Standard Random Variables

Notation: P{X} / P(X) , E[X] / E(X), assume ***independent*** *and identical distribution (iid).* Python: *np.random*

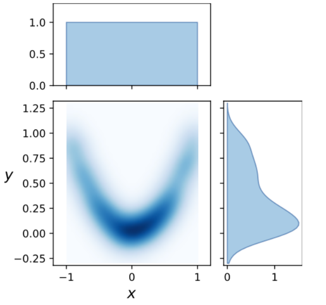
Discrete Distribution $X \in N$ *Categorical* $X \sim Cat([p_1, p_2, ..]) \rightarrow$	**PMF** *Prob. Mass Function* *Valid* $i. \forall x_i.\ P\{X = x_i\} \geq 0$ $ii. \sum_{i=0}^{\infty} P\{X = x_i\} = 1$ (density sum to 1)	**CDF** *Cumulative Distribution* $F_X(x) = P\{X \leq \lfloor x \rfloor\},\ 1 - P\{X > x\},$ $P\{X = K\} = P\{X \leq k\} - P\{X \leq k - 1\}$	$\boldsymbol{E}[\boldsymbol{X}] =$ $\sum_{i=0}^{\infty} x_i P\{X = x_i\}$	$\boldsymbol{Var}[\boldsymbol{X}] =$ $E[X^2] - E[X]^2$ *LOTUS,* $E[g(X)] =$ $\sum g(x)\boldsymbol{P}\{X = k\}$
Bernoulli trial $X \sim Bern(p)$	$P\{X\} = p,\ \ P\{\bar{X}\} = q$	$q = (1 - p)$	p	pq
Binomial *with* *replace* $X \sim Bin(n, p)$ #successes in n $Bern(p)$ trials $X \sim Bin(1, p)$, *(0-1) distribution* if *n=1*	$P\{X = k\} = \binom{n}{k} p^k q^{n-k}$ $P\{X = k\} = p^k q^{1-k}$	*Normal Approximation* *Poisson* $n \rightarrow \infty,\ p \rightarrow 0,\ \ \lambda = np$ *is moderate*	np	npq
Geometric / Negative Binomial $X \sim Geom(p),\ \ X \sim \boldsymbol{NegBin}(r,\ p)$ .... in n $Bern(p)$ trials until 1^{st}/**r** successes	$P\{X = k\} = q^k p, \qquad k = \#failures$ $P\{X = k\} = \binom{\mathbf{n-1}}{\mathbf{r-1}} q^{k-1} p^r,\ k = \#trials$	$1 - q^{k+1},\ x > 0$ $[Exp(\lambda)]$ *Approximation*	$\frac{q}{p}, \ \ \frac{1 \cdot \boldsymbol{r}}{p}$	$\frac{(1-p) \cdot \boldsymbol{r}}{p^2}$
Poisson $X \sim Pois(\lambda),\ \lambda = np > 0$ ***memoryless*** #events in a fixed interval of time t	$P\{X = k\} = \frac{e^{-\lambda} \lambda^k}{k!}$ $Pois(\lambda t)$ *given* $Exp(\lambda)$ *as waiting time interval*	*by def*	λ	λ
[Negative] HyperGeometric *no* *replace* $X \sim \boldsymbol{N}HGemo(w, b, n), total\ N = w + b$ #successes in n draws / until $\boldsymbol{n}$ **failures**	$P\{X = k\} = \frac{\binom{w}{k}\binom{b}{n-k}}{\binom{w+b}{n}}$	/	$np = n\frac{w}{N}$ $n\frac{w}{\boldsymbol{b+1}}$	$\frac{N-n}{N-1} npq$
Joint Prob	$P_{ij} = P\left\{X = x_i,\ Y = y_j\right\}$	$F(x, y) = \sum_0^{\lfloor x_i \rfloor} \sum_0^{\lfloor y_j \rfloor} P_{ij}$ $= P\left\{X \leq x_i, Y \leq y_j\right\}$	*Valid* $\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} P_{ij} = 1$	
Marginal Prob marginalize over another variable	$P\{X = x_i\} = \sum_y P\left\{X = x_i,\ Y = y_j\right\}$ $= \sum_{j=1}^{\infty} P_{ij}$	$F_X(x) = F(x,\ \infty)$ $= \sum_{x_i \leq x} \sum_{j=0}^{\infty} P_{ij}$	$\forall i, j.\ P_{ij} \geq 0$	

X \ Y	1	2	3	
1	0.32	0.03	0.01	0.36
2	0.06	0.24	0.02	0.32
3	0.02	0.03	0.27	0.32
	0.40	0.30	0.30	1

Marginals for X $g(x) = \sum_y f(x, y)$

Marginals for Y $h(y) = \sum_x f(x, y)$

$\sum_x \sum_y f(x, y) = 1$



the joint density function

Joint Prob	$f(x,y) = \frac{\partial}{\partial x \partial y} F(x,y)$	$F(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(u,\ v)dudv$ $= \iint_B \ f(x,y)dxdy$	*Valid* $F(-\infty, \infty) = 1$	
Marginal Prob	$f_X(x) = \int_{y=-\infty}^{\infty} f(x,y)dy$	$F_X(x) = F(x, \infty),\ y \to \infty$ $= \int_{-\infty}^{x} f_X(x)dx$	$\forall x, y.\ f(x,y) \geq 0$	
Continuous Distribution $X \in R$	**PDF** *Prob Density Function* *Valid* $i. \forall x.\ f_x(x) \geq 0$ $ii. \int_{-\infty}^{\infty} f_X(x)dx = 1$ (density sum to 1)	**CDF** *Cumulative Distribution* $F_X(x) = \int_{-\infty}^{x} f_X(t)dt$ *, complement, LoTP* $f_X(x) = \frac{d}{dx} F_X(x)$	$\boldsymbol{E[X]} =$ $\int_{-\infty}^{\infty} x \cdot f(x)dx$	$\boldsymbol{Var[X]}$ *LOTUS*, $E[g(X)] =$ $\int_{-\infty}^{\infty} g(x)f(x)dx$
Uniform $X \sim U(a,b) = \mathrm{e}^{-\lambda Exp(\lambda)}$ a completely random point in $[low, high]$	$f_X(x) = \begin{cases} \frac{1}{b-a}, & x \in [a,b] \\ 0, & otherwise \end{cases}$	$F_X(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & x \in [a,b] \\ 1, & x > b \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential $X \sim Exp(\lambda)$, $rate\ \lambda = \frac{1}{\theta} > 0$ ***memoryless*** waiting time between 2 successive events	$f_X(x) = \begin{cases} \lambda\, e^{-\lambda x}, & x > 0 \\ 0, & otherwise \end{cases}$	$F_X(x) = \begin{cases} 1 - e^{-\lambda x}, & x > 0 \\ 0, & otherwise \end{cases}$	$\theta = \frac{1}{\lambda}$ *integrate by part*	$\theta^2 = \frac{1}{\lambda^2}$ */ tabular*
Pareto $X \sim Pareto(shape = \alpha) = e^{Exp(\lambda)}$ cascade events, wealth, $x_m = 1$. ***Loglinearity***	$f_X(x) = \begin{cases} \alpha x_m^{\alpha} x^{-(\alpha+1)}, & x \geq x_m \\ 0, & x < x_m \end{cases}$	$F_X(x) = \begin{cases} 1 - \left(\frac{x_m}{x}\right)^{\alpha}, & x \geq x_m \\ 0, & x < x_m \end{cases}$	$\begin{cases} \frac{\alpha x_m}{\alpha - 1}, & \alpha > 1 \\ \infty, & \alpha \leq 1 \end{cases}$	$\begin{cases} valid, & \alpha > 2 \\ \infty, & \alpha \leq 2 \end{cases}$
Normal / Gaussian $X \sim N(0,1)$ *Standard* $X \sim N(\mu, \sigma^2)$ $normal(loc = \mu,\ \ scale = \sigma)$	$f_X(x) = ce^{-\frac{x^2}{2}},\ \ x \in R,\ \ c = \frac{1}{\sqrt{2\pi\sigma}}$ $f_X(x) = ce^{-\frac{(x-\mu)^2}{2\sigma^2}} = ce^{-\frac{\left(\frac{x-\mu}{\sigma}\right)^2}{2}}$	$\Phi_X(x) = c\int_{-\infty}^{x} e^{-\frac{x^2}{2}}\, dx$ $\Phi_{\frac{X-\mu}{\sigma}}(z)$ CLT	μ	σ^2
Beta $X \sim Beta(\alpha, \beta) = \frac{\Gamma(\alpha,1)}{\Gamma(a,1)+\Gamma(\beta+1)}$ As prior for Bayesian	$f_X(x) = \begin{cases} \frac{1}{B} x^{\alpha-1}(1-x)^{\beta-1}, & x \in [0,1] \\ 0, & otherwise \end{cases}$	$B = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} = \frac{(\alpha-1)!(\beta-1)!}{(\alpha+\beta-1)!}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Gamma $X \sim \Gamma(k, 1)$, *shape k>0* $X \sim \Gamma(k, \lambda)$, *scale=1/λ>0* $Y \mid X \sim N(\mu, \frac{1}{\lambda X})$	$f_X(x) = \frac{1}{\Gamma(k)} x^{k-1} e^{-x} \quad , x > 0$ $f_X(x) = \frac{1}{\Gamma(k)} \frac{(\lambda x)^k}{x} e^{-\lambda x} \ , x > 0$	$\Gamma(k) = (k-1)! \qquad , k \in N$ $\Gamma(k) = \int_0^{\infty} e^{-x}\, x^{k-1} dx\, ,\ k \in R^+$ $\Gamma(1, \lambda) = Exp(\lambda), -\frac{1}{\lambda} \log(U)$	$\frac{k}{\lambda}$	$\frac{k}{\lambda^2}$