y2017 p3 q4. CPS & Defunctionalisation

```
type expr = | Integer of int | Pair of expr * expr | Apply of string * expr   (* apply a named function: ADD *)
type value = | INT of int | PAIR of value * value
```

```
let rec eval = function
| Integer n -> INT n
| Pair (e1, e2) -> PAIR (eval e1, eval e2)
| Apply (f, e) -> eval_function (f, eval e)
```

let V = eval e in C (eval_function (f, V))

1. Add a continuation parameter c to each function, return value.

(a)

```
let rec eval_cps C = function
| Integer n -> C (INT n)
| Pair (e1,e2) -> eval_cps (fun V1 -> (* PAIR1 *) λV1,e2,C
                    eval_cps (fun V2 -> (* PAIR2 *) λV1,V2,C C (PAIR (V1,V2)) ) e2 )
                  e1)
| Apply (f, e) -> eval_cps (fun V -> C (eval_function (f, V)) ) e
                  (* FUNC *) λf,C
```

let V1 = eval e1 in V2 = eval e2 in C (PAIR (V1, V2))

Thus

```
let eval_2 e = eval_cps (fun x -> x) e .   (* ID *)
```

(b) Eliminate high-order continuations.

1. Add a constructor to cnt for each fun ∃ (* Cntr *) λ... (free variables)

```
type cnt =
| ID
| PAIR1 of expr * cnt
| PAIR2 of value * cnt
| FUNC of string * cnt
```

Call apply_cnt at every application of continuation.

```
let rec eval_cps_dfn C = function
| Integer n -> apply_cnt C (INT n)
| Pair (e1, e2) -> eval_cps_dfn ($PAIR1 (e2, C)) e1
| Apply (f, e) -> eval_cps_dfn (FUNC (f, C)) e
```

Mutually recursive.

```
and apply_cnt = function
| (ID, V) -> V
| (PAIR1 (e1, C), V1) -> eval_cps_dfn (PAIR2 (V1, C)) e1
| (PAIR2 (V1, C), V2) -> apply_cnt (C, PAIR (V1, V2))
| (FUNC (f, C), V) -> apply_cnt (C, eval_function (f, V))
```

```
let eval_3 e = eval_cps_dfn ID e .
```