# 3: Statistical Properties of Language

Machine Learning and Real-world Data (MLRD)

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# Last session: You implemented a Naive Bayes classifier

- Smoothed vs Unsmoothed
- The accuracy of the un-smoothed classifier was seriously affected by unseen words.
- We implemented add-one (Laplace) smoothing:

$$\hat{P}(w_i|c) = \frac{count(w_i, c) + 1}{\sum_{w \in V}(count(w, c) + 1)} = \frac{count(w_i, c) + 1}{(\sum_{w \in V} count(w, c)) + |V|}$$

- Smoothing helped!

# Today: frequency distributions in language

Questions:

- Why did smoothing help? (or in other words:)
- What is it about the distribution of words in a language that affected the performance of the un-smoothed classifier?
- Two Laws: Zipf's Law and Heap's Law

# Zipf’s Law: Word frequency distributions obey a power law

- There are a small number of very high-frequency words
- There are a large number of low-frequency words
- Zipf’s law: the $n$th most frequent word has a frequency proportional to $1/n$

*“a word’s frequency in a corpus is inversely proportional to its rank”*

# The parameters of Zipf's law are language-dependent

Zipf's law:

$$f_w \approx \frac{k}{{r_w}^{\alpha}}$$

where

$f_w$: frequency of word $w$

$r_w$: frequency rank of word $w$

$\alpha$, $k$: constants (which vary with the language)

e.g. $\alpha$ is around $1$ for English but $1.3$ for German

# The parameters of Zipf's law are language-dependent

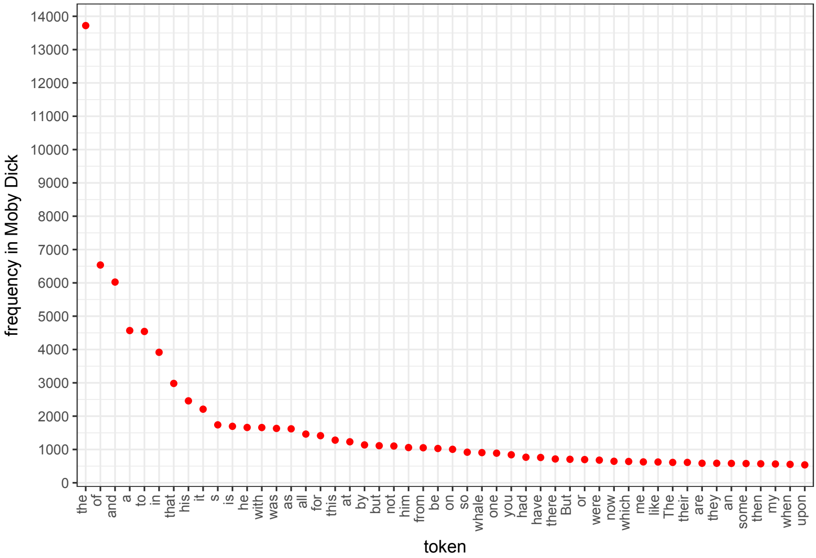
Actually...

$$f_w \approx \frac{k}{(r_w + \beta)^\alpha}$$

where

- $\beta$: a shift in the rank
- see summary paper by Piantadosi `https://link.springer.com/article/10.3758/s13423-014-0585-6`
- we won't worry about the rank-shift today

# There are a small number of high-frequency words...




Moby Dick has 206,052 words in total.

# Similar sorts of high-frequency words across languages

Top 10 most frequent words in some large language samples:

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Top 10 most frequent words in some large language samples:

**English**

| | | |
|---|---|---|
| 1 | the | 6,187,267 |
| 2 | of | 2,941,444 |
| 3 | and | 2,682,863 |
| 4 | a | 2,126,369 |
| 5 | in | 1,812,609 |
| 6 | to | 1,620,850 |
| 7 | it | 1,089,186 |
| 8 | is | 998,389 |
| 9 | was | 923,948 |
| 10 | to | 917,579 |

BNC,
100Mw

# Similar sorts of high-frequency words across languages

Top 10 most frequent words in some large language samples:

| | English | | | German | |
|---|---|---|---|---|---|
| 1 | the | 6,187,267 | 1 | der | 7,377,879 |
| 2 | of | 2,941,444 | 2 | die | 7,036,092 |
| 3 | and | 2,682,863 | 3 | und | 4,813,169 |
| 4 | a | 2,126,369 | 4 | in | 3,768,565 |
| 5 | in | 1,812,609 | 5 | den | 2,717,150 |
| 6 | to | 1,620,850 | 6 | von | 2,250,642 |
| 7 | it | 1,089,186 | 7 | zu | 1,992,268 |
| 8 | is | 998,389 | 8 | das | 1,983,589 |
| 9 | was | 923,948 | 9 | mit | 1,878,243 |
| 10 | to | 917,579 | 10 | sich | 1,680,106 |

BNC, 100Mw

“Deutscher Wortschatz”, 500Mw

# Similar sorts of high-frequency words across languages

Top 10 most frequent words in some large language samples:

| English | | | German | | | Spanish | | |
|---|---|---|---|---|---|---|---|---|
| 1 | the | 6,187,267 | 1 | der | 7,377,879 | 1 | que | 32,894 |
| 2 | of | 2,941,444 | 2 | die | 7,036,092 | 2 | de | 32,116 |
| 3 | and | 2,682,863 | 3 | und | 4,813,169 | 3 | no | 29,897 |
| 4 | a | 2,126,369 | 4 | in | 3,768,565 | 4 | a | 22,313 |
| 5 | in | 1,812,609 | 5 | den | 2,717,150 | 5 | la | 21,127 |
| 6 | to | 1,620,850 | 6 | von | 2,250,642 | 6 | el | 18,112 |
| 7 | it | 1,089,186 | 7 | zu | 1,992,268 | 7 | es | 16,620 |
| 8 | is | 998,389 | 8 | das | 1,983,589 | 8 | y | 15,743 |
| 9 | was | 923,948 | 9 | mit | 1,878,243 | 9 | en | 15,303 |
| 10 | to | 917,579 | 10 | sich | 1,680,106 | 10 | lo | 14,010 |
| BNC, 100Mw | | | "Deutscher Wortschatz", 500Mw | | | subtitles, 27.4Mw | | |

# Similar sorts of high-frequency words across languages

Top 10 most frequent words in some large language samples:

| English | | German | | Spanish | | Italian | |
|---|---|---|---|---|---|---|---|
| 1 the | 6,187,267 | 1 der | 7,377,879 | 1 que | 32,894 | 1 non | 25,757 |
| 2 of | 2,941,444 | 2 die | 7,036,092 | 2 de | 32,116 | 2 di | 22,868 |
| 3 and | 2,682,863 | 3 und | 4,813,169 | 3 no | 29,897 | 3 che | 22,738 |
| 4 a | 2,126,369 | 4 in | 3,768,565 | 4 a | 22,313 | 4 è | 18,624 |
| 5 in | 1,812,609 | 5 den | 2,717,150 | 5 la | 21,127 | 5 e | 17,600 |
| 6 to | 1,620,850 | 6 von | 2,250,642 | 6 el | 18,112 | 6 la | 16,404 |
| 7 it | 1,089,186 | 7 zu | 1,992,268 | 7 es | 16,620 | 7 il | 14,765 |
| 8 is | 998,389 | 8 das | 1,983,589 | 8 y | 15,743 | 8 un | 14,460 |
| 9 was | 923,948 | 9 mit | 1,878,243 | 9 en | 15,303 | 9 a | 13,915 |
| 10 to | 917,579 | 10 sich | 1,680,106 | 10 lo | 14,010 | 10 per | 10,501 |
| BNC, 100Mw | | “Deutscher Wortschatz”, 500Mw | | subtitles, 27.4Mw | | subtitles, 5.6Mw | |

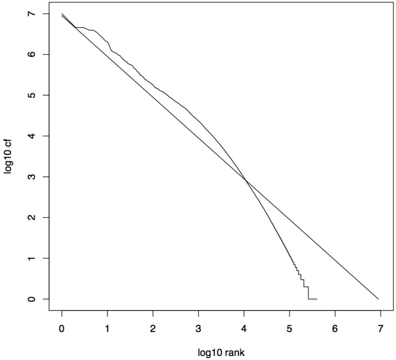
# Similar sorts of high-frequency words across languages

Top 10 most frequent words in some large language samples:

| | English | | German | | Spanish | | Italian | | Dutch | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | the | 6,187,267 | der | 7,377,879 | que | 32,894 | non | 25,757 | de | 4,770 |
| 2 | of | 2,941,444 | die | 7,036,092 | de | 32,116 | di | 22,868 | en | 2,709 |
| 3 | and | 2,682,863 | und | 4,813,169 | no | 29,897 | che | 22,738 | het/'t | 2,469 |
| 4 | a | 2,126,369 | in | 3,768,565 | a | 22,313 | è | 18,624 | van | 2,259 |
| 5 | in | 1,812,609 | den | 2,717,150 | la | 21,127 | e | 17,600 | ik | 1,999 |
| 6 | to | 1,620,850 | von | 2,250,642 | el | 18,112 | la | 16,404 | te | 1,935 |
| 7 | it | 1,089,186 | zu | 1,992,268 | es | 16,620 | il | 14,765 | dat | 1,875 |
| 8 | is | 998,389 | das | 1,983,589 | y | 15,743 | un | 14,460 | die | 1,807 |
| 9 | was | 923,948 | mit | 1,878,243 | en | 15,303 | a | 13,915 | in | 1,639 |
| 10 | to | 917,579 | sich | 1,680,106 | lo | 14,010 | per | 10,501 | een | 1,637 |
| | BNC, 100Mw | | "Deutscher Wortschatz", 500Mw | | subtitles, 27.4Mw | | subtitles, 5.6Mw | | subtitles, 800Kw | |

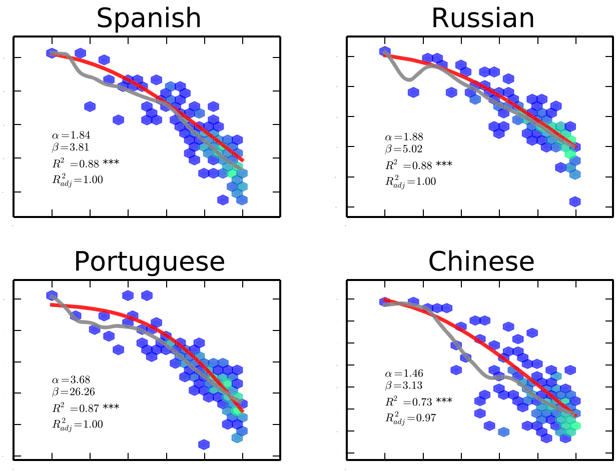
# It is helpful to plot Zipf curves in log-space

Reuters dataset: taken from `https://nlp.stanford.edu/IR-book/pdf/irbookonlinereading.pdf` – chapter 5




By fitting a simple line to the data in log-space we can estimate the language specific parameters $\alpha$ and $k$ (we will do this today!)

# In log-space we can more easily estimate the language specific parameters

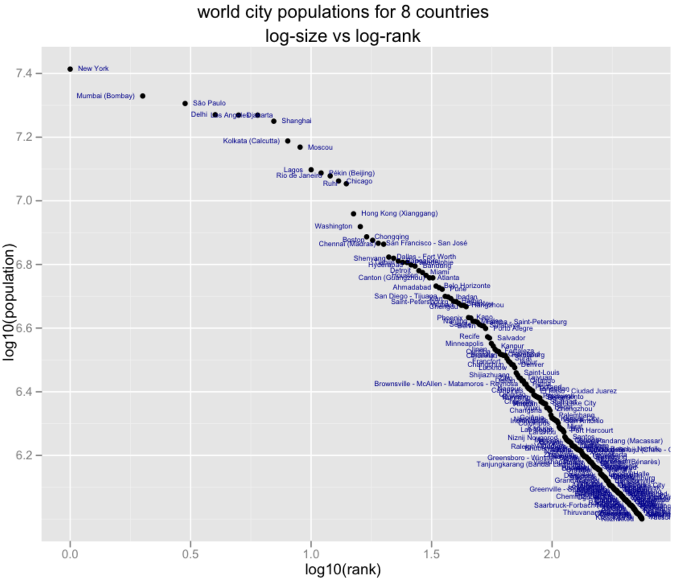



From Piantadosi `https://link.springer.com/article/10.3758/s13423-014-0585-6`

# Zipfian (or near-Zipfian) distributions occur in many collections

- Sizes of settlements
- Frequency of access to web pages
- Size of earthquakes
- Word senses per word
- Notes in musical performances
- machine instructions
- ...

# Zipfian (or near-Zipfian) distributions occur in many collections



# There is also a relationship between vocabulary size and text length

So far we have been thinking about frequencies of particular words:

- we call any unique word a type: *the* is a word type
- we call an instance of a type a token: there are 13721 *the* tokens in Moby Dick
- the number of types in a text is the size of the vocabulary (also called dictionary)

Today you will also explore this relationship.

# Heaps' law describes the relationship between vocabulary and text-length

Heaps' Law:

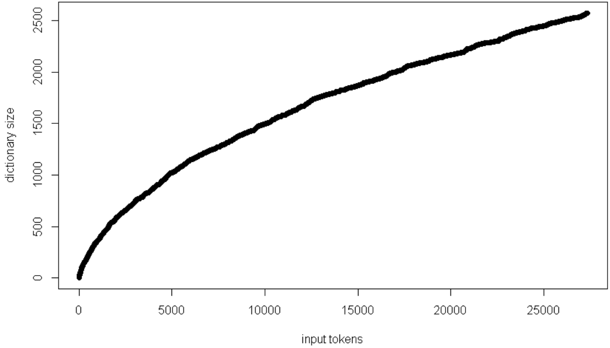
The relationship between the size of a vocabulary and the size of text that gave rise to it is

$$u_n = kn^\beta$$

where

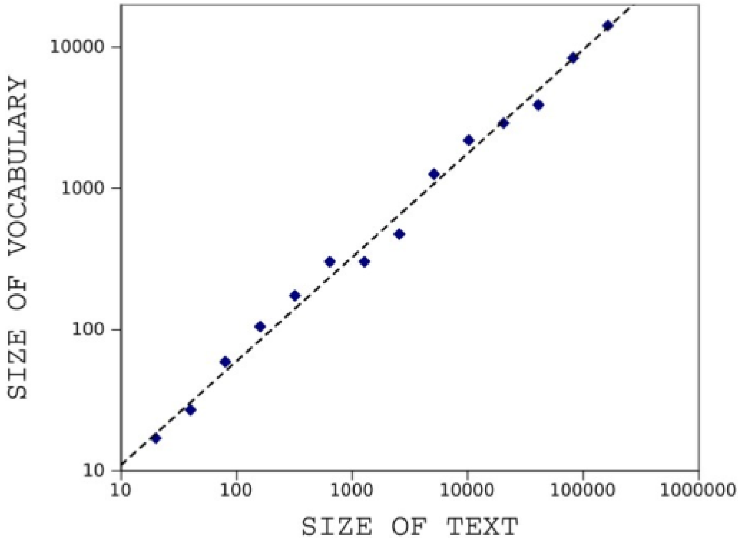
- $u_n$: number of types (unique items), i.e. vocabulary size
- $n$: total number of tokens, i.e.text size
- $\beta$, $k$: constants (language-dependent)
  - $\beta$ is around $\frac{1}{2}$
  - $30 \leq k \leq 100$

# Heaps' Law



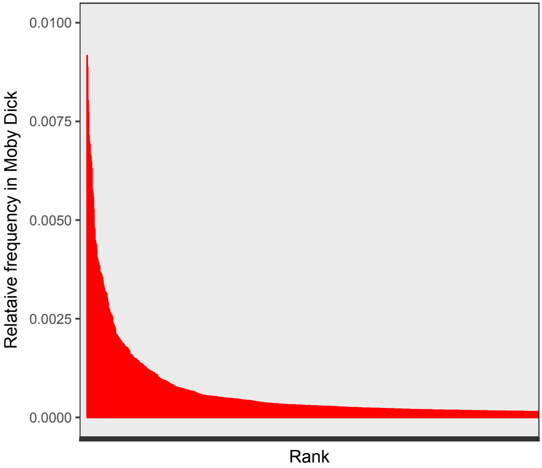

- No saturation: there will always be more new types
- As we progress through a text it takes longer and longer to encounter a new type

# It is helpful to plot Heaps' law in log-space

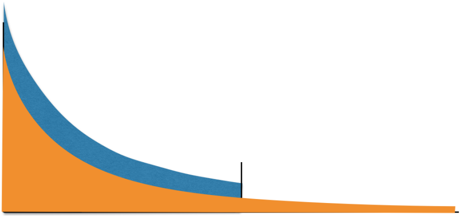


# Zipf's law and Heaps' law affected our classifier

- The Zipfian curve has a lot of probability mass in the long tail.
- By Heaps' law, we need increasing amounts of text to see new word types in the tail



# Zipf's law and Heaps' law affected our classifier



- With MLE, only seen types receive a probability estimate:

e.g. we used:

$$\hat{P}_{MLE}(w_i|c) = \frac{count(w_i, c)}{\sum_{w \in V_{training}} count(w, c)}$$

- True probability (e.g. for NEG class): orange; MLE: blue
- Total probabilities must sum to 1; in MLE all that probability mass is given to seen types
- MLE overestimates the probability of seen types (as opposed to unseen)

# Smoothing redistributes the probability mass

- Add-one smoothing redistributes the probability mass.

e.g. we used:

$$\hat{P}(w_i|c) = \frac{count(w_i, c) + 1}{\sum_{w \in V}(count(w, c) + 1)} = \frac{count(w_i, c) + 1}{(\sum_{w \in V} count(w, c)) + |V|}$$

- It takes some portion away from the MLE overestimate.
- It redistributes this portion to the unseen types.
- Better estimate; still not perfect.

# Today we will investigate Zipf's and Heaps' law in movie reviews

Follow task instructions on moodle to:

- Plot a frequency vs rank graph for larger set of movie reviews (you are given chart plotting code)
- Plot a log frequency vs log rank graph
- Indicate the location of your 10 chosen words from Tick 1, e.g. in colour, on this plot.
- Use least-squares algorithm to fit a line to the log-log plot (you are given `best-fit` code)
- Estimate the parameters of the Zipf equation
- Plot type vs token graph for the movie reviews

# Ticking for Task 3

There is no automatic ticker for Task 3

- Write everything in your lab book
- Save all your graphs (as screenshots or otherwise)