

Standard Random Variables

Notation: $P\{X\} / P(X)$, $E[X] / E(X)$, assume **independent** and identical distribution (iid). Python: `np.random`

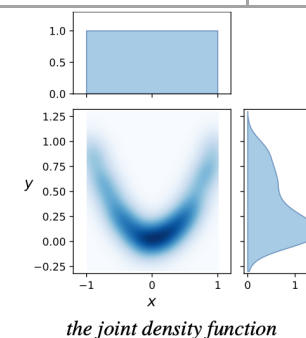
<u>Discrete Distribution</u> $X \in \mathbb{N}$ Categorical $X \sim \text{Cat}([p_1, p_2, \dots]) \rightarrow$	PMF Prob. Mass Function Valid i. $\forall x_i, P\{X = x_i\} \geq 0$ ii. $\sum_{i=0}^{\infty} P\{X = x_i\} = 1$ (density sum to 1)	CDF Cumulative Distribution $F_X(x) = P\{X \leq \lfloor x \rfloor\}$, $1 - P\{X > x\}$, $P\{X = K\} = P\{X \leq K\} - P\{X \leq K - 1\}$	$E[X] = \sum_{i=0}^{\infty} x_i P\{X = x_i\}$	$\text{Var}[X] = E[X^2] - E[X]^2$ LOTUS, $E[g(X)] = \sum g(x)P\{X = k\}$
Bernoulli trial $X \sim \text{Bern}(p)$	$P\{X\} = p$, $P\{\bar{X}\} = q$	$q = (1 - p)$	p	pq
Binomial with replace $X \sim \text{Bin}(n, p)$ #successes in n Bern(p) trials $X \sim \text{Bin}(1, p)$, (0-1) distribution if $n=1$	$P\{X = k\} = \binom{n}{k} p^k q^{n-k}$ $P\{X = k\} = p^k q^{1-k}$	Normal Approximation Poisson $n \rightarrow \infty, p \rightarrow 0, \lambda = np$ is moderate	np	npq
Geometric / Negative Binomial $X \sim \text{Geom}(p)$, $X \sim \text{NegBin}(r, p)$ in n Bern(p) trials until 1 st / r successes	$P\{X = k\} = q^k p$, $k = \# \text{failures}$ $P\{X = k\} = \binom{n-1}{r-1} q^{k-1} p^r$, $k = \# \text{trials}$	$1 - q^{k+1}$, $x > 0$ [Exp(λ)] Approximation	$\frac{q}{p}$, $\frac{1 \cdot r}{p}$	$\frac{(1-p) \cdot r}{p^2}$
Poisson $X \sim \text{Pois}(\lambda)$, $\lambda = np > 0$ memoryless #events in a fixed interval of time t	$P\{X = k\} = \frac{e^{-\lambda} \lambda^k}{k!}$ $\text{Pois}(\lambda t)$ given $\text{Exp}(\lambda)$ as waiting time interval	by def	λ	λ
[Negative] HyperGeometric no replace $X \sim \text{NHGemo}(w, b, n)$, total $N = w + b$ #successes in n draws / until n failures	$P\{X = k\} = \frac{\binom{w}{k} \binom{b}{n-k}}{\binom{w+b}{n}}$	/	$np = n \frac{w}{N}$ $n \frac{w}{b+1}$	$\frac{N-n}{N-1} npq$
Joint Prob	$P_{ij} = P\{X = x_i, Y = y_j\}$	$F(x, y) = \sum_0^{\lfloor x_i \rfloor} \sum_0^{\lfloor y_j \rfloor} P_{ij}$ $= P\{X \leq x_i, Y \leq y_j\}$	<u>Valid</u> $\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} P_{ij} = 1$	
Marginal Prob marginalize over another variable	$P\{X = x_i\} = \sum_y P\{X = x_i, Y = y_j\}$ $= \sum_{j=1}^{\infty} P_{ij}$	$F_X(x) = F(x, \infty)$ $= \sum_{x_i \leq x} \sum_{j=0}^{\infty} P_{ij}$	$\forall i, j. P_{ij} \geq 0$	

		Y			
		1	2	3	
X	1	0.32	0.03	0.01	0.36
	2	0.06	0.24	0.02	0.32
	3	0.02	0.03	0.27	0.32
		0.40	0.30	0.30	1

Marginals for X $g(x) = \sum_y f(x, y)$

Marginals for Y $h(y) = \sum_x f(x, y)$

$\sum_x \sum_y f(x, y) = 1$



Joint Prob	$f(x, y) = \frac{\partial}{\partial x \partial y} F(x, y)$	$F(x, y) = \int_{-\infty}^x \int_{-\infty}^y f(u, v) du dv$ $= \iint_B f(x, y) dx dy$	<u>Valid</u> $F(-\infty, \infty) = 1$	
Marginal Prob	$f_X(x) = \int_{y=-\infty}^{\infty} f(x, y) dy$	$F_X(x) = F(x, \infty), y \rightarrow \infty$ $= \int_{-\infty}^x f_X(x) dx$	$\forall x, y. f(x, y) \geq 0$	
Continuous Distribution $X \in R$	PDF Prob Density Function <u>Valid</u> i. $\forall x. f_X(x) \geq 0$ ii. $\int_{-\infty}^{\infty} f_X(x) dx = 1$ (density sum to 1)	CDF Cumulative Distribution $F_X(x) = \int_{-\infty}^x f_X(t) dt$, complement, LoTP $f_X(x) = \frac{d}{dx} F_X(x)$	$E[X] = \int_{-\infty}^{\infty} x \cdot f(x) dx$	$Var[X]$ LOTUS, $E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x) dx$
Uniform $X \sim U(a, b) = e^{-\lambda Exp(\lambda)}$ a completely random point in [low, high]	$f_X(x) = \begin{cases} \frac{1}{b-a}, & x \in [a, b] \\ 0, & otherwise \end{cases}$	$F_X(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & x \in [a, b] \\ 1, & x > b \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential $X \sim Exp(\lambda)$, rate $\lambda = \frac{1}{\theta} > 0$ memoryless waiting time between 2 successive events	$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & otherwise \end{cases}$	$F_X(x) = \begin{cases} 1 - e^{-\lambda x}, & x > 0 \\ 0, & otherwise \end{cases}$	$\theta = \frac{1}{\lambda}$ integrate by part	$\theta^2 = \frac{1}{\lambda^2}$ / tabular
Pareto $X \sim Pareto(shape = \alpha) = e^{Exp(\lambda)}$ cascade events, wealth, $x_m = 1$. Loglinearity	$f_X(x) = \begin{cases} \alpha x_m^\alpha x^{-(\alpha+1)}, & x \geq x_m \\ 0, & x < x_m \end{cases}$	$F_X(x) = \begin{cases} 1 - \left(\frac{x_m}{x}\right)^\alpha, & x \geq x_m \\ 0, & x < x_m \end{cases}$	$\begin{cases} \frac{\alpha x_m}{\alpha-1}, & \alpha > 1 \\ \infty, & \alpha \leq 1 \end{cases}$	$\begin{cases} valid, & \alpha > 2 \\ \infty, & \alpha \leq 2 \end{cases}$
Normal / Gaussian $X \sim N(0,1)$ Standard $X \sim N(\mu, \sigma^2)$ normal(loc = μ , scale = σ)	$f_X(x) = c e^{-\frac{x^2}{2}}, x \in R, c = \frac{1}{\sqrt{2\pi\sigma}}$ $f_X(x) = c e^{-\frac{(x-\mu)^2}{2\sigma^2}} = c e^{-\frac{(\frac{x-\mu}{\sigma})^2}{2}}$	$\Phi_X(x) = c \int_{-\infty}^x e^{-\frac{x^2}{2}} dx$ $\Phi_{\frac{X-\mu}{\sigma}}(z)$ CLT	μ	σ^2
Beta $X \sim Beta(\alpha, \beta) = \frac{\Gamma(\alpha, 1)}{\Gamma(\alpha, 1) + \Gamma(\beta, 1)}$ As prior for Bayesian	$f_X(x) = \begin{cases} \frac{1}{B} x^{\alpha-1} (1-x)^{\beta-1}, & x \in [0,1] \\ 0, & otherwise \end{cases}$	$B = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} = \frac{(\alpha-1)!(\beta-1)!}{(\alpha+\beta-1)!}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Gamma $X \sim \Gamma(k, 1)$, shape $k > 0$ $X \sim \Gamma(k, \lambda)$, scale $= 1/\lambda > 0$ $Y X \sim N(\mu, \frac{1}{\lambda X})$	$f_X(x) = \frac{1}{\Gamma(k)} x^{k-1} e^{-x}, x > 0$ $f_X(x) = \frac{1}{\Gamma(k)} \frac{(\lambda x)^k}{x} e^{-\lambda x}, x > 0$	$\Gamma(k) = (k-1)!, k \in N$ $\Gamma(k) = \int_0^\infty e^{-x} x^{k-1} dx, k \in R^+$ $\Gamma(1, \lambda) = Exp(\lambda), -\frac{1}{\lambda} \log(U)$	$\frac{k}{\lambda}$	$\frac{k}{\lambda^2}$