

## Physics-dynamics coupling with element-based high-order Galerkin methods: quasi equal-area physics grid

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### ABSTRACT

Atmospheric modeling with element-based high-order Galerkin methods presents a unique challenge to the conventional physics-dynamics coupling paradigm, due to the highly irregular distribution of nodes within an element and the distinct numerical characteristics of the Galerkin method. The conventional coupling procedure is to evaluate the physical parameterizations (*physics*) on the dynamical core grid. Evaluating the physics at the nodal points exacerbates numerical noise from the Galerkin method, enabling and amplifying local extrema at element boundaries. Grid imprinting may be substantially reduced through the introduction of an entirely separate, approximately isotropic finite-volume grid for evaluating the physics forcing. Integration of the spectral basis over the control-volumes provides an area average state to the physics, which is more representative of the state in the vicinity of the nodal points rather than the nodal point itself, and is more consistent with the notion of a ‘large-scale state’ required by conventional physics packages. This study documents the implementation of a quasi-equal area physics grid into NCAR’s Community Atmosphere Model with Spectral Elements, and is shown to be effective at mitigating grid imprinting in the solution. The physics grid is also appropriate for coupling to other components within the Community Earth System Model, since the coupler requires component fluxes to be defined on a finite-volume grid, and one can be certain that the fluxes on the physics grid are indeed, volume-averaged.

### 1. Introduction

An increasing number of numerical methods publications in the atmospheric science literature concern trans-

port, shallow-water, and three-dimensional models employing element-based high-order Galerkin discretizations such as finite-element and discontinuous Galerkin methods (for an introduction to these methods see, e.g., Durran 2010; Nair et al. 2011; Ullrich 2014). Some global models based on Galerkin methods have reached a level of maturity for which they are being considered for next generation climate and weather models due to their in-

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FIG. 1. Example of CAM-SE GLL quadrature grids, marked with red filled circles, (a & c) on the cubed-sphere and (b & d) in an element. (a)-(b) and (c)-(d) use  $4 \times 4$  ( $np = 4$ ) and  $8 \times 8$  ( $np = 8$ ) GLL quadrature points in each element, respectively. (a) and (c) have the same average grid-spacing at the Equator ( $7.5^\circ$ ) which is obtained by using (a)  $4 \times 4$  ( $ne = 4$ ) and (b)  $2 \times 2$  ( $ne = 2$ ) elements on each cubed-sphere face/panel, respectively. The element boundaries are marked with thick light blue lines. The grid configurations shown on (a) and (c) are referred to as  $ne4np4$  and  $ne2np8$ , respectively.

herent conservation properties, high-order accuracy (for smooth problems), high parallel efficiency, high processor efficiency, and geometric flexibility facilitating mesh-refinement applications. NCAR's Community Atmosphere Model (CAM; Neale et al. 2012) offers a dynamical core based on continuous Galerkin finite elements (Taylor and Fournier 2010), referred to as CAM-SE (CAM Spectral Elements; Taylor et al. 2008; Dennis et al. 2012; Lauritzen et al. 2018). CAM-SE is, in particular, being used for high resolution climate modeling (e.g., Small et al. 2014; Reed et al. 2015; Bacmeister et al. 2018) and static mesh-refinement applications (e.g., Fournier et al. 2004; Zarzycki et al. 2014a,b; Guba et al. 2014b; Rhoades et al. 2016). Other examples of models based on high-order Galerkin methods that are being considered for ‘operational’ weather-climate applications are Giraldo and Restelli (2008), Nair et al. (2009), Brdar et al. (2013) and Rasch et al. (in progress).

Assumptions inherent to the physical parameterizations (also referred to as *physics*) require the state passed by the dynamical core represent a ‘large-scale state’, for example, in quasi-equilibrium-type convection schemes (Arakawa and Schubert 1974; Plant 2008). In finite-volume methods (e.g., Lin 2004), one may think of the dynamical core state as the average state of the atmosphere over a control volume, and for resolutions typical of climate simulations is entirely consistent with the notion of a ‘large-scale state’. For finite-difference methods (e.g., Suarez et al. 1983) the point value is thought of as

representative for the atmospheric state in the vicinity of the point value and one can usually associate a volume with the grid-point. Hence the physics grid (the grid on which the state of the atmosphere is evaluated and passed to physics) and the dynamics grid (the grid the dynamical core uses) coincide. Having the physics and dynamics grids coincide is obviously convenient since no interpolation is needed (which could disrupt conservation properties) and the number of degrees of freedom on both grids is exactly the same.

For the regular latitude-longitude, cubed-sphere and icosahedral grids the distance between the grid-points is gradually varying for finite-volume/finite-difference discretizations. Examples of models that use these grids are CAM-FV (latitude-longitude grid, Lin 2004), FV3 (cubed-sphere grid, Putman and Lin 2007) and ICON (icosahedral grid, Wan et al. 2013). For high-order element-based Galerkin methods, the dynamical core grid is defined by the quadrature points. In CAM-SE, these are the Gauss-Lobatto-Legendre (GLL) quadrature nodes. A unique aspect of the high-order quadrature rules is that the nodes within an element are located at the roots of the basis set, which may be irregularly spaced. For example, Figure 1 shows GLL points on an individual element of a cubed-sphere grid for degree 3 ( $np \times np = 4 \times 4$  quadrature points) and degree 7 ( $np \times np = 8 \times 8$  quadrature points) Lagrange polynomial basis used in CAM-SE. The higher the order of the quadrature rule, the greater variance in distance between GLL quadrature points within an each element. GLL quadrature points cluster near the edges and, in particular, the corners of the elements.

The resolved scales of motion are not determined by the distance between quadrature nodes, but rather the degree of the polynomial basis in each element. The nodes may be viewed as irregularly spaced samples of an underlying spectrally truncated state. From this perspective, one might expect the nodal solutions to be independent of location within an element. While the interior quadrature nodes are  $C^\infty$  in CAM-SE (i.e. the basis representation is infinitely smooth and all derivatives are continuous), the smoothness of boundary nodes are constrained by the need to patch neighboring solutions together to form the global basis set, an operation known as the direct stiffness summation (DSS; Maday and Patera 1987; Canuto et al. 2007). The DSS operation is attractive because it allows for high-order accuracy with minimal communication between elements, but degrades the solution to  $C^0$  at element boundaries (i.e., all derivatives are discontinuous). Through evaluating the physics at the nodal points, strong grid-scale forcing or oscillatory behavior near an element boundary may exacerbate the discontinuity, and our initial expectation, that the nodal solutions are independent of within-element location, is unlikely for non-smooth problems, e.g., the presence of rough topography or moist physics grid-scale forcing.

It is the purpose of this paper to document the implementation of an entirely separate, quasi-equal area finite-volume physics grid into CAM-SE. In this framework, the dynamical core state is integrated over control volumes to provide a volume averaged state to the physics, thereby minimizing the influence of any one particular nodal value on the physics forcing. Section 2 provides a thorough explanation of how grid imprinting manifests in high-order Galerkin methods for non-smooth problems. The implementation of the physics grid configuration into CAM-SE is presented in Section 3. Idealized model configurations with and without topography are presented in Section 4, illustrating the physics grid is effective at mitigating undesirable grid imprinting in the solution. Section 5 contains a discussion of results and concluding remarks.

## 2. The Quadrature Node Problem

Figure 2 is a one-dimensional schematic illustrating of how grid-imprinting is enabled by the physics, when using high-order Galerkin methods for the dynamics. The schematic depicts a time-step, starting from smooth initial conditions (Figure 2a), and subsequently advancing the dynamics one Runge-Kutta time-step (Figure 2b). Since the boundary nodes of adjacent elements overlap one-another, there are now two solutions for each boundary node. The DSS operator, effectively a numerical flux applied to the element boundaries such that overlapping nodal values agree, is applied (Figure 2c), degrading the boundary nodes to  $C^0$  in the process. This discontinuity may be exacerbated if, e.g., the physics updates the state at an element boundary (Figure 2d,e), resulting in characteristically tighter gradients on the boundary nodes compared to if the physics forcing were applied to an interior node (Figure 2g,h).

To test the degree to which nodal solutions depend on within-element position, an aqua-planet simulation (Neale and Hoskins 2000; Medeiros et al. 2016), which consists of an ocean covered planet in perpetual equinox, with fixed, zonally symmetric sea surface temperatures idealized after the present day climatology, is carried out using CAM-SE, with CAM4 physics (Neale et al. 2010) and run for one year. The nominal low resolution  $ne30np4$  grid is used, pertaining to an average equatorial grid spacing of 111.2km. The probability density distribution of the upward vertical pressure velocity ( $\omega$ ), conditionally sampled based on three categories - ‘interior’, ‘edge’ and ‘corner’ nodes - is provided in Figure 3a. The rationale for concentrating on the  $\omega$  field is that it closely follows divergent modes, which are sensitive to the discontinuous pressure gradients that characterize boundary nodes. There is an apparent dependence on nodal location, with interior nodes being characteristically sluggish, and corner and

edge nodes having systematically larger magnitude vertical motion. This behavior is consistent with the smoothness properties of the different nodal locations, with discontinuous pressure gradients resulting in greater vertical motion at ‘edge’ and ‘corner’ nodes. The main division of solutions shown in Figure 3a is primarily between whether a node is, or is not situated on an element boundary, and is a nuanced signature of high-order element-based Galerkin methods for non-smooth problems.

If the conventional physics-dynamics coupling paradigm is applied to CAM-SE, then the physics are to be evaluated at the GLL nodes, and a volume associated with the quadrature point should be defined. One approach to construct this grid is to decompose each spectral element into  $(np-1) \times (np-1)$  subcells and then take the dual grid of this subcell grid. For cubed-sphere meshes, this dual grid will have a control volume associated with each quadrature point. These control volumes will be triangles for the cube corner quadrature points and quadrilaterals for all remaining quadrature points. Newton iteration can than be used to adjust the corners of these control volumes so that their spherical area exactly match the Gaussian weight multiplied by the metric term (these weights are used for integrating the basis functions over the elements and can therefore, in this context, be interpreted as areas). For cubed-sphere meshes, the Newton iteration can be replaced by a direct method if some of the quadrilaterals are replaced by pentagons giving additional flexibility in matching the spherical area to the quadrature weights. Such a dual grid is shown in Figure 4. This grid is used in the NCAR CESM (Community Earth System Model) coupler for passing states between ocean, atmosphere and land components since the current remapping method is finite-volume based and therefore requires control volumes (it is noted that methods exist that do not require control volumes for conservative interpolation, e.g., Ullrich and Taylor (2015)). Hence the components ‘see’ an irregular atmospheric grid. Similarly, the parameterizations in the atmosphere ‘see’ a state that is anisotropically sampled in space (see Figure 1 and 5 in Kim et al. 2008).

The quadrature grid in element-based Galerkin methods is defined to perform mathematical operations on the basis functions, e.g., computing gradients and integrals, rather than evaluating the state variables for physics-dynamics coupling. One may argue that it would be more consistent to integrate the basis functions over quasi-equal area control volumes within each element and pass those control volume average values to physics rather than irregularly spaced quadrature point values. In this case when integrating basis functions over control volumes a grid-cell average value is more representative of the values near the extrema at the element boundary than the quadrature point value. The relationship between the nodal values, the basis functions and the proposed control volumes is illustrated

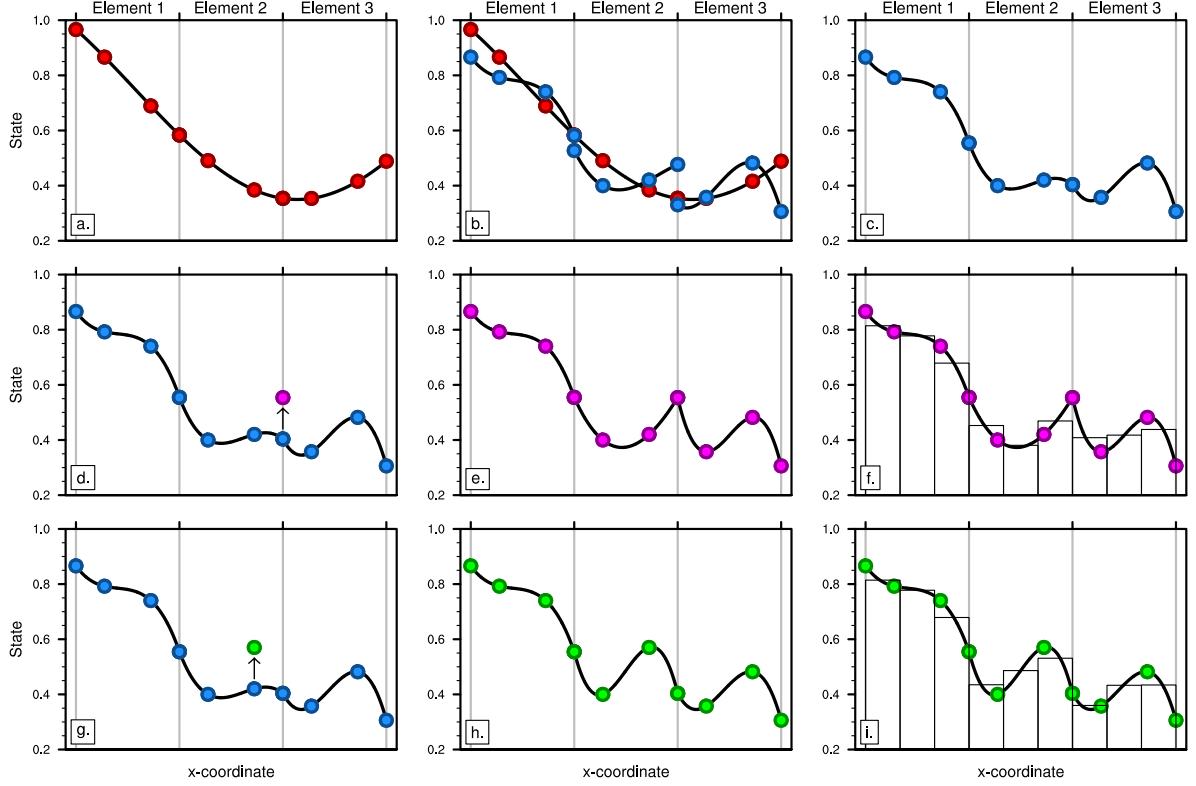


FIG. 2. A one-dimensional schematic showing the relationship between the basis functions, the quadrature nodes and the proposed physics grid, over the coarse of a time-step. The filled circles are the GLL quadrature points in each element, which are connected by a Lagrange polynomials basis (curves). (a) Smooth initial condition are (b) advanced by the dynamics one Runge-Kutta step (blue), and (c) shows the solution after applying the DSS operator. Applying (d) grid-scale forcing to an element boundary node, (e) the basis representation is clearly  $C^0$  at the element boundary. In contrast, (d) applying grid-scale forcing to an interior node (e) results in a smooth,  $C^{inf}$  continuous field. (f),(i) Vertical bars represent the physics grid, found through integrating the basis over the control volumes.

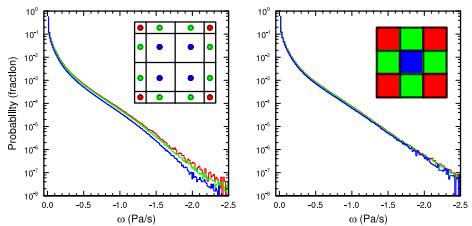


FIG. 3. Probability density distribution of instantaneous upward  $\omega$  in a pair of aqua-planet simulations using CAM4 physics. Figure is constructed from one year of six hourly data, at all vertical levels. (Left)  $ne30np4$  configuration conditionally sampled for interior, edge and corner node control volumes, and (Right)  $ne30pg3$  configuration, but sampled by within-element physics grid-cell location. Note the consistently larger magnitude  $\omega$  for boundary nodes compared with interior nodes, and that the bias is eliminated through mapping to a quasi-equal area physics grid.

schematically in one-dimension in parts (f) and (i) in Figure 2.

### 3. Methods

Here we focus on CAM-SE, however, in principle the methods apply to any element-based high-order Galerkin model. The physics grid in CAM-SE is defined by sub-dividing each element using equi-angular gnomonic coordinate lines to define the sides of the physics grid control volumes (see the Appendix for details). Note that the element boundaries are defined by equi-angular gnomonic grid lines. The notation  $pg = 3$  refers to the configuration where the elements are divided into  $pg \times pg = 3 \times 3$  equi-angular physics grid cells (see Figure 5), and is therefore not an equal-area grid, but rather a quasi-equal area grid. Defining the physics grid by sub-dividing elements makes it possible to use the same element infrastructure as already used in CAM-SE, thereby facilitating its implementation. Here we make use of the  $ne30np4$  and  $ne30pg3$

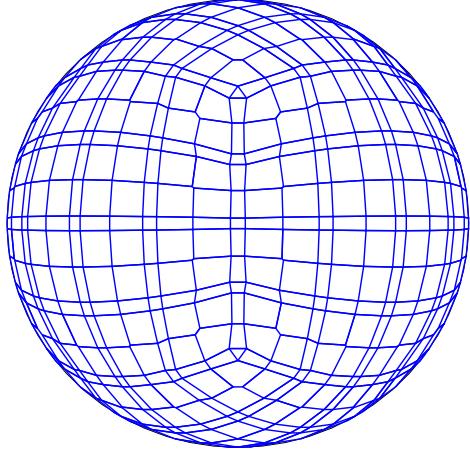


FIG. 4. An example of control volumes constructed around GLL quadrature points ( $ne4np4$ ) so that the spherical area of the control volumes exactly match the quadrature weight multiplied by the metric factor.

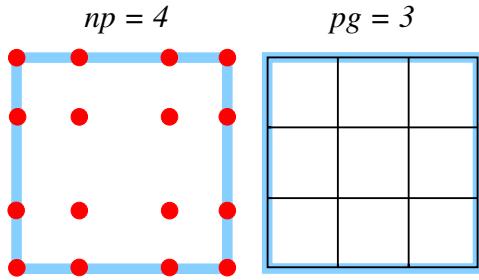


FIG. 5. A schematic illustration of an element, indicating the relationship between (left) the dynamical core grid, and (right) the proposed quasi-equal area physics grid. The physics grid contains  $pg \times pg = 3 \times 3$  grid cells in each element.

grids that use GLL quadrature point physics grid (physics and dynamics grid coincide), and the same ( $pg = 3$ ) resolution quasi equal-area physics grids, respectively. In all configurations we use degree three Lagrange basis ( $np = 4$ ) and  $ne \times ne = 30 \times 30$  elements on each cubed-sphere panel.

A consequence of separating physics and dynamics grids is that the atmospheric state must be mapped to the physics grid and the physics tendencies must be mapped back to the dynamics grid. This is discussed in separate sections below. When separating physics and dynamics grids it is advantageous to use a vertical coordinate that is static during physics-dynamics coupling. This was one motivation to switch to a dry-mass vertical coordinate in CAM-SE (Lauritzen et al. 2018); since dry mass remains constant throughout physics the dry-mass vertical coordinate remains fixed during physics-dynamics coupling.

The dry mass coordinate subsequently evolves by the dynamics as floating Lagrangian layers (Lin 2004) periodically mapped back to a reference hybrid-sigma-pressure coordinate after Simmons and Burridge (1981).

*a. Mapping state from dynamics grid (GLL) to physics grid (pg)*

The dynamics state is defined on the GLL grid in terms of temperature  $T^{(gll)}$ , zonal wind component  $u^{(gll)}$ , meridional wind component  $v^{(gll)}$ , and dry pressure level thickness  $\Delta p^{(gll)}$ . In the mapping of the atmospheric state to the physics grid it is important that the following properties are met:

1. conservation of scalar quantities such as mass and dry thermal energy,
2. for tracers; shape-preservation (monotonicity), i.e. the mapping method must not introduce new extrema in the interpolated field, in particular, negatives,
3. consistency, i.e. the mapping preserves a constant,
4. linear correlation preservation.

Other properties that may be important, but not pursued here, includes total energy conservation and axial angular momentum conservation. We argue that the most consistent method for mapping scalar state variables from the GLL grid to the physics grid is to integrate the Lagrange basis function representation (used by the SE dynamical core) over the physics grid control volumes, i.e. integrate the basis function representation of  $\Delta p^{(gll)} \times T^{(gll)}$  and  $\Delta p^{(gll)}$  over the physics grid control volume (see, e.g., Lauritzen et al. 2017; Ullrich and Taylor 2015)

$$\Delta p^{(pg)} = \frac{1}{A^{(pg)}} \int_{A^{(pg)}} \Delta p^{(gll)} dA, \quad (1)$$

$$T^{(pg)} = \frac{1}{A^{(pg)} \Delta p^{(pg)}} \int_{A^{(pg)}} T^{(gll)} \Delta p^{(gll)} dA, \quad (2)$$

where  $A^{(pg)}$  is the physics grid area. The integrals are numerically computed using the GLL quadrature rule. Thermal energy and dry air mass is conserved and the mapping is consistent. For the wind, which is a vector, the zonal and meridional wind components are mapped by transforming to contra-variant wind components, evaluating the basis function representation thereof at the equi-angular center of the physics grid control volumes and then transformed back to latitude-longitude coordinate system winds. All of the operations are local to the element and do not require communication between elements.

The mapping of tracers is more problematic since the SE basis function representation is oscillatory although the shape-preserving filter guarantees shape-preservation at the GLL nodes (Guba et al. 2014a). To avoid this

issue we use the CAM-SE-CSLAM version of CAM-SE (Conservative Semi-Lagrangian Multi-tracer transport scheme Lauritzen et al. 2017), where tracers are advected on the  $pg = 3$  physics grid. Note that in CAM-SE-CSLAM the dry mass internally predicted by CSLAM,  $\Delta p^{(cslam)}$ , is, by design, equal to  $\Delta p^{(gll)}$  integrated over the CSLAM/physics grid control volume (Lauritzen et al. 2017). Since the tracer grid and physics grids are co-located and  $\Delta p^{(pg)} = \Delta p^{(cslam)}$  then the mass conservation, correlation preservation, consistency and shape-preservation constraints are inherently fulfilled.

### b. Mapping tendencies from physics grid ( $pg$ ) to dynamics grid (GLL)

The physics tendencies are computed on the finite-volume physics grid and are denoted  $f_T^{(pg)}, f_u^{(pg)}, f_v^{(pg)}$ , and  $f_m^{(pg)}$ . Note that dry air mass is not modified by physics and hence there is no tendency for dry mass,  $f_{\Delta p} \equiv 0$ . Also, it is important to map tendencies and not state from the physics grid to GLL grid otherwise one will get spurious tendencies from mapping errors when the actual physics tendency is zero (unless a reversible map is used).

It is important that this process:

1. for tracers; mass tendency is conserved,
2. for tracers; in each tracer grid cell the mass tendency from physics must not exceed tracer mass available in tracer grid cell (it is assumed that the physics tendency will not drive tracer mixing ratio negative on the physics grid),
3. linear correlation preservation,
4. consistency, i.e. the mapping preserves a constant tendency.

Other properties that may be important, but not pursued here, includes total energy conservation (incl. components of total energy) and axial angular momentum conservation. Scalar variables are mapped from the physics grid to GLL grid using a tensor-product Lagrange interpolation. The local coordinates on a cubed-sphere are discontinuous at the element edges so the interpolation requires special attention at the cube corners and edges. The details are provided in the Appendix. Lagrange interpolation preserves a constant (including zero) and linear correlations. Tracer and physics grids are co-located so tracer mass, tracer shape, and tracer correlations are trivially preserved on the tracer grid; and the inconsistency in point 2 above will not appear.

We do, however, need to map water tracers (such as water vapor, cloud liquid and cloud ice) to the GLL grid to account for moist effects in the equations of motion solved on the GLL grid. The CSLAM water tracer mixing ratios updated by physics tendencies are mapped to the GLL

grid using the same tensor cubic interpolation as is used for temperature and velocity components. In between the calls to physics (i.e. in the dynamical core sub-stepping) the water tracers are advected on the GLL grid with the SE method (see section 3.5 in Lauritzen et al. 2018). Water tracer mass is not conserved in the mapping from tracer/physics grid to GLL grid. This procedure makes sure that the water tracers on the GLL grid are 'nudged' to the CSLAM solution for water tracers and the mass budget is closed on the tracer/physics grid.

In the mapping algorithm from physics to dynamics it was found (a) important to use an algorithm that is smooth across element boundaries and (b) that obtaining mass-conservation without excessive grid imprinting at element edges is difficult. In regard to (a), using an algorithm that only uses information from an element of control volumes will (at best) be  $C^0$  at the element boundaries where most of the GLL points are located. A stencil that extends beyond one element is necessary. Mass-conservation requires a control volume to be defined around the GLL points (see, e.g., Figure 4 in this paper or Figure 8b in Ullrich et al. 2016). These volumes are artificial and not consistent with the SE method. Integrating the CSLAM reconstruction of water tracers of such artificial control volumes led to GLL node grid imprinting in the mapping and will not preserve a constant mixing ratio since the mapping of  $\Delta p^{(pg)}$  to GLL will not yield the GLL node value for dry pressure-level thickness (i.e. the maps are not reversible). Hence, after much experimentation, the best results in terms of grid-imprinting were obtained with tensor-cubic interpolation and by using the CAM-SE-CSLAM configuration.

### c. Time splitting and physics-dynamics coupling

The physics and dynamics are integrated in time using a sequential-update approach (Lauritzen et al. 2018). The dynamical core is sub-cycled over smaller time-steps, e.g., the vertical remapping time-step  $\Delta t_{remap}$  is cycled  $rsplit$  times over the coarse of the longer physics time-step,  $\Delta t_{phys}$ . In CAM-SE, a fraction of the physics forcing, e.g.,  $f_q \Delta t_{remap}$  is applied at the beginning of each  $rsplit$  vertical remap subcycles, such that the full forcing ( $f_q \Delta t_{phys}$ ) is realized over the course of a physics time-step. This approach of dribbling the tendencies over sub-intervals has the advantage of reducing gravity wave noise (Thatcher and Jablonowski 2016), but may disrupt tracer mass conservation. In CAM-SE-CSLAM, all but the tracer mass quantaties are dribbled, with tracer mass receiving the full physics update, e.g.,  $f_q \Delta t_{remap}$ , applied only at the beginning of the first remap sub-cycle, and thereby conserving tracer mass.

## 4. Results

The CAM aqua-planet reference configuration (Neale and Hoskins 2000; Medeiros et al. 2016) consists of an ocean covered planet in a perpetual equinox, with fixed, zonally symmetric sea surface temperatures idealized after the present day climatology. Two year long aqua-planet simulations are performed using CAM-SE in the *ne30np4* configuration, and CAM-SE-CSLAM in a *ne30pg3* configuration (CAM-trunk revision 88685). A plot similar to Figure 3 is constructed for the *ne30pg3* simulation, in which a probability density distribution of upward  $\omega$  is conditionally sampled based on location within the element. In the *ne30pg3* configuration, the sampling is based on a grid cell index 1-9, corresponding to the control volume location within the element. Through the use of the physics grid, the dynamical core state appears independent of location within the element, a marked improvement over the *ne30np4* configuration (Figure 3). Since the state is independent of in-element location, it follows that the physics forcing, which is evaluated from the state, should also be independent of in-element location.

The low-level, mean and variance of the physics tendencies in the two aqua-planet simulations are shown in Figure 6. The mean states in *ne30np4* and *ne30pg3* resemble one another, consistent with the similar climatological zonal mean precipitation rate in the simulations (not shown). The mean physics tendencies contains modest grid imprinting in the *ne30np4* configuration (barely visible near the storm-track regions), while in the variance field, grid imprinting is both ubiquitous and unmistakable. The variance is larger on boundary nodes, manifesting as a ‘stitching’ pattern resembling the cube-sphere grid. In *ne30pg3*, the grid imprinting is all but eliminated based on the mean and variance of the physics tendencies (Figure 6), consistent with our expectation.

As stated in Section 3, the mapping of the state to the physics grid and the reverse interpolation of physics tendencies to the GLL grid is not total energy conserving. CAM has a global energy fixer (Williamson et al. 2015) which can be used to estimate the errors associated with the mapping algorithms. To do so, it is presumed that there are no compensating mapping errors in going to and from the physics and dynamics grids, and that CAM-SE-CSLAM and CAM-SE have the same energy dissipation rates. Under these assumptions the spurious globally integrated total energy errors due to the mapping algorithm is estimated to be approximately  $0.0025 \text{ W/m}^2$  in the aqua-planet simulations. In comparison, the dynamical core total energy dissipation is on the order of  $0.1 \text{ W/m}^2$  (Lauritzen et al. 2018).

Grid imprinting associated with the flow around obstacles is more problematic than that encountered on the aqua-planets. In order to diagnose grid imprinting due to topographic flow, an idealized held-suarez configura-

tion (Held and Suarez 1994) is outfitted with real world topography after Fox-Rabinovitz et al. (2000); Baer et al. (2006), and run for 1200 days in the *ne30np4* and *ne30pg3* configurations. Figure 7 shows the mean  $\omega$  at two different vertical levels in the middle troposphere. The data are plotted on the control volume grid (e.g., Figure 4), in order to delineate whether a particular value is associated with an interior, edge or element boundary node.

At higher latitudes (e.g., the southern Andes), the flow is smooth, conforming reasonably to the underlying topography. At lower latitudes (within 20 to 30 degrees from the equator), over the Andes or the Himalayas, there is a clear preference for extrema to occur at the element boundaries (Figure 7). The vertical structure of  $\omega$  in regions of strong grid-imprinting indicates full-troposphere upward/downward motion (not shown). Grid imprinting is therefore more common in regions of weak stratification, such as occurs in the deep tropics, with forced up-slope flow facilitating the release of gravitational instability. Resolved updrafts/downdrafts often align with the element boundaries due to its systematically tighter pressure gradients, as explained in Figure 2. This type of grid imprinting has been previously identified in CAM-SE in an AMIP configuration (Figure 7 in Lauritzen et al. (2015)), manifesting as a band of precipitation near the foot of the Himalayas.

Through the use of the quasi-equal area physics grid, grid imprinting due to topographic flow is reduced (Figures 7). Figure 7 also shows the state in CAM-SE-CSLAM, but on the dynamical core grid. Arguably, grid imprinting due to topography in CAM-SE-CSLAM is not much of an improvement over CAM-SE, from the perspective of the dynamical core grid. Some regions (e.g., Andes) appear to have a larger grid imprinting signal, relative to CAM-SE, while other regions (e.g., Himalayas) indicate an improvement. The native topography lives on the physics grid, and the topography is mapped to the nodal points at run-time in CAM-SE-CSLAM. Mapping topography to the quadrature nodes ensures that no new extrema will be introduced to the boundary nodes, where the solution is least smooth. This effect can not be very large, since the grid noise is similar in CAM-SE and CAM-SE-CSLAM from the perspective of the dynamical core grid. From the perspective of the physics grid, the CAM-SE-CSLAM solution clearly mitigates the influence of grid-induced extrema on the state (Figure 7). The reduction in grid imprinting in CAM-SE-CSLAM is therefore almost entirely a result of the smoothing effect of integrating the basis functions over the control volumes of the physics grid.

## 5. Conclusions

Element-based high-order Galerkin Methods possess many of the attractive qualities recommended for next

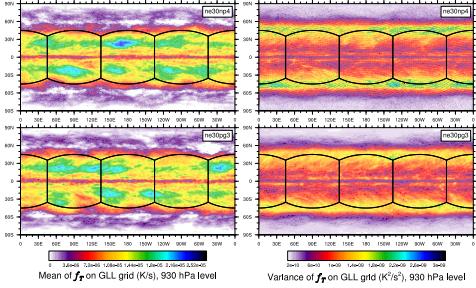


FIG. 6. Mean (left) and variance (right) of the low level temperature tendencies from the physical parameterizations on the GLL grid, with the *ne30np4* configuration, (top row) and *ne30pg3* configuration (bottom row), in a pair of year-long aqua-planet simulations after Medeiros et al. (2016). Grid imprinting is observed along the element boundaries in *ne30np4*, but is absent from the *ne30pg3* simulation.

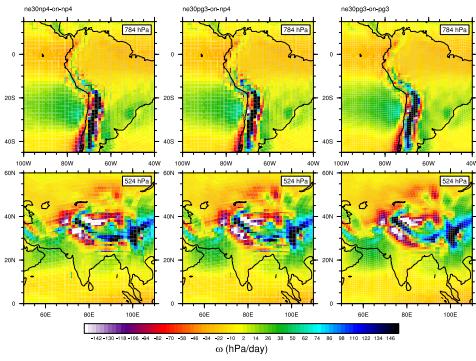


FIG. 7. Mean  $\omega$  at two model levels in the middle troposphere, in a Held-Suarez configuration outfitted with real world topography. (Left) CAM-SE state on the GLL grid, *ne30np4*, (Middle) CAM-SE-CSLAM state on the GLL grid, *ne30np4* and (Right) CAM-SE-CSLAM state on the physics grid, *ne30pg3*. The  $\omega$  fields are computed from a 1200 day Held-Suarez simulation. The data are contoured according to a ‘cell fill’ approach, in which the coupler grids (e.g., Figure 4) are used to delineate the vertices of the control volumes.

generation global atmospheric models. Among these, high-order accuracy is achieved with minimal communication between elements, allowing for near perfect scaling on massively parallel systems. Element communication amounts to a numerical flux applied to the element boundaries, reconciling overlapping solutions of adjacent elements but degrading the smoothness of the boundary nodes in the process (to  $C^0$ ). For non-smooth problems, gradients are systematically tighter at the element boundaries, and local extrema often characterize the boundary nodes. This behavior is illustrated using NCAR’s Community Atmosphere Model with Spectral Elements dynamics (CAM-SE) in an aqua-planet configuration, and in a Held-Suarez configuration with real-world topography.

The authors argue that the conventional physics-dynamics coupling paradigm, in which the physical parameterizations are evaluated on the dynamical core grid, exacerbates grid imprinting. A separate physics grid is proposed and implemented in CAM-SE, and referred to as CAM-SE-CSLAM, through dividing the elements into quasi-equal areas with equivalent degrees of freedom. The

state is mapped to the physics grid with high-order accuracy through integrating CAM-SE’s Lagrange basis functions over the control volumes. Control volumes near element boundaries now represent a state in the vicinity of the extrema produced through the boundary exchange, as opposed to the nodal value itself. These control volumes are also compatible with a ‘large-scale state’ as required by the physical parameterizations. The physical parameterizations are evaluated on the finite volume grid, and the forcing terms are mapped back to the dynamical core grid using a cubic tensor-product Lagrange interpolation. In aqua-planet simulations, evaluating the parameterizations on the physics grid removes any obvious dependence of proximity to the element boundary, resulting in a more realistic state with negligible grid imprinting. The mapping algorithm does not conserve total energy, but it is estimated that these errors are one to two orders of magnitude less than the total energy dissipation from the dynamical core.

In CAM-SE-CSLAM, the physics grid replaces the default CAM-SE quadrature point-based coupler grid (Fig-

ure 4) to compute fluxes between model components in the Community Earth System Model (CESM). The appeal here is two-fold. Through integrating the Lagrange basis functions over control volumes, one can be certain that the fluxes computed from this grid are a volume averaged flux. The same can not be said for CAM-SE, where the nodal values are effectively assigned to each control volume. The second advantage of the new coupler grid is that extrema occurring on boundary nodes may no longer influence other model components, in simulations without rough topography. While grid imprinting is reduced almost entirely in the aqua-planets, experiments with real-world topography reduces, but does not eliminate imprinting from the mean state. The quasi-equal area physics grid is nonetheless effective at mitigating numerical nuances associated with high-order element-based Galerkin methods, for non-smooth problems.

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## APPENDIX

The mapping of the physics tendencies from the physics grid to the GLL grid is done with tensor-cubic Lagrange interpolation. The elements of the cubed-sphere in SE are created from an equi-angular gnomonic projection. Consider one element  $(\alpha, \beta) \in [\alpha_1^{(elem)}, \alpha_2^{(elem)}] \times [\beta_1^{(elem)}, \beta_2^{(elem)}]$ , where  $(\alpha, \beta)$  are central angle coordinates and  $\alpha_1^{(elem)}$  and  $\alpha_2^{(elem)}$  are the minimum and maximum central angles in the  $\alpha$ -coordinate direction, respectively, and similarly for  $\beta$ . Let  $\Delta\alpha^{(elem)} = \alpha_2^{(elem)} - \alpha_1^{(elem)}$

and  $\Delta\beta^{(elem)} = \beta_2^{(elem)} - \beta_1^{(elem)}$ . The physics grid cell central angle centers are located at

$$(\alpha_i^{(pg)}, \beta_j^{(pg)}) = \left[ \alpha_1^{(elem)} + (i - \frac{1}{2}) \Delta\alpha^{(pg)}, \beta_1^{(elem)} + (j - \frac{1}{2}) \Delta\beta^{(pg)} \right], \quad (\text{A1})$$

where  $\Delta\alpha^{(pg)} = \Delta\beta^{(pg)} = \frac{\Delta\alpha^{(elem)}}{pg} = \frac{\Delta\beta^{(elem)}}{pg}$ . The interpolation is performed in central-angle coordinates using tensor product cubic interpolation. For elements located on a cubed-sphere edge or corner the coordinate system for neighboring elements may be on a different panel. To take into account this coordinate change the central angle locations of physics grid cell centers located on other panels are transformed to the coordinate system of the panel the element in question is located on (the transformations are given in, e.g., Nair et al. 2005). An illustration is given in Figure A1 for an element located in the lower left corner of a panel. The element in question is  $(\xi, \chi) \in (-1, 1)^2$  where, for simplicity, we have transformed the element coordinates into normalized coordinates  $(\xi, \chi) = \left( \frac{2(\alpha^{(pg)} - \alpha_1^{(elem)})}{\Delta\alpha^{(elem)}} - 1, \frac{2(\beta^{(pg)} - \beta_1^{(elem)})}{\Delta\beta^{(elem)}} - 1 \right)$ ; also used internally in the SE dynamical core (see, e.g., section 3.3 in Lauritzen et al. 2018). The GLL points are located at  $-1, -\sqrt{5}/5, \sqrt{5}/5$ , and  $1$  in each coordinate direction. Near the edges/corners of an element cubic extrapolation is used if the centered stencil expands beyond the panel.

## References

- Arakawa, A., and W. H. Schubert, 1974: Interaction of a cumulus cloud ensemble with the large-scale environment, Part I. *J. Atmos. Sci.*, **31**, 674–701.
- Bacmeister, J. T., K. A. Reed, C. Hannay, P. Lawrence, S. Bates, J. E. Truesdale, N. Rosenbloom, and M. Levy, 2018: Projected changes in tropical cyclone activity under future warming scenarios using a high-resolution climate model. *Climatic Change*, **146**, 547–560, doi:10.1007/s10584-016-1750-x, URL <http://dx.doi.org/10.1007/s10584-016-1750-x>.
- Baer, F., H. Wang, J. J. Tribbia, and A. Fournier, 2006: Climate modeling with spectral elements. *Mon. Wea. Rev.*, **134**, 3610–3624, doi:10.1175/MWR3360.1, URL <http://dx.doi.org/10.1175/MWR3360.1>.
- Brdar, S., M. Baldauf, A. Dedner, and R. Klöfkorn, 2013: Comparison of dynamical cores for nwp models: comparison of cosmo and dune. *Theoretical and Computational Fluid Dynamics*, **27** (3-4), 453–472, doi:10.1007/s00162-012-0264-z.
- Canuto, C., M. Y. Hussaini, A. Quarteroni, and T. Zang, 2007: *Spectral Methods: Evolution to Complex Geometries and Applications to Fluid Dynamics*. 1st ed., Springer.
- Dennis, J. M., and Coauthors, 2012: CAM-SE: A scalable spectral element dynamical core for the Community Atmosphere Model. *Int. J. High. Perform. C.*, **26** (1), 74–89, doi:10.1177/

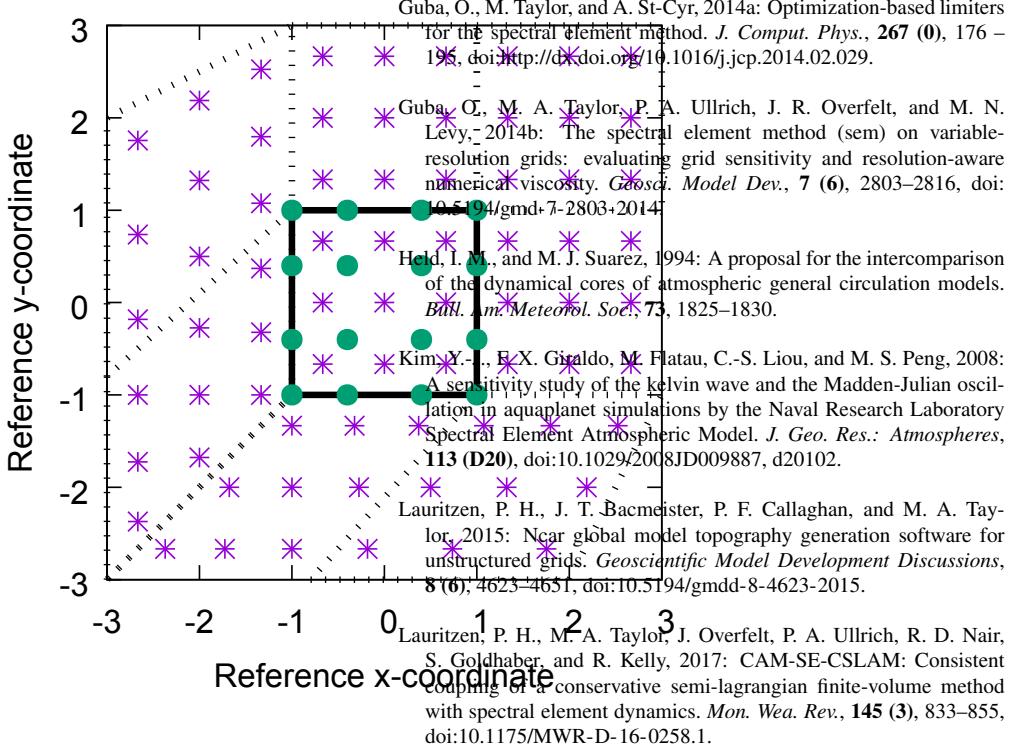


FIG. A1. Schematic of the coordinate system in which the dimensionally split cubic Lagrange interpolation is computed. The physics grid centers are marked with asterisks and the GLL points, we are interpolating to, with solid filled circles. The element in which the GLL points are located is bounded by thick black lines and located in the lower left corner of a panel. The stippled lines mark the boundaries of the remaining elements. For simplicity we are using the normalized coordinate centered at the element on which the GLL points we are interpolating to are located. Note that the coordinates for points on neighboring panels (using a different local coordinate system) must be transformed to the coordinate system of the element in question.

1094342011428142, URL <http://hpc.sagepub.com/content/26/1/74.abstract>, <http://hpc.sagepub.com/content/26/1/74.full.pdf+html>.

Durran, D., 2010: *Numerical Methods for Fluid Dynamics: With Applications to Geophysics*, Texts in Applied Mathematics, Vol. 32. 2nd ed., Springer, 516 p.

Fournier, A., M. A. Taylor, and J. J. Tribbia, 2004: The spectral element atmosphere model (SEAM): High-resolution parallel computation and localized resolution of regional dynamics. *Mon. Wea. Rev.*, **132** (3), 726–748.

Fox-Rabinovitz, M. S., G. L. Stenchikov, M. J. Suarez, L. L. Takacs, and R. C. Govindaraju, 2000: A uniform- and variable-resolution stretched-grid gem dynamical core with realistic orography. *Mon. Wea. Rev.*, **128**, 1883–1898.

Giraldo, F., and M. Restelli, 2008: A study of spectral element and discontinuous galerkin methods for the Navier-Stokes equations in nonhydrostatic mesoscale atmospheric modeling: Equation sets and test cases. *J. Comput. Phys.*, **227** (8), 3849 – 3877, doi:<http://dx.doi.org/10.1016/j.jcp.2007.12.009>.

- Neale, R. B., and Coauthors, 2010: Description of the NCAR Community Atmosphere Model (CAM 4.0). NCAR Technical Note, National Center of Atmospheric Research.
- Neale, R. B., and Coauthors, 2012: Description of the NCAR Community Atmosphere Model (CAM 5.0). NCAR Technical Note NCAR/TN-486+STR, National Center of Atmospheric Research.
- Plant, G. C. C., R. S., 2008: A stochastic parameterization for deep convection based on equilibrium statistics. *J. Atmos. Sci.*, **65**, 87–105, doi:10.1175/2007JAS2263.1, URL <http://dx.doi.org/10.1175/2007JAS2263.1>.
- Putman, W. M., and S.-J. Lin, 2007: Finite-volume transport on various cubed-sphere grids. *J. Comput. Phys.*, **227** (1), 55–78.
- Rasch, P., S. Xie, P.-L. Ma, W. Lin, E. Roesler, H. Wang, and Coauthors, in progress: An overview of the atmospheric component of the energy exascale earth system model. *J. Adv. Model. Earth Syst.*
- Reed, K. A., J. T. Bacmeister, N. A. Rosenbloom, M. F. Wehner, S. C. Bates, P. H. Lauritzen, J. E. Truesdale, and C. Hannay, 2015: Impact of the dynamical core on the direct simulation of tropical cyclones in a high-resolution global model. *Geophys. Res. Lett.*, **42** (9), 3603–3608, doi:10.1002/2015GL063974.
- Rhoades, A. M., X. Huang, P. A. Ullrich, and C. M. Zarzycki, 2016: Characterizing sierra nevada snowpack using variable-resolution cesm. *Journal of Applied Meteorology and Climatology*, **55** (1), 173–196, doi:10.1175/JAMC-D-15-0156.1, URL <https://doi.org/10.1175/JAMC-D-15-0156.1>.
- Simmons, A. J., and D. M. Burridge, 1981: An energy and angular-momentum conserving vertical finite-difference scheme and hybrid vertical coordinates. *Mon. Wea. Rev.*, **109** (4), 758–766.
- Small, R. J., and Coauthors, 2014: A new synoptic scale resolving global climate simulation using the community earth system model. *Journal of Advances in Modeling Earth Systems*, **6** (4), 1065–1094, doi:10.1002/2014MS000363.
- Suarez, M. J., A. Arakawa, and D. A. Randall, 1983: The parameterization of the planetary boundary layer in the ucla general circulation model: Formulation and results. *Mon. Wea. Rev.*, **111** (11), 2224–2243.
- Taylor, M., J. Edwards, and A. St-Cyr, 2008: Petascale atmospheric models for the community climate system model: new developments and evaluation of scalable dynamical cores. *J. Phys.: Conf. Ser.*, **125**, doi:10.1088/1742-6596/125/1/012023.
- Taylor, M. A., and A. Fournier, 2010: A compatible and conservative spectral element method on unstructured grids. *J. Comput. Phys.*, **229** (17), 5879 – 5895, doi:10.1016/j.jcp.2010.04.008.
- Thatcher, D. R., and C. Jablonowski, 2016: A moist aquaplanet variant of the Held–Suarez test for atmospheric model dynamical cores. *Geoscientific Model Development*, **9** (4), 1263–1292, doi:10.5194/gmd-9-1263-2016, URL <https://www.geosci-model-dev.net/9/1263/2016/>.
- Ullrich, P. A., 2014: A global finite-element shallow-water model supporting continuous and discontinuous elements. *Geosci. Model Dev.*, **7** (6), 3017–3035, doi:10.5194/gmd-7-3017-2014.
- Ullrich, P. A., D. Devendran, and H. Johansen, 2016: Arbitrary-order conservative and consistent remapping and a theory of linear maps: Part ii. *Mon. Wea. Rev.*, **144** (4), 1529–1549, doi:10.1175/MWR-D-15-0301.1.
- Ullrich, P. A., and M. A. Taylor, 2015: Arbitrary-order conservative and consistent remapping and a theory of linear maps: Part I. *Mon. Wea. Rev.*, **143**, 2419–2440, doi:10.1175/MWR-D-14-00343.1.
- Wan, H., and Coauthors, 2013: The icon-1.2 hydrostatic atmospheric dynamical core on triangular grids, part i: formulation and performance of the baseline version. *Geosci. Model Dev.*, **6**, 735–763.
- Williamson, D. L., J. G. Olson, C. Hannay, T. Toniazzo, M. Taylor, and V. Yudin, 2015: Energy considerations in the community atmosphere model (cam). *J. Adv. Model. Earth Syst.*, **7** (3), 1178–1188, doi:10.1002/2015MS000448, URL <http://dx.doi.org/10.1002/2015MS000448>.
- Zarzycki, C. M., C. Jablonowski, and M. A. Taylor, 2014a: Using variable-resolution meshes to model tropical cyclones in the community atmosphere model. *Mon. Wea. Rev.*, **142** (3), 1221–1239, doi:10.1175/MWR-D-13-00179.1.
- Zarzycki, C. M., M. N. Levy, C. Jablonowski, J. R. Overfelt, M. A. Taylor, and P. A. Ullrich, 2014b: Aquaplanet experiments using cam’s variable-resolution dynamical core. *J. Climate*, **27** (14), 5481–5503, doi:10.1175/JCLI-D-14-00004.1.