Memory Calculation

Reference:

- OS, CompArch, Ad CompArch @ Cambridge
- CS 152: Computer Architecture and Engineering Final Exam @ UC Berkeley
- Cache: wiki page

By default, the unit used is 1 bit. Note that 1 Byte = 8 bits, i.e. 0x _ _ in HEX.

Given page size of 1MB (2^{20}), Frame Offset (FO) = Page O. (PO) = $\log_2(\text{page size})$ = $\log_2 1\text{MiB}$ = 20.

Physical memory $P = \{1, ..., M\}$ (RAM) is divided into frames.

- If the physical address space is 32-bit, then the physical address space is 2^{32}.
- Physical address = Physical Frame No. (PFN) + Frame Offset (FO),

```
~ 1-level Page Table

| addr (32) | PFN (12) | FO (20) |
| --------- | -------- | ------- |
| 0x _ _ *4 | pfn      | o       |



~ 2-level Page Table

| addr (48) | PFN1 (14) | PFN2 (14) | FO (20) |
| --------- | --------- | --------- | ------- |
| 0x _ _ *4 | pfn1      | pfn2      | o       |
```

Secondary storage memory $\{1, ..., K\}$, e.g. on disk.

Virtual memory $V = \{1, ..., N\}$, including physical and secondary storage memory, is divided into pages. Usually $N \gg M \gg K$.

- If the virtual address space is 48-bit, then the virtual address space is 2^{48}.
- Virtual address = Virtual Page No. (VPN) + Page Offset (PO),

```
| addr (48) | VPN (28) | PO (20) |
| --------- | -------- | ------- |
| 0x _ _ *6 | vpn      | o       |
```

Page Table maps the virtual pages to the physical frames $V \to P$, where the offset is the same in both cases, i.e. PO=FO.

- PTE is indexed by virtual page number (VPN).
- PTE (16) = PFN (12) + RWXI (4),

```
| PT index  | PFN (12) | Metadata (4) |
| --------- | -------- | ------------ |
|  VPN (28) | pfn      | 0 W 0 I      |
```

Translation Lookaside Buffer (TLB) $V \rightarrow P \cup (K \cup \emptyset)$ (hit or miss).

- fully associative cache with a small number of Page Table Entries (PTE) for faster access.
- TLB entry (44) = VPN (28) + PFN (12) + Metadata (4),

```
|   TLB    | VPN (28) | PFN (12) | Metadata (4) |
| -------- | -------  | -------- | ------------ |
| parallel | vpn      | pfn      | 0 W 0 I      |
```

Summary

Consider a system with the following specifications:

OS

- 32-bit physical address space
- 48-bit virtual address space
- Page size 1MB (2^{20})
- assume: 1-level page table
- PTE and TLB entries contain 4 bits of metadata [R W X I]

Cache

- 16KiB directed-mapped / fully-associative L1 cache with 16 B cache-lines.
- 64KiB 4-way set-associative L1 cache with 16 B cache-lines.
- assume: all accesses are 4 bytes long and entirely complete before the next one starts.

Cache: assume all accesses are 4 bytes long and entirely complete before the next one starts. *
*

Warning: distinguish between data address and value, e.g. load a byte (or word) from memory address 0x FFFF FFFF,

```
READBYTE 0x FFFF FFFF
READWORD 0x FFFF FFFC
```

Two types of problems for virtual memory:

- synonyms: two VAes map to the same PA (index aliasing).
  - coherence problem, e.g., conflicting cache states, whether cached, are flushed.
  - solved by using non-overlapping physical memory for different virtual addressing.
  - or (part of) the cache must be flushed when the virtual-to-physical mapping changes.
- homonyms: same VA map to different PAes.
  - e.g. after context switch, virtual-to-physical mapping changes.
  - solved by adding process ID to the VA or VIPT.

Virtual addressing and caching,

- TLB + Physically-Indexed Physically-Tagged (PIPT),
  - ✓ I/O mostly uses physical addresses.
  - ✓ ~~homonyms and synonyms~~.
  - ✗ TLB delay for all accesses.
- Virtually-Indexed Virtually-Tagged (VIVT).
  - ✓ Faster, TLB delay only for misses, bringing data from PA.
  - ✗ IO mostly uses physical addresses.
  - ✗ homonyms and synonyms.
  - ✗ require copy permission info for each cache entry.
- Virtually-Indexed Physically-Tagged (VIPT)
  - Physical tag: index into the physical cache.
    - parallel with TLB lookup VPN -> PFN if, and check the tag,
    - index bits taken within the part of not translated PO (rule).
    - limitation: cache size < page size * associativity (# ways).
  - ✓ Faster / lower latency.
  - ✓ ~~homonyms~~, where the physical tag PPN + PO can help detecting.
  - ✗ synonyms, when (rule) is not satisfied.
    - multiple VPNs map to the same PFN

      ```
      |----VPN----|----PO----| virtual addressing
      |--PFN--|---INDEX+BO---| cache addressing
      ```

    - solved by storing part of the VPN in the shared cache.
  - hit ✓; TLB miss ✗;
- Physically-Indexed Virtually-Tagged (PIVT)

Cache address = Tag [+ Index] + Offset, where

- set index selects the cache line for non-fully associative caches,
- tag checks validity of the cache line (AND gate),
- offset selects the word within the cache line (MUX gate).

```
| addr (48)    | VPN (36)      | PO (12)                     |
| ------------ | ------------- | --------------------------- |
| cache addr   | tag (48-i)      | index (i)  | offset (2)  |
| ------------ | --------------- | ---------- | ----------- |
| 0x _ _ *4    | upper addr bits | index      | word select |
```

Set index is the number of bits needed to select a cache line.

- 16KiB cache with 16 B cache-lines.
- total number of cache lines = cache size / block size = 16KiB / 16B = 2^{10}.
- number of lines / way = number of cache lines / number of ways = 2^{10}.
- set index is $\log_2(\text{number of lines / way})$ = 10 bits.

Hence,

$$\text{cache size} = \#\text{lines} \times \text{block size (bytes / line)}.$$
$$= \text{associativity (\# ways)} \times 2^{\text{index}}(\#\text{ sets}) \times \text{block size (bytes / line)}.$$
$$= \text{associativity (\# ways)} \times 2^{\text{index}+\text{block offset}}(\text{bytes / way})$$

Cache line (or block) = Tag + MESI + Word 1 + ... + Word 4, which contains 4 words (16 bytes) in total, for spatial locality.

Block offset is the number of bits needed to select a word within a cache line.

- $\log_2 4 = 2$ bits, if we select a word (by default).
- $\log_2 16 = 4$ bits, if we select a byte.

Tag is the upper address bits, i.e. the remaining bits after index and offset.

Direct-mapped cache, also known as 1-way set associative.

Each block only maps to a single cache line.

```
| addr (32)    | tag (18)        | index (10) | offset (2)  |
| ------------ | --------------- | ---------- | ----------- |
| 0x _ _ *4    | upper addr bits | index      | word select |
```

For each cache line = tag + MSI + Word 1 + ... + Word 4,

```
| cache          | tag (20) | MSI | word 1     | ... | word 4     |
| -------------- | -------- | --- | --------- | --- | --------- |
| set index (10) | tag      | S   | 0x _ _ *4 | ... | 0x _ _ *4 |
```

Fully-associative cache, N*M-way set associative.

Each block can be placed any line in the cache and the hardware checks all cache lines in parallel, i.e. no index.

```
| addr (32)   | tag (30)          | block offset (2) |
| ----------- | ----------------- | ---------------- |
| 0x _ _ *4   | upper address bits | offset          |
```

For each cache line = tag + MSI + Word 1 + ... + Word 4,

```
| cache    | tag (30) | MSI | word 1     | ... | word 4     |
| -------- | -------- | --- | --------- | --- | --------- |
| parallel | ...      | .   | ...       | ... | ...       |
| way i    | tag      | S   | 0x _ _ *4 | ... | 0x _ _ *4 |
| parallel | ...      | .   | ...       | ... | ...       |
```

N-way set-associative cache

Each memory block maps to a set among M sets, each containing N lines, which is a compromise between the two above types of caches.

64KiB (4x) 4-way set-associative L1 cache with the same cache line setup.

- number of cache lines = 64KiB/16 = 4KiB, number of ways = 4,
- number of sets in each way = 4KiB/4 = 1KiB ~ 2^{10},
- set index = $\log_2 2^{10} = 10$ bits.
- i.e. exactly the same as the 64/4=16KiB direct-mapped cache above.

```
| addr (32)   | tag (20)          | index (10)  | offset (2)  |
| ----------- | ----------------- | ----------- | ----------- |
| 0x _ _ *4   | upper address bits | index      | word select |
```

With 4 ways, to be checked in parallel,

```
| cache          | way 1              | ... | way 4              |
| -------------- | ------------------ | --- | ------------------ |
| set index (10) | tag + MESI + Word*4 | ... | tag + MESI + Word*4 |
```