MONTHLY WEATHER REVIEW

Physics-dynamics coupling with element-based high-order Galerkin methods: quasi equal-area physics grid

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ABSTRACT

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- We should compute TKE of tendencies and show that we ar removing scales with nc¡2

1. Introduction

An increasing number of numerical methods publications in the atmospheric science literature concern transport, shallow-water, and three-dimensional models employing element-based high-order Galerkin discretizations such as finite-element and discontinuous Galerkin methods (for an introduction to these methods see, e.g., **??**). Some global models based on Galerkin methods have reached a level of maturity for which they are being considered for next generation climate and weather models due to their inherent conservation properties, high-order accuracy (for smooth problems), high parallel efficiency, high processor efficiency, and geometric flexibility facilitating mesh-refinement applications. NCAR's Community Atmosphere Model (CAM; **?**) offers a dynamical core based on continuous Galerkin finite elements (**?**), referred to as CAM-SE (CAM Spectral Elements; **??**). CAM-SE is, in particular, being used for high resolution climate modeling (e.g., **???**) and static mesh-refinement applications (e.g., **?????**). Other examples of models based on high-order Galerkin methods that are being considered for 'operational' weather-climate applications are **?**, **?** and **?**.

Traditionally the state of the atmosphere passed to the sub-grid-scale parameterizations (also referred to as *physics*) for models based on finite-volume and finite-difference methods has been the cell-averaged state in each control volume and the grid-point value, respectively. For the regular latitude-longitude, cubed-sphere and icosahedral grids the distance between the grid-points is gradually varying for finite-volume/finite-difference discretiza-

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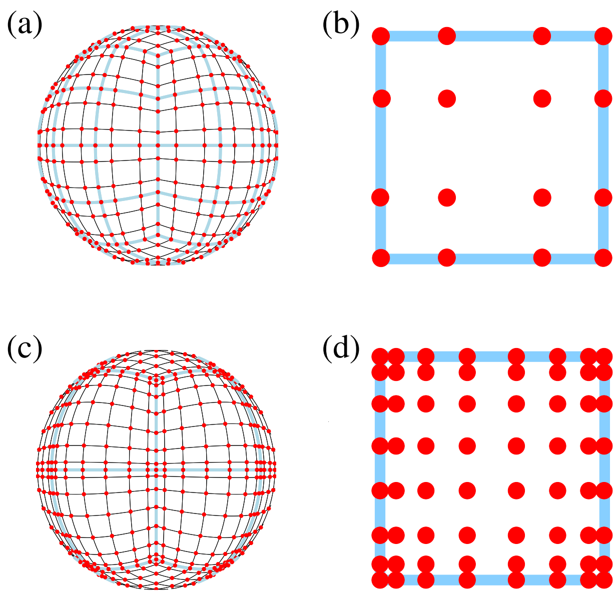

FIG. 1. Example of CAM-SE GLL quadrature grids, marked with red filled circles, (a & c) on the cubed-sphere and (b & d) in an element. (a)-(b) and (c)-(d) use 4×4 ($np = 4$) and 8×8 ($np = 8$) GLL quadrature points in each element, respectively. (a) and (c) have the same average grid-spacing at the Equator (7.5°) which is obtained by using (a) 4×4 ($ne = 4$) and (b) 2×2 ($ne = 2$) elements on each cubed-sphere face/panel, respectively. The element boundaries are marked with thick light blue lines. The grid configurations shown on (a) and (c) are referred to as *ne*4*np*4 and *ne*2*np*8, respectively.

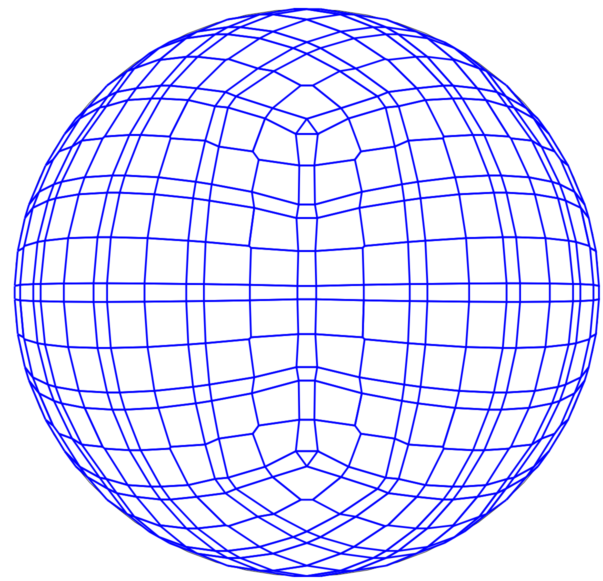

FIG. 2. An example of control volumes constructed around GLL quadrature points (NE4NP4) so that the spherical area of the control volumes exactly match the quadrature weight multiplied by the metric factor.

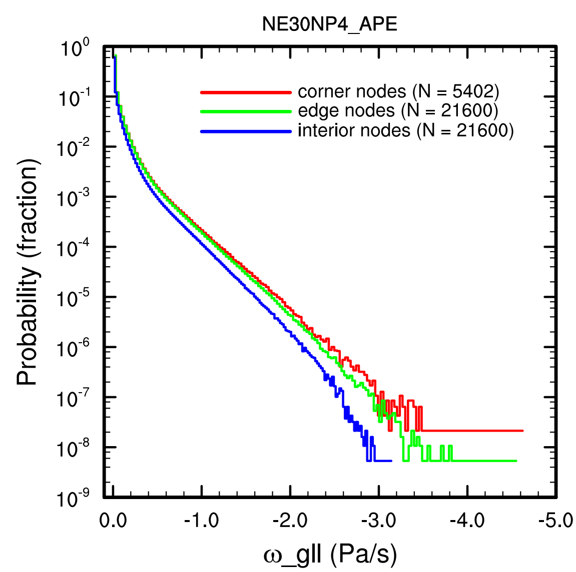

FIG. 3. PDF of instantaneous ω (1 month) classifying the points into small, medium, and large volumes/GLL weights. Note the consistent higher *omega* values for smaller areas compared to ω associated with larger volumes (which makes sense). The question is how parameterizations respond to that.

tions. If the same physics-dynamics coupling paradigm is applied to high-order element-based Galerkin methods, the state of the atmosphere passed to physics would be evaluated at the quadrature points. In the case of CAM-SE these are the Gauss-Lobatto-Legendre (GLL) quadrature points. Having the physics and dynamics grids coincide is obviously convenient since no interpolation is needed (which could disrupt conservation properties) and the number of degrees of freedom on both grids is exactly the same. A unique aspect of the high-order quadrature rules is that the nodes within an element are not equally spaced. For example, Figure 1 shows GLL points on an individual element of a cubed-sphere grid for degree 3 ($np = 4$ quadrature points) and degree 7 ($np = 8$ quadrature points) Lagrange polynomial basis in CAM-SE. Both grids have the same average resolution on the sphere (due to different number of elements), however, the higher the order of the quadrature rule the less equi-distant are the quadrature points. GLL quadrature points cluster near the edges and, in particular, the corners of the elements.

Parameterizations use the state of the atmosphere from the dynamical core as the large-scale state for computing sub-grid-scale processes. For example, the dynamical core state defines the large-scale environment in a mass-flux based convection scheme. One may think of the dynamical core state as the average state of the atmosphere over a control volume as inherent to finite-volume methods. For finite-difference methods the point value is thought of as representative for the atmospheric state in the vicinity of the point value and one can usually associate a volume with the grid-point. Hence the physics grid (the grid on which the state of the atmosphere is evaluated and passed to physics) and the dynamics grid (the grid the dynamical core uses) coincide. If we apply this concept to GLL quadrature values then a volume associated with the quadrature point should be defined. An example of that is shown on Figure 2 where control volumes have been defined around the quadrature points so that the spherical area of the control volumes exactly match the Gaussian weight multiplied by the metric term (these weights are used for integrating the basis functions over the elements and can therefore, in this context, be interpreted as areas). [Mark: could we be mathematically more rigorous? perhaps an appendix describing the iterative algorithm?] This grid is used in the NCAR CESM (Community Earth System Model) coupler for passing states between ocean, atmosphere and land-ice components since the current remapping method is finite-volume based and therefore requires control volumes [1]. Hence the components 'see' an irregular atmospheric grid. Similarly, the parameterizations in the atmosphere 'see' a state that is anisotropically sampled in space (see Figure 1 and 5 in **?**). The effect of this grid anisotropy can be seen in moist simulations, such as aqua-planet experiments (**??**). Figure 3 show a PDF of instantaneous upward vertical velocity ω for one year of an aqua-planet experiment using CAM-SE. The vertical velocity is classified by GLL weights pertaining to nodes lying on element corners ('corner nodes'), nodes along the element edges ('edge nodes') and nodes within the interior of an element ('interior nodes'). Consistently, the magnitude of ω is larger for smaller grid cell areas, due to the ability of smaller grid cells areas to support tighter horizontal gradients. [The question is then how will the physics respond to this?]

The quadrature grid in high-order element-based Galerkin methods is defined to perform mathematical operations on the basis functions, e.g., computing gradients and integrals, rather than evaluating the state variables for physics-dynamics coupling. Within each element there is a high-order C^∞ representation of state variables. One may argue that it would be more consistent to integrate the basis functions over quasi-equal area control volumes within each element and pass those control volume average values to physics rather than irregularly spaced quadrature point values. The relationship between the nodal values, the basis functions and the proposed control volumes is illustrated schematically in one-dimension in Figure 4).

[1] it is noted that methods exist that do not require control volumes for conservative interpolation (**?**)

Integrating basis functions over control volumes may be beneficial in reducing the intrinsic oscillatory behavior of high-order basis functions within each element. Moreover, if there is a strong grid-scale forcing or oscillatory behavior near an element boundary, the solution will be least smooth near that element boundary. The element boundaries are least smooth since the boundary exchange of the continuous Galerkin method results in a degradation to C^0 along the element edges. This is illustrated in the context of CAM-SE on Figure 5. In this case when integrating basis functions over control volumes a grid-cell average value is more representative of the values near the extrema at the element boundary than the quadrature point value.

It is the purpose of this paper to formulate the CAM-SE-physgrid version in which we separate physics and dynamics grids as illustrated in one dimension above. We show simulations from Held-Suarez .... CAM5 aquaplanet. ...

2. Methods

Here we focus on CAM-SE, however, in principle the methods apply to any element-based high-order Galerkin model. The physics grid in CAM-SE is defined by sub-dividing each element using equi-angular gnomonic coordinate lines to define the sides of the physics grid control volumes. Note that the element boundaries are defined by equi-angular gnomonic grid lines. The notation $nc = 3$ refers to the configuration where the elements are divided into $nc \times nc = 3 \times 3$ quasi equal-area physics grid cells (see Figure 8). Defining the physics grid by subdiving elements makes it possible to use the same infrastructure as used for the quadrature point values thereby facilitating its implementation in CAM-SE. Here we make use of the $NE30NP4$, $NE30NP4NC2$, $NE30NP4NC3$, and $NE30NP4NC4$ grids that use GLL quadrature point physics grid (physics and dynamics grid coincide), coarser ($nc = 2$), same ($nc = 3$) and finer ($nc = 4$) resolution quasi equal-area physics grids, respectively, compared to the GLL point resolution. In all configurations we use degree 3 Lagrange basis ($np = 4$) and $ne \times ne = 30 \times 30$ elements on each cubed-sphere panel resulting in an average GLL quadrature point spacing at the Equator of 1°. Vertical grid spacing is the standard CAM5 configuration ($nlev = 30$).

A consequence of separating physics and dynamics grids is that the atmospheric state must be mapped to the physics grid and the physics tendencies must be mapped back to the dynamics grid. Note that tendencies and not an updated state is mapped back to the dynamics grid. If one were to map an updated state the errors in the mapping process may adversely affect the simulation, e.g., in the case of no physics forcing there will be a non-zero 'physics forcing' entirely due to the errors in the mapping algorithm.

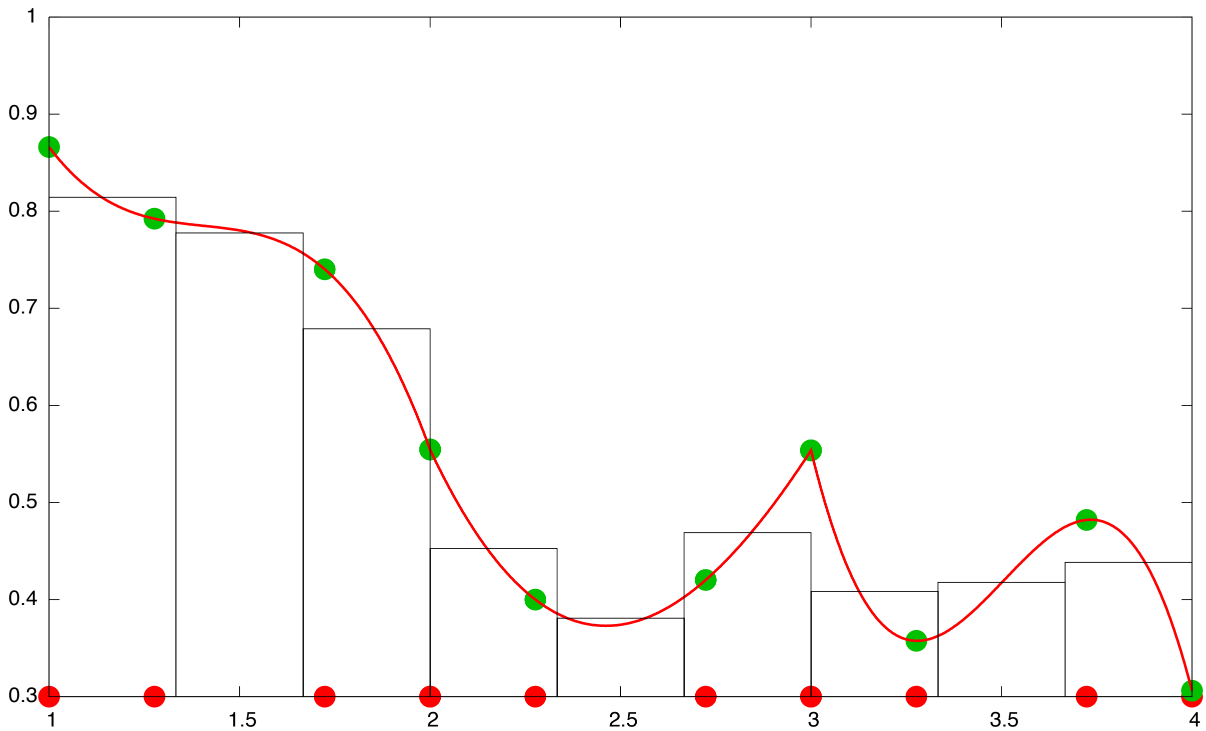

FIG. 4. A graphical illustration of the physics grid in one dimension. Three elements are shown and the filled red circles are the GLL quadrature points in each element. The red curve is the basis function representation of the field and the green filled circles are the quadrature point values. The physics grid divides each element into 3 equal-area control volumes. The histogram shows the average values over the physics grid control volumes resulting from integrating the basis functions over the respective control volumes.

In a climate model setting it is important that this process does not violate important conservation properties such as:

- mass-conservation,
- shape-preserving (monotone), i.e. the mapping method does not introduce new extrema in the interpolated field, in particular, negatives,
- consistency, i.e. the mapping preserves a constant.

Other properties that may be important, but not pursued here, is energy conservation and axial angular momentum conservation. It may be desirable to preserve the high-order of the basis functions during the mapping process so that the mapping is high-order accurate for smooth fields and less information is lost during the mapping process.

- description of PHIS

a. Remapping state: GLL grid → physics grid

The state variables that need to be mapped from the GLL grid to the physics grid are temperature $T^{(GLL)}$ and velocity components $(u^{(GLL)}, v^{(GLL)})$. Temperature is mapped by integrating the SE basis function representation of $T^{(GLL)} \times \Delta p^{(GLL)}$ and $\Delta p^{(GLL)}$ over the physics grid control volumes. The temperature on the physics grid is recovered from $\frac{T^{(phys)} \times \Delta p^{(phys)}}{\Delta\ p^{(phys)}}$. This mapping method conserves dry thermal energy $c_p^{(d)} T^{(GLL)} \times \Delta p^{(GLL)}$, where $c_p^{(d)}$ is the heat capacity for dry air at constant pressure, in each element. The velocity vectors are transformed from spherical coordinates to contravariant components (see, e.g., section 3.2 in **?**) and the basis function representation of each contravariant velocity component is evaluated at the centerpoint of the physics grid control volumes. Thereafter the vectors are transformed back to spherical coordinates. Since the atmospheric state mapped from dynamics to physics grid is based on the high-order SE basis functions, the loss of accuracy in transferring the state from GLL to physics grid is minimized.

b. Remapping: physics grid → GLL grid

The CAM physics package returns tendencies for temperature $f_T^{(phys)}$, velocity components $(f_u, f_v)^{(phys)}$, water vapor $f_Q^{(phys)}$, other tracers $f_q^{(phys)}$, and surface pressure

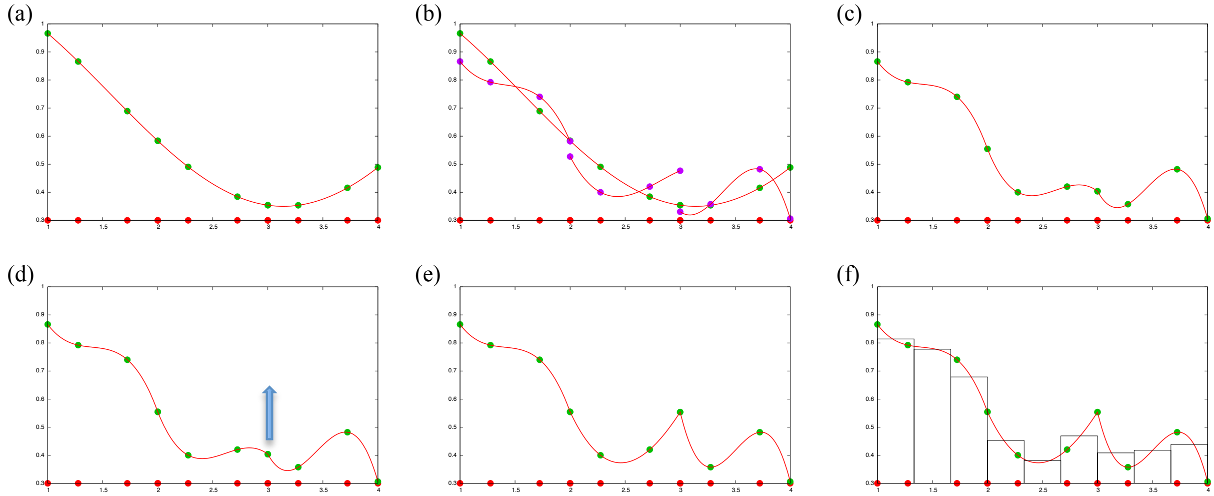

FIG. 5. A 1D schematic illustration on how CAM-SE advances the solution to the equations of motion in time. Consider 3 elements. The red filled circles are the GLL quadrature points in each element ($np = 4$). Note that the quadrature points on the boundary are shared between elements. (a) Assume a degree 3 global Lagrange polynomial initial condition (red curve) which can be represented exactly by the degree 3 Lagrange basis in each element. (b) The solution to the equations of motion are advanced in time (one Runga-Kutta step) independently in each element leading to the quadrature values marked with filled purple circles. The Lagrange basis is shown with red curves connecting the purple circles. There are now two solutions, one from left and one from right, for the quadrature points at the element end points. In CAM-SE the values are averaged so that the solution is C^0. Note that the averaging changes the Lagrange polynomials throughout except at the internal quadrature points. (c) shows the solution after averaging. (d) Assume there is a grid-scale forcing that increases the quadrature value located at $x = 3$. (e) The solution is now clearly C^0 at the element boundary at $x = 3$. (f) Histogram shows the average values resulting in integrating the basis functions over the control volumes.

$f_{PS}^{(phys)}$. The latter forcing is due to the vertical coordinate in CAM-SE being based on 'wet' pressure (dry air mass plus the weight of water vapor) so if there is a change in moisture in the column then the 'wet' surface pressure PS changes whereas the dry air mass (surface pressure) remains constant (see section 3.1.8 'Adjustment of pressure to include change in mass of water vapor' in **?**).

As for the dynamics to physics grid mapping, conservation is important and we therefore mass-weight the variables being mapped. For that Δp on the physics and dynamics grid is needed. Mapping the updated surface pressure on the physics grid to the dynamics grid is not desirable: first of all, if there is no tendency on surface pressure then the mapped surface pressure on the GLL grid will be different from the surface pressure on the GLL grid before calling physics. As mentioned before, this is equivalent to having a spurious forcing on PS entirely due to errors in the mapping algorithm. Secondly, conservation properties will result unless the GLL grid surface pressure is overwritten by the mapped PS from the physics grid to the dynamics grid. To ensure conservation and spurious forcing due to mapping errors the following algorithm is adopted for the mass-weighting.

Let Δp_{phys} be the updated pressure level thickness returned by physics. Map water-vapor mass $\Delta p_{phys} f_Q^{(phys)}$ from the physics to the dynamics grid using a conservative, consistent, and shape-preserving method (see below) resulting in $\Delta p_{phys} f_Q^{(phys)}$. This variable is the surface pressure tendency on the GLL grid, $f_{PS}^{(GLL)}$. Now take the surface pressure on the GLL grid before calling physics $p_s^{(GLL)}$ and add the surface pressure tendency $\Delta t PS^{(GLL)}$. This updated surface pressure $p_s^{(GLL)}$ defines the updated pressure-level thicknesses $\Delta p^{(GLL)}$. We use the physics updated pressure level thickness on the physics grid $\Delta p^{(phys)}$ for the mass-weighting of the physics tendencies $f_i^{(phys)}$, where $i = T, Q, q, u_x. u_y, u_z$, and $\Delta p^{(GLL)}$ for recovering the tendencies after mapping. The velocity forcing is transformed into a Cartesian coordinate system vector as for the dynamics to physics grid mapping.

1) MAPPING ALGORITHM

[Paul: we are just using low-order map? would it be easy to switch to higher-order?] To build the non-monotone "first guess" map from the finite volume physics grid to the finite element dynamics grid, a continuous polynomial reconstruction of degree $n_c - 1$ (and order n_c) is built that exactly interpolates the volume averaged values in each physics grid element. For the monotone "first guess" map, a second-order bilinear reconstruction is instead employed that interpolates the density field at the

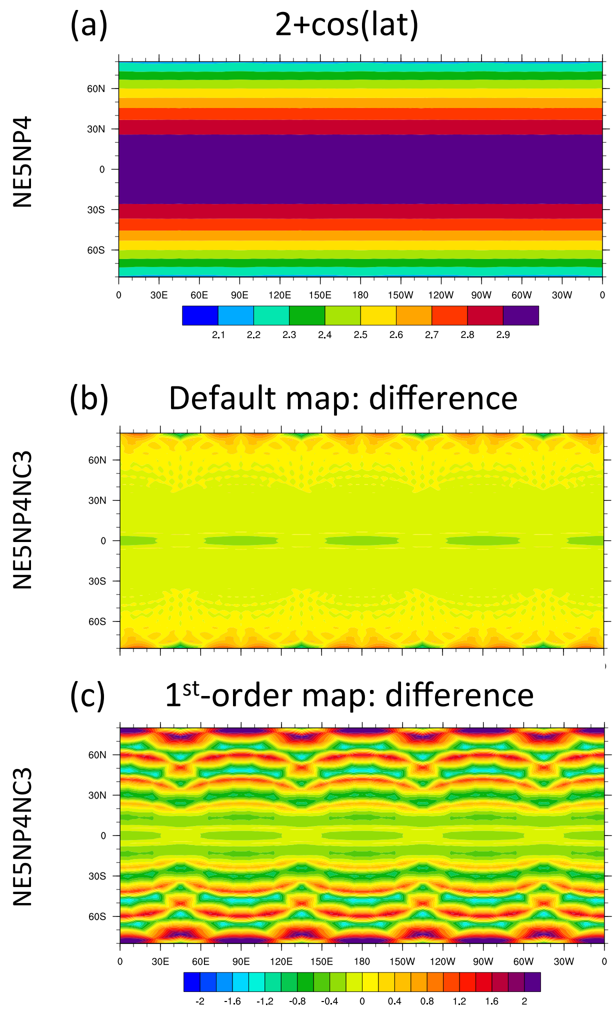

FIG. 6. (a) Smooth function $(2+\cos(\theta))$ initialized on the *NE5NP4* GLL grid. (b) and (c) show the difference between the interpolated field and the analytical value at the physics grid cell center. The interpolation is from the *NE5NP4* GLL grid to the NE5NP4NC3 physics grid (both have an approximate grid spacing of 6°). In (b) the interpolation algorithm is the default algorithm that is higher-order for smooth fields, shape-preserving, consistent, and mass-conservative. (c) is the same as (b) but using the first-order mapping method. All data has been bilinearly interpolated to a 1° regular latitude-longitude grid for plotting.

center of each finite volume. In each case the reconstruction is then sampled at each of the Gauss-Lobotto-Legendre nodes of the dynamics grid.

3. Results

Acknowledgments. NCAR is sponsored by the National Science Foundation (NSF).

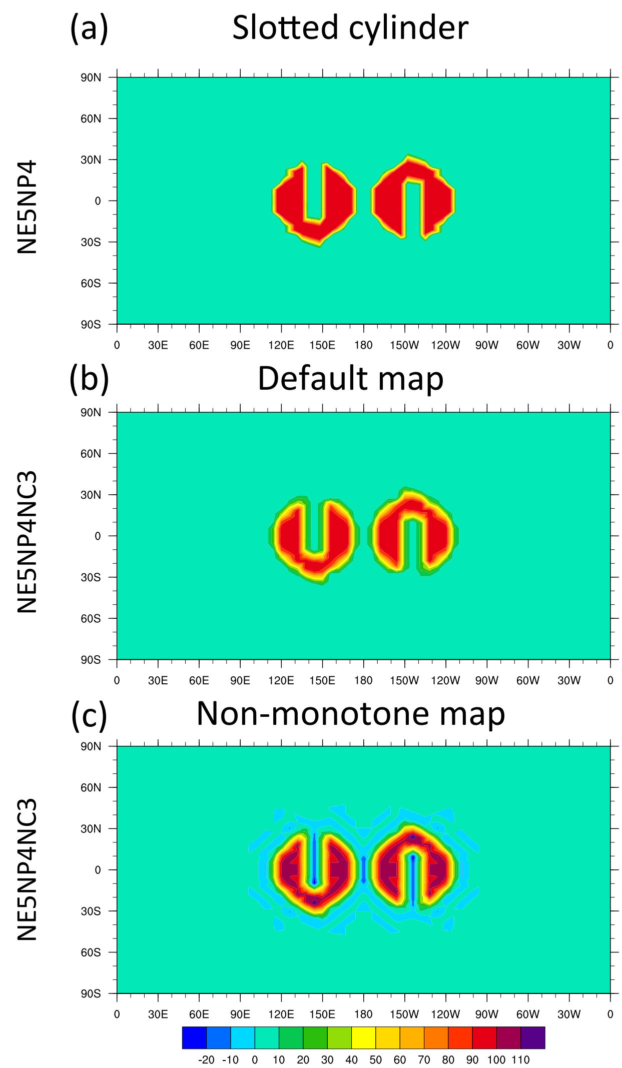

FIG. 7. (a) Slotted-cylinder distribution initialized on on the *NE5NP4* GLL grid (approximately 6° resolution). (b) Default mapping of the *NE5NP4* GLL grid data to the physics grid *NE5NP4NC3*. (c) Same as (b) but using the non-monotone map. All data has been bilinearly interpolated to a 1° regular latitude-longitude grid for plotting.

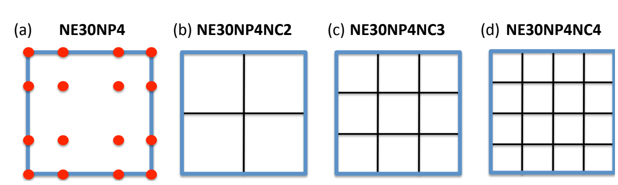

FIG. 8. A graphical illustration of the different physics column configurations: (a) Gauss-Lobatto-Legendre quadrature grid for $np = 4$ (filled circles) and (b-d) 'equal-area' finite-volume grids of different resolutions ($nc = 2,3,4$).

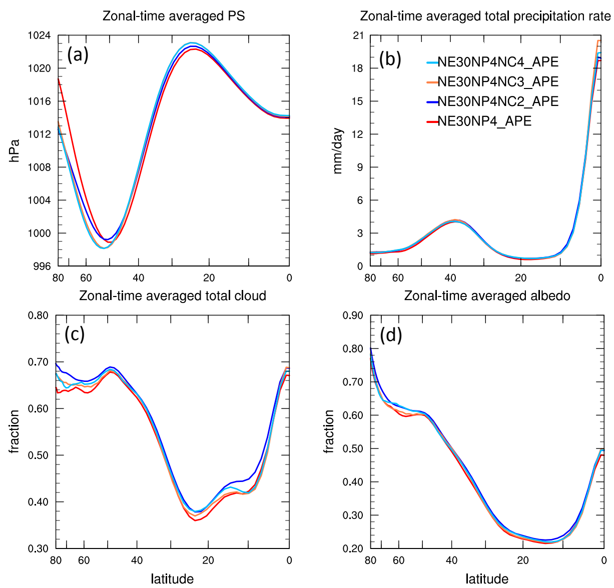

FIG. 9. Zonal-time average (a) surface pressure PS, (b) total precipitation rate $PRECT$, (c) total cloud fraction $CLDTOT$, and (d) albedo as a function of latitude (from Equator to 80°N) for the different model configurations. The data has been averaged over a period of 30 months and mapped to a $1.5^{\circ} \times 1.5^{\circ}$ regular latitude-longitude grid for analysis.