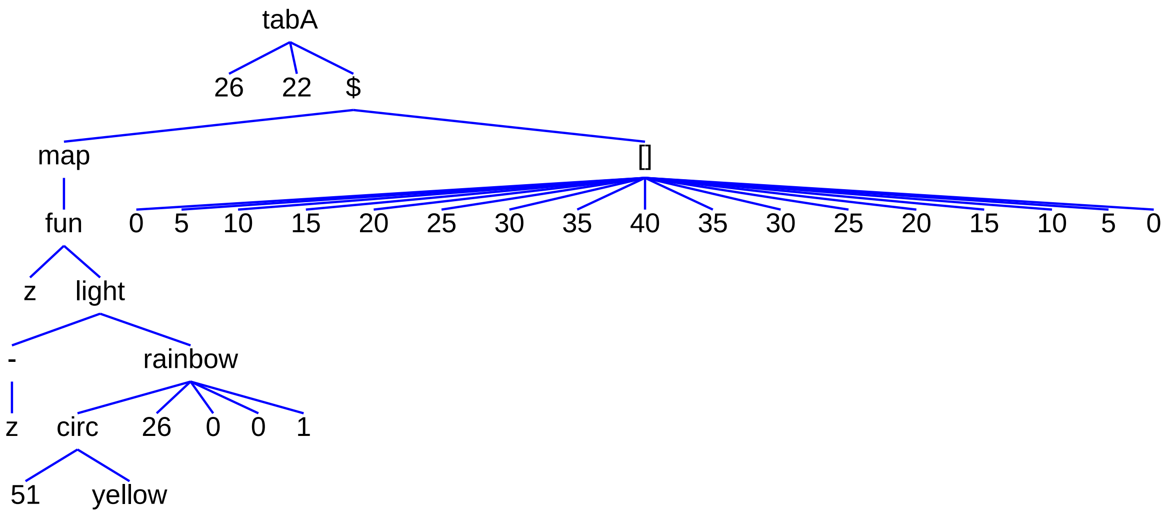
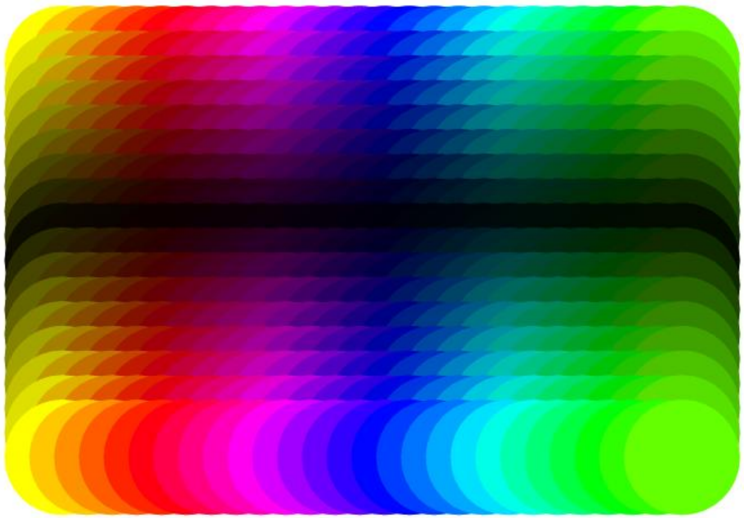
Example: Yin & Yang (Examples/yinyangRR2)



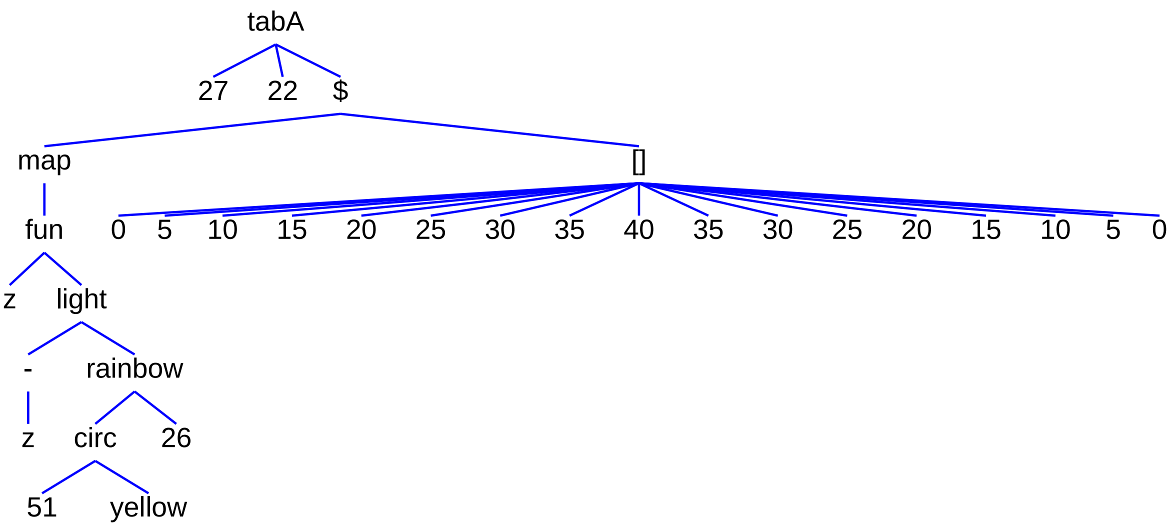


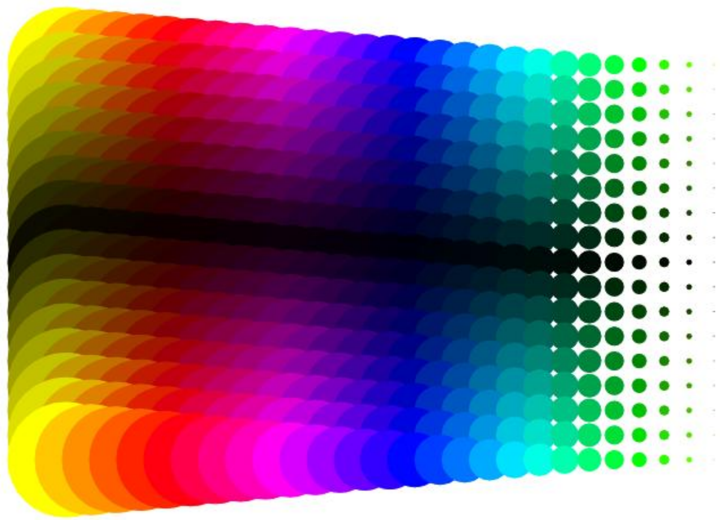
Example: Concatenation (Examples/circtab)



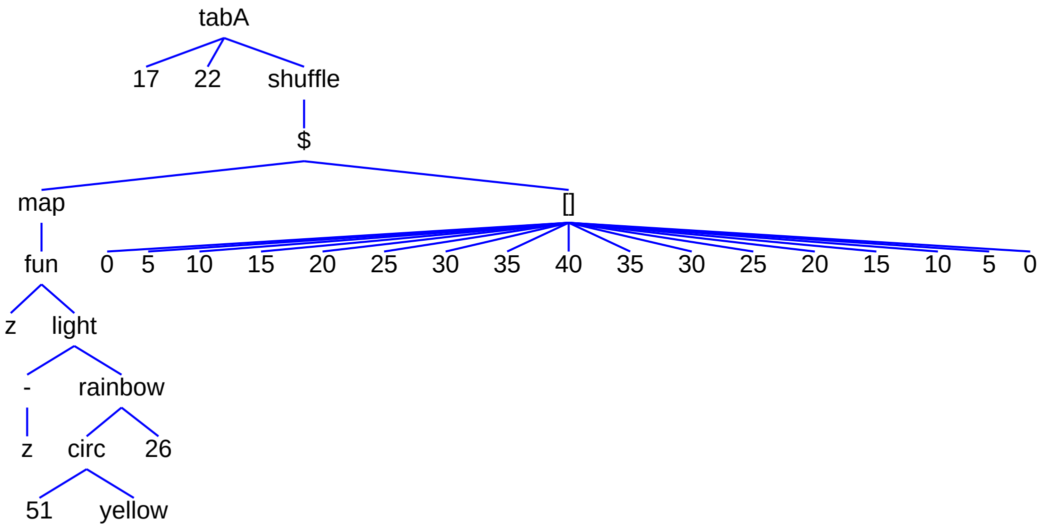


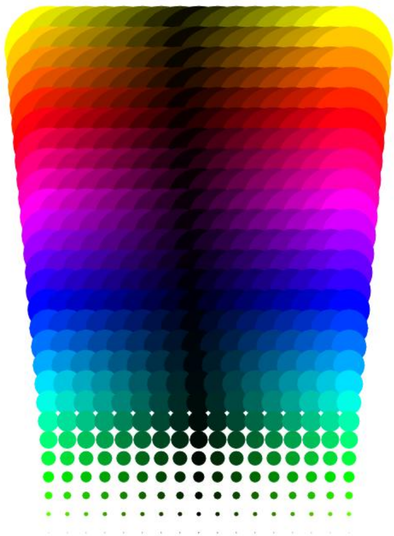
Example: Concatenation (Examples/circtabA)



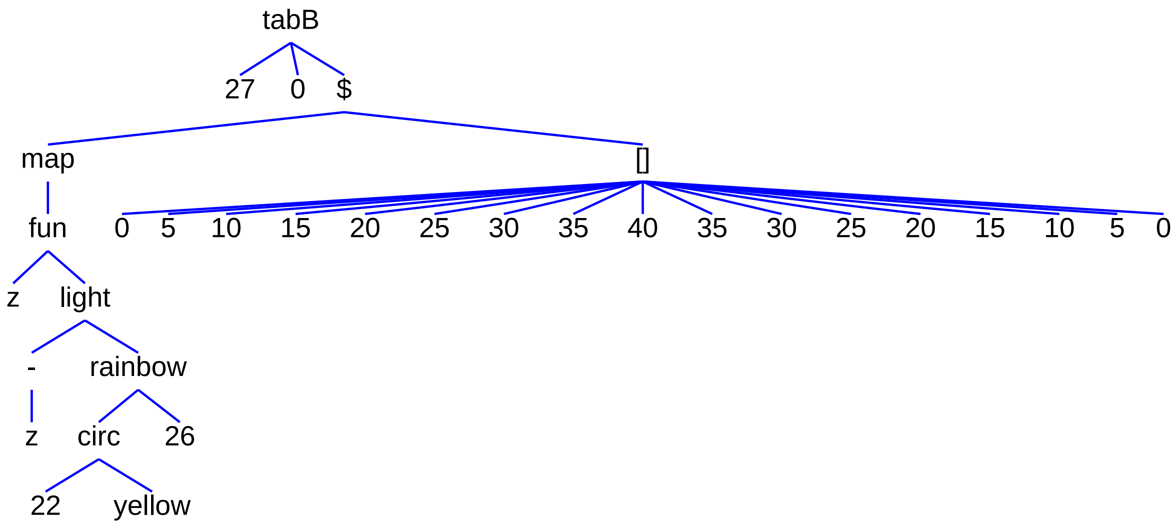


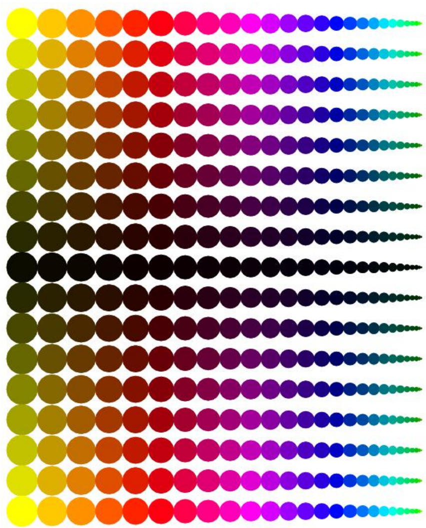
Example: Shuffling (Examples/circtabAS)



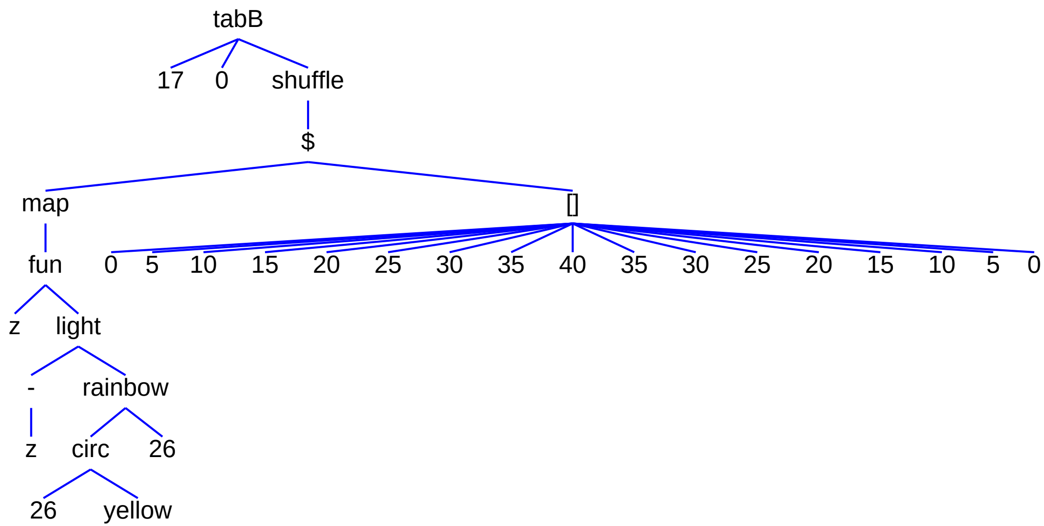


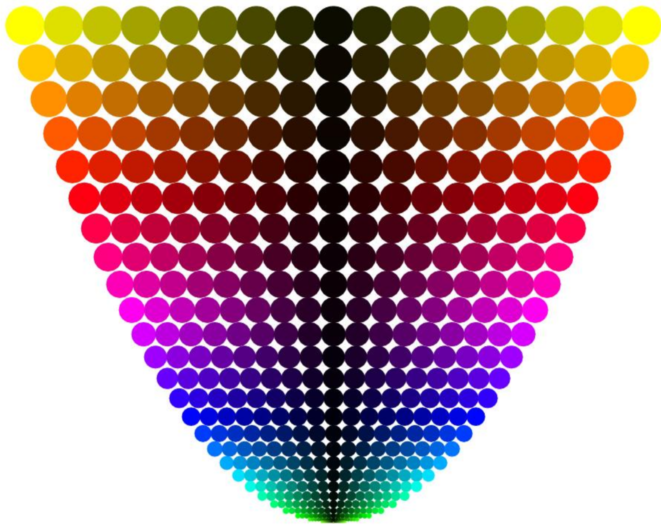
Example: Concatenation (Examples/circtabB)



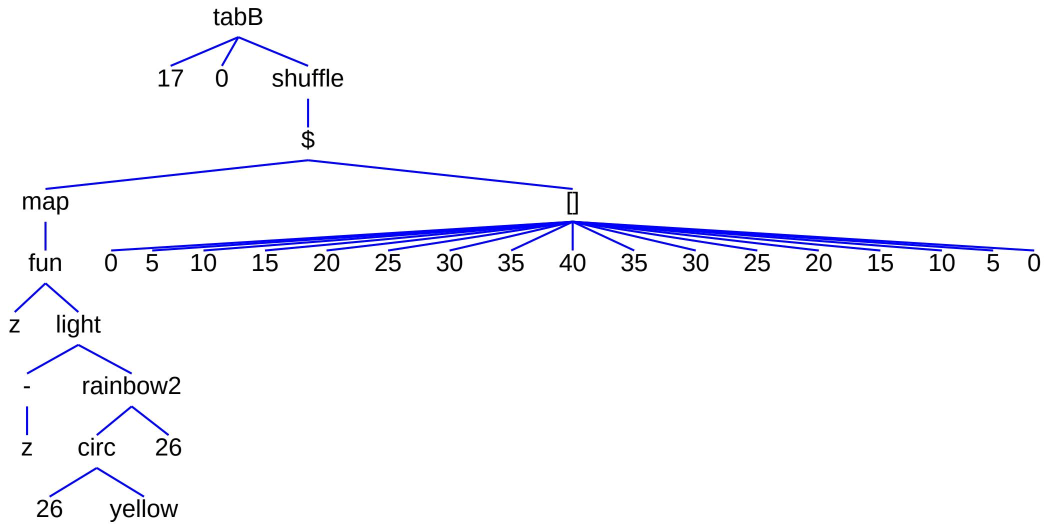


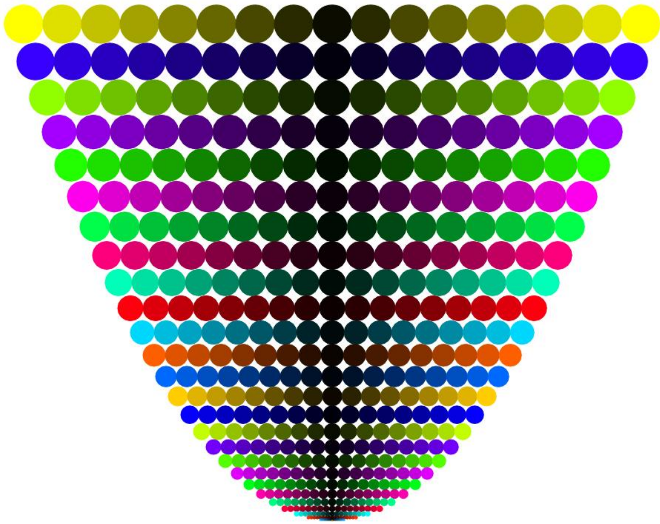
Example: Shuffling (Examples/circtabBS)





Example: Shuffling (Examples/circtabBR2S)





Example: Reversal and shuffling (Examples/circtabBSR)