Standard Random Variable

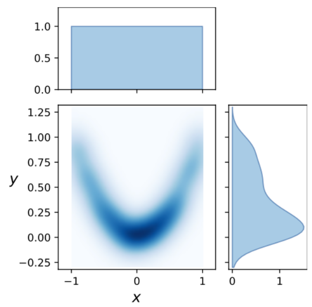
Notation: P{X} / P(X) , E[X] / E(X), assume ***independent*** *and identical distribution (iid).*

Discrete Distribution $X \in N$	**PMF** *Prob. Mass Function* *Valid* $i.\, \forall x, P\{X = k\} \geq 0$ $ii.\, \sum_0^\infty P\{X = k\} = 1$, *density sum to 1*	**CDF** *Cumulative Distribution* $F_X(x) = P\{X \leq \lfloor x \rfloor\}$ $= 1 - P\{X > x\}$, *complement*	$\boldsymbol{E[X]} =$ $\sum_{i=0}^\infty x_i P\{X = x_i\}$	$\boldsymbol{Var[X]} =$ $E[X^2] - E[X]^2$ LOTUS, $E[g(X)] =$ $\sum g(x)\boldsymbol{P}\{X = k\}$
Bernoulli trial $X \sim Bern(p)$	$P\{X\} = p,\ P\{\bar{X}\} = q$	$q = (1 - p)$	p	pq
Binomial *with replace* $X \sim Bin(n, p)$ #successes in n $Bern(p)$ trials $X \sim Bin(1, p)$, *(0-1) distribution* if *n=1*	$P\{X = k\} = \binom{n}{k} p^k q^{n-k}$ $P\{X = k\} = p^k q^{1-k}$	*Normal Approximation* *Poisson* $n \to \infty,\ p \to 0,\ \lambda = np$ *is moderate*	np	npq
Geometric / Negative Binomial $X \sim Geom(p),\ X \sim NegBin(r,\ p)$ …. in n $Bern(p)$ trials until 1^{st}/**r** success(es)	$P\{X = k\} = q^k p$, $k = \#failures$ $P\{X = k\} = \binom{\mathbf{n-1}}{\mathbf{r-1}} q^{k-1} p^r$, $k = \#trials$	$1 - q^{k+1},\ x > 0$ *Exp Approximation*	$\frac{q}{p}, \ \frac{1 \cdot \boldsymbol{r}}{p}$	$\frac{(1-p) \cdot \boldsymbol{r}}{p^2}$
Poisson *memoryless* $X \sim Pois(\lambda),\ \lambda = np$ #events in a fixed interval of time t	$P\{X = k\} = \frac{e^{-\lambda}\lambda^k}{k!}$ $Pois(\lambda t)$ *given* $Exp(\lambda)$ *as waiting time interval*	*by def*	λ	λ
[Negative] HyperGeometric *no replace* $X \sim NHGemo(w, b, n), total\ N = w + b$ #successes in n draws #successes until $\boldsymbol{n}$ **failures**	$P\{X = k\} = \frac{\binom{w}{k}\binom{b}{n-k}}{\binom{w+b}{n}}$	/	$np = n\frac{w}{N}$ $n\frac{w}{\boldsymbol{b+1}}$	$\frac{N-n}{N-1} npq$
Joint Prob	$P_{ij} = P\left\{X = x_i,\ Y = y_j\right\}$	$F(x, y) = \sum_0^{\lfloor x_i \rfloor} \sum_0^{\lfloor y_j \rfloor} P_{ij}$ $= P\{X \leq x_i, Y \leq y_j\}$	*Valid* $\sum_{i=0}^\infty \sum_{j=0}^\infty P_{ij} = 1$	
Marginal Prob marginalize over another variable	$P\{X = x_i\} = \sum_y P\{X = x_i,\ Y = y_j\}$ $= \sum_{j=1}^\infty P_{ij}$	$F_X(x) = F(x,\ \infty)$ $= \sum_{x_i \leq x} \sum_{j=0}^\infty P_{ij}$	$\forall i, j.\ P_{ij} \geq 0$	

Marginals for X $g(x) = \sum_y f(x, y)$

X \ Y	1	2	3	
1	0.32	0.03	0.01	0.36
2	0.06	0.24	0.02	0.32
3	0.02	0.03	0.27	0.32
Marginals for Y $h(y) = \sum_x f(x, y)$	0.40	0.30	0.30	1

$\sum_x \sum_y f(x, y) = 1$



the joint density function

Joint Prob	$f(x,y) = \frac{\partial}{\partial x \partial y} F(x,y)$	$F(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(u, v) du dv$ $= \iint_B f(x,y) dx dy$	Valid $F(-\infty, \infty) = 1$	
Marginal Prob	$f_X(x) = \int_{-\infty}^{\infty} f(x,y) dy$	$F_X(x) = F(x, \infty),\ y \to \infty$ $= \int_{-\infty}^{x} f_X(x) dx$	$\forall x, y.\ f(x,y) \geq 0$	
Continuous Distribution $X \in R$	**PDF** *Prob Density Function* Valid $i. \forall x,\ f_X(x) \geq 0$ $ii. \int_{-\infty}^{\infty} f_X(x) dx = 1$, *density sum to 1*	**CDF** *Cumulative Distribution* $F_X(x) = \int_{-\infty}^{x} f_X(t) dt$, *complement* $f_X(x) = \frac{d F_X(t)}{dt}$	$\boldsymbol{E[X]} =$ $\int_{-\infty}^{\infty} x \cdot f_X(x) dx$	$\boldsymbol{Var[X]}$ LOTUS, $E[g(X)] =$ $\int_{-\infty}^{\infty} g(x) f_X(x) dx$
Uniform $X \sim U(a,b)$ a completely random point in $[a,b]$	$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & otherwise \end{cases}$	$F_X(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x > b \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential $X \sim Exp(\lambda)$, $rate\ \lambda = \frac{1}{\theta}$ ***memoryless*** waiting time between 2 successive events	$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & otherwise \end{cases}$	$F_X(x) = \begin{cases} 1 - e^{-\lambda x}, & x > 0 \\ 0, & otherwise \end{cases}$	$\theta = \frac{1}{\lambda}$	$\theta^2 = \frac{1}{\lambda^2}$
Normal / Gaussian $X \sim N(0,1)$ *Standard* $X \sim N(\mu, \sigma^2)$	$f_X(x) = ce^{-\frac{x^2}{2}}$ $c = \frac{1}{\sqrt{2\pi}}$ $f_X(x) = ce^{-\frac{(\frac{x-\mu}{\sigma})^2}{2}}$ $c = \frac{1}{\sqrt{2\pi}\,\sigma}$	$\Phi_X(x) = c \int_{-\infty}^{x} e^{-\frac{x^2}{2}} dx$ $\Phi_{\frac{X-\mu}{\sigma}}(z)$	μ	σ^2
Beta $X \sim Beta(\alpha, \beta)$	$f_X(x) = \begin{cases} \frac{1}{B} x^{\alpha-1}(1-x)^{\beta-1}, & 0 \leq x \leq 1 \\ 0, & otherwise \end{cases}$	$B = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Gamma $X \sim \Gamma(k, 1)$ $X \sim \Gamma(k, \lambda)$	$f_X(x) = \frac{1}{\Gamma(k)} x^{k-1} e^{-x}$ $, x > 0$ $f_X(x) = \frac{1}{\Gamma(k)} \frac{(\lambda x)^k}{x} e^{-\lambda x}$ $, x > 0$	$\Gamma(k) = (k-1)!$ $, k \in N$ $\Gamma(k) = \int_0^{\infty} e^{-x} x^{k-1} dx$ $, k \in R^+$	$\frac{k}{\lambda}$	$\frac{k}{\lambda^2}$