PEREGRINE
SECURITIES

DERIVATIVE AND QUANTITATIVE RESEARCH

Currency Hedge Design: Accounting for Uncertain Correlation and Volatility

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SUMMARY

- Management of currency risk embedded in offshore assets has been made easier by substantially improved liquidity in currency derivative markets and the availability of flexible hedging strategies.
- However, fund managers are still faced with the difficult problem of selecting an appropriate currency hedge.
- In previous Peregrine Securities work, it was shown that currency hedge selection can be approached in an optimization framework and that the particular choice of hedge is strongly dependent on the correlation between the exchange rate and the returns of the foreign asset.
- Unfortunately, correlations between assets are generally unstable and it can be difficult to forecast an appropriate value to use as input to the optimizer.
- In this work we outline an approach to the determination of an optimal currency hedge in the presence of non-constant volatility and correlation. It is shown that implementation of the dynamic conditional correlation (DCC) model in a simulation framework allows one to incorporate the effects of time-varying parameters into the hedge selection process in a systematic and quantitative manner.
- The impact of daily changing volatility on certain statistical properties of the final cumulative return for an asset is examined. For example, we show that GARCH(1,1) volatility dynamics lead to an increase in tail risk, particularly for shorter investment horizons.
- The correlation between final cumulative returns is found to be dependent on both the long-run average correlation as well as the current value. Shorter terms display a higher dependence on the initial value, with its influence reduced for longer terms.
- The incorporation of an assumption of time varying volatility and correlation into the determination of an optimal currency hedge is demonstrated with selected examples.
- It is shown that the particular choice of short-term hedge is strongly dependent on the current value of correlation, emphasising the need for a modelling framework that provides accurate estimates of time-varying correlation.
- An important application of the framework described in this work is the determination of an optimal overall asset allocation and currency hedge at the same time.
- In the examples considered, a zero-cost collar with the bought put option struck at 95% of the prevailing exchange rate is found to be the favoured currency hedge.
- This finding emphasised the importance of considering currency options as they allow for a risk-reduction via two mechanisms. Negative correlation between assets in the portfolio and the exchange rate makes it beneficial to maintain some degree of currency exposure, while the elimination of extreme returns above the cap and below the floor leads to further reductions in risk.

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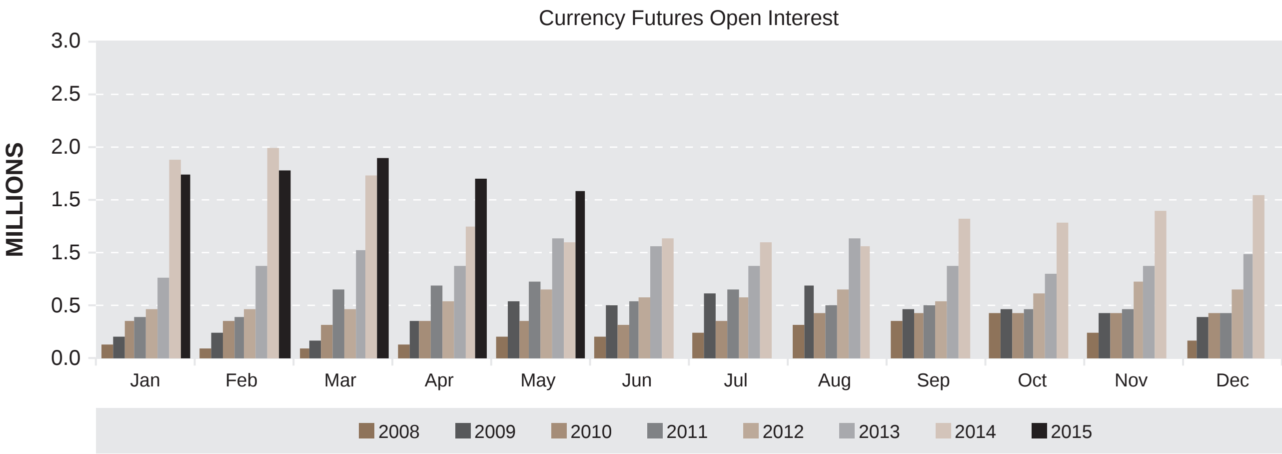
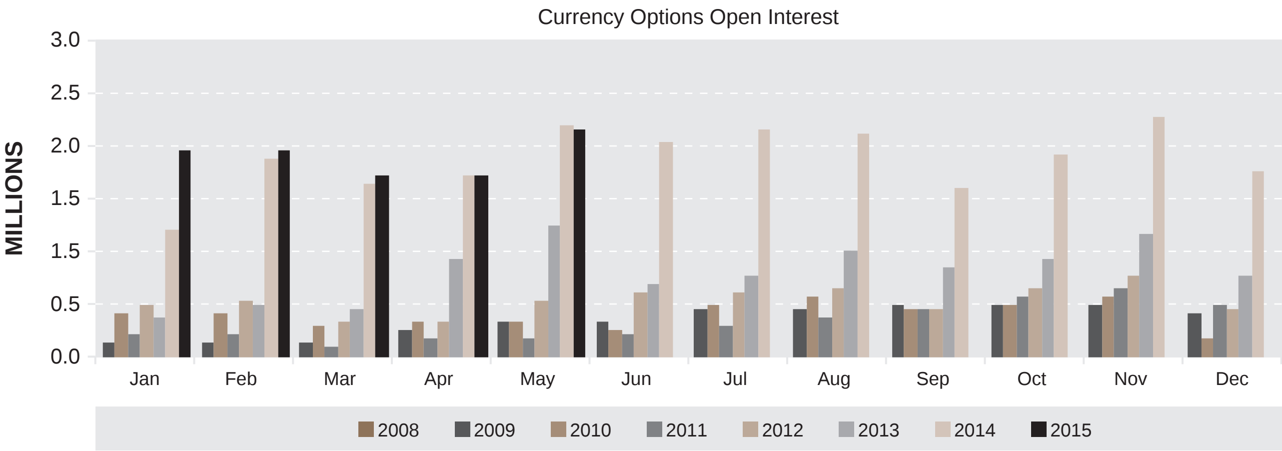
1 INTRODUCTION

The impact of the exchange rate on the performance of portfolios holding offshore assets is an important consideration for fund managers. Recent years have seen increasing interest in instruments that facilitate the management of currency exposure, be it in the form of linear currency futures contracts or non-linear currency option overlays. Domestically, currency derivative activity has continued to grow since the inception of the JSE's Currency Derivatives Market in 2007. Graphical evidence of this trend can be seen in figure 1 which shows monthly open interest comparisons per year for both futures and options contracts. Contributing to the positive growth in the use of such instruments has been improved pricing efficiency, in line with a global migration from over-the-counter (OTC) to listed currency derivatives. Delta-1 execution is nearly instantaneous and fund managers have the ability to trade directly with authorised dealers. However, optionality still requires careful sourcing of the most efficient price.

FIGURE 1: INCREASING CURRENCY DERIVATIVE ACTIVITY





(Source: Peregrine Securities June 15 Close-Out Report)

Even though the management of currency risk embedded in offshore assets is made easier by the availability of flexible hedging strategies, fund managers are still faced with the difficult problem of selecting an appropriate hedge. Possible approaches to the currency hedge decision were reviewed in a previous Peregrine Securities Report (Seymour, Chikurunhe and Flint, 2015). In summary, early published work focussed on finding an optimal fixed proportion of currency exposure to hedge (Perold and Schulman, 1988, Black, 1990, Froot, 1993, and Gastineau, 1995), while Jorion (1994) advocated the use of a portfolio optimization approach in which the optimal hedge is solved for *at the same time* as the allocation to all the assets in the portfolio. The two main advantages of this joint optimization approach are:

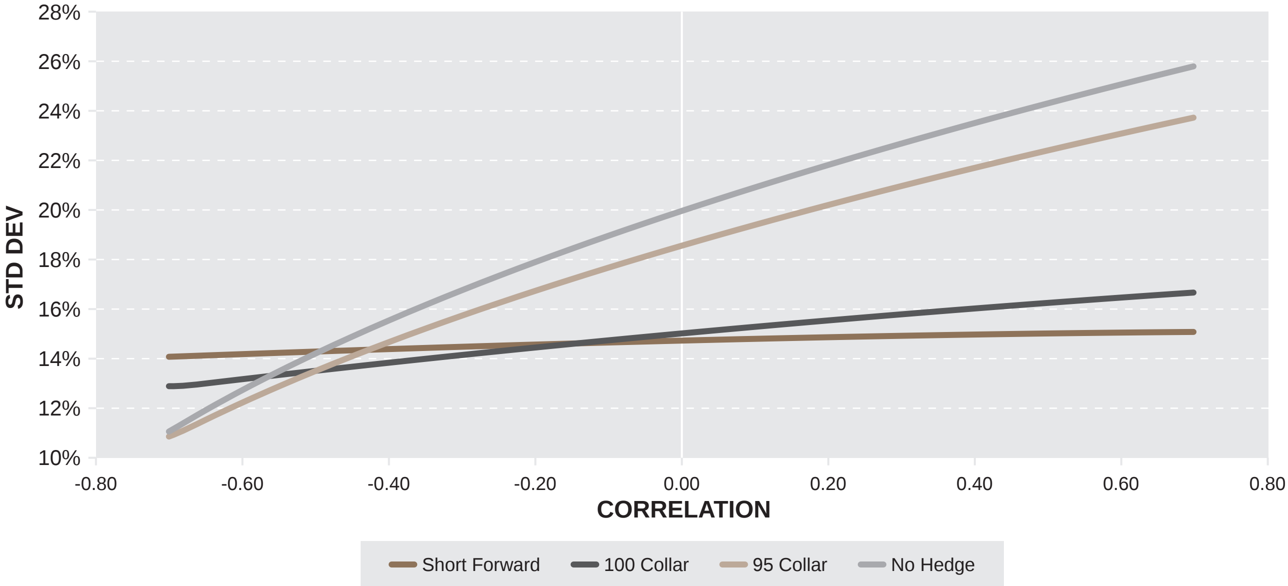
- The interaction between the exchange rate and non-foreign assets in the portfolio is taken into account, leading to further potential risk-reduction benefits.
- One derives in a systematic manner a currency hedge and overall portfolio allocation that is consistent with the views and risk preferences of the fund manager.

An implementation of the joint optimization method of Jorion (1994) via a simulation and optimization framework was described by Seymour et al. (2015). An important extension that was discussed was the use of *currency options*, which are typically difficult, if not impossible, to include in commonly-used analytical portfolio optimization frameworks. However, the calculation of underlying asset return distributions numerically via simulation makes the effect of their inclusion in portfolios straightforward to quantify. A similar analysis on optimal currency hedging strategies featuring optionality has subsequently been published (Chen, Kritzman and Turkington, 2015), although the assumed objective of the fund manager is different. While Seymour et al. (2015) focus on finding a currency hedge that reduces the standard deviation of the portfolio, Chen et al. (2015) follow a full-scale optimization approach which aims to maximize the expected value of a kinked utility function. It should be noted, however, that the simulation framework described in the current report has been successfully applied to the construction of portfolios via full-scale optimization (Seymour, Chikurunhe and Flint, 2014) with the computational demands of the method addressed via parallel computing methods. A currency hedge selection approach based on the maximization of expected utility can therefore also be easily considered.

An important finding from most published works on currency exposure management is the impact of the correlation between the returns of the exchange rate and the returns of the foreign asset (in the original currency). As described by Seymour et al. (2015), when considering the risk of a holding in a foreign asset in isolation (i.e. ignoring other possible constituents in the portfolio), the currency hedge leading to the lowest standard deviation is a function of the assumed exchange rate-foreign asset correlation. This finding is summarised in figure 2 (repeated from Seymour et al. (2015)) which shows the forecast standard deviation of a holding in the S&P 500 combined with various currency hedging strategies.

For very negative values of correlation (< -0.5), a position with no currency hedge is seen to have less risk than a position in which currency exposure is completely removed via linear currency futures contracts. However, a zero-cost collar in which the bought put option is struck at 95% of the prevailing exchange rate is seen to lead to further risk reduction. Higher values of correlation favour a collar strategy with reduced exposure to currency movements, or complete removal of currency exposure via linear instruments.

FIGURE 2: IMPACT OF EXCHANGE RATE-FOREIGN ASSET CORRELATION ON CURRENCY HEDGE PERFORMANCE



Seymour et al. (2015) outline a systematic procedure for selecting an appropriate currency hedge which can be summarised as follows:

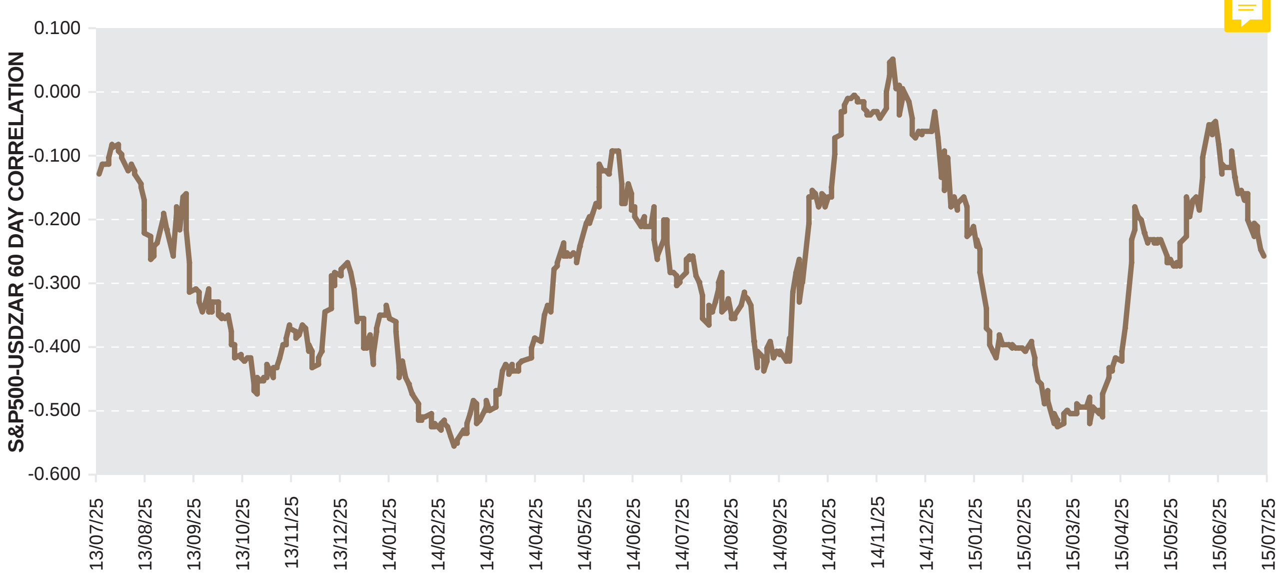
- Define a set of candidate hedges (e.g. collars with different put strike levels, linear futures contracts)
- For a specified value of exchange rate-foreign asset correlation, estimate via simulation the standard deviation of the holding for each currency hedge under consideration
- Choose the hedge which results in the lowest standard deviation

In the context of foreign assets in combination with domestic assets, the approach based on the joint optimization method of Jorion (1994) can be used to determine the optimal currency hedge. An important point to remember is that the optimal currency hedge in this case is not necessarily the same as when considering the foreign asset in isolation, as the interaction between the domestic assets and the exchange rate may make it beneficial to maintain currency exposure, particularly if the correlations between the domestic assets and the exchange rate are generally negative.

It is thus clear that given a view on the future correlation between the exchange rate and both foreign and domestic asset returns, one can follow a systematic procedure to determine a currency hedge that is consistent with one's risk preferences. Unfortunately, correlations between assets are generally unstable and it can be difficult to forecast an appropriate value to use as input to the optimizer. The time-varying nature of correlation can be seen graphically in figure 3, which shows the correlation between the S&P 500 and the USDZAR exchange rate between July 2013 and July 2015.

Values were calculated based on a rolling window of 60 trading days' returns, and can be seen to vary dramatically between a low of -0.56 and a high of +0.05. The main issue is that one needs to specify a correlation value that will apply to cumulative returns at the end of a given time horizon, but correlation estimated from history does not give an indication of what that correlation will be.

FIGURE 3: ROLLING 60-DAY S&P 500-USDZAR HISTORICAL CORRELATION



In the current work we address the problem of uncertain future correlation in the context of finding the best currency hedge. The approach is based on considering a more sophisticated model for the evolution of underlying asset prices that incorporates the time-varying properties of both volatility and correlation. Although the model is obviously more complex than assuming a constant volatility and correlation structure throughout the future time horizon, it allows one to examine the effects of time-varying correlation and volatility on the optimal currency hedge in a systematic way. The remainder of the report is structured as follows. In section 2 we describe certain empirical features of historical asset returns and discuss how these can be modelled via the dynamic conditional correlation (DCC) model. The impact of non-constant volatility on the final return distribution of an asset, as well the relationship between current correlation and final realised correlation are discussed in section 3. Section 4 considers the issue of minimizing the risk of a foreign asset holding in the presence of uncertain correlation, while section 5 describes how the joint optimization approach of Jorion (1994) for general asset and currency hedge allocation can be successfully applied under the assumption of DCC dynamics. Section 6 concludes.

2 MODELLING TIME-VARYING VOLATILITY AND CORRELATION

2.1 Introduction

A requirement for any analysis involving portfolio risk is a forecast of the portfolio return distribution at the end of the time horizon of interest, which will clearly depend on the assumed statistical properties of the constituents. An important issue that needs to be considered is the estimation of these properties, given available data. Ideally, one would use historical returns measured over the same length of time. For example, annual return statistics could be measured from historical annual returns. This method is unfortunately not feasible as the available number of observations would generally be very small, leading to inaccurate parameter estimates. Furthermore, one would be forced to include data from different market regimes, placing doubt on the validity of the estimates.

The typical approach used to overcome such difficulties is to estimate the properties of higher frequency return data (such as daily, weekly or monthly) and then use these to derive longer-term properties based on certain assumptions. The most common assumption made is that the observed returns are *independent and drawn from the same distribution*, and thus all returns have the same volatility and expected return. In this case, simple scaling rules can be applied to convert statistics corresponding to higher frequency returns (e.g. daily) to values for longer-term returns (e.g. annual). A well-known example is the estimation of annual volatility as the standard deviation of observed daily returns multiplied by the square root of 252 (the "square root of time" rule). In standard mean-variance analysis, it is further assumed that all asset returns are normally distributed so that the distribution of each asset is completely specified by knowing only the expected return and volatility.

An alternative method of using daily return properties to estimate longer-term return distributions is *simulation*. In this case, complete asset price paths are generated for a large number of scenarios based on an assumed model that describes the random behaviour of an asset's price. A simple example is the lognormal random walk where the log price on a particular date is given as the previous day's log price plus a random return X_t:

$$\ln(S_t) = \ln(S_{t-1}) + X_t$$

For a simulation procedure in which prices are simulated daily, the statistical properties of X_t would correspond to daily data. Simulation therefore allows one to estimate a return distribution corresponding to a longer time horizon by taking into account the cumulative effect of shorter term returns.

The simulation framework developed at Peregrine securities has allowed for two specifications for the random properties of X_t, namely

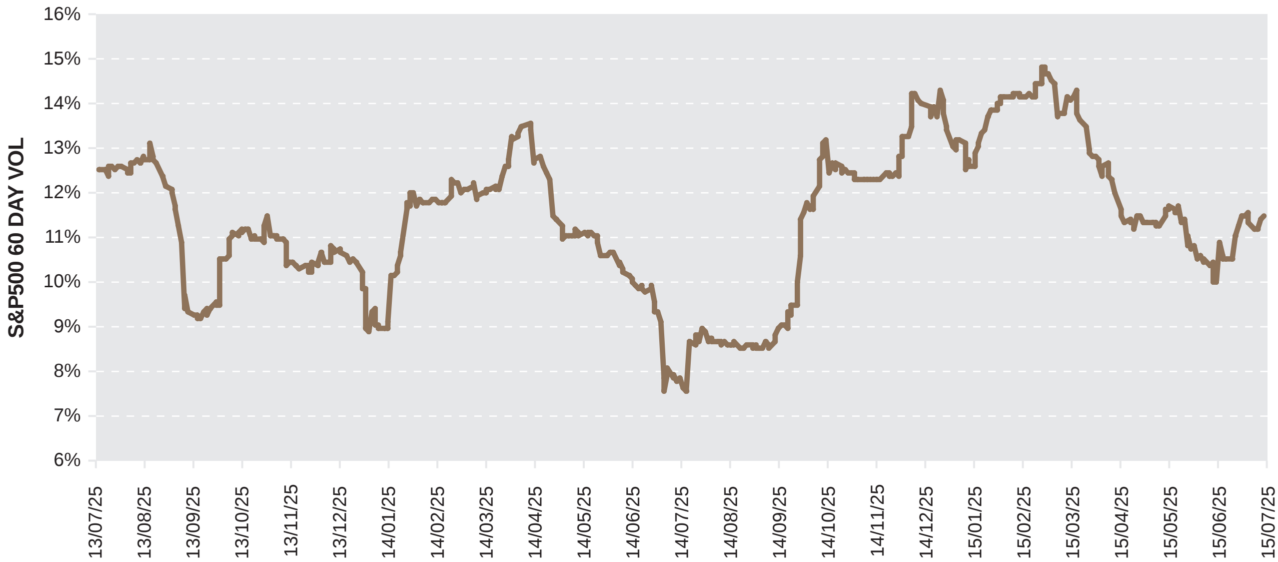
- Normally distributed returns
- Returns sampled from historical data.

In both cases, it is assumed that the expected return and volatility are constant throughout the simulation period. Although this generally leads to the same optimal portfolios when considering mean-variance efficient combinations of traditional assets such as equities and bonds, certain factors make simulation worthwhile:

- Non-linear instruments such as options can be included in the portfolio, making a systematic and quantitative analysis of the effect of these instruments straightforward.
- Additional properties of the portfolio return distribution such as VaR and CVaR are easily calculated.
- Path-dependent risk measures, which would typically be impossible to forecast, can be estimated (Seymour, Chikurunhe, and Flint, 2012).

The simple simulation model described above is straightforward to use in that one need only specify expected returns and volatilities for each asset, as well as corresponding correlations in order to generate a forecast return distribution for a wide variety of portfolios. In this regard, it is similar to the input requirements for traditional mean-variance analysis, but with the added benefit of increased flexibility in terms of the types of instruments that can be included in the portfolio and the risk measures that can be computed. However, there are well-known empirical features of historical returns that are not captured by the model. One of these was discussed in section 1, namely that of *time-varying correlation*. Another distributional property that is assumed constant, but in reality is not, is *volatility*. Figure 4 demonstrates this clearly with values of standard deviation for the S&P 500 calculated for rolling 60 trading day windows between July 2013 and July 2015.

FIGURE 4: ROLLING 60-DAY S&P 500 HISTORICAL VOLATILITY



Given that observed returns exhibit potentially complex dynamics, it is natural to consider what impact features such as non-constant volatility and correlation will have on the distribution of portfolio returns at the end of the time horizon, and in particular, the difference made to various risk measures, compared to the constant volatility-correlation case. The analysis of the effect of time-varying properties can be easily achieved in a simulation framework simply by using an asset price model that explicitly accounts for these dynamics. Note that the idea of using properties of high frequency data to derive the properties of longer term data is still the same.

Models aimed at incorporating empirically observed return characteristics have been extensively studied, both in the context of option pricing and risk management. A well-known example in the field of option pricing is the Heston stochastic volatility model, and its application to the pricing of exotic options has been described in a previous Peregrine Securities report (Seymour, 2011). It should be noted, however, that models developed for option pricing are not directly applicable for the issues to be addressed in this report. A particular issue is *parameter estimation*, where models are calibrated to give prices as close as possible to observed vanilla option prices, rather than to give an accurate reflection of real-world return data.

Models widely used in risk management specifically aimed at capturing real-world dynamics in a straightforward way are more appropriate, and can be easily incorporated into our simulation framework. One of the most popular models employed to describe time-varying volatility and correlation is the dynamic conditional correlation (DCC) model of Engle (2002), and is the model that forms the basis of the current research. A key feature of the DCC model that contributes to its computational tractability is that volatilities and correlations are modelled separately. The main steps involved in the estimation of the DCC model are as follows:

- Estimate a time-varying volatility model for each asset.
- Calculate standardised returns by dividing observed returns by estimated volatilities for each date.
- Estimate a recursive relationship for the evolution of the covariance matrix of the standardised returns.

The time-varying volatility model typically employed is the well-known generalized autoregressive conditional heteroscedasticity (GARCH) of Bollerslev (1986), while the dynamics of the correlation matrix are specified in a similar manner.

The aim of the current section is to provide an overview of the DCC model and describe its implementation in a simulation framework. Furthermore, we wish to demonstrate the effect of more complicated asset price dynamics on the final portfolio return distribution and determine whether or not any significant differences arise relative to the constant volatility-correlation model. Section 2.2 describes the univariate volatility component of the DCC model, namely the GARCH(1,1) model, while section 2.3 examines the complete DCC specification. A discussion of the simulation of asset price dynamics under the DCC model as well as an analysis of the effect of non-constant parameters on the final portfolio distribution is presented in section 3.

2.2 The GARCH(1,1) Model

As described In section 1, the assumed model for asset price dynamics in the Peregrine Securities simulation framework is based on a lognormal random walk:

$$\ln(S_t) = \ln(S_{t-1}) + X_t$$

In the original formulation, drift and volatility are assumed to be constant so that the return X_t can be written as:

$$X_t = \mu + \sigma Z_t$$

where Z_t is a mean zero-variance one process, either assumed to be normally distributed or derived from historical data. Importantly, the drift μ and volatility σ are assumed to constant for all returns. In extending the model to take into account time-varying volatility, the return equation can be rewritten with volatility showing a clear dependence on time:

$$X_t = \mu + \sigma_t Z_t$$

What is then required is a description of the behaviour of σ_t. The GARCH(1,1) model is such a description and is summarised by the following formula:

$$\sigma_t^2 = \gamma V_L + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2$$

where V_L is the long-term variance and u_{t-1} is the observed return on day $t{-}1$. The parameters γ, α and β are weights to each component of the equation and sum to one. When estimating parameters, γV_L is generally replaced by a single parameter ω. In essence, the model states that the variance on a particular date is some multiple of the previous variance plus a random disturbance resulting from the previous return (the "(1,1)" refers to the fact that only the previous day's values are used). The inclusion of a long-term variance term ensures that the model displays the key feature of mean reversion where over time variance tends towards some long-run value.

The derivation of the GARCH(1,1) model is based on the idea that when estimating volatility, more weight should be given to recent return observations, rather than the uniform weighting scheme implicit in the standard volatility estimation formula. Consequently, one finds that volatilities estimated via the GARCH(1,1) model respond more quickly to return shocks. This feature has led to GARCH(1,1) models being especially suited to the calculation of accurate one-day ahead value-at-risk (VaR) forecasts (see Seymour and Polakow, 2003).

The parameters of the model are estimated from observed historical return data via maximum likelihood estimation. The estimation can sometimes be problematic and in such cases it may be preferable to use a special case of the GARCH(1,1) model, namely the exponentially weighted moving average (EWMA) model. In this case, the model is defined by a single parameter λ such that $\gamma = 0$, $\alpha = 1 - \lambda$, and $\beta = \lambda$. The EWMA model forms the basis of the original RiskMetrics™ system developed at JP Morgan. Note, however, that the GARCH(1,1) model is theoretically more appealing as it features volatility mean reversion, whereas the EWMA model does not.

In addition to mean reversion, an important empirical characteristic captured by the GARCH(1,1) model is that of *volatility clustering*. An examination of returns over a sufficiently long period generally shows that large return shocks tend to occur in groups or clusters. An example of this is seen in the daily return history of the S&P 500 between March 2002 and July 2015 (figure 5). Volatilities derived from the fitted GARCH(1,1) model clearly show volatility spikes corresponding to the larger returns, and like the returns themselves, these tend to occur in clusters.

FIGURE 5: LONG-TERM S&P 500 RETURN HISTORY (MARCH 2002 TO JULY 2015)

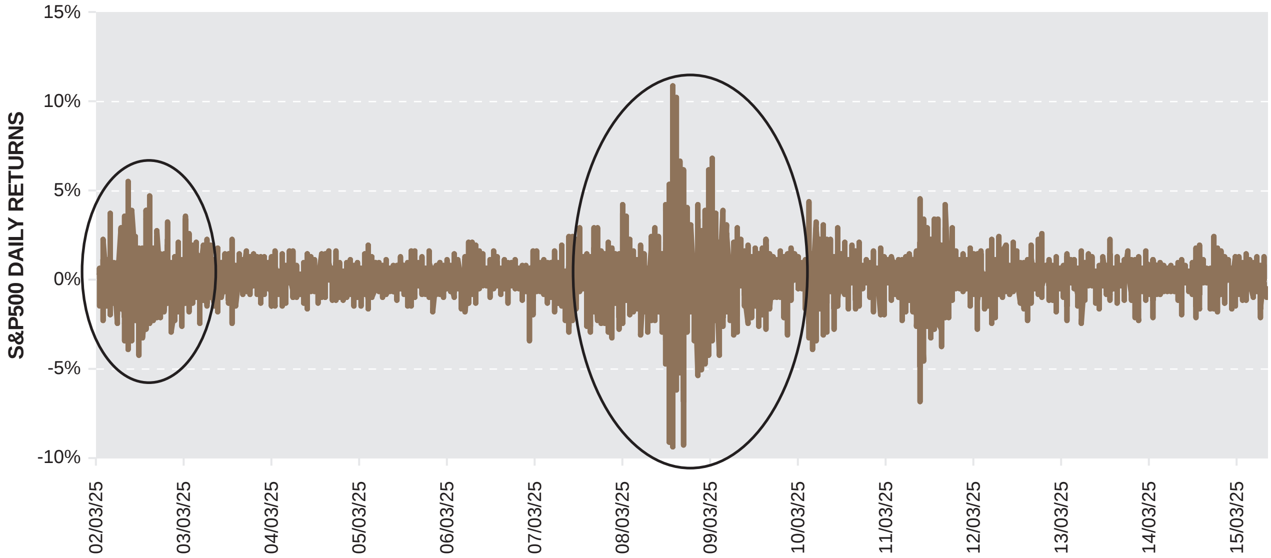
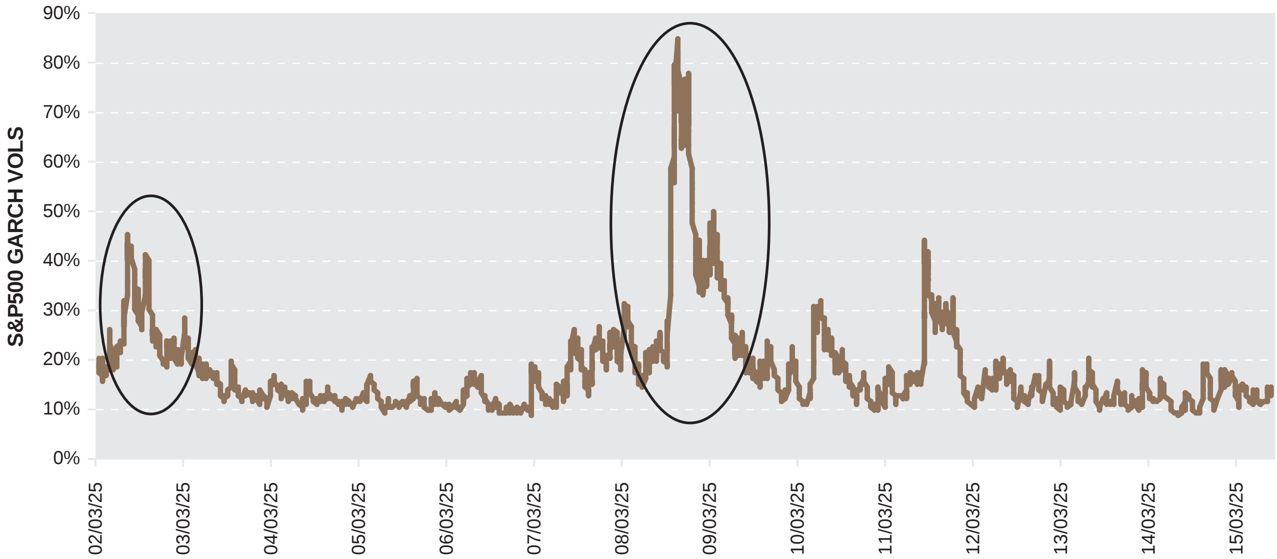


FIGURE 6: S&P 500 VOLATILITY CLUSTERING



For the examples considered in this report, GARCH parameters for all assets were estimated from returns observed between July 2013 and July 2015, and are shown in table 1. A shorter-term period was selected as the correlation of interest in this work (S&P 500 – USDZAR) has been seen to exhibit different types of behaviour over different periods of history, as will be shown in section 2.4. Consequently, all parameters were estimated from the most recent regime, rather than a long-term data set.

TABLE 1: ESTIMATED GARCH PARAMETERS

Stock Name	Omega	Alpha	Beta	Long-Term Vol	Current Vol
S&P500	7.14E-06	0.1240	0.7370	11.38%	11.27%
DOLLAR	2.05E-06	0.0574	0.9095	12.49%	12.03%
TOP40	2.18E-06	0.0687	0.9062	14.78%	18.28%
BONDS	3.94E-07	0.0263	0.9480	6.21%	6.02%
PROPERTY	6.57E-06	0.1095	0.7909	12.90%	12.55%

Historical volatilities derived from the estimated GARCH parameters are shown in figures 7 and 8. It is clear that the S&P 500 series is more variable than that of the Rand-Dollar exchange rate, demonstrating a higher sensitivity to return shocks. This is a consequence of the significantly lower value of β for the S&P 500 series (0.74 vs 0.91) which results in a series that reverts more quickly to the long-run mean.

FIGURE 7: DERIVED S&P 500 GARCH(1,1) VOLATILITY SERIES

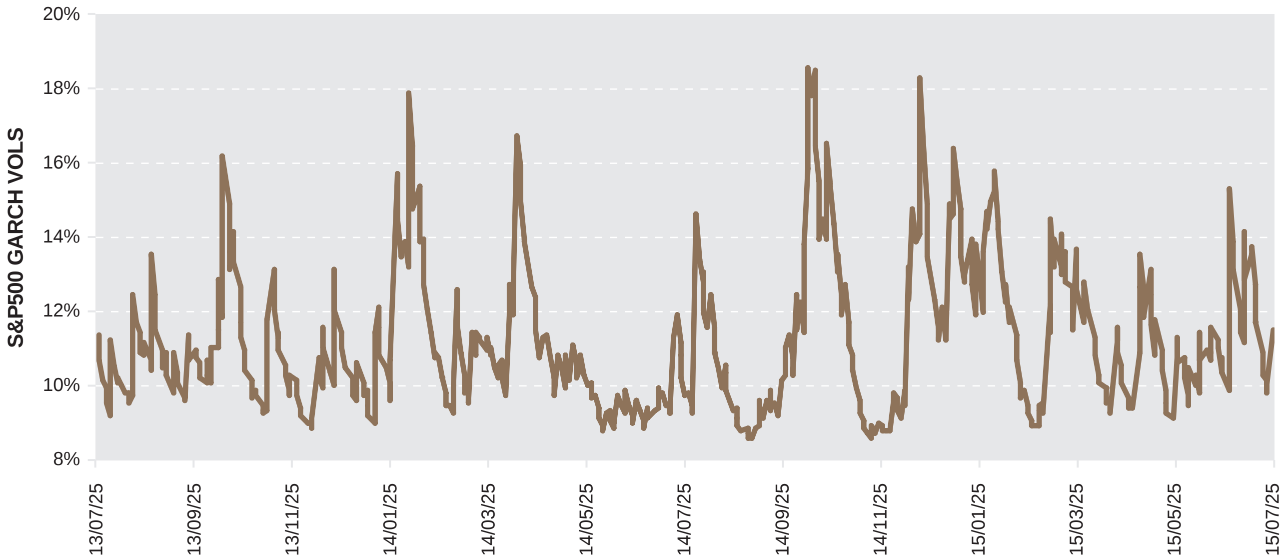
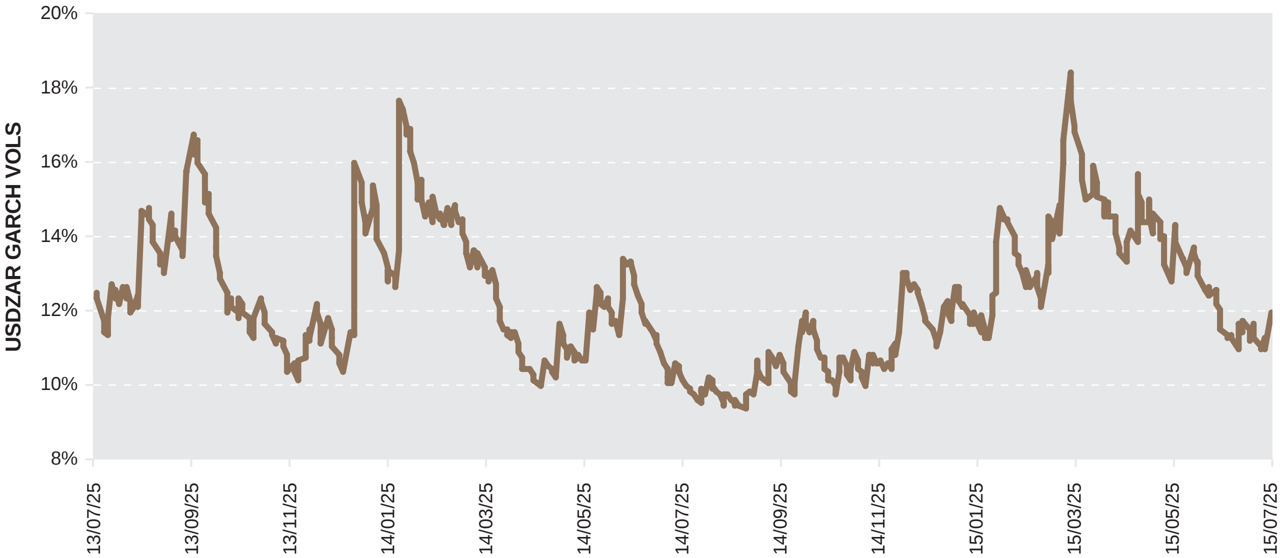


FIGURE 8: DERIVED USDZAR GARCH(1,1) VOLATILITY SERIES



2.3 The Dynamic Conditional Correlation (DCC) Model

When considering a portfolio of assets, the multivariate version of the GARCH model provides a possible means by which to capture the properties of time varying volatility for each asset, as well as the changing correlations between their returns. Extending the GARCH(1,1) model to accommodate an arbitrary number of dependent assets is, however, not straightforward and leads to complicated description of asset return dynamics. The volatility of each asset depends not only on its own previous return and volatility, but also on the cross products of returns of all other assets, as well as the volatilities of all other assets. The number of parameters to be estimated is thus very large and as a result, the model cannot be generally applied to portfolios (Engle, 2002). The DCC model of Engle (2002) is a simpler alternative in which the number of parameters to be estimated for the assumed correlation process is independent of the number of assets. As stated in section 2.1, this is achieved by modelling the volatilities of all assets separately and then modelling the correlation matrix of the residual series. Thus, once GARCH(1,1) volatilities have been estimated, historical residual returns are calculated as

$$Z_t = \frac{X_t - \mu}{\sigma_t}$$

where X_t is the observed return, μ the drift rate, and σ_t the GARCH volatility. Parameters α and β for a recursive relationship for the entire covariance matrix Q_t are then estimated:

$$Q_t = (1 - \alpha - \beta)S + \alpha Z_{t-1} Z'_{t-1} \ + BQ_{t-1}$$

where S is an estimate of the long-term average covariance matrix, typically found by calculating the covariance matrix for the entire residual series. It is clear that there is some similarity to the GARCH(1,1) description of the evolution of volatility – the covariance matrix on a particular date is equal to some proportion of the previous matrix with an update due to new returns as well a weighting towards the long-run average matrix. Correlations are then derived from Q_t via the following standard transformation:

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}\, q_{jj,t}}}$$

An important decision to be made when implementing any model is the historical period on which to base parameter estimates. One option is to use a long-term (more than 10 years) data set in order to obtain an as accurate as possible description of the process under consideration. However, as shown in figure 9, S&P 500-USDZAR correlation appears to have switched between different regimes between 2002 and 2015. It was therefore decided to base model parameters on the most recent regime in order to capture the dynamics of current behaviour. While GARCH(1,1) parameters for all assets were successfully estimated, estimation of the α and β correlation parameters using returns observed between July 2013 and July 2015 proved to be problematic. Optimization of the maximum likelihood objective function failed to converge to a feasible solution. and it thus appears as if DCC parameter estimation can present the same difficulties known to exist in the estimation of GARCH parameters (see Zumbach (2000) for a discussion of challenges involved in GARCH fitting) Local maxima may be an issue, and it is therefore possible that the estimation procedure could benefit from the application of a global random search optimization algorithm as employed in the full-scale optimization portfolio construction method (Seymour et al. 2014). However, this was not considered further and is left for future research. Consequently, a strategy similar to that employed in the case of unstable GARCH models was employed, namely an EWMA model with specified parameters. DCC α and β were set to 0.03 and 0.96 respectively. As shown in figure 10, these values led to a derived DCC correlation similar to the rolling 60 day correlation estimate. Notice, however, that DCC correlation leads the value derived from a rolling window as the DCC model responds more quickly to return shocks.

FIGURE 9: CORRELATION REGIMES FOR LONG-TERM DATA

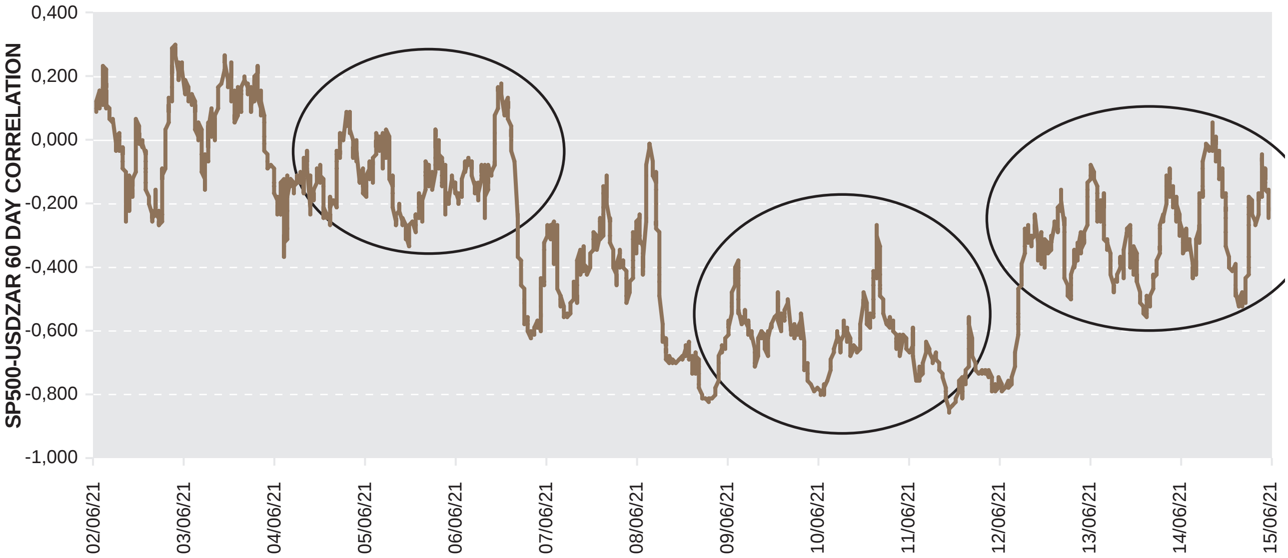
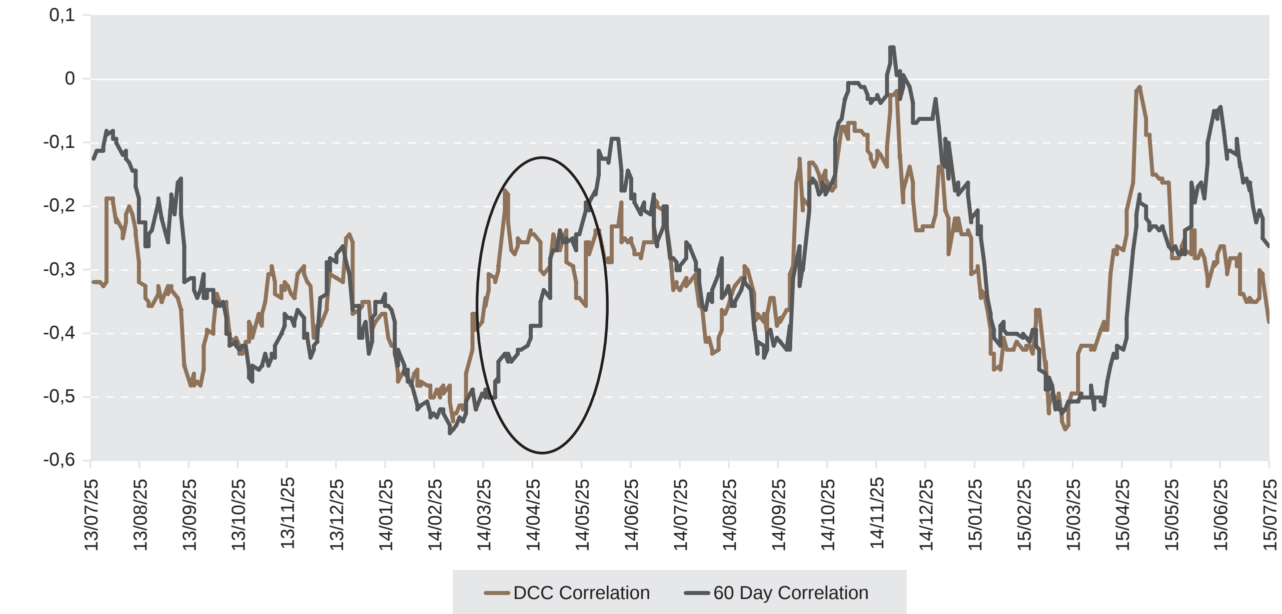


FIGURE 10: COMPARISON BETWEEN DCC AND ROLLING WINDOW CORRELATION



3 EFFECT OF DCC DYNAMICS ON OVERALL RETURN DISTRIBUTION

3.1 Introduction

We now consider the effect of non-constant volatility and correlation on the final distribution of asset returns and in particular, examine whether any differences exist relative to the assumption of invariant values. In section 3.2 we first consider the return properties of a single asset whose volatility follows a GARCH(1,1) process. Section 3.3 then discusses how the correlation between the cumulative returns of a pair of assets at the end of the investment horizon is influenced by constantly changing interim correlation.

3.2 Effect of GARCH(1,1) Dynamics on the Final Distribution of a Single Asset

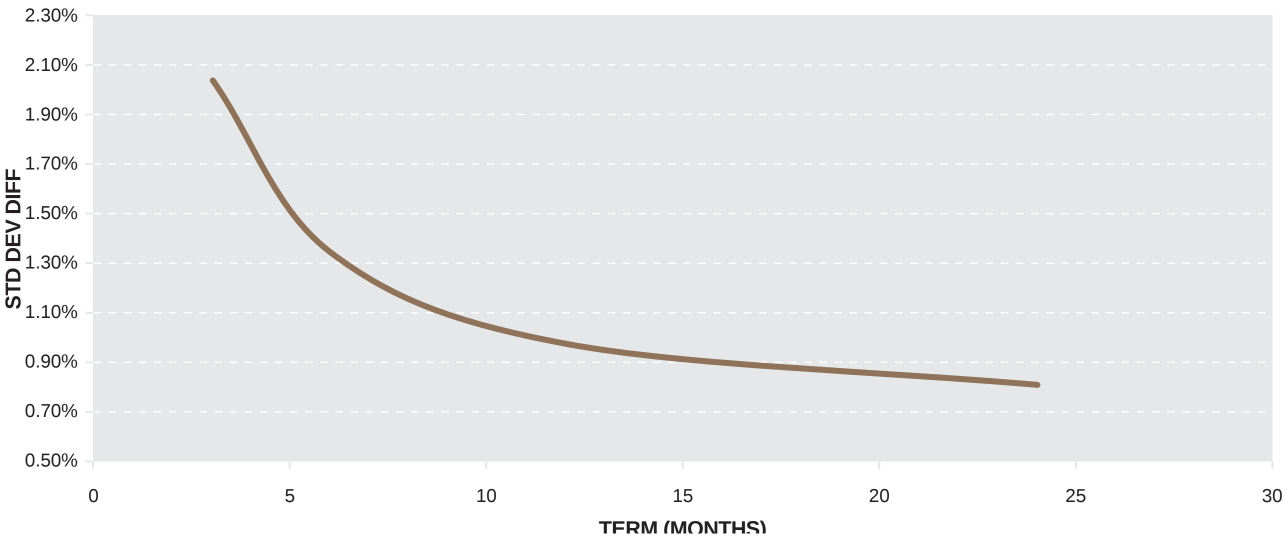
The main questions considered in this section are:

- Is the effect of GARCH(1,1) dynamics on the final distribution influenced by the length of the time horizon?
- How dependent is the standard deviation of the final overall return dependent on the current level?
- Does a time-varying volatility process lead to significant deviations from normality?

The examples in this section are based on distributions derived for the Top40 index via simulation using the GARCH(1,1) parameters presented in table 1. In the constant volatility case, volatility was set equal to the long-term value used for the GARCH(1,1) model. Final distributions are for simple returns, i.e. measured as final price / initial price – 1.

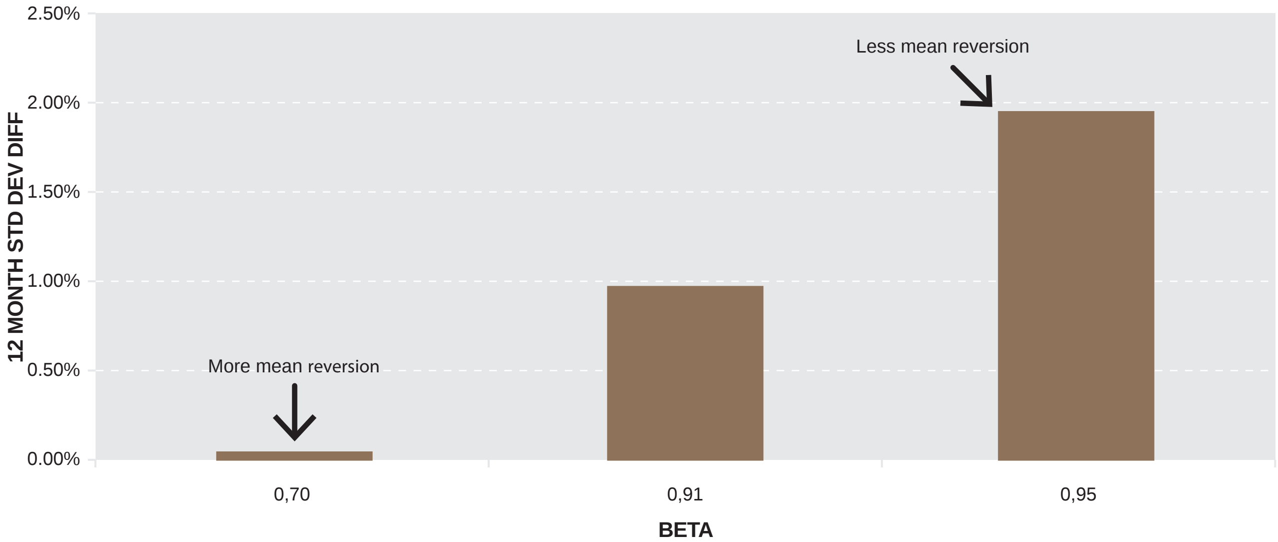
An initial indication of the influence of term on the impact of GARCH(1,1) dynamics is given figure 11 which shows the difference in annualised standard deviation between the non-constant and constant volatility final return distributions. It is clear that the difference between the models decreases as the length of the time horizon is increased. It is important to note that the initial volatility in the GARCH(1,1) case is 18.3%, significantly higher than the long-run value of 14.8% and the value used in the constant volatility model (also 14.8%). It therefore makes sense that shorter-term GARCH volatility is higher, owing to the higher starting value. However, the effect of the initial value decreases with increasing term as mean reversion ensures that future volatilities oscillate around the long-run value.

FIGURE 11: DIFFERENCE BETWEEN GARCH(1,1) AND CONSTANT VOL FINAL DISTRIBUTION STANDARD DEVIATIONS



One would expect that the difference in final overall standard deviation for the two models would also be dependent on the speed of mean reversion, as determined by the β parameter. This is confirmed by figure 12 which shows the difference in standard deviation at 12 months for values of 0.7 (higher mean reversion), 0.91 (original), and 0.95 (higher mean reversion). It is clear that for a higher degree of mean reversion, the effect of the initial volatility estimate is significantly reduced.

FIGURE 12: EFFECT OF MEAN REVERSION ON INFLUENCE OF CURRENT VOLATILITY



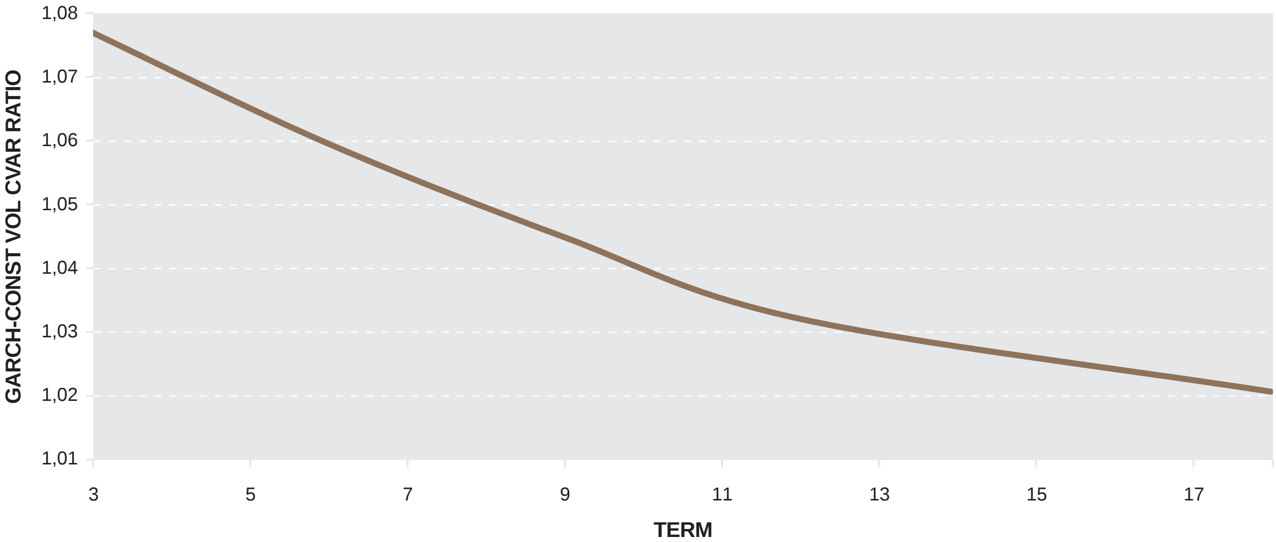
With the starting value of volatility set equal to the long-run average, one finds that across all terms, the standard deviation of the final return distribution is the same as in the constant volatility case. This suggests that the effect of time-varying daily volatility on the volatility of returns corresponding to the entire investment horizon is averaged out owing to oscillation around the long-term mean. However, it is important to consider other properties of the return distribution, such as tail risk. Table 2 shows selected statistics for the three-month return distribution of the Top40 assuming both GARCH(1,1,) and constant volatility dynamics. In the GARCH(1,1) case, the starting volatility was set equal to the long-run value, leading to an end-of-term standard deviation equal to that of the constant volatility model. The difference between the distributions is observed in the tails, with an expected shortfall (CVaR) more than 1% higher in magnitude when GARCH(1,1) dynamics are considered. It is thus clear that GARCH(1,1) dynamics result in a non-normal terminal return distribution with a higher likelihood of extreme losses. This is confirmed by the Lilliefors test which rejects the hypothesis that the GARCH(1,1) continuously compounded three-month returns are from a normal distribution at the 5% significance level.

TABLE 2: THREE-MONTH RETURN STATISTICS (VAR AT 1% CONFIDENCE LEVEL)

	GARCH	Constant Vol
Average	4.34%	4.34%
Std Dev	7.65%	7.62%
VaR	-12.44%	-12.07%
Expected Shortfall	-15.23%	-14.14%

Of interest is the effect of time horizon on the non-normality of the final continuously compounded cumulative returns. It was found that distributions corresponding to terms longer than two years passed the Lilliefors test, suggesting a decreasing difference between GARCH(1,1) and constant volatility distributions with increasing term. Further evidence for this can be seen by comparing the ratio of the tail risk measures corresponding to the two asset return models. As shown in figure 13, the ratio tends towards unity with increasing term, indicating convergence between the tails for the two types of distribution.

FIGURE 13: CONVERGENCE OF GARCH AND CONSTANT VOLATILITY TAIL RISK FOR INCREASING TERM



3.3 Correlation between Final Cumulative Returns

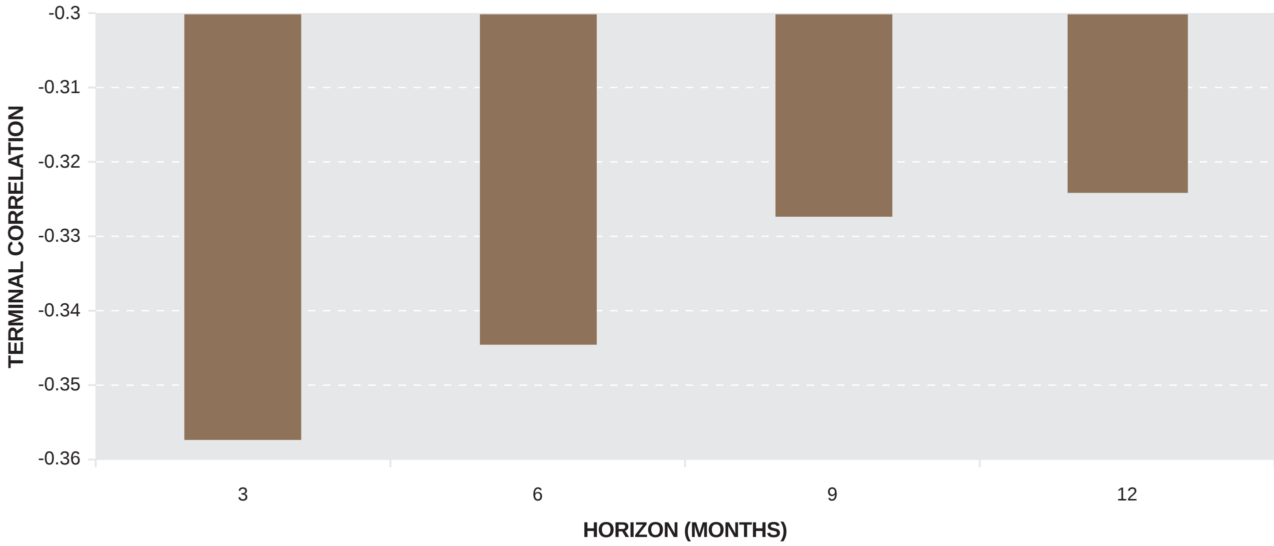
As discussed previously, the correlation between the returns of a foreign asset and those of the exchange rate is a crucial determinant of the optimal currency hedge. Unfortunately, the time-varying nature of correlation makes it difficult to know how the final cumulative returns will be correlated. The DCC model gives one a way to determine this in a systematic and quantitative manner.

We consider in this section an example based on the S&P 500 and the Rand-Dollar exchange rate. A return distribution for each series was generated by jointly simulating returns assuming DCC dynamics. In other words, the two series followed their own GARCH(1,1) processes, but with the random residuals correlated according to a daily changing correlation matrix, as defined by the DCC specification. The cumulative effect of changing correlation was estimated by calculating the correlation between cumulative whole-period returns measured at the end of the time horizon.

Because the DCC model specified in this study features mean-reversion around a long-term mean, it was found that an initial correlation equal to the long-run value led to a final realised correlation indistinguishable from the constant correlation assumption for all terms. The interesting case is when current correlation is different from the long-run average. As in the discussion on volatility in section 3.2, the final realised value is somewhere between the initial and long-run values, with increasing term giving rise to the long-term average. Based on the parameters estimated for the example, the current S&P 500-USDZAR correlation was found to be -0.37, with a long-run average of -0.32. As shown in figure 14, the influence of current correlation decreases as the time horizon increases. The two main points from this analysis are:

- The current estimate of correlation is important for shorter-term (e.g. 3 months) horizons.
- The final realised correlation that is an important determinant of the risk associated with a foreign asset holding is dependent on both the current estimate of correlation and the long-run average value.

FIGURE 14: DECREASING EFFECT OF CURRENT CORRELATION WITH INCREASING HORIZON



4 RISK MANAGEMENT OF A FOREIGN ASSET HOLDING: OPTIMAL CURRENCY HEDGE SELECTION UNDER NON-CONSTANT CORRELATION

4.1 Introduction

In the current section, we revisit the problem of choosing a currency hedging strategy where one's objective is to minimize the standard deviation of a position in a foreign asset. An approach to achieving this goal was described by Seymour et al. (2015) and simply involves specifying a set of candidate currency hedges and selecting the one leading to the lowest standard deviation. It was shown that the optimal hedge is strongly dependent on the assumed correlation between the foreign asset returns (in the original currency) and the exchange rate. Very negative correlation (<-0.5) favours some or full currency exposure, whereas higher correlation (> -0.3) requires little to no currency exposure.

As discussed in section 3.3, specifying a single correlation value that is expected to describe the dependency between final cumulative returns is difficult owing to the non-static nature of correlation. *The DCC model provides a mechanism by which to incorporate the assumption of time-varying correlation into the hedge selection process*.

The main aim of the current section is to compare the risk properties of selected hedging strategies under the assumption of constant and non-constant correlation. An important property of the DCC model discussed in section 3.3 is that the realised correlation between final cumulative returns is dependent on the current level of correlation. We therefore consider hedge performance for three different current levels and for each, compare the resulting risk under the assumption of DCC dynamics estimates obtained under the assumption of constant volatility and correlation.

The parameters for the time-invariant model were based on sample statistics for the selected historical period (July 2013 to July 2015) and led to a correlation value of -0.32. The long-term average of the DCC model was also set equal to the sample value and hedge performance for current correlation values above (-0.1), equal to (-0.32) and below (-0.6) the long-term average was examined.

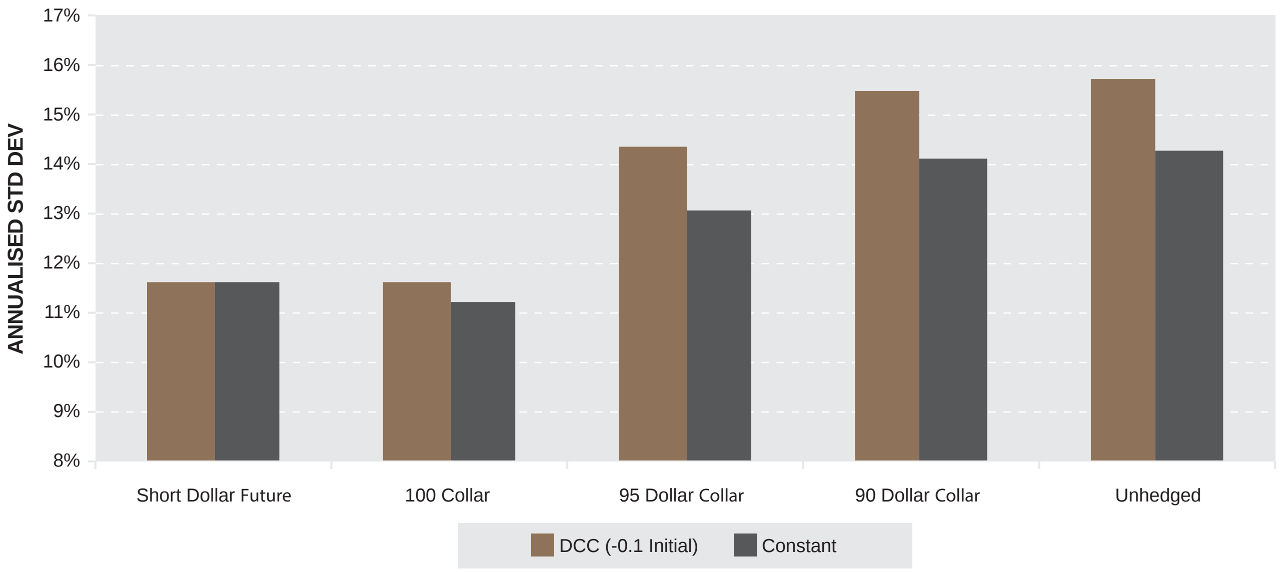
The set of candidate currency management strategies consisted of the following:

- 3-Month short Dollar future (i.e. zero currency exposure)
- 3-Month zero-cost collars with the bought put option struck at 90%, 95% and 100% of the prevailing exchange rate
- No currency hedge

4.2 Current Correlation above the Long-Term Average

One finds that with a current correlation set at a higher value than the long-term average, the DCC model leads to significantly higher risk estimates for the currency hedges featuring exposure to the exchange rate, compared to the constant parameter lognormal random walk model. This is because the realised correlation under the DCC model was found to be -0.16, leading to less offsetting of risk from movements in the exchange rate. The influence of the current correlation value over the relatively short investment horizon is clearly highlighted by the similarity to the realised whole-period correlation. Strategies featuring less or no currency exposure (100 collar, short future) are less affected by assumptions regarding the dynamics of correlation, with DCC and constant parameter models giving very similar values for standard deviation. These results are summarised in figure 15.

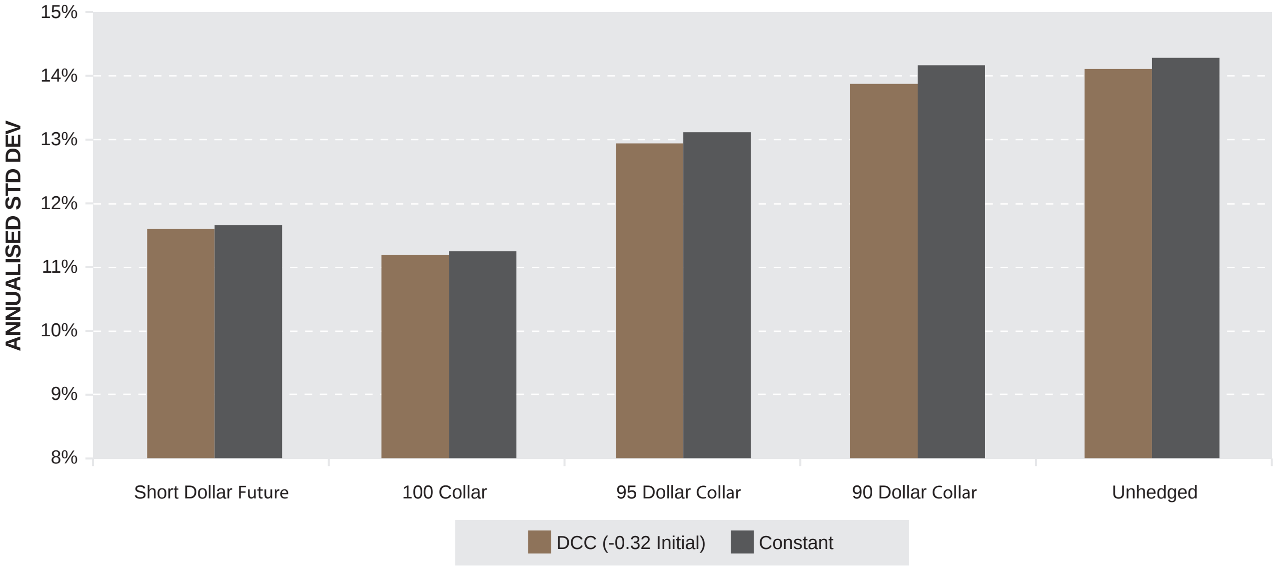
FIGURE 15: RISK ESTIMATES FOR CANDIDATE CURRENCY HEDGING STRATEGIES: HIGH CURRENT CORRELATION



4.3 Current Correlation Equal to Long-Term Average

The analysis presented in section 3 demonstrated that a current DCC correlation equal to the long-term average leads to correlation between final cumulative returns that is indistinguishable from that obtained under a constant volatility-correlation assumption. Clearly, the overall riskiness of a foreign asset position would also depend on the initial volatility values assumed for each individual GARCH process. In the examples under consideration, initial volatilities were left equal to their values derived from the estimate of the models' parameters. It turned out that both the S&P 500 and USDZAR series had current volatilities very similar to their corresponding long-term averages, as shown in table 1. This, combined with a current correlation set equal to the long-run average, leads to very similar risk estimates across all hedging strategies for the two asset return models, as shown in figure 16. Although the correlation in this case is negative (-0.32), it is not sufficient to lead to an offset of the risk introduced by the embedded currency exposure. Consequently, strategies with little (100 collar) to zero (short future) currency exposure display noticeably less risk than the higher exposure strategies.

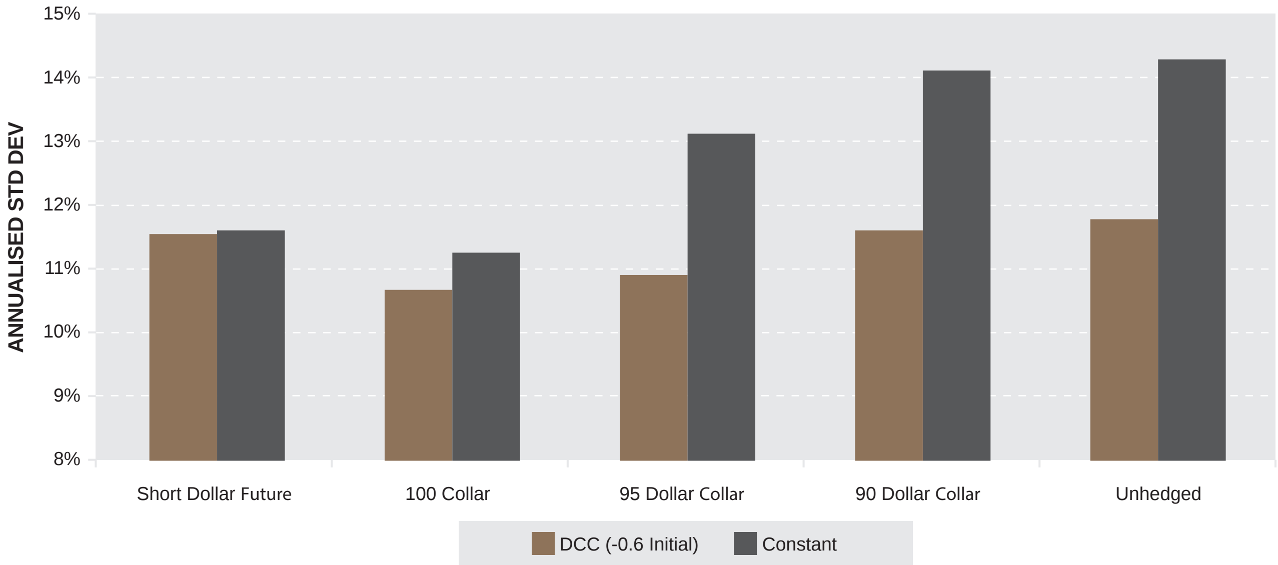
FIGURE 16: RISK ESTIMATES FOR CANDIDATE CURRENCY HEDGING STRATEGIES: CURRENT CORRELATION EQUAL TO LONG-TERM AVERAGE



4.4 Current Correlation Less than the Long-Term Average

An assumption of initial DCC correlation equal to -0.6 leads to a whole-period return correlation equal to -0.53, significantly less than the historical sample correlation of -0.32. One thus finds that all strategies featuring some degree of currency exposure give rise to significantly less risk under the assumption of DCC dynamics compared to the constant correlation case, as shown in figure 17. In this particular example, the realised correlation of -0.53 is still not sufficiently negative to lead to lower risk than a zero-exposure strategy. However, both the 100 and 95 collar strategies show superior risk reduction. The importance of considering currency options is thus clearly highlighted, with dual risk-reduction benefits arising from both exposure to negatively correlated exchange rate movements as well as elimination of returns above the cap and below the floor.

FIGURE 17: RISK ESTIMATES FOR CANDIDATE CURRENCY HEDGING STRATEGIES: VERY NEGATIVE CURRENT CORRELATION



5 ASSET ALLOCATION WITH NON-CONSTANT VOLATILITY AND CORRELATION

5.1 Introduction

The joint optimization approach of Jorion (1994) is an effective method for determining an appropriate currency hedge. Instead of considering the foreign asset and associated currency hedge in isolation, the determination of the optimal currency hedge is incorporated into the overall asset allocation decision for the portfolio. As discussed in section 1, a major benefit of such an approach is that one takes into account the interaction between the exchange rate and all assets in the portfolio, leading to potentially enhanced risk reduction.

An important characteristic of the DCC model is that it can be implemented for large portfolios, thus making it possible to incorporate the effect of time-varying correlation and volatility into an asset allocation framework. In the current section we compare optimal allocations derived from an assumption of constant volatilities and correlations with those based on an assumption of DCC dynamics. Unfortunately, such a comparison is potentially very complicated owing to the dependence of final asset return distributions on current volatilities as well as the influence of current correlations on final realised correlations. Given that the quantity of interest in the current study is the correlation between a foreign asset and the exchange rate, only the effect of the current S&P 500-USDZAR correlation was considered. All other parameters such as current GARCH(1,1) volatilities and all current correlations for the remaining asset pairs were fixed at their estimated values.

The examples are based on a portfolio selected to be representative of a balanced portfolio and consisted of domestic equity, bonds and cash, as well as foreign equity (represented by the S&P 500). It was assumed that the aim was to find an optimal allocation to these assets *as well as a possible currency hedge*. Candidate currency hedges were included as available assets for the optimization routine and consisted of the same set as the examples presented in section 4: a short dollar future and zero-cost collars with the bought put option struck at 90%, 95% and 100% of the prevailing exchange rate.

As in any optimization problem, optimal allocations are dependent on input assumptions regarding expected returns and volatility. For the current study, volatility parameters (constant and DCC) were based on history. Expected returns were then set such that the Sharpe ratio of each asset was 0.75. In the DCC case, the long-run volatility and correlation values were set equal to the historical sample values as used in the constant correlation model. It was further assumed that the investment horizon was 3 months and that the risk tolerance for the portfolio was an annualised standard deviation of 6%.

The following sections consider the impact of the current level of S&P 500-USDZAR correlation by examining portfolio allocations resulting from different assumed current values. Note that no cap was placed on the allocation to foreign equity so as not to mask the effects of changes in the parameters under investigation.

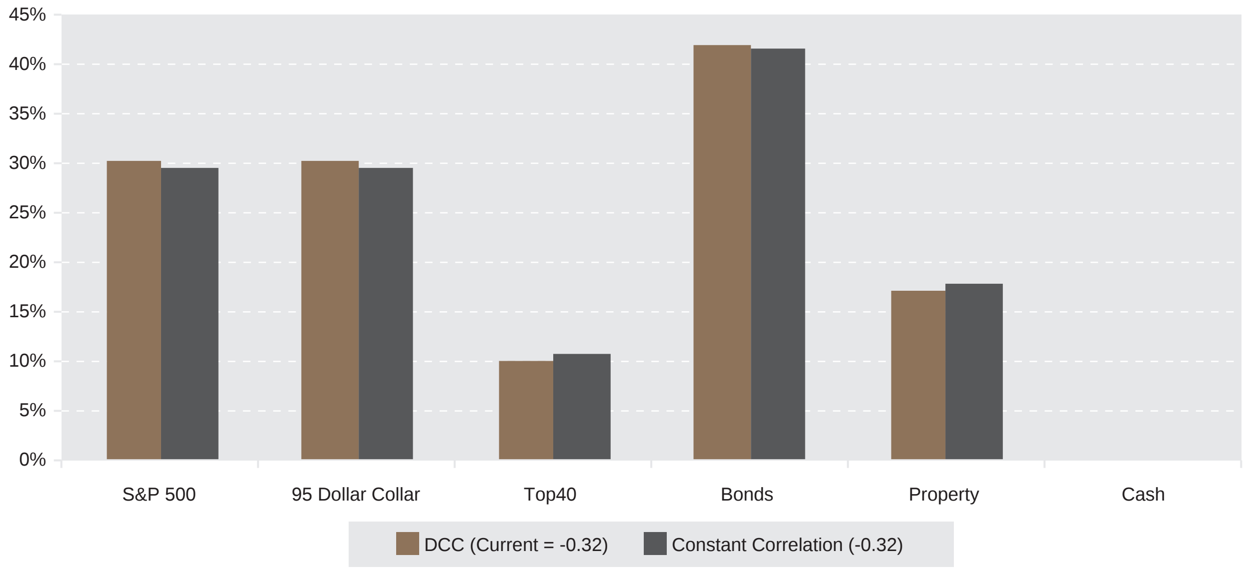
5.2 Efficient Asset Allocations for Current S&P 500-USDZAR Correlation Equal to Long-Run Average

Sections 3 and 4 demonstrated that with current DCC correlation equal to the long-run average, one obtains a correlation between the final cumulative returns that is nearly indistinguishable from the constant parameter model. It is therefore unsurprising that optimal portfolio allocations for the two models are practically the same, as shown in figure 18. Two particularly interesting features of the allocations shown in figure 18 are:

- Foreign equity is favoured over domestic equity.
- The optimal currency hedge is a 95 Dollar collar, which is different to the optimal hedge when considering the foreign asset in isolation.

Both of these points are likely due to the negative correlation between the exchange rate and all assets in the portfolio. Gaining currency exposure via the holding in the S&P 500 leads to a reduction in risk and thus the optimizer favours an increased allocation to the foreign asset in this instance. Even though S&P 500 risk is reduced the most by a short Dollar future, such a strategy removes the risk reduction benefits from interactions between the exchange rate and the domestic assets in the portfolio. Consequently, a currency hedge featuring some degree of currency exposure is favoured, namely the 95 Dollar collar.

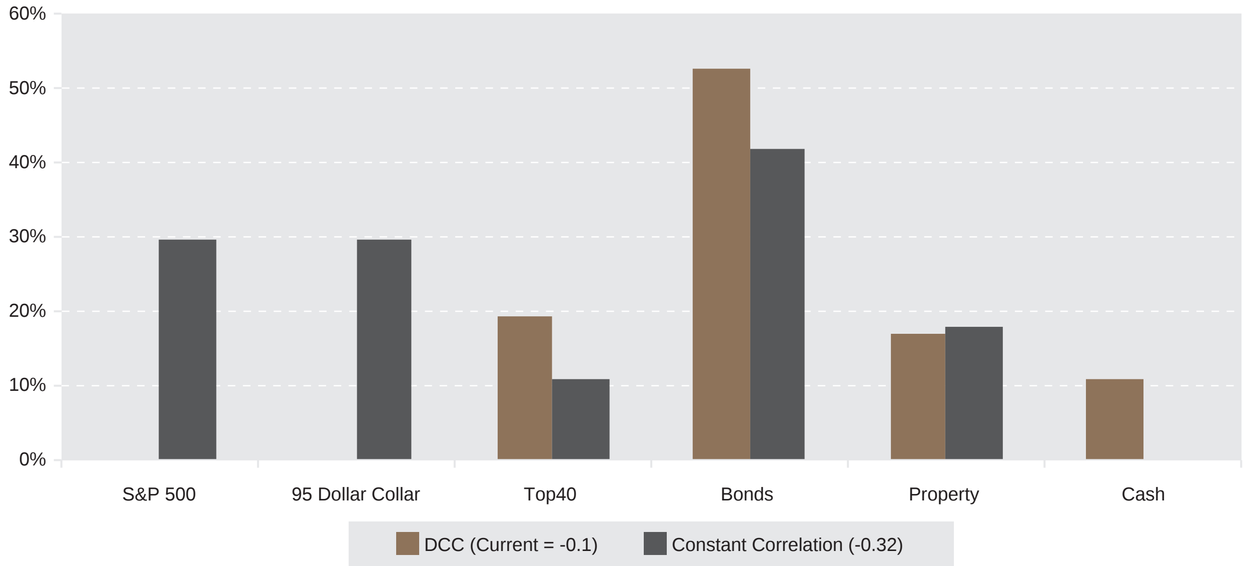
FIGURE 18: OPTIMAL ALLOCATIONS FOR CURRENT S&P 500-USDZAR CORRELATION EQUAL TO LONG-RUN AVERAGE



5.3 Effect of High S&P 500-USDZAR Current Correlation on Optimal Allocations

Figure 19 shows that setting the current correlation to -0.1 has a dramatic impact on the overall allocations for the portfolio. One sees that a high current correlation leads to a zero allocation to foreign equity. Even though the correlations between the exchange rate and the remaining assets have been kept fixed at their original negative values, the associated risk reduction benefit is insufficient to outweigh the increase in risk of an S&P 500 position with some degree of currency exposure. As shown in figure 15, a current correlation of -0.1 increases the standard deviation of an S&P 500 position featuring currency exposure significantly. The optimizer thus shifts allocations away from foreign equity into domestic equity, bonds and cash.

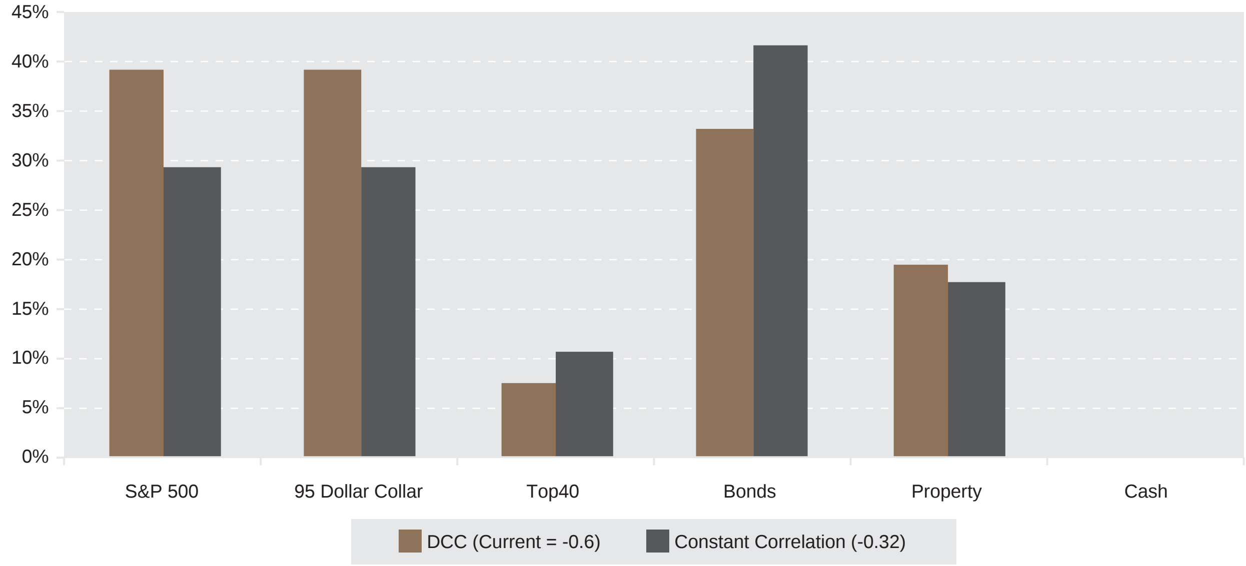
FIGURE 19: OPTIMAL ALLOCATIONS FOR CURRENT S&P 500-USDZAR CORRELATION EQUAL ABOVE LONG-RUN AVERAGE



5.4 Effect of Very Negative S&P 500-USDZAR Current Correlation on Optimal Allocations

In contrast to what was seen in section 5.3, a very negative correlation of -0.6 leads to a significant increase in the allocation to foreign equity at the expense of domestic equity and bonds. Essentially, the effects of negative correlation discussed in section 5.2 are amplified, making it even more beneficial to include foreign equity with some degree of currency exposure.

FIGURE 20: OPTIMAL ALLOCATIONS FOR VERY NEGATIVE CURRENT S&P 500-USDZAR CORRELATION



6 CONCLUSION

In this work we outlined an approach to the determination of an optimal currency hedge in the presence of non-constant volatility and correlation. It was shown that implementation of the dynamic conditional correlation (DCC) model in a simulation framework allows one to incorporate the effects of time-varying parameters into the hedge selection process in a systematic and quantitative manner.

Given that the optimal hedge is dependent on the forecast return distribution, the impact of daily changing volatility on certain statistical properties of the final cumulative return for an asset was studied. It was found that the annualised standard deviation of the final distribution is dependent on the current level of volatility, as determined by the estimated model. Interestingly, if the current level is equal to the long-run volatility estimate, then the final annualised standard deviation obtained under an assumption of non-constant volatility is the same as that obtained for the constant volatility model. Although this may suggest that the impact of time-varying volatility is averaged out owing to oscillations around the long term mean volatility, it was found that the time-varying volatility model led to an increase in tail risk, particularly for shorter investment horizons (e.g. 3 months).

As in the case of volatility, the correlation between final cumulative returns was found to be dependent on both the long-run average correlation as well as the current value. Shorter terms displayed a higher dependence on the initial value, with its influence reduced for longer terms.

The incorporation of an assumption of time varying volatility and correlation into the determination of an optimal currency hedge was demonstrated with selected examples. It was shown that the particular choice of short-term hedge is strongly dependent on the current value of correlation, emphasising the need for a modelling framework that provides accurate estimates of time-varying correlation.

An important application of the framework described in this work was the determination of an optimal overall asset allocation and currency hedge *at the same time*. This joint optimization approach leads to greater efficiency as one takes into account the interaction between the exchange rate and all assets in the portfolio. In the examples considered, a zero-cost collar with the bought put option struck at 95% of the prevailing exchange rate was found to be the favoured currency hedge. This finding emphasised the importance of considering currency options as they allow for a risk-reduction via two mechanisms. Negative correlation between portfolio assets in the portfolio and the exchange rate makes it beneficial to maintain some degree of currency exposure, while the elimination of extreme returns above the cap and below the floor leads to further reductions in risk.

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