LL(1) CFG $\langle T, NT, P, S\rangle$

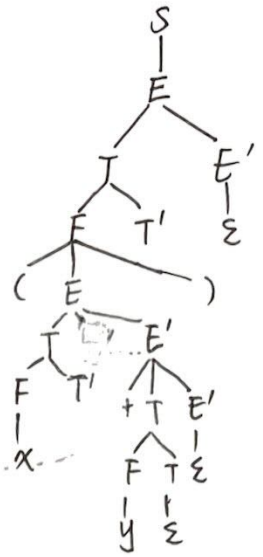
exp. tokens: $(x+y)\$$

P: $S \to E\$$, $E \to TE'$, $E' \to +TE' \mid \varepsilon$, $T \to FT'$, $T' \to *FT' \mid \varepsilon$, $F \to (E) \mid id$

Notation: $a \in T$, $A \in NT$, $\alpha, \beta \in (T \cup NT)^*$, $w \in T^*$

input (remaining)	stack (dfs)	action (predict / match)	reason
$(x+y)\$$	S	$M[S,(] = \{E\$\}$	$S \to E\$$
	$E\$$	$M[E,(] = \{TE'\}$	$E \to TE'$
	$TE'\$$	$M[T,(] = \{FT'\}$	$T \to FT'$
	$FT'E'\$$	$M[F,(] = \{(E)\}$	$F \to (E) \mid id$
$(x+y)\$$	$(E)T'E'\$$	match ✓	
$x+y)\$$	$E)T'E'\$$	$M[E,id] = \{TE'\}$	$E \to TE'$
	$TE')T'E'\$$	$M[T,id] = \{FT'\}$	$T \to FT'$
	$FT'E')T'E'\$$	$M[F,id] = \{id\}$	$F \to id$
$x+y)\$$	$id\,T'E')T'E'\$$	match ✓	
$+y)\$$	$T'E')T'E'\$$	$M[T',+] = \{\varepsilon\}$	$T' \to *FT' \mid \varepsilon$
	$E')T'E'\$$	$M[E',+] = \{+TE'\}$	$E' \to +TE'$

AST for $(x+y)\$$



Action table

M (NT \ T)	id	+	*	(	)	$	reason
E	TE'			TE'			$E \to TE'$
E'		$+TE'$			ε	ε	$E' \to +TE' \mid \varepsilon$
T	FT'			FT'			$T \to FT'$
T'		ε	$*FT'$		ε	ε	$T' \to *FT' \mid \varepsilon$
F	id			(E)			$F \to (E) \mid id$
S	$E\$$			$E\$$			$S \to E\$$

Top of stack

- Non-determinism exists when there are multiple actions!
- Need to LL(k) with larger k. or fix the grammar.

FIRST(α)

	$E\$$	TE'	FT'	$+TE'$	$*FT'$	(E)	id	ε
FIRST	(id	(id	(id	+	*	(	id	ε

$FIRST(\alpha) \triangleq \{a \in T \mid \alpha \Rightarrow^* a\beta\} \cup \{\varepsilon \mid \alpha \Rightarrow^* \varepsilon\}$

init $FIRST(a) := \{a\}$ $FIRST(NT) := \{\}$

$A \to \alpha\beta$

while changes:

$A \to B\beta$

for all $\{\} \to \{\varepsilon\}$

Reason: $S \to E \to TE' \to FT'E' \to (\text{ or } id$

FOLLOW(NT)

S	E	E' ✓	T	T' ✓	F
$\$$	$)\$$	$)\$$	$)\$$	$+)\$$	$+*)\$$

$FOLLOW(NT) \triangleq \{a \in T \mid S \Rightarrow^+ [\alpha]\,NT\,a[\beta]\}$

e.g. $S \Rightarrow E')$

init $FOLLOW(S) := \{\$\}$ other $NT := \{\}$

while changes:

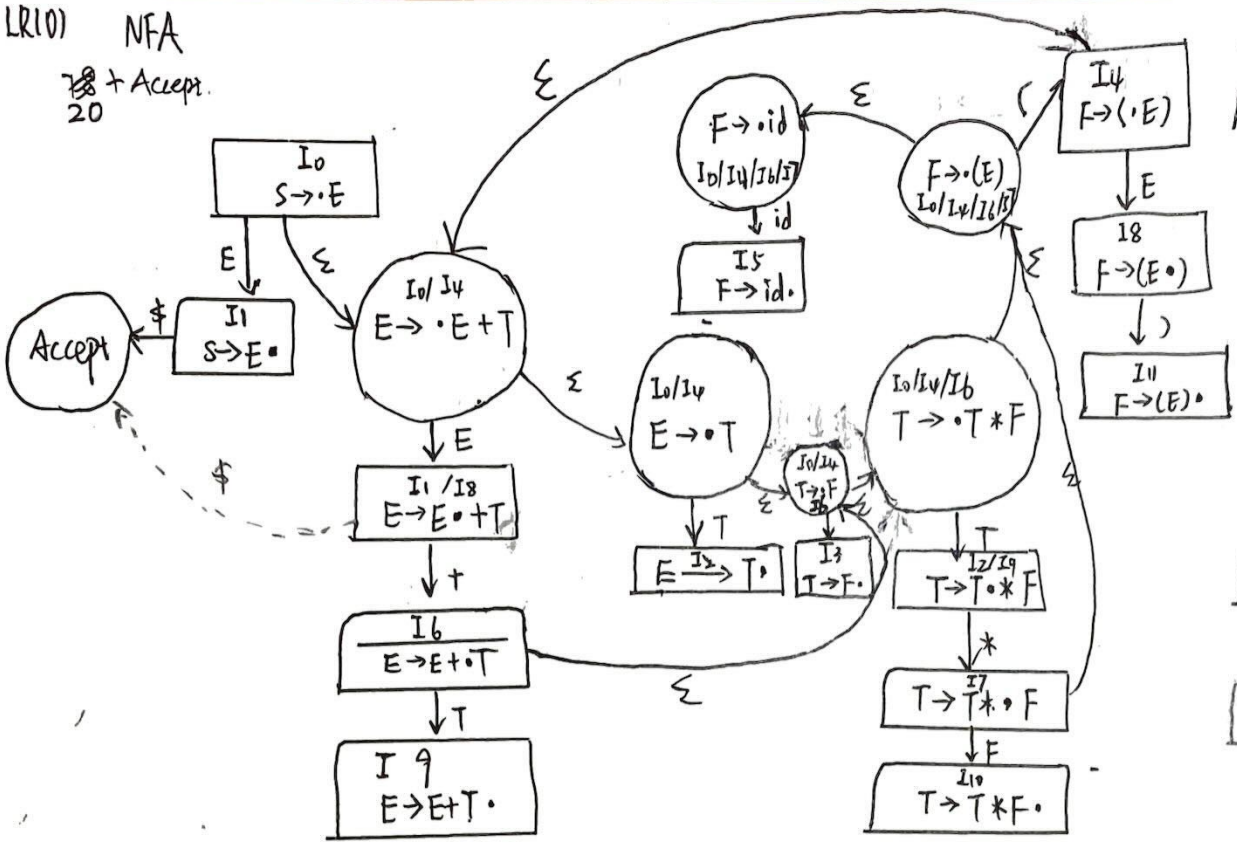
$A \to \alpha B[\beta]$

$FIRST(\beta) - \{\varepsilon\}$

Reason: $T \to FT'$ ↑ larger!

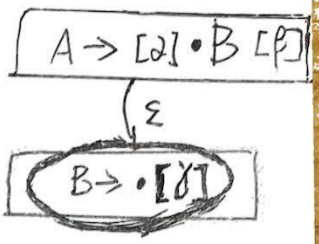
LR(0) NFA

7+Accept
20



CFG <T, NT, P2, S>

P2: S→E$
E→E+T | T
T→T*F | F
F→(E) | id

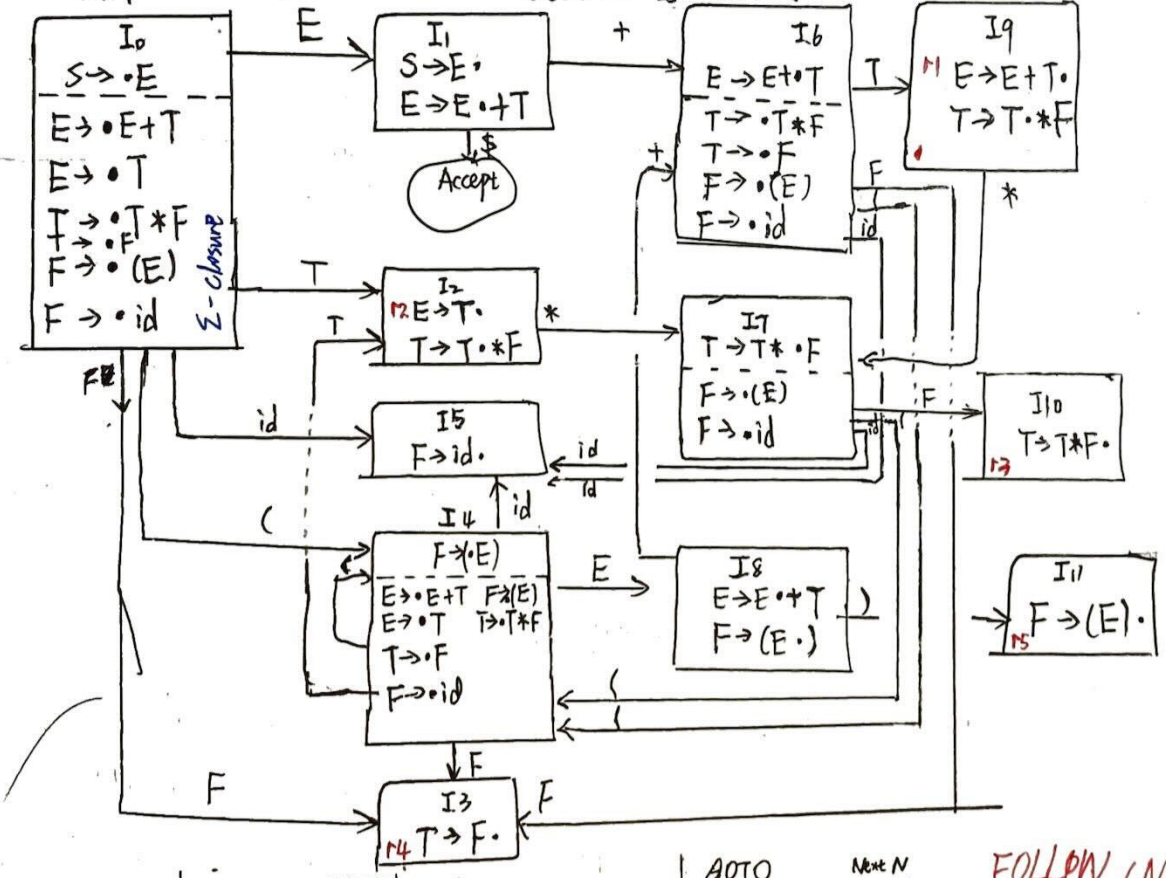


SLR(1), LR(1)

「shift: A→α·aβ
δ(Ii, a) = Ij : Sj
δ(Ii, A) = Ij : GOTO j

Reduce B→β·
[A→α·]

DFA 11+Accept



STATE	Next token: id	+	*	(	)	$	GOTO: E	T	F	Next N
0	S5			S4			1	2	3	
1		S6				acc				
2		r2	S7/r2		r2	r2				
3		r4	r4		r4	r4				
4	S5			S4			8	2	3	
5		r6	r6		r6	r6				
6	S5			S4				9	3	
7	S5			S4					10	
8		S6			S11					
9		r1	S7/r1		r1	r1				
10		r3	r3		r3	r3				
11		r5	r5		r5	r5				

Top of Stack

FOLLOW(NT)

S	E	T	F
$	+)$	*+)$	

∀a∈FOLLOW(B).
shift / reduce conflict : LR(0)
> Solved by LR(1) / fix Grammar
reduce / reduce conflict
∃a. a∈FOLLOW(A), a∈FOLLOW(B)
r1/r2

[S] LR(1) CFG $\langle T, NT', P_2, S \rangle$
ex. tokens (x+y) $

P_2: $S \to E\$$; $E \to E+T \mid T$; $T \to T*F \mid F$; $F \to (E) \mid id$

input	stack = State + Symbol	action[x, a] (reduce / shift)	reason : ACTION / GOTO
(x+y)$	$ 0	shift	$F \to \cdot(E) \in \delta(I_0, \varepsilon) = I_0$: S_4 = ACTION[0, (]
x+y)$	$(04	shift	$F \to \cdot id \in \delta(I_0, () = I_4$: S5
+y)$	$(id 045	reduce $F \to id$	"+" ∈ FOLLOW(F) ∴ GOTO[4, F] = 3
	$(F 043	reduce $T \to F$	"+" ∈ FOLLOW(T) : GOTO[4, T] = 2
	$(T 042	reduce $E \to T$	"+" ∈ FOLLOW(E) GOTO[4, E] = 8
+y)$	$(E 048	shift	$E \to E \cdot +T \in \delta(I_0, (E) = I_8$: S6
y)$	$(E+ 0486	shift	$F \to \cdot id \in \delta(I_0, (E+) = I_6$: S5
)$	$(E+id 04865	reduce $F \to id$	")" ∈ FOLLOW(F) GOTO[6, F] = 3
	$(E+F 04863	reduce $T \to F$	")" ∈ FOLLOW(T) GOTO[6, T] = 9
	$(E+T 04869	reduce $E \to E+T$	")" ∈ FOLLOW(E) GOTO[4, E] = 8
)$	$(E 0488	shift	$E \to (E \cdot) \in \delta(I_0, (E) = I_8$: S11
$	$(E) 04811	reduce $F \to (E)$	"$" ∈ FOLLOW(F) GOTO[0, F] = 3
	$F 0431	reduce $T \to F$	"$" ∈ FOLLOW(T) GOTO[0, T] = 2
	$T 02	reduce $E \to T$	"$" ∈ FOLLOW(E) GOTO[0, E] = 1
	$E 01	reduce $S \to E$	"$" ∈ FOLLOW(S) GOTO ACTION[1, $] = accept
	$S 0	accept	accept.

A ⇒ T
(x+y)$

↑ Production Rule.

LR(1) $A \to \alpha \cdot [a\beta], \underline{b}$

ACTION [i, b] = reduce $A \to \alpha$

Define NFA

tag

$A \to \alpha\alpha \cdot a\beta, b$ —a→ $A \to \alpha a \cdot \beta, b$

$A \to \alpha \cdot B\beta, b$ —B→ $A \to \alpha B \cdot \beta, b$

add new look-ahead For each $b' \in FIRST(\beta b)$

$A \to \alpha \cdot B\beta, b$ —ε→ $B \to \cdot\gamma, b'$