A detailed total energy analysis of the Community Atmosphere Model (CAM)

P.H. Lauritzen 1* , and D.L. Williamson 1

¹National Center for Atmospheric Research, Boulder, Colorado, USA

Key Points:

• ..

^{*1850} Table Mesa Drive, Boulder, Colorado, USA

Abstract

1 Introduction

In coupled climate modeling with prognostic atmosphere, ocean, land, land-ice, and sea-ice components, it is important to conserve total energy to a high degree to avoid spurious long term trends in the simulated Earth system. Conservation of total energy in this context refers to having a closed total energy budget. For example, the total energy change in a column in the atmosphere is exactly balanced by the net sources/sinks given by the fluxes through the column. The fluxes into the atmospheric component from the surface models must be balanced by the fluxes in the respective surface components and so on. Henceforth we will focus only on the atmospheric component which, in a numerical model, is split into a resolved-scale component (the dynamical core) and a sub-grid-scale component (parameterizations or, in modeling jargon, physics).

The atmospheric equations of motion conserve total energy but the discretizations used in climate and weather models are usually not inherently total energy conservative. Exact conservation is probably not necessary but conservation to within $0.01\ W/m^2$ has been considered sufficient to avoid spurious trends in century long simulations [Boville, 2000; Williamson et al., 2015]. Spurious sources and sinks of total energy can be introduced by the dynamical core, physics, physics-dynamics coupling as well as discrepancies between the total energy of the continuous and discrete equations of motion and for the physics. Hence the study of total energy conservation in comprehensive models of the atmosphere quickly becomes a quite complex and detailed matter. In addition there can easily be compensating errors in the system as a whole.

Here we focus on versions of the Community Atmosphere Model (CAM) that use the spectral-element [SE, *Lauritzen et al.*, 2018] and finite-volume [FV, *Lin*, 2004] dynamical cores. These dynamical cores couple with physics in a time-split manner, i.e. physics receives a state updated by dynamics [see *Williamson*, 2002, for a discussion of time-split versus process split physics-dynamics coupling in the context of CAM].

In its pure time-split form the physics tendencies are added to the state previously produced by the dynamical core and the resulting state provides the initial state for the subsequent dynamical core calculation. We refer to this as state-updating. Alternatively, when the dynamical core adopts a shorter time step than the physics, say N sub-steps, then (1/N)th of the physics-calculated tendency is added to the state before each dynamics sub-step. We refer to this modification of time-splitting as *dribbling*. CAM-FV uses the state-update approach while CAM-SE has options to use state-update, *dribbling* or a combination of the two (i.e. some variables use state-updating and others use *dribbling*). The dribbling variants can lead to spurious sources or sinks of total energy referred to as physics-dynamics coupling errors.

The dynamical core usually has implicit or explicit filters to control spurious noise near the grid scale which will lead to energy dissipation [Thuburn, 2008; Jablonowski and Williamson, 2011]. Similarly models often have sponge layers to control the solution near the top of the model that may be a sink of total energy. There are examples of numerical discretizations of the adiabatic frictionless equations motion that are designed so that total energy is conserved in the absence of time-truncation and filtering errors, e.g., mimetic spectral-element discretizations such as the one used in the horizontal in CAM-SE [Taylor, 2011]. These provide consistency between the discrete momentum and thermodynamic equations leading to global conservation associated with the conversion of potential to kinetic energy. In spectral transform models it is customary to add the energy change due to explicit diffusion on momentum back as heating (referred to as frictional heating), so that the diffusion of momentum does not affect the total energy budget [see, e.g., p.71 in Neale et al., 2012]. This is also done in CAM-SE [Lauritzen et al., 2018].

It is the purpose of this paper to provide a detailed total energy analysis of the CAM-SE and CAM-FV. \dots

2 Method

2.1 Defining total energy

In the following it is assumed that the model top and bottom are coordinate surfaces and that there is no flux of mass through the model top and bottom. In a dry atmosphere the total energy equation integrated over the entire sphere is given by

$$\frac{d}{dt} \int_{z=z_s}^{z=z_{top}} \iint_{\Omega} E_{\nu} \rho^{(d)} dA dz = \int_{z=z_s}^{z=z_{top}} \iint_{\Omega} F_{net} \rho^{(d)} dA dz, \tag{1}$$

[e.g., Kasahara, 1974] where F_{net} is net fluxes calculated by the parameterizations (e.g., heating and momentum forcing), d/dt the total/material derivative, z_s is the height of the surface, Ω the sphere, $\rho^{(d)}$ the density of dry air and E_v is the total energy. E_v can be split into kinetic energy $K = \frac{1}{2}\vec{v}^2$ (v is the wind vector), internal energy $c_v^{(d)}T$, where $c_v^{(d)}$ is the heat capacity of dry air at constant volume, and potential energy $\Phi = gz$

$$E_{\nu} = K + c_{\nu}^{(d)} T + \Phi.$$
 (2)

If the vertical integral is performed in a mass-based vertical coordinate, e.g., pressure, then the integrated total energy equation for a dry atmosphere can be written as

$$\frac{d}{dt} \int_{p=p_s}^{p=p_{top}} \iint_{\Omega} E_p \rho^{(d)} dA dp + \frac{d}{dt} \iint_{\Omega} \Phi_s p_s dA = \int_{p=p_s}^{p=p_{top}} \iint_{\Omega} F_{net} \rho dA dp, \tag{3}$$

[e.g., Kasahara, 1974] where

$$E_p = K + c_p^{(d)} T. (4)$$

In a moist atmosphere, however, there are several definitions of total energy used in the literature related to what heat capacity is used for water vapor and whether or not condensates are accounted for in the energy equation. To explain the details of that we focus on the energy equation for CAM-SE.

CAM-SE is formulated using a terrain-following hybrid-sigma vertical coordinate η but the coordinate levels are defined in terms of dry air mass $(M^{(d)})$ instead of total air mass; $\eta^{(d)}$ [see *Lauritzen et al.*, 2018, for details]. In such a coordinate system it is convenient to define the tracer state in terms of a dry mixing ratio instead of moist mixing ratio

$$m^{(\ell)} \equiv \frac{\rho^{(\ell)}}{\rho^{(d)}}, \text{ where } \ell = \text{`wv', 'cl', 'ci', 'rn', 'sw',}$$

$$(5)$$

where $\rho^{(d)}$ is the mass of dry air per unit volume of moist air and $\rho^{(\ell)}$ is the mass of the water substance of type ℓ per unit volume of moist air. Moist air refers to air containing dry air ('d'), water vapor ('wv'), cloud liquid ('cl'), cloud ice ('ci'), rain amount('rn') and snow amount('sw'). For notational purposes define the set of all components of air

$$\mathcal{L}_{all} = \{ 'd', 'wv', 'cl', 'ci', 'rn', 'sw' \},$$
 (6)

Define associated heat capacities at constant pressure $c_p^{(\ell)}$. Using the $\eta^{(d)}$ vertical coordinate and dry mixing ratios the total energy that the frictionless adiabatic equations of motion in the CAM-SE dynamical core conserves is

$$\widehat{E}_{dyn} = \int_{\eta=0}^{\eta=1} \iint_{\mathcal{S}} \left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}} \right) \sum_{\ell \in \Gamma, \dots} \left[m^{(\ell)} \left(K + c_p^{(\ell)} T + \Phi_s \right) \right] dA d\eta^{(d)}, \tag{7}$$

where Φ_s is the surface geopotential and $\widehat{(\cdot)}$ refers to the global integral.

In the CAM physical parameterizations a different definition of total energy is used. Due to the evolutionary nature of the model development, the parameterizations have not yet been converted to match the SE dynamical core. For the computation of total energy condensates are assumed to be zero and the heat capacity of moisture is the same as for dry air. This is equivalent to using a moist mass (dry air plus water vapor) but c_p of dry air.

$$\widehat{E}_{phys} = \int_{\eta=0}^{\eta=1} \iint_{\mathcal{S}} \left(\frac{\partial M^{(d)}}{\partial \eta^{(d)}} \right) \left(1 + m^{(wv)} \right) \left[\left(K + c_p^{(d)} T + \Phi_s \right) \right] dA d\eta^{(d)}. \tag{8}$$

We note that earlier versions of CAM using the spectral transform dynamical core used c_p of moist air. One can make the adiabatic, frictionless equations of motion in the dynamical core conserve $E^{(physics)}$ by not including condensates in the mass/pressure field as well as energy conversion term in the thermodynamic equation and setting the heat capacity for moisture to $c_p^{(d)}$ [Taylor, 2011]. We will also make use of this configuration in the analysis presented in this paper.

2.2 Spurious energy sources and sinks

In a weather/climate model total energy conservation errors can appear in many places throughout the algorithm. Below is a general list of where conservation errors can appear with specific examples from CAM:

- 1. Parameterization errors: Individual parameterizations may not have a closed energy budget. However, CAM parameterizations are required to have a closed energy budget under the assumption that pressure remains constant during the computation of the subgrid-scale parameterization tendencies. In other words, the total energy change in the column is exactly balanced by the net sources/sinks given by the fluxes through the column.
- Pressure work: That said, if parameterizations update specific humidity then the surface pressure changes (e.g., moisture leaving the column). In that case the pressure changes which, in turn, changes total energy, here referred to as pressure work [section 3.1.8 in Neale et al., 2012].
- 3. *Physics-dynamics coupling (PDC)*: Assume that physics computes a tendency. Usually the tendency is passed to the dynamical core which is responsible for adding the tendencies to the state before advancing the dynamics. PDC energy errors can be split into two types:
 - 'Dribbling' errors (or, equivalently, temporal PDC errors): If the total energy increment from the parameterizations does not match the change in total energy when the tendencies are added to the state in the dynamical core, then there will be a spurious PDC error. This will not happen with the state-update approach in which the tendencies are added immediate after physics and before the dynamical core advances the solution in time, but it does happen with dribbling.
 - Change of vertical grid/coordinate errors: If the vertical coordinate in physics and in the dynamical core are different then there can be spurious PDC energy errors even when using the state-update method for adding tendencies to the dynamical core state. For example, many non-hydrostatic dynamical cores [Skamarock et al., 2012, e.g. MPAS,] use a terrain-following height coordinate whereas physics uses pressure.
 - Change of horizontal grid errors: If the physics tendencies are computed on a
 different horizontal grid than the dynamical core then there can be spurious energy errors from mapping tendencies between horizontal grids [e.g., Herrington
 et al., 2018].
- 4. *Dynamical core errors*: Energy conservation errors in the dynamical core, not related to PDC errors, can arise in multiple parts of the algorithms used to solve the equations of motion. For dynamical cores employing implicit filtering [e.g., limiters

in flux operators *Lin*, 2004] and/or possessing inherent damping to control small scales, it is hard to diagnose what their energy dissipation is compared to other errors in the discretization. If explicit filtering is used, e.g., hyperviscosity on momentum, then one can diagnose the energy dissipation from filtering and add a corresponding heating to balance it. There may also be energy loss from viscosity applied to other variables such a temperature or surface pressure which are harder to compensate. Here is a beak-down relevant to CAM-SE using a floating Lagrangian vertical coordinate:

- Horizontal inviscid dynamics: Energy errors resulting from solving the inviscid, adiabatic equations of motion.
- Hyperviscosity: Filtering errors.
- Vertical remapping: The vertical remapping algorithm does not conserve total energy.
- · Near round-off negative values
- 5. Continuous total energy formula discrepancy: If the continuous equations of motion for the dynamical core conserve a total energy different from the one used in the parameterizations then an energy inconsistency is present in the system as a whole. This is the case with the new version of CAM-SE that conserves a total energy that is more accurate and comprehensive than the CAM physics package as discussed above. As also noted above, this mismatch arose from the evolutionary nature of the model development and not by deliberate design.

To avoid total energy conservation errors which could accumulate and ultimately lead to a climate drift, it is customary to use an energy fixer to restore total energy conservation. Since the spatial distribution of energy errors, in general, is not known, global fixers are used. In CAM a uniform increment is added to the temperature field to compensate for total energy loss in the dynamical core, physics-dynamics coupling and energy change due to pressure work.

3 Results

A series of simulations have been performed with CESM2.0 using CAM version 6 physics (https://doi.org/10.5065/D67H1H0V). Various configurations are used and referred to in terms of the *COMPSET* (Component Set) value used in CESM2.0:

- FSH94: Dry dynamical core configuration based on Held-Suarez forcing which relaxes temperature to a zonally symmetric equilibrium temperature profile and simple linear drag at the lower boundary [Held and Suarez, 1994].
- *QPC6*: The QPC6 configuration refers to an aqua-planet setup; i.e. an ocean covered planet in perpetual equinox, with fixed, zonally symmetric sea surface temperatures [*Neale and Hoskins*, 2000; *Medeiros et al.*, 2016].
- F2000climo: The F2000climo configuration refers to a 'real-world' AMIP (Atmospheric Model Intercomparison Project) type simulations using perpetual year 2000 SST (Sea Surface Temperature) boundary conditions.

3.1 Diagnostics

The discrete global integrals $\widehat{(\cdot)}$ are computed consistent with the dynamical core as outlined in section 2.2. of *Lauritzen et al.* [2014]. The total energy tendency is denoted

$$\partial \widehat{E} \equiv \frac{d\widehat{E}}{dt}.\tag{9}$$

Describe temporal averaging and 2 By placing output calls in appropriate locations in the code, we can directly compute $\partial \widehat{E}$ due to various steps in the algorithm refer section 2.2. Define the following energy tendencies (corresponding to itemized list in section 2.2):

1. $\partial \widehat{E}^{(param)}$: Total energy tendency due to parameterizations. In CAM the total energy budget for each parameterization is closed (assuming constant pressure) so $\partial \widehat{E}^{(param)}$ are balanced by net fluxes in/out of the physics columns. Note that this is the only energy tendency that is not spurious since CAM parameterizations have a closed energy budget. This total energy tendency is discretely computed as

$$\partial \widehat{E}_{phys}^{(param)} = \frac{\widehat{E}_{pAP} - \widehat{E}_{pBP}}{\Delta t_{phys}},$$
(10)

where Δt_{phys} is the physics time-step (1800s) and the subscript phys refers to the energy tendency being computed in CAM physics.

2. $\partial \widehat{E}^{(pwork)}$: Total spurious energy tendency due to pressure work

$$\partial \widehat{E}_{phys}^{(pwork)} = \frac{\widehat{E}_{pAM} - \widehat{E}_{pAP}}{\Delta t_{phys}}.$$
 (11)

The total forcing from physics (at least in CAM) consists of parameterizations, pressure work and total energy fixer, with associated energy tendency

$$\partial \widehat{E}_{phys}^{(efix)} = \frac{\widehat{E}_{pBP} - \widehat{E}_{pBF}}{\Delta t_{phys}}.$$
 (12)

For notational convenience we refer to the associated energy tendency from all of physics as

$$\partial \widehat{E}_{phys}^{(phys)} \equiv \partial \widehat{E}_{phys}^{(param)} + \partial \widehat{E}_{phys}^{(pwork)} + \partial \widehat{E}_{phys}^{(efix)} = \frac{\widehat{E}_{pAM} - \widehat{E}_{pBF}}{\Delta t_{phys}}.$$
 (13)

3. $\partial \widehat{E}^{(pdc)}$: Total spurious energy tendency due to physics-dynamics coupling errors is the difference between the energy tendency from physics and the energy tendency in the dynamics resulting from adding the physics increment to the dynamical core state

$$\partial \widehat{E}^{(pdc)} = \partial \widehat{E}_{phys}^{(phys)} - \partial \widehat{E}_{dyn}^{(phys)}, \tag{14}$$

where

$$\partial \widehat{E}_{dyn}^{(param)} = \frac{\widehat{E}_{dAD} - \widehat{E}_{dAF}}{\Delta t_{pdc}},$$
(15)

and Δt_{pdc} is the time-step between physics increments being added to the dynamical core. In CAM-SE there are 3 physics-dynamics coupling algorithms described in detail in section 3.6 in *Lauritzen et al.* [2018]. One is state-update in which the entire physics increments is added to the dynamics state at the beginning of dynamics (referred to as ftype = 1), in which case $\Delta t_{pdc} = \Delta t_{phys}$, one is 'dribbling' in which the physics tendency is split into nsplit equal chunks and added throughout dynamics (more precisely after every vertical remapping; referred to as ftype = 0 resulting in $\Delta t_{pdc} = \frac{1}{nsplit}\Delta t_{phys}$), and then a combination of the two where tracers use ftype = 1 and all other physics tendencies used ftype = 0 (referred to as ftype = 2).

- 4. The total energy tendency from the dynamical core is split into several terms:
 - Horizontal adiabatic dynamics (dynamics excluding physics forcing tendency)

$$\partial \widehat{E}_{dyn}^{(2D)} = \frac{\widehat{E}_{dAD} - \widehat{E}_{dBD}}{\Delta t_{dyn}},\tag{16}$$

where $\Delta t_{dyn} = \frac{\Delta t_{phys}}{nsplit \times rsplit}$ In CAM-SE the viscosity is explicit to we can compute the total energy tendency due to hyperviscosity

$$\partial \widehat{E}_{dyn}^{(hvis)} = \frac{\widehat{E}_{dAH} - \widehat{E}_{dBH}}{\Delta t_{hvis}},\tag{17}$$

which, in CAM-SE, includes a frictional heating term (viscosity on momentum has been added to $\partial \widehat{E}_d y n_{dyn}^{(hvis)}$) with associate energy tendency

$$\partial \widehat{E}_{dyn}^{(fheat)} = \frac{\widehat{E}_{dAH} - \widehat{E}_{dCH}}{\Delta t_{hvis}},$$
(18)

where $\Delta t_{hvis} = \frac{\Delta t_{phys}}{nsplit \times nsplit \times hypervis_subcycle}$. The residual

$$\partial \widehat{E}_{dyn}^{(res)} = \partial \widehat{E}_{dyn}^{(2D)} - \partial \widehat{E}_{dyn}^{(hvis)}, \tag{19}$$

is energy errors due to inviscid dynamics and time-truncation errors.

The energy tendency due to vertical remapping is

$$\partial \widehat{E}_{dyn}^{(remap)} = \frac{\widehat{E}_{dAR} - \widehat{E}_{dAD}}{\Delta t_{remap}},$$
(20)

where $\Delta t_{remap} = \frac{\Delta t_{phys}}{nsplit}$.

The 3D adiabatic dynamical core (no physics forcing) energy tendency is denoted

$$\partial \widehat{E}_{dyn}^{(adiab)} = \partial \widehat{E}_{dyn}^{(2D)} + \partial \widehat{E}_{dyn}^{(remap)}. \tag{21}$$

split parameters not defined

- Errors associated with a discrepancy in energy formulas are more tricky to assess. more text needed
- 3.1.0.1 A couple of observations regarding the energy budget when both physics and dynamics uses the same total energy formula : It is important to note that the energy fixer fixes energy dissipation in the dynamical core, pressure work and physics-dynamics coupling errors

$$-\partial \widehat{E}_{phys}^{(efix)} = \partial \widehat{E}_{phys}^{(pwork)} + \partial \widehat{E}_{dyn}^{(adiab)} + \partial \widehat{E}_{dyn}^{(pdc)}. \tag{22}$$

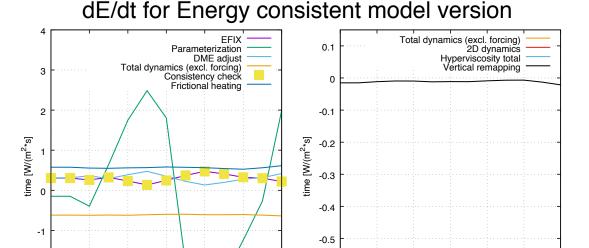
The forcing from the parameterizations, $\partial \widehat{E}_{phys}^{(param)}$, does not appear in this budget (although the dynamical core state does 'feel' the parameterization forcing) as the energy cycle for the parameterizations is, by design in CAM, closed. Furthermore, one can use (22) to diagnose energy dissipation in the dynamical core and physics-dynamics coupling from quantities computed in physics

$$\partial \widehat{E}_{dyn}^{(adiab)} + \partial \widehat{E}_{dyn}^{(pdc)} = -\partial \widehat{E}_{phys}^{(efix)} - \partial \widehat{E}_{phys}^{(pwork)}. \tag{23}$$

This is useful if the diagnostics are not implemented in the dynamical core.

The and the total energy is computed using the dynamical cores definition of total energy (7).

'The discrepancy between the more comprehensive energy formula (7) and the CAM physics formula for total energy is about 0.5 W/m^2 [Taylor, 2011]. By only including dry air and water vapor in ρ and setting $c_p^{(wv)} = c_p^{(d)}$ in the equations of motion, the dynamical core (in the absence of truncation errors) will conserve the energy used in CAM physics.'



-0.6

-0.7 C

2

time [months]

10

12

Figure 1. [in note form] Note that the parameterizations have a closed energy budget so the fluctuations in the energy change due to parameterizations is balanced by fluxes in/out of the physics columns. The purpose of this Figure is to show that the energy tendency in the dynamical core is quite constant (to within $0.2 W/m^2$ or less); so only one month simulation may be enough to assess energy diagnostics for the dynamical core. DME adjust fluctuates with the physics forcing; obviously the energy fixers fluctuates with DME adjust. The *consistency check* (triangles) shows that the energy fixer exactly compensates for total energy loss in dynamical core and dme adjust (there are no physics dynamics coupling errors in this configuration).

12

10

3.2 Configuration 1: Consistent total energy definitions in parameterizations and dynamical core, and total energy conserving physics-dynamics coupling

This configuration is most energetically consistent in that the physical parameterizations and the continuous equations of motion on which the dynamical is based, conserve the same total energy $\left\langle E^{(physics)} \right\rangle$ defined in equation (8); and there are no spurious sources/sinks in physics-dynamics coupling. Energetic consistency in dynamics and physics is obtained by setting $c_p^{(\ell)} \equiv c_p^d \equiv \text{and } \mathcal{L}_{all} = \{ `d`, `wv` \}$. If the parameterizations effectively update the model state then there are no physics-dynamics coupling errors [ftype = 1 setup described in detail in *Lauritzen et al.*, 2018]. Namelist changes resulting in this configuration are lcp_moist = .true., se_qsize_condensate_loading = 1, and ftype = 1.

4 Conclusions

A: ...

-2

0

2

6

time [months]

Acknowledgments

The National Center for Atmospheric Research is sponsored by the National Science Foundation. We thank NCAR's Computational and Information Systems Laboratory (CISL) and NCAR's Climate and Global Dynamics division (CGD) for computational resources.

```
do nt=1,ntotal
  PARAMETERIZATIONS:
  output 'pBF'
  Energy fixer
  output 'pBP'
  Physics updates the state and state saved for energy fixer
  output 'pAP'
  Dry mass correction
  output 'pAM'
  DYNAMICAL CORE:
  output 'dED'
  do ns=1,nsplit
    output 'dAF'
    Update state with (1/nsplit) of the physics tendencies
    output 'dBD'
    do nr=1,rsplit
       Advance the adiabatic frictionless equations of motion
       in floating Lagrangian layer.
       do ns=1,hypervis_subcycle
           output 'dBH'
           Advance hyperviscosity operators.
           output 'dCH'
           Add frictional heating to temperature.
           output 'dAH'
       end do
    end do
    output 'dAD'
    Vertical remapping from floating Lagrangian levels to Eulerian levels
  end do
  output 'dBF'
end do
```

Figure 2. Pseudo-code ...

References

Boville, B. (2000), *Numerical Modeling of the Global Atmosphere in the Climate System*, chap. Toward a complete model of the climate system, in Numerical Modeling of the Global Atmosphere in the Climate System, pp. 419–442, first ed., Springer Netherlands.

Held, I. M., and M. J. Suarez (1994), A proposal for the intercomparison of the dynamical cores of atmospheric general circulation models, *Bull. Amer. Meteor. Soc.*, 75, 1825–1830.

Herrington, A. R., P. H. Lauritzen, M. A. Taylor, S. Goldhaber, B. E. Eaton, K. A. Reed, and P. A. Ullrich (2018), Physics-dynamics coupling with element-based high-order Galerkin methods: quasi equal-area physics grid, *Mon. Wea. Rev.*

Jablonowski, C., and D. L. Williamson (2011), The pros and cons of diffusion, filters and fixers in atmospheric general circulation models., in: P.H. Lauritzen, R.D. Nair, C.
Jablonowski, M. Taylor (Eds.), Numerical techniques for global atmospheric models, Lecture Notes in Computational Science and Engineering, Springer, 2010, in press., 80.

- Kasahara, A. (1974), Various vertical coordinate systems used for numerical weather prediction, *Mon. Wea. Rev.*, 102(7), 509–522.
- Lauritzen, P. H., J. T. Bacmeister, T. Dubos, S. Lebonnois, and M. A. Taylor (2014), Held-Suarez simulations with the Community Atmosphere Model Spectral Element (CAM-SE) dynamical core: A global axial angular momentum analysis using Eulerian and floating Lagrangian vertical coordinates, *J. Adv. Model. Earth Syst.*, 6, doi: 10.1002/2013MS000268.
- Lauritzen, P. H., R. Nair, A. Herrington, P. Callaghan, S. Goldhaber, J. Dennis, J. T. Bacmeister, B. Eaton, C. Zarzycki, M. A. Taylor, A. Gettelman, R. Neale, B. Dobbins, K. Reed, and T. Dubos (2018), Ncar release of cam-se in cesm2.0: A reformulation of the spectral-element dynamical core in dry-mass vertical coordinates with comprehensive treatment of condensates and energy, *J. Adv. Model. Earth Syst.*, doi: 10.1029/2017MS001257.
- Lin, S.-J. (2004), A 'vertically Lagrangian' finite-volume dynamical core for global models, *Mon. Wea. Rev.*, *132*, 2293–2307.
- Medeiros, B., D. L. Williamson, and J. G. Olson (2016), Reference aquaplanet climate in the community atmosphere model, version 5, *J. Adv. Model. Earth Syst.*, 8(1), 406–424, doi:10.1002/2015MS000593.
- Neale, R. B., and B. J. Hoskins (2000), A standard test for agcms including their physical parametrizations: I: the proposal, *Atmos. Sci. Lett*, 1(2), 101–107, doi:10.1006/asle.2000. 0022.
- Neale, R. B., C.-C. Chen, A. Gettelman, P. H. Lauritzen, S. Park, D. L. Williamson, A. J. Conley, R. Garcia, D. Kinnison, J.-F. Lamarque, D. Marsh, M. Mills, A. K. Smith, S. Tilmes, F. Vitt, P. Cameron-Smith, W. D. Collins, M. J. Iacono, R. C. Easter, S. J. Ghan, X. Liu, P. J. Rasch, and M. A. Taylor (2012), Description of the NCAR Community Atmosphere Model (CAM 5.0), NCAR Technical Note NCAR/TN-486+STR, National Center of Atmospheric Research.
- Skamarock, W. C., J. B. Klemp, M. G. Duda, L. Fowler, S.-H. Park, and T. D. Ringler (2012), A multi-scale nonhydrostatic atmospheric model using centroidal Voronoi tesselations and C-grid staggering, *Mon. Wea. Rev.*, 240, 3090–3105, doi:doi:10.1175/MWR-D-11-00215.1.
- Taylor, M. A. (2011), Conservation of mass and energy for the moist atmospheric primitive equations on unstructured grids, in: P.H. Lauritzen, R.D. Nair, C. Jablonowski, M. Taylor (Eds.), Numerical techniques for global atmospheric models, *Lecture Notes in Computational Science and Engineering, Springer, 2010, in press.*, 80, 357–380, doi: 10.1007/978-3-642-11640-7_12.
- Thuburn, J. (2008), Some conservation issues for the dynamical cores of NWP and climate models, *J. Comput. Phys.*, 227, 3715 3730.
- Williamson, D. L. (2002), Time-split versus process-split coupling of parameterizations and dynamical core, *Mon. Wea. Rev.*, *130*, 2024–2041.
- Williamson, D. L., J. G. Olson, C. Hannay, T. Toniazzo, M. Taylor, and V. Yudin (2015), Energy considerations in the community atmosphere model (cam), *J. Adv. Model. Earth Syst.*, 7(3), 1178–1188, doi:10.1002/2015MS000448.