Standard Random Variable
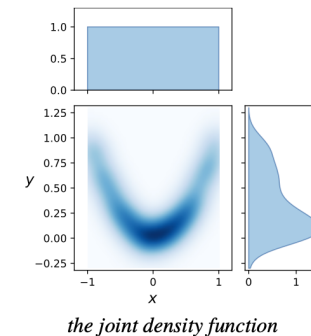
Notation: P{X} / P(X) ,   E[X] / E(X), assume **independent** and identical distribution (iid).

Discrete Distribution $X \in N$	**PMF** Prob. Mass Function Valid $i. \forall x, P\{X = k\} \geq 0$ $ii. \sum_0^\infty P\{X = k\} = 1$, density sum to 1	**CDF** Cumulative Distribution $F_X(x) = P\{X \leq x\}$ $= 1 - P\{X > x\}$, complement	$E[X] =$ $\sum_{i=0}^\infty x_i P\{X = x_i\}$	$Var[X] =$ $E[X^2] - E[X]^2$ LOTUS, $E[g(X)] =$ $\sum g(x) \boldsymbol{P}\{X = k\}$
Bernoulli trial $X \sim Bern(p)$	$P\{X\} = p, \; P\{\bar{X}\} = q$	$q = (1 - p)$	p	pq
Binomial $\qquad$ **with** <u>replace</u> $X \sim Bin(n, p)$ #successes in $n\ Bern(p)$ trials $X \sim Bin(1, p)$, *(0-1) distribution* if n=1	$P\{X = k\} = \binom{n}{k} p^k q^{n-k}$ $P\{X = k\} = p^k q^{1-k}$	*Normal Approximation* *Poisson* $n \to \infty,\ p \to 0,\ \lambda = np$ is moderate	np	npq
Geometric / Negative Binomial $X \sim Geom(p),\ X \sim NegBin(r, p)$ .... in $n\ Bern(p)$ trials until 1st/**r** success(es)	$P\{X = k\} = q^k p, \qquad k = \#failures$ $P\{X = k\} = \binom{n-1}{r-1} q^{k-1} p^r,\ k = \#trials$	$1 - q^{k+1},\ x > 0$ *Exp Approximation*	$\dfrac{q}{p}, \dfrac{1 \cdot r}{p}$	$\dfrac{(1-p) \cdot r}{p^2}$
Poisson $X \sim Pois(\lambda),\ \lambda = np$ $\qquad$ **memoryless** #events in a fixed interval of time t	$P\{X = k\} = \dfrac{e^{-\lambda} \lambda^k}{k!}$ $Pois(\lambda t)$ given $Exp(\lambda)$ as waiting time interval	*by def*	λ	λ
[Negative] HyperGeometric $\quad$ **no** <u>replace</u> $X \sim NHGemo(w, b, n), total\ N = w + b$ #successes in n draws #successes until **n failures**	$P\{X = k\} = \dfrac{\binom{w}{k}\binom{b}{n-k}}{\binom{w+b}{n}}$	/	$np = n\dfrac{w}{N}$ $n\dfrac{w}{\boldsymbol{b+1}}$	$\dfrac{N-n}{N-1} npq$
Joint Prob	$P_{ij} = P\left\{X = x_i, Y = y_j\right\}$	$F(x, y) = \sum_{x_i \leq x} \sum_{y_j \leq y} P_{ij}$ $= P\{X \leq x_i, Y \leq y_j\}$	<u>Valid</u> $\sum_{i=1}^\infty \sum_{j=1}^\infty P_{ij} = 1$	
Marginal Prob marginalize over another variable	$P\{X = x_i\} = \sum_y P\{X = x_i, Y = y_j\}$ $= \sum_{j=1}^\infty P_{ij}$	$F_X(x) = F(x, \infty)$ $= \sum_{x_i \leq x} \sum_{j=1}^\infty P_{ij}$	$\forall i, j.\ P_{ij} \geq 0$	

the joint density function

Joint Prob	$f(x,y) = \frac{\partial}{\partial x \partial y} F(x,y)$	$F(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(u,v)dudv$ $= \iint_B f(x,y)dxdy$	*Valid* $F(-\infty, \infty) = 1$	
Marginal Prob	$f_X(x) = \int_{-\infty}^{\infty} f(x,y)dy$	$F_X(x,y) = F(x,\infty), \; y \to \infty \uparrow$ $= \int_{-\infty}^{x} f_X(x)dx$	$\forall x, y. \; f(x,y) \geq 0$	
  $X \in R$	**PDF** Prob Density Function *Valid* i. $\forall x, f_x(x) \geq 0$ ii. $\int_{-\infty}^{\infty} f_X(x)dx = 1$, *density sum to 1*	**CDF** Cumulative Distribution $F_X(x) = \int_{-\infty}^{x} f_X(t)dt$, *complement* $f_X(x) = \frac{d F_X(t)}{dt}$	$E[X] =$ $\int_{-\infty}^{\infty} x \cdot f_X(x)dx$	$Var[X]$ LOTUS, $E[g(X)] =$ $\int_{-\infty}^{\infty} g(x)f_X(x)dx$
Uniform $X \sim U(a,b)$ a completely random point in $[a,b]$	$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & otherwise \end{cases}$	$F_X(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x > b \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential $X \sim Exp(\lambda)$, rate $\lambda = \frac{1}{\theta}$ **memoryless** waiting time between 2 successive events	$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & otherwise \end{cases}$	$F_X(x) = \begin{cases} 1 - e^{-\lambda x}, & x > 0 \\ 0, & otherwise \end{cases}$	$\theta = \frac{1}{\lambda}$	$\theta^2 = \frac{1}{\lambda^2}$
Normal / Gaussian $X \sim N(0,1)$ *Standard* $X \sim N(\mu, \sigma^2)$	$f_X(x) = ce^{-\frac{x^2}{2}} \quad c = \frac{1}{\sqrt{2\pi}}$ $f_X(x) = ce^{-\frac{(\frac{x-\mu}{\sigma})^2}{2}} \quad c = \frac{1}{\sqrt{2\pi}\,\sigma}$	$\Phi(x) = c \int_{-\infty}^{x} e^{-\frac{x^2}{2}} dx$ $\Phi(z), \; Z = \frac{X-\mu}{\sigma}$	μ	σ^2
Beta $X \sim Beta(\alpha, \beta)$	$f_X(x) = \begin{cases} \frac{1}{B} x^{\alpha-1}(1-x)^{\beta-1}, & 0 \leq x \leq 1 \\ 0, & otherwise \end{cases}$	$B = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Gamma $X \sim \Gamma(k,1)$ $X \sim \Gamma(k,\lambda)$	$f_X(x) = \frac{1}{\Gamma(k)} x^{k-1} e^{-x}, \; x > 0$ $f_X(x) = \frac{1}{\Gamma(k)} \frac{(\lambda x)^k}{x} e^{-\lambda x}, \; x > 0$	$\Gamma(k) = (k-1)! \quad\quad, \; k \in N$ $\Gamma(k) = \int_0^{\infty} e^{-x} x^{k-1} dx, \; k \in R^+$	$\frac{k}{\lambda}$	$\frac{k}{\lambda^2}$