

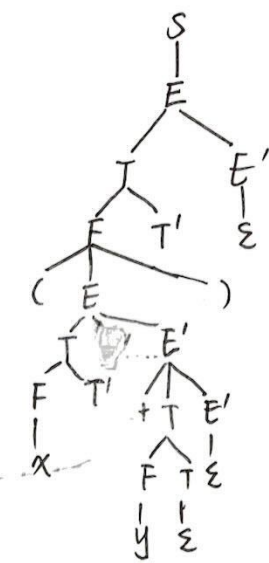
LL(1) CFG $\langle T, NT, P, S \rangle$ exp. tokens: $(x+y)\$$

P3: $S \rightarrow E\$$
 $E \rightarrow TE'$
 $E' \rightarrow +TE' \mid \epsilon$

$T \rightarrow FT'$
 $T' \rightarrow *FT' \mid \epsilon$
 $F \rightarrow (E) \mid id.$

Notation: $a \in T, A \in NT$
 $\alpha, \beta \in (T \cup NT)^*$
 $w \in T^*$

inputs	remaining	stack	action	predict match	reason
$(x+y)\$$		S	$M[S, (] = \{E\}$		$S \rightarrow E\$$
		E\$	$M[E, (] = \{TE'\}$		$E \rightarrow TE'$
		TE'\$	$M[T, (] = \{FT'\}$		$T \rightarrow FT'$
		FT'E'\$	$M[F, (] = \{(E)\}$		$F \rightarrow (E) \mid id$
$(x+y)\$$		(E)T'E'\$	match ✓		
$x+y)\$$		E)T'E'\$	$M[E, id] = \{TE'\}$		$E \rightarrow TE'$
		TE')T'E'\$	$M[T, id] = \{FT'\}$		$T \rightarrow FT'$
		FT'E')T'E'\$	$M[F, id] = \{id\}$		$F \rightarrow id$
$x+y)\$$		idT'E')T'E'\$	match ✓		
$+y)\$$		T'E')T'E'\$	$M[T', +] = \{\epsilon\}$		$T' \rightarrow *FT' \mid \epsilon$
		↑ E')T'E'\$	$M[E', +] = \{+TE'\}$		$E' \rightarrow +TE'$



AST for $(x+y)\$$

Action table

M	T	Next token	id	+	*	()	\$	reason
E		TE'				TE'			$E \rightarrow TE'$
E'				+TE'			ϵ	ϵ	$E' \rightarrow +TE' \mid \epsilon$
T		FT'				FT'			$T \rightarrow FT'$
T'				ϵ	*FT'		ϵ	ϵ	$T' \rightarrow *FT' \mid \epsilon$
F		id				(E)			$F \rightarrow (E) \mid id$
S		E\$				E\$			$S \rightarrow E\$$

Left-recursion
 $S \rightarrow E \mid F$
 $E \rightarrow TE' \mid \epsilon$
 $T \rightarrow FT' \mid \epsilon$
 $F \rightarrow (E) \mid id$
 choice in grammar
 if E then E_1 else $E_2 \mid \epsilon$
 Non-determinism exists when there are multiple actions!
 Need to LL(k) with larger k or fix the grammar.

FIRST(α)

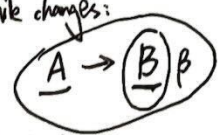
	E\$	TE'	FT'	+TE'	*FT'	(E)	id	ϵ
FIRST	(id	(id	(id	+	*	(id	ϵ

$FIRST(\alpha) \triangleq \{a \in T \mid \alpha \Rightarrow^* a\beta\} \cup \{\epsilon \mid \alpha \Rightarrow^* \epsilon\}$

init $FIRST(a) := \{a\}$ $FIRST(NT) := \{\epsilon\}$

$A \Rightarrow a\beta$

while changes:



Reason: $S \rightarrow E \rightarrow TE' \rightarrow FT'E' \xrightarrow{id}$

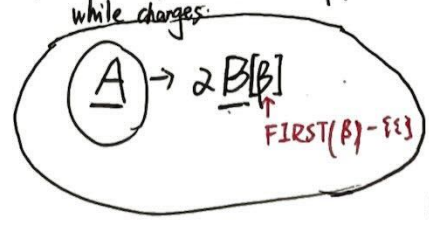
FOLLOW(NT)

S	E	E'	T	T'	F
\$)\$)\$)\$	+\$	*)\$

$FOLLOW(NT) \triangleq \{a \in T \mid S \Rightarrow^+ [\alpha] NT a[\beta]\}$
 e.g. $S \Rightarrow E'$

init $FOLLOW(S) := \{\$ \}$

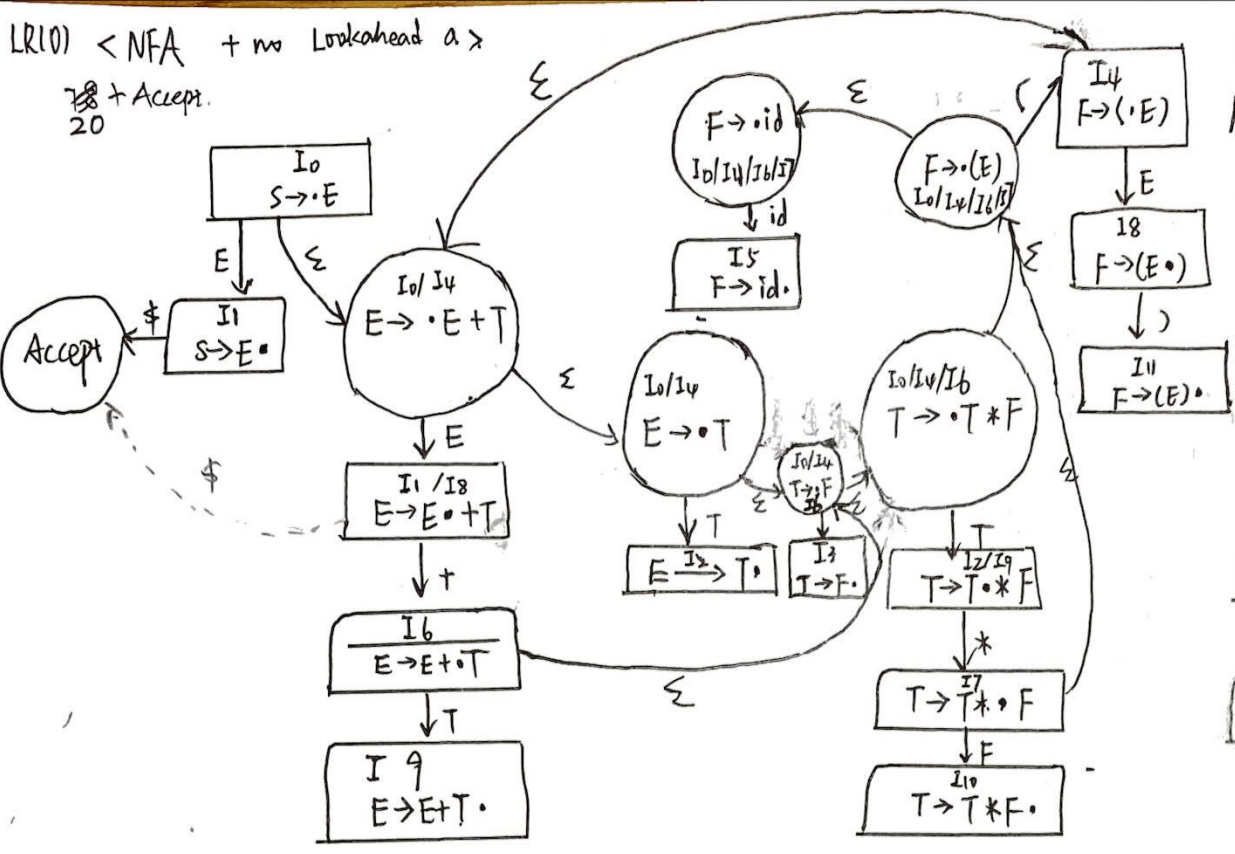
other NT := $\{\epsilon\}$



Reason: $T \rightarrow FT' \xrightarrow{larger!}$

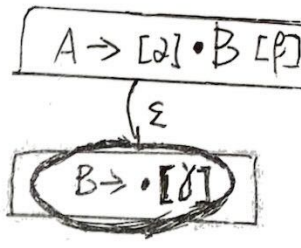
LR(0) < NFA + no Lookahead a >

18 + Accept.
20



CFG < T, NT, P2, S >

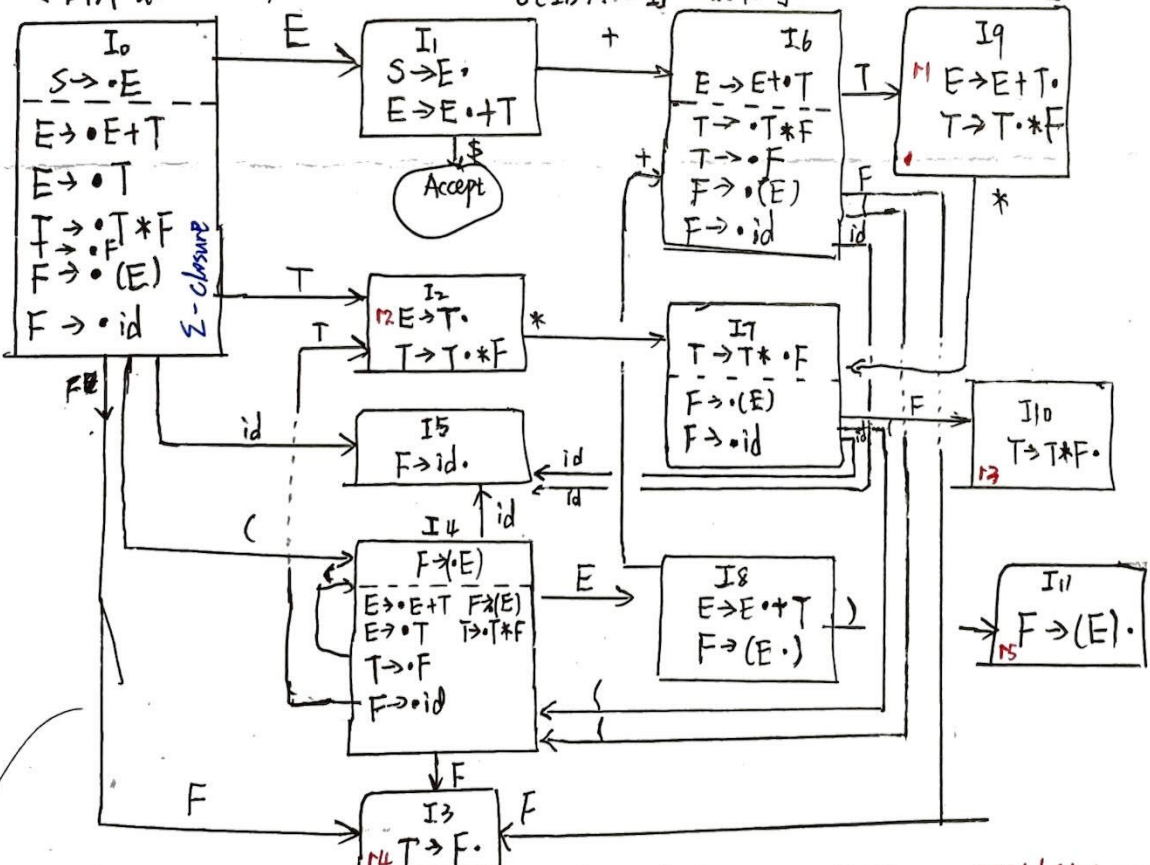
P2: $S \rightarrow E \$$
 $E \rightarrow E + T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow (E) \mid id$



SLR(1), ~~LR(0)~~
 < DFA + a > 11 + Accept

shift: $A \rightarrow \alpha \cdot a \beta$
 $\delta(I_i, a) = I_j : S_j$
 $\delta(I_i, A) = I_j : \text{GOTO } j$

Reduce $B \rightarrow \beta \cdot$
 $[A \rightarrow \alpha \cdot]$



Top of stack

STATE	Next token	ACTION					goto			Next N
		id	+	*	()	\$	E	T	
0		S5				S4				
1			S6				acc			
2			R2	S7	S7		R2	R2		
3			R4	R4			R4	R4		
4		S5				S4			8	2 3
5			R6	R6			R6	R6		
6		S5				S4			9	3
7		S5				S4				10
8			S6				S11			
9			R1	S7	S7		R1	R1		
10			R2	R2			R2	R2		
11			R5	R5			R5	R5		

FOLLOW (NT)

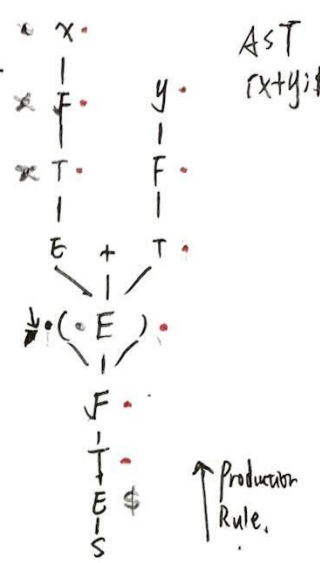
	S	E	T	F
	\$	+	*)
	()	*	+

Non-determinism
 shift / reduce conflict: LR(0)
 Solved by SLR(1) / fix Grammar
 reduce / reduce conflict
 Ja. $a \in \text{FOLLOW}(A)$, $a \in \text{FOLLOW}(B)$
 r1/r2

SLR(1) CFG $\langle T, NT, P_2, S \rangle$
 LR(0) + lookahead DFA
 ex. tokens $(x+y) \&$

$P_2: S \rightarrow E \&$
 $E \rightarrow E + T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow (E) \mid id$

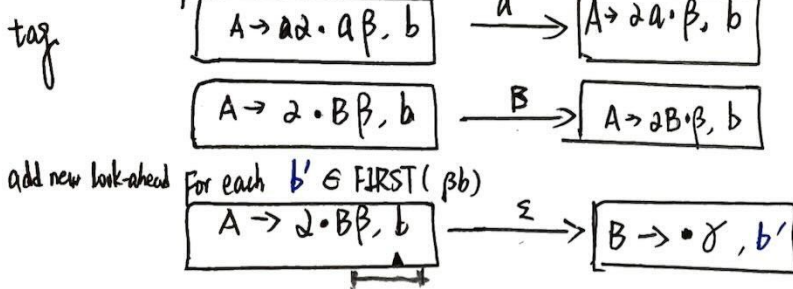
input	stack = State + Symbol	action [X, a]	reason (promise)	same as action.
$(x+y) \&$	$\$ 0$	$a[0, (] = \text{shift } 4$	$F \rightarrow \cdot (E) \in \delta(I_0, \epsilon) = I_4 / S_0: S_4 = \text{ACTION}[0, (]$	
$x+y) \&$	$\$ (04$	$a[4, id] = \text{shift } 5$	$F \rightarrow \cdot id \in \delta(I_4, \epsilon) = I_5 / S_5: S_5 = \text{ACTION}[4, id]$	
$+y) \&$	$\$ (id 045$	$a[5, +] = \text{reduce } F \rightarrow id$	"+" $\in \text{FOLLOW}(F) \sim \text{GOTO}[4, F] = 3$	
	$\$ (F 043$	$a[3, +] = \text{reduce } T \rightarrow F$	"+" $\in \text{FOLLOW}(T) : \text{GOTO}[4, T] = 2$	
	$\$ (T 042$	$a[2, +] = \text{reduce } E \rightarrow T$	"+" $\in \text{FOLLOW}(E) \text{ GOTO}[4, E] = 8$	
$+y) \&$	$\$ (E 048$	$a[8, +] = \text{shift } 6$	$E \rightarrow E \cdot + T \in \delta(I_0, (E)) = I_8 / S_8$	
$y) \&$	$\$ (E + 0486$	$a[6, id] = \text{shift } 8$	$F \rightarrow \cdot id \in \delta(I_6, (E+)) = I_6 / S_6$	
$) \&$	$\$ (E + id 04865$	$a[5, +] = \text{reduce } F \rightarrow id$	"+" $\in \text{FOLLOW}(F) \text{ GOTO}[6, F] = 3$	
	$\$ (E + F 048653$	$a[3, +] = \text{reduce } T \rightarrow F$	"+" $\in \text{FOLLOW}(T) \text{ GOTO}[6, T] = 2$	
	$\$ (E + T 04869$	$a[9, +] = \text{reduce } E \rightarrow E + T$	"+" $\in \text{FOLLOW}(E) \text{ GOTO}[6, E] = 8$	
$) \&$	$\$ (E 04868$	$a[8, +] = \text{shift } 11$	$E \rightarrow (E \cdot) \in \delta(I_8, (E)) = I_8 / S_8$	
$\&$	$\$ (E) 04811$	$a[11, \&] = \text{reduce } F \rightarrow (E)$	"\$" $\in \text{FOLLOW}(F) \text{ GOTO}[10, F] = 3$	
	$\$ F 048113$	$a[3, \&] = \text{reduce } T \rightarrow F$	"\$" $\in \text{FOLLOW}(T) \text{ GOTO}[10, T] = 2$	
	$\$ T 02$	$a[2, \&] = \text{reduce } E \rightarrow T$	"\$" $\in \text{FOLLOW}(E) \text{ GOTO}[10, E] = 1$	
	$\$ E 01$	$a[1, \&] = \text{reduce } S \rightarrow E$	"\$" $\in \text{FOLLOW}(S) \text{ GOTO} \text{ / } \leftarrow$	
	$\$ S 0$	accept	$\text{ACTION}[1, \&] = \text{accept}$	



LR(1) $A \rightarrow \alpha \cdot [a\beta], b$

$\text{ACTION}[i, b] = \text{reduce } A \rightarrow \alpha$ with lookahead!

Define NFA



shift when $b = a$.

$A \rightarrow \alpha \cdot \underline{b}$ Reduce $A \rightarrow \alpha$
 $S \rightarrow \alpha \cdot \underline{\$}$ Accept.

