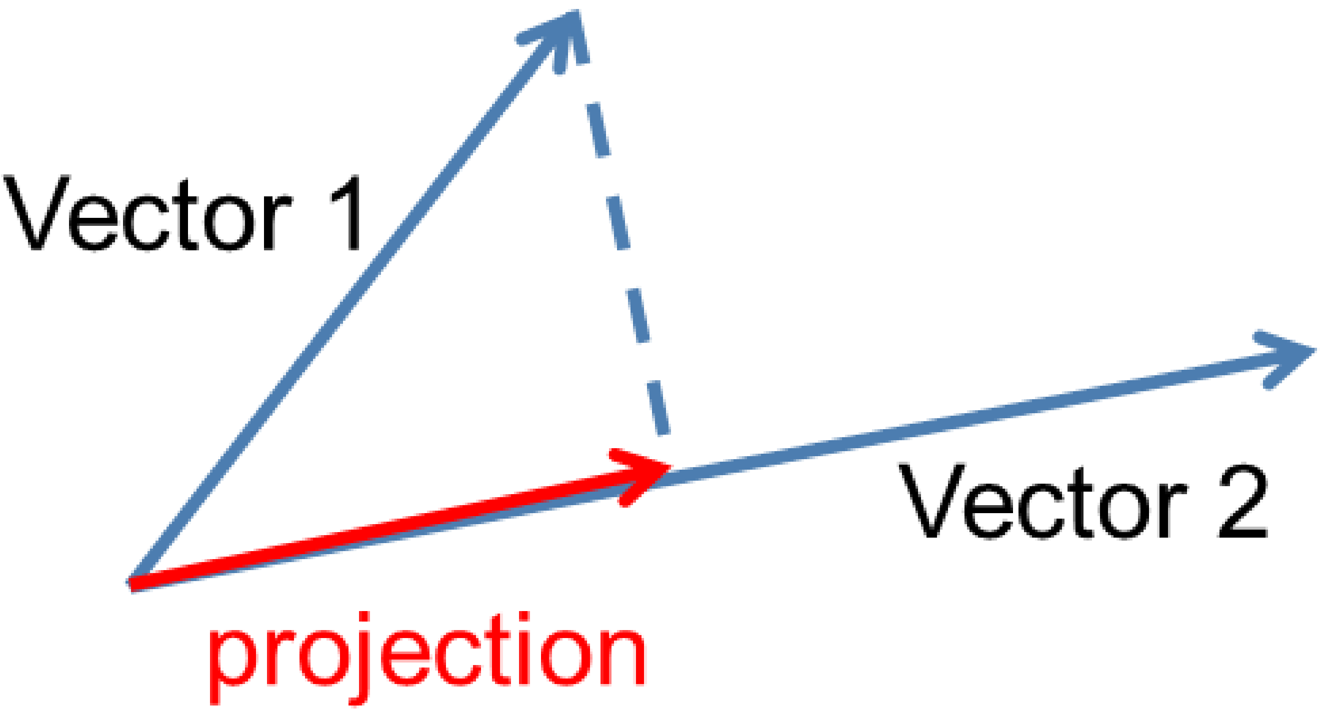

5. 投影向量(Projection)

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Knowledge is power.

This is also an optimization problem. Are you familiar with this equation?1.

$$x^* = (A^TA)^{-1}A^TY$$

1. Projection a Vector b onto a line

p is the vector from the origin to the closest points; a is the line direction;

So,

$$p = \hat{x}a. e = b - p$$

which is perpendicular to p, a .

Thus,

$$a * (b - \hat{x}a) = 0$$
$$=> a * b = \hat{x} * a * a$$
$$=> \hat{x} = \frac{a*b}{a*a}$$
$$=> p = \frac{a^Tb}{a^Ta}a = \frac{a*b}{||a||^2}a$$

2. Projection onto a Subspace

Suppose V is a subspace of R^m with basis $a_1, ..., a_n$. The projection of b onto V is the vector in V closest to b . This projection vector p , will be by definition a linear combination of the basis vectors of V :

$m*1=$ $m*n$ — $n*1$

$p=(a1,a2,\ldots an)$ $\begin{pmatrix} x1 \\ x2 \\ \ldots \\ xn \end{pmatrix}$

If A is a matrix with column vectors $a_1, \ldots, a_n$ and $\hat{x}$ is a vector with components $\widehat{x_1}, \ldots \widehat{x_n}$, the $p = A\hat{x}$. This error vector, the vector that points from p to b, will be $e = b - p = b - A\hat{x}$. This error vector will be perpendicular to the subspace V, which is equivalent to being perpendicular to the basis vectors $a_1, \ldots, a_n$. Stated mathematically, this is

$$a_1^T(b - A\hat{x}) = 0,$$
$$a_n^T(b - A\hat{x}) = 0$$

We can write all of the above in matrix form:

$$A^T(b - A\hat{x}) = 0$$

This in turn can be written as

$$A^TA\hat{x} = A^Tb$$

If A^TA is invertible we can solve for $\hat{x}$ to get:

$$\hat{x} = (A^TA)^{-1}A^Tb$$

Now, the projection vector p is the vector $A\hat{x}$, so

$$p = A\boxed{(A^TA)^{-1}}A^Tb$$

In general, to calculate the projection of any vector onto the space W we multiply the vector by the projection matrix

$$P = A(A^TA)^{-1}A^T$$

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