LL(1) CFG $\langle T, NT, P, S\rangle$

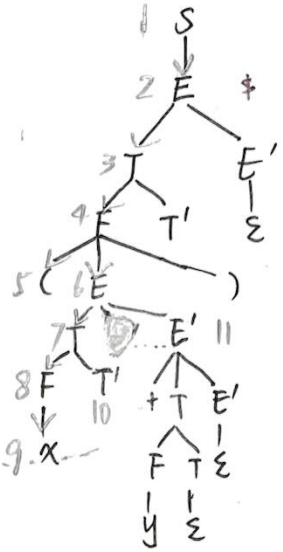
exp. tokens: $(x+y)\$$

P3: $S \to E\$$ | $E \to TE'$ | $E' \to +TE' \mid \varepsilon$ | $T \to FT'$ | $T' \to *FT' \mid \varepsilon$ | $F \to (E) \mid id$

Notation: $a \in T$, $A \in NT$, $\alpha, \beta \in (T \cup NT)^*$, $w \in T^*$

input (remaining)	stack (dfs)	action (predict / match)	reason
$(x+y)\$$	S	$M[S,(] = \{E\$\}$	$S \to E\$$
	$E\$$	$M[E,(] = \{TE'\}$	$E \to TE'$
	$TE'\$$	$M[T,(] = \{FT'\}$	$T \to FT'$
	$FT'E'\$$	$M[F,(] = \{(E)\}$	$F \to (E) \mid id$
$(x+y)\$$	$(E)T'E'\$$	match ✓	
$x+y)\$$	$E)T'E'\$$	$M[E,id] = \{TE'\}$	$E \to TE'$
	$TE')T'E'\$$	$M[T,id] = \{FT'\}$	$T \to FT'$
	$FT'E')T'E'\$$	$M[F,id] = \{id\}$	$F \to id$
$x+y)\$$	$id\,T'E')T'E'\$$	match ✓	
$+y)\$$	$T'E')T'E'\$$	$M[T',+] = \{\varepsilon\}$	$T' \to *FT' \mid \varepsilon$
	$E')T'E'\$$	$M[E',+] = \{+TE'\}$	$E' \to +TE'$
	...		

AST for $(x+y)\$$



Action table

Top of stack: M[NT, T]	id	+	*	(	)	$	reason
E	TE'			TE'			$E \to TE'$
E'		$+TE'$			ε	ε	$E' \to +TE' \mid \varepsilon$
T	FT'			FT'			$T \to FT'$
T'		ε	$*FT'$		ε	ε	$T' \to *FT' \mid \varepsilon$
F	id			(E)			$F \to (E) \mid id$
S	$E\$$			$E\$$			$S \to E\$$

S	E	E'	T	T'	F
(, id	(, id	+	(, id	*	(, id

Left-recursion: $S \to EOF \mid$ left | ε choice in grammar

if E then C_1 (else E_2 / ε) end D

- Non-determinism exists when there are multiple actions!
- Need to LL(k) with larger k. or fix the grammar.

FIRST(2)

	$E\$$	TE'	FT'	$+TE'$	$*FT'$	(E)	id	ε
FIRST	(, id	(, id	(, id	+	*	(	id	ε

$FIRST(\alpha) \triangleq \{a \in T \mid \alpha \Rightarrow^* a\beta\} \cup \{\varepsilon \mid \alpha \Rightarrow^* \varepsilon\}$

init $FIRST(a) := \{a\}$ $FIRST(NT) := \{\}$

$A \to \alpha\beta$

while changes:

$A \to B\beta$

for all $\{\} \to \{\varepsilon\}$

Reason: $S \to E \to TE' \to FT'E' \to (, id$ ($\{A \to B_1, A \to B_2\}$)

FOLLOW(NT)

	S	E	E' ✓	T	T' ✓	F
	$\$$	$)\$$	$)\$$	$+)\$$	$+)\$$	$+*)\$$

$FOLLOW(NT) \triangleq \{a \in T \mid S \Rightarrow^+ [\alpha]\, NT\, a\, [\beta]\}$

e.g. $S \Rightarrow E')$

init $FOLLOW(S) := \{\$\}$ other $NT := \{\}$

while changes:

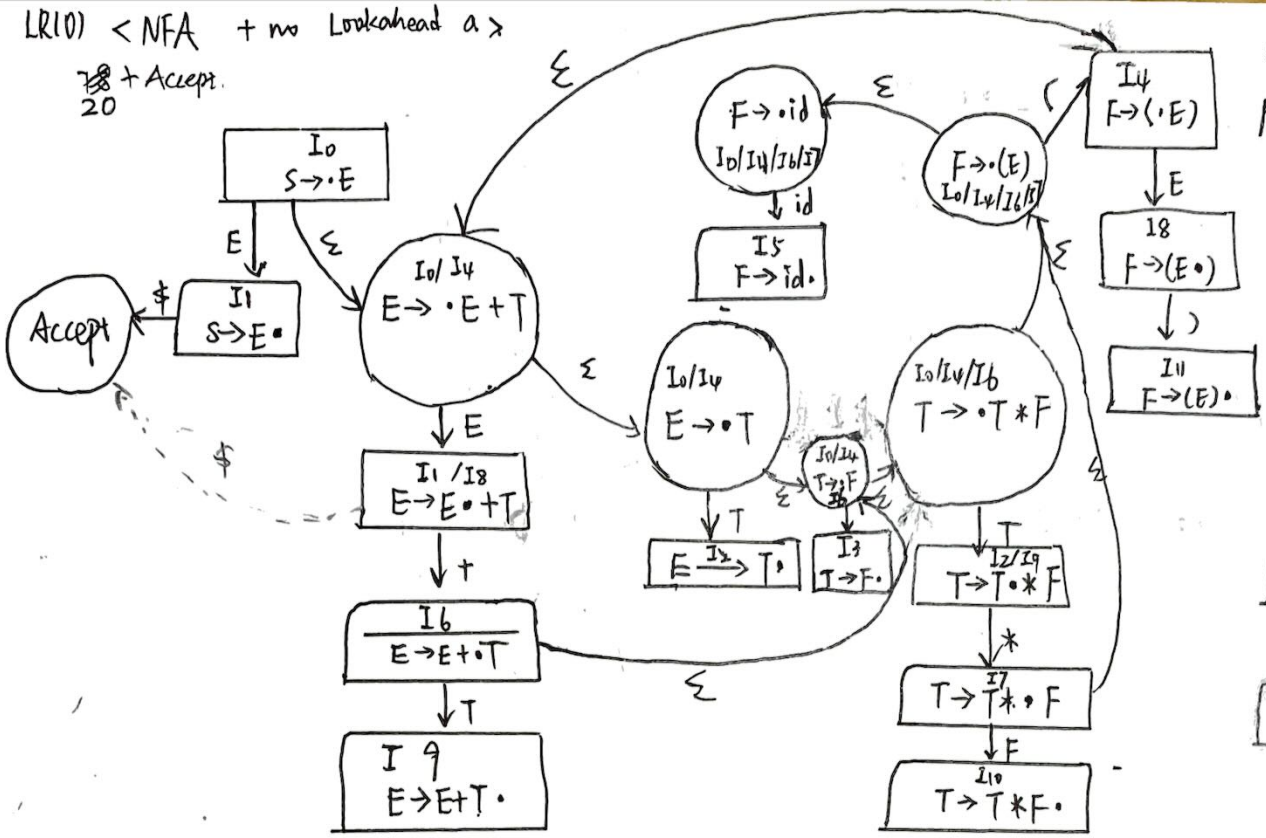
$A \to \alpha B[\beta]$

$FIRST(\beta) - \{\varepsilon\}$

Reason: $T \to FT'$ ↑ larger!

LR(0) < NFA + no Lookahead a >

78 + Accept.
20



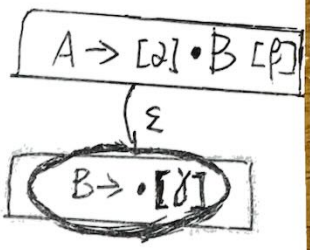
CFG < T, NT, P2, S >

P2: S → E $

E → E+T | T

T → T*F | F

F → (E) | id



SLR(1), ~~LR(0)~~

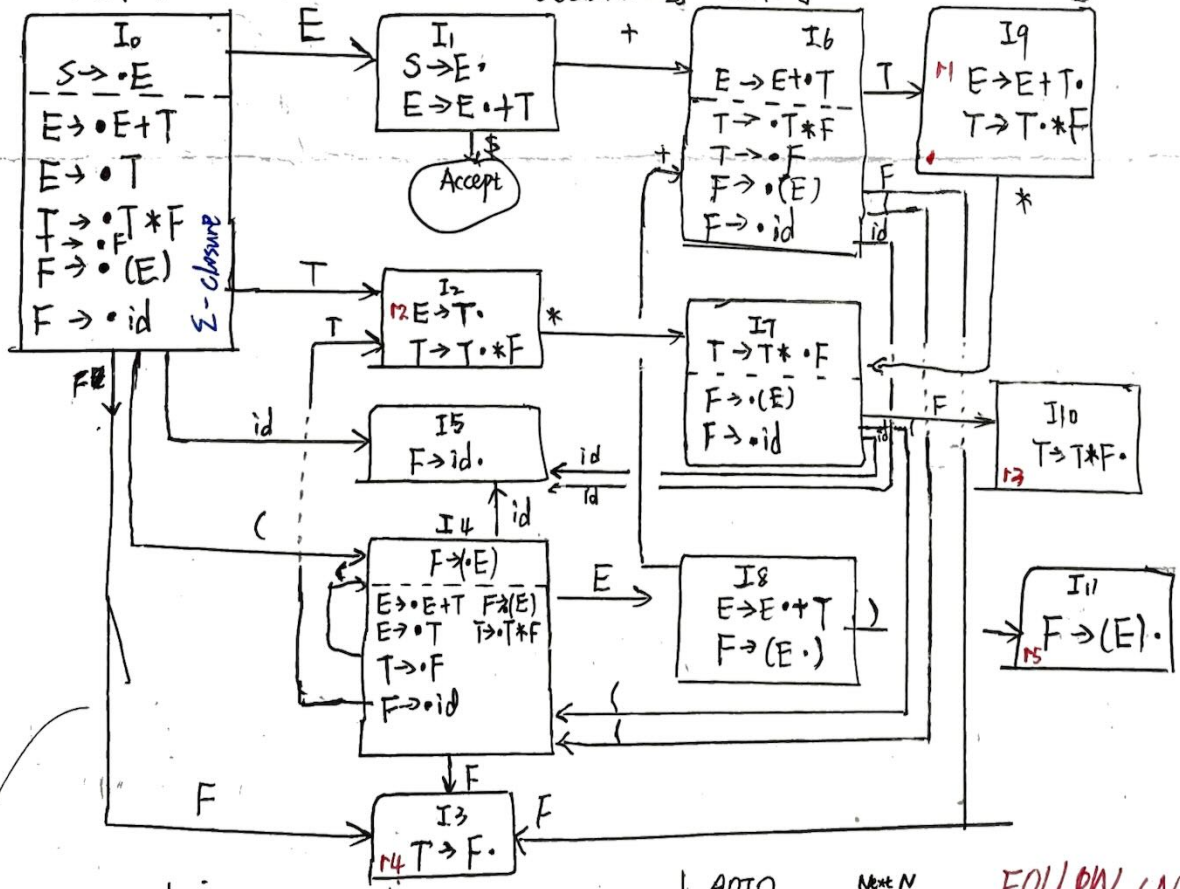
< DFA + lookahead a > 11 + Accept

Shift: A → α · aβ

δ(Ii, a) = Ij : Sj

δ(Ii, A) = Ij : GOTO j

Reduce B → β · [A → α ·]



STATE	Next token: id	+	*	(	)	$	GOTO E	T	F
0	S5			S4			1	2	3
1		S6				acc			
2		r2	S7/r2		r2	r2			
3		r4	r4		r4	r4			
4	S5			S4			8	2	3
5		r6	r6		r6	r6			
6	S5			S4				9	3
7	S5			S4					10
8		S6			S11				
9		r1	S7/r1		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Top of Stack

FOLLOW(NT)

	S	E	T	F
	$	+)$	*+)$	

∃a ∈ FOLLOW(B). Non-determinism

shift / reduce conflict : LR(0)

> Solved by ~~SLR(1)~~ / fix Grammar

reduce / reduce conflict

∃a. a ∈ FOLLOW(A), a ∈ FOLLOW(B)

r1/r2

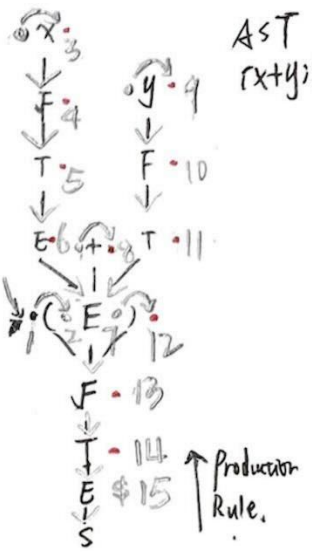
LR(1)

↳ LR(0) + lookahead DFA

CFG ⟨T, NT', P2, S⟩ ex. tokens (x+y) $

P2: S→E$; E→E+T | T ; T→T*F | F ; F→(E) | id

input	stack = State + Symbol	action[X, a] (reduce ← / shift →)	reason (premise) : ACTION / GOTO (same as action ←)
(x+y)$	$ 0	a[0,(] = shift 4	F→•(E) ∈ δ(I0, ε) = I0/S0 : S4 = ACTION[0,(]
x+y)$	$(04	a[4,id] = shift 5	F→•id ∈ δ(I0,() = I4/S4 : S5 = ACTION[4,id]
+y)$	$(id 045	a[5,+] = reduce F→id	"+" ∈ FOLLOW(F) ⇝ GOTO[4,F] = 3
	$(F 043	a[3,+] = reduce T→F	"+" ∈ FOLLOW(T) : GOTO[4,T] = 2
	$(T 042	a[2,+] = reduce E→T	"+" ∈ FOLLOW(E) GOTO[4,E] = 8
+y)$	$(E 048	a[8,+] = shift 6	E→E•+T ∈ δ(I0,(E) = I8 / S6 (Action ←)
y)$	$(E+ 0486	a[6,id] = shift 8	F→•id ∈ δ(I0,(E+) = I6/S6
)$	$(E+id 04865	a[5,)] = reduce F→id	")" ∈ FOLLOW(F) GOTO[6,F] = 3
	$(E+F 0486 3	a[3,)] = reduce T→F	")" ∈ FOLLOW(T) GOTO[6,T] = 9
	$(E+T 0486 9	a[9,)] = reduce E→E+T	")" ∈ FOLLOW(E) GOTO[4,E] = 8
)$	$(E 048 8	a[8,)] = shift 11	E→(E•) ∈ δ(I0,(E) = I8 / S11
$	$(E) 0 4 11	a[11,$] = reduce F→(E)	"$" ∈ FOLLOW(F) GOTO[0,F] = 3
	$F 0 3	a[3,$] = reduce T→F	"$" ∈ FOLLOW(T) GOTO[0,T] = 2
	$T 02	a[2,$] = reduce E→T	"$" ∈ FOLLOW(E) GOTO[0,E] = 1
	$E 01	a[1,$] = reduce S→E	"$" ∈ FOLLOW(S) GOTO ACTION[1,$] = accept ↩
	$S 0	accept	accept.



LR(1) $A \to \alpha \cdot [a\beta], b$ ACTION[i, b] = reduce $A \to \alpha$ with lookahead!

Define NFA

tag

$A \to a\alpha \cdot a\beta, b \xrightarrow{a} A \to a\alpha a \cdot \beta, b$

$A \to \alpha \cdot B\beta, b \xrightarrow{B} A \to \alpha B \cdot \beta, b$

add new look-ahead: For each $b' \in FIRST(\beta b)$

$A \to \alpha \cdot B\beta, b \xrightarrow{\varepsilon} B \to \cdot\gamma, b'$

shift when b = a.

$A \to \alpha \cdot, b$ ($b \in$ first(b)) Reduce $A \to \alpha$.

$S \to \alpha \cdot, \$$ Accept.