ELECTRICALENGINEERING

Power consumed by a CPU

Asked 11 years ago Modified 5 years, 8 months ago Viewed 13k times

9

I think the power for a CPU with current *I* and voltage *U* is $I \cdot U$.

I wonder how the following conclusion from Wikipedia is derived?

> The power consumed by a CPU, is approximately proportional to CPU frequency, and to the square of the CPU voltage:
>
> $P = CV^2 f$
>
> (where C is capacitance, f is frequency and V is voltage).

power capacitor mosfet cpu cmos

Share Cite Follow

edited Jun 6, 2012 at 14:23
clabacchio
13.5k 4 39 80

asked Jun 5, 2012 at 3:46
Tim
349 2 4 10

2 Is it more suitable on Electronic.SE or Physics.SE or here? Please consider migration instead of closing – Tim Jun 5, 2012 at 12:05

1 `C` in that equation is just some constant, not capacitance. It kinda-sorta could be "effective capacitance", since it has the right units for capacitance, but the factor is wrong. As others have noticed, there's a `1/2` missing, but importantly, there's a load coefficient missing, related to the fraction of gates which switch each clock cycle. Call it a proportionality constant and leave it at that. – Ben Voigt Jun 5, 2012 at 18:06

1 @Ben - The line `(where C is capacitance, f is frequency and V is voltage).` *is* quoted from the WP page, though. – stevenvh Jun 5, 2012 at 18:09

3 @stevenvh, please tell me you are editing and posting a new verison of the post you just deleted, I was about to give you a +1 and a comment just asking that you remove the historical artifacts and make one clear concise post. – Kortuk Jun 5, 2012 at 19:07

1 @Kortuk - I have a much better and more detailed answer in my head, no time now, I'll post it tomorrow. – stevenvh Jun 5, 2012 at 19:58

6 Answers

Sorted by: Highest score (default)

15

MSalters answer is 80% correct. The estimate comes from the average power necessary to charge and discharge a capacitor at constant voltage, through a resistor. This is because a CPU, as well as every integrated circuit, is a big ensemble of switches, each one driving another one.

Basically you can model a stage as a MOS inverter (it can be more complicate, but the power remains the same) charging the input gate capacitance of the following one. So it all comes down to a resistor charging a capacitor, and another one discharging it (not at the same time of course :)).

The formulas that I'm going to show are taken from *Digital Integrated Circuits - A design perspective* from Rabaey, Chakandrasan, Nikolic.

Consider a capacitor charged by a MOS:

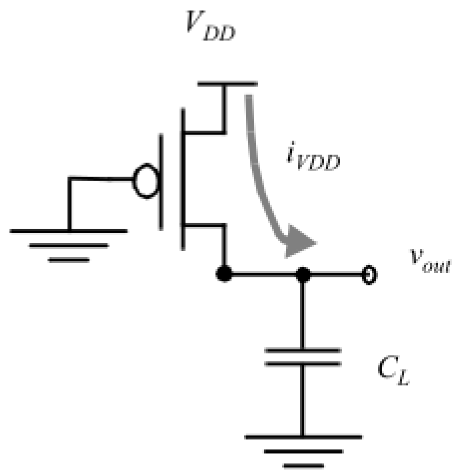


Figure 5.23 Equivalent circuit during the low-to-high transition.

the energy taken from the supply will be

$$E_{VDD} = \int_0^{\infty} i_{VDD}(t) V_{DD}\, dt = V_{DD} \int_0^{\infty} C_L \frac{dv_{out}}{dt} dt = C_L V_{DD} \int_0^{VDD} dv_{out} \\ = C_L {V_{DD}}^2$$

While the energy stored in the capacitor at the end will be

$$E_C = \int_0^{\infty} i_{VDD}(t) v_{out} dt = \ldots = \frac{C_L {V_{DD}}^2}{2}$$

Of course, we don't wait an infinite time to charge and discharge the capacitor, as Steven points out. But it's not even dependent on the resistor, because its influence is on the final voltage of the capacitor. But that aside, we want a certain voltage un the following gate before considering the transient over. So let's say that it's 95% Vdd, and we can factor it out.

So, independently on the output resistance of the MOS, it takes half of the energy that you store in the capacitor to charge it at constant voltage. The energy stored in the capacitor will be dissipated on the pMOS in the discharge phase.

If you consider that in a switching cycle there are a L->H and a H->L transition, and define f_S the frequency at which this inverter completes a cycle, you have that the power dissipation of this simple gate is:

$$P = \frac{E_{VDD}}{t} = E_{VDD} \cdot f_S = C_L V_{DD}{}^2 f_S$$

Note that if you have N gates, it's sufficient to multiply the power by N. Now, for a complex circuit the situation is slightly more complicated, as not all the gates will commute at the same frequency. You can define a parameter $\alpha < 1$ as the average fraction of gates that commute at every cycle.

So the formula becomes

$$P_{TOT} = \alpha N C_L V_{DD}{}^2 f_S$$

Small demonstration of the reason because R factors out: as Steven writes, the energy in the capacitor will be:

$$E_C = \frac{V_{DD}^2 \cdot C}{2}\left(1 - e^{\frac{-2T_{charge}}{RC}}\right)$$

so apparently, R is a factor of the energy stored in the capacitor, due to the finite charging time. But if we say that a gate must be charged to 90% Vdd in order to complete a transition, than we have a fixed ratio between Tcharge and RC, which is:

$$T_{charge} = \frac{-log(0.1)RC}{2} = kRC$$

one chosen it, we have again an energy which is independent of R.

Note that the same is obtained integrating from 0 to kRC instead of infinite, but the calculations become slightly more complicated.

Share Cite Follow edited Sep 30, 2017 at 10:39 answered Jun 5, 2012 at 19:34

clabacchio
13.5k 4 39 80

great answer except it misses any pictures for me to verify the technical accuracy with. – Kortuk Jun 5, 2012 at 19:45

Thanks! (1) Do you still mean E_{VDD} by E? (2) Where is dividing by 2 in the formula for P? (3) In the circuit, is the current direct current, or alternating current? – Tim Jun 5, 2012 at 19:52

@Tim yes, the energy in a cycle is Evdd because it's the charge necessary to charge the capacitor; the stored half will dissipated in discharge. The current is neither of both, is variable current that will have the exponential characteristic (fin-like) of charging and discharging the cap. – clabacchio Jun 5, 2012 at 20:01

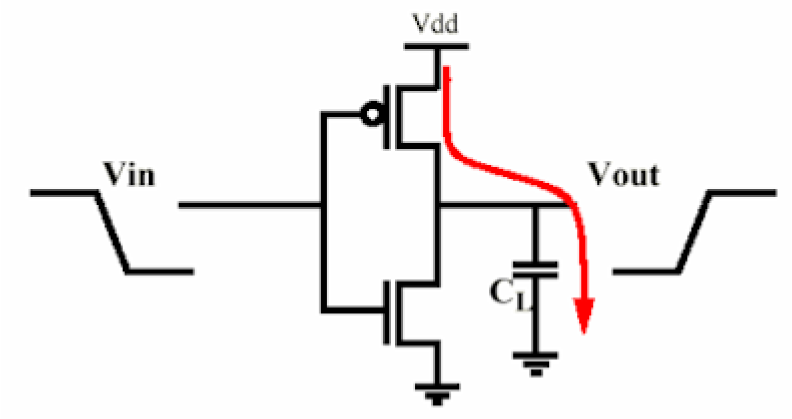
Thanks! (1) I still don't quite understand there is no dividing by 2 in formula of E_C, while there is in formula for E_VDD. (2) I looked at Wikipedia, but couldn't figure the concepts of DC and AC well enough to understand your last sentence in your comment. Could you explain them and why the current here is neither of them? – Tim Jun 5, 2012 at 20:06

@Tim Ec is divided by 2, for reason that come from physics and that you can derive from the equation (that I cut for brevity). The signal is variating over time, hence (t), and is neither AC or DC, but eventually more similar to the former. It's unpredictable since depends on the operation of the gate. – clabacchio Jun 5, 2012 at 20:27

10

I posted another answer before, but it wasn't good, also improper language, and I want to apologize to markrages.

I've been thinking this over and I think that my problem here is that to me the quoted text suggests that the capacitance is responsible for the power dissipation. Which isn't so. **It's resistive.**



Voilà une paire complémentaire MOS. The MOSFETs together with the capacitor form a charge pump. When the output goes high the P-MOSFET conducts it will charge the capacitor from V_{DD}, when it goes low the capacitor will be discharged to V_{SS} through the N-MOSFET. Both MOSFETs have an on resistance which makes that they dissipate power dureing the charging/discharging. Now Ben suggests the resistance value doesn't matter, while I say the opposite. Well, we're both right, so also both wrong.

First Ben: both capacitor voltage and current vary exponentially during charging. The current

$$I = \frac{V_{DD}}{R} e^{\frac{-t}{RC}}$$

$$P = I^2 R = \frac{V_{DD}^2}{R} e^{\frac{-2t}{RC}}$$

and integrating over time gives us energy dissipated in the resistor:

$$U = \frac{V_{DD}^2}{R} \int_{t=0}^{\infty} e^{\frac{-2t}{RC}} \mathrm{d}t = \frac{V_{DD}^2}{R} \frac{RC}{2} = \frac{V_{DD}^2 \cdot C}{2}$$

which is indeed independent of R. So it looks like Ben is right.

Now me. "Infinity!? Are you out of your mind? This job has to be done in 0.3ns!" At school we seemed to have ages to charge a capacitor. If t is finite we get

$$U = \frac{V_{DD}^2}{R} \int_{t=0}^{t1} e^{\frac{-2t}{RC}} \mathrm{d}t = \frac{V_{DD}^2 \cdot C}{2} \left(1 - e^{\frac{-2t}{RC}} \right)$$

and then R is still a factor.
In practice it won't matter however since $RC \ll T_{CLOCK}$.

I cut some corners here assuming that R is constant. But it's not easy. $R(t)$ depends on the gate's voltage, which depends on the gate's capacitance's charge curve, which depends on R. Easy if it's a linear system, but this isn't, so I chose for the exponential as an approximation.

Conclusion: while the dissipation is expressed in terms of C it happens in R, which on first sight seems to have nothing to do with it.

What can be done about it? Lowering R is no use. Can we decrease C? It would help to decrease the charge being drained from V_{DD} to V_{SS} , but we need C. The gate capacitance is what makes a MOSFET work!

What if R were zero, absolute zero? Then we wouldn't have dissipation, right? In that case switching would give an infinite di/dt, which would cause the switching energy to be radiated instead of dissipated, but the amount of energy would be the same. Your CPU would get less hot, but would be a wideband 100W RF noise transmitter.

Share Cite Follow

edited Jun 6, 2012 at 6:47
clabacchio
13.5k 4 39 80

answered Jun 6, 2012 at 5:55
stevenvh
145k 21 454 667

No agree :). Your paragraph about the finite time is right, but it assumes that we fix the time that we give for the transition, while what is fixed is the voltage at which we assume the transition finished. So the resistor goes away again, because it determines the maximum speed of the CPU, and it's why is better to scale down the capacitance (one of the reasons) – clabacchio Jun 6, 2012 at 6:56

Note that I usually leave a big margin for errors in my answers, but this is - almost - copied from a very expensive book :). I trust its (conceptual) accuracy more than any other, typos apart. – clabacchio Jun 6, 2012 at 6:59

@clabacchio - Ben is Ben Voigt, who commented on my other answer. The resistor goes away again because of the short RC time. But there's no reason why you shouldn't break off the charging at a higher clock speed if a 90% charge would be sufficient. My very expensive book is my head (sometimes with the help of Mathematica) :-) – stevenvh Jun 6, 2012 at 7:07

My reasoning is different: I say that it's not because t>>RC (it would be a waste of resources), but that t=kRC, where k is a design constraint that ensures enough voltage swing to be robust. If you always use the same k, then that factor goes away (also with rhyme). The thing about the book was to make clear that I don't support my claim just for arrogance – clabacchio Jun 6, 2012 at 7:11

Better the way it is :-). I even hid the content from the +10k rep users. I think Kortuk was too positive about it. About the RC, I think we're saying the same thing. If your k=2.3 then you end up at my 90%. – stevenvh Jun 6, 2012 at 7:18

5

The main power draw in CPU's is caused by the charging and discharging of capacitors during calculations. These electrical charges are dissipated in resistors, turning the associated electrical energy into heat.

The amount of energy in each capacitor is $C_i/2 \cdot V^2$. If this capacitor is charged and discharged *f* times per second, the energy going in and out is $C_i/2 \cdot V^2 \cdot f$. Sum for all switching capacitors and substituting $C = \Sigma C_i/2$, you get $C \cdot V^2 \cdot f$

Share Cite Follow

answered Jun 5, 2012 at 9:49

MSalters
561 3 7

Thanks! WHy C = ΣCi/2, not C = ΣCi? In other words, how do you make dividing by 2 disappear? – Tim Jun 5, 2012 at 11:25

1 @Tim: Pure a matter of definition. In practice, the *C* value of a CPU is measured directly. – MSalters Jun 5, 2012 at 11:52

In series, 1/C = \sum_i 1/C_i; in parallel, C = \sum_i, C_i. Neither is your formula C = 1/2 \time \sum_i C_i. This is my confusion. – Tim Jun 5, 2012 at 11:55

1 @Tim: That's assuming the capacitors are hardwired in parallel anyway (`sum_i`). With all the gates switching on a CPU, this isn't a given anyway. But the main reason I dropped the 1/2 is because I'm using an engineering approach, not a pure physics approach. A CPU isn't acting as a capacitor anyway. The `C` value isn't related to `(dV/dt)/I` ; it's merely an observed contant relating *P*, *V* and *f*. – MSalters Jun 5, 2012 at 12:07

@Tim: If you keep the 1/2, it will just cancel out, you'll just get a different value for capacitance. For example, solve for C, you get either `V^2·F/P` or `(1/2)·V^2·F/P` . Now, let's say you change the voltage, frequency, and power. With the first equation, you get `V1^2·F1/P1 = V2^2·F2/P2` and in the other case you get `(1/2)V1^2·F1/P1 = (1/2)V2^2·F2/P2` which is the same thing. – David Schwartz Jun 5, 2012 at 12:07

3

[Capacitance](https://en.wikipedia.org/wiki/Capacitance) is measured in [Farads](https://en.wikipedia.org/wiki/Farad), which is [Coulombs](https://en.wikipedia.org/wiki/Coulomb) per Volt.

Frequency is measured in Hertz, which is units per second.

Reducing we get Coulomb-Volts per second, more commonly known as **[Watts](https://en.wikipedia.org/wiki/Watt)**, a unit of power.

Share Cite Follow

answered Jun 5, 2012 at 4:35

Ignacio Vazquez-Abrams
48.3k 4 72 102

1 This does not really answer why though. – Kortuk Jun 6, 2012 at 10:59

0

Generally the current consumed by a device is proportional to the voltage. Since the power is voltage*current, the power becomes proportional to the square of the voltage.

Share Cite Follow

answered Jun 5, 2012 at 3:51
Mark Ransom
349 1 7

1 That is far from "generally". In fact there's a special name for such devices: Ohmic loads (From Ohm's law, V = I · R) – MSalters Jun 5, 2012 at 9:42

0

Your equation is correct for the power drawn at any particular instant. But the current drawn by the CPU is not constant. The CPU is running at some frequency and changing states on a regular basis. It uses a certain amount of power for each state change.

If you understand I as the RMS current (the square root of the average of the square of the current) then your equation is correct. Putting these together, you get:

V · I(Rms) = C · V^2 · F
I(Rms) = C · V · F

So the average current varies linearly with the voltage, frequency, and capacitance. The power varies with the square of the DC supply voltage .

Share Cite Follow

edited Sep 30, 2017 at 11:42
Autistic
13.9k 2 27 64

answered Jun 5, 2012 at 8:34
David Schwartz
884 5 9

Thanks! My question is why V · I(Rms) = C · V^2 · F? Do you have some references for that formula? – Tim Jun 5, 2012 at 11:27

I'm don't quite get what you want to know. – David Schwartz Jun 5, 2012 at 11:36

Why is V · I(Rms) = C · V^2 · F true? WHere do you learn it from? – Tim Jun 5, 2012 at 11:47

It's true because it combines two power equations, each of which is correct and which measure the same thing. That `I` has to be RMS power for `P=V·I` to give you average power can be trivially proven with calculus from `P = I^2·R` . – David Schwartz Jun 5, 2012 at 11:54

1 @Tim: If you divide by two, you just have to double the capacitance and the equation works the same. If you want to divide by two, you can. You'll just use capacitance numbers that are double what everyone else uses and you will get the same answers. (We use 12 inch feet, but you could

use 6 inch feet if you wanted to. You can still design cars, buildings, and bridges. You'll just call them different sizes from everyone else.) – David Schwartz Jun 5, 2012 at 12:34