

Standard Random Variable

Notation: $P\{X\} / P(X)$, $E[X] / E(X)$, assume **independent** and identical distribution (iid).

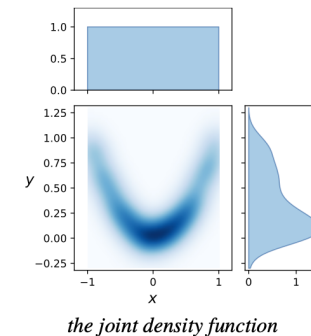
<u>Discrete Distribution</u> $X \in \mathbb{N}$	PMF Prob. Mass Function Valid i. $\forall x, P\{X = k\} \geq 0$ ii. $\sum_0^\infty P\{X = k\} = 1$, density sum to 1	CDF Cumulative Distribution $F_X(x) = P\{X \leq x\}$ $= 1 - P\{X > x\}$, complement	$E[X] = \sum_{i=0}^\infty x_i P\{X = x_i\}$	$\text{Var}[X] = E[X^2] - E[X]^2$ LOTUS, $E[g(X)] = \sum g(x)P\{X = k\}$
Bernoulli trial $X \sim \text{Bern}(p)$	$P\{X\} = p, P\{\bar{X}\} = q$	$q = (1 - p)$	p	pq
Binomial <i>with replace</i> $X \sim \text{Bin}(n, p)$ #successes in $n \text{ Bern}(p)$ trials $X \sim \text{Bin}(1, p)$, (0-1) distribution if $n=1$	$P\{X = k\} = \binom{n}{k} p^k q^{n-k}$ $P\{X = k\} = p^k q^{1-k}$	Normal Approximation Poisson $n \rightarrow \infty, p \rightarrow 0, \lambda = np$ is moderate	np	npq
Geometric / Negative Binomial $X \sim \text{Geom}(p)$, $X \sim \text{NegBin}(r, p)$ in $n \text{ Bern}(p)$ trials until 1 st / r success(es)	$P\{X = k\} = q^k p$, $k = \# \text{failures}$ $P\{X = k\} = \binom{n-1}{r-1} q^{k-1} p^r$, $k = \# \text{trials}$	$1 - q^{k+1}, x > 0$ Exp Approximation	$\frac{q}{p}, \frac{1 \cdot r}{p}$	$\frac{(1-p) \cdot r}{p^2}$
Poisson $X \sim \text{Pois}(\lambda), \lambda = np$ <i>memoryless</i> #events in a fixed interval of time t	$P\{X = k\} = \frac{e^{-\lambda} \lambda^k}{k!}$ $\text{Pois}(\lambda t)$ given $\text{Exp}(\lambda)$ as waiting time interval	by def	λ	λ
[Negative] HyperGeometric <i>no replace</i> $X \sim \text{NHGemo}(w, b, n)$, total $N = w + b$ #successes in n draws #successes until n failures	$P\{X = k\} = \frac{\binom{w}{k} \binom{b}{n-k}}{\binom{w+b}{n}}$	/	$np = n \frac{w}{N}$ $n \frac{w}{b+1}$	$\frac{N-n}{N-1} npq$
Joint Prob	$P\{X = x_i, Y = y_j\} = P_{ij}$	$F(x, y) = \sum_{x_i \leq x} \sum_{y_j \leq y} P_{ij}$ $= P\{X \leq x_i, Y \leq y_j\}$	<u>Valid</u> $\sum_{i=1}^\infty \sum_{j=1}^\infty P_{ij} = 1$	
Marginal Prob marginalize over another variable	$P\{X = x_i\} = \sum_y P\{X = x_i, Y = y_j\}$ $= \sum_{j=1}^\infty P_{ij}$	$F_X(x) = F(x, \infty)$ $= \sum_{x_i \leq x} \sum_{j=1}^\infty P_{ij}$	ii. $\forall i, j. P_{ij} \geq 0$	

		Y			
		1	2	3	
X	1	0.32	0.03	0.01	0.36
	2	0.06	0.24	0.02	0.32
	3	0.02	0.03	0.27	0.32
		0.40	0.30	0.30	1

Marginals for X
 $g(x) = \sum_y f(x, y)$

Marginals for Y
 $h(y) = \sum_x f(x, y)$

$\sum_x \sum_y f(x, y) = 1$



Joint Prob	$f(x, y) = \frac{\partial}{\partial x \partial y} F(x, y)$	$F(x, y) = \int_{-\infty}^x \int_{-\infty}^y f(u, v) du dv$ $= \iint_B f(x, y) dx dy$	<u>Valid</u> $F(-\infty, \infty) = 1$	
Marginal Prob	$f_X(x) = \int_{-\infty}^{\infty} f(x, y) dy$	$F_X(x, y) = F(x, \infty), y \rightarrow \infty$ $= \int_{-\infty}^x f_X(x) dx$	$\forall x, y. f(x, y) \geq 0$	
Continuous Distribution $X \in R$	PDF Prob Density Function <u>Valid</u> i. $\forall x, f_X(x) \geq 0$ ii. $\int_{-\infty}^{\infty} f_X(x) dx = 1$, density sum to 1	CDF Cumulative Distribution $F_X(x) = \int_{-\infty}^x f_X(t) dt$, complement $f_X(x) = \frac{d F_X(t)}{dt}$	$E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x) dx$	Var[X] LOTUS, $E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x) dx$
Uniform $X \sim U(a, b)$ a completely random point in $[a, b]$	$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}$	$F_X(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x > b \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential $X \sim \text{Exp}(\lambda)$, rate $\lambda = \frac{1}{\theta}$ <i>memoryless</i> waiting time between 2 successive events	$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$	$F_X(x) = \begin{cases} 1 - e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$	$\theta = \frac{1}{\lambda}$	$\theta^2 = \frac{1}{\lambda^2}$
Normal / Gaussian $X \sim N(0, 1)$ Standard $X \sim N(\mu, \sigma^2)$	$f_X(x) = c e^{-\frac{x^2}{2}} \quad c = \frac{1}{\sqrt{2\pi}}$ $f_X(x) = c e^{-\frac{(\frac{x-\mu}{\sigma})^2}{2}} \quad c = \frac{1}{\sqrt{2\pi} \sigma}$	$\Phi(x) = c \int_{-\infty}^x e^{-\frac{x^2}{2}} dx$ $\Phi(z), Z = \frac{X-\mu}{\sigma}$	μ	σ^2
Beta $X \sim \text{Beta}(\alpha, \beta)$	$f_X(x) = \begin{cases} \frac{1}{B} x^{\alpha-1} (1-x)^{\beta-1}, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$	$B = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Gamma $X \sim \Gamma(k, \lambda)$ $X \sim \Gamma(k, 1)$	$f_X(x) = \frac{1}{\Gamma(k)} x^{k-1} e^{-x}, x > 0$ $f_X(x) = \frac{1}{\Gamma(k)} \frac{(\lambda x)^k}{x} e^{-\lambda x}, x > 0$	$\Gamma(k) = (k-1)!, k \in N$ $\Gamma(k) = \int_0^{\infty} e^{-x} x^{k-1} dx, k \in R^+$	$\frac{k}{\lambda}$	$\frac{k}{\lambda^2}$