

Let hailstone  $n$  have position  $p_n = \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix}$  and velocity  $v_n = \begin{pmatrix} vx_n \\ vy_n \\ vz_n \end{pmatrix}$   
 and the new stone have position  $p = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$  and velocity  $v = \begin{pmatrix} vx \\ vy \\ vz \end{pmatrix}$

Assume by construction that  $\forall n, \exists t_n \in \mathbb{N}$  such that

$$p + t_n v = p_n + t_n v_n$$

$$\Rightarrow p - p_n = -t_n (v - v_n)$$

$$\Rightarrow (p - p_n) \times (v - v_n) = 0 \quad \forall n$$

Consider  $n=1$  and  $n=2$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$$

$$(y - y_1)(vz - vz_1) - (z - z_1)(vy - vy_1) = (y - y_2)(vz - vz_2) - (z - z_2)(vy - vy_2)$$

$$\cancel{y}vz - yvz_1 - \cancel{y_1}vz + y_1vz_1 - \cancel{z}vy + zvy_1 + \cancel{z_1}vy - z_1vy_1 = \cancel{y}vz - yvz_2 - \cancel{y_2}vz + y_2vz_2 - \cancel{z}vy + zvy_2 + \cancel{z_2}vy - z_2vy_2$$

$$(vz_2 - vz_1)y + (vy_1 - vy_2)z + (z_1 - z_2)vy + (y_2 - y_1)vz = y_2vz_2 - z_2vy_2 + z_1vy_1 - y_1vz_1$$

$$(z - z_1)(vx - vx_1) - (x - x_1)(vz - vz_1) = (z - z_2)(vx - vx_2) - (x - x_2)(vz - vz_2)$$

$$\cancel{z}vx - zvz_1 - \cancel{z_1}vx + z_1vz_1 - \cancel{x}vz + xvz_1 + \cancel{x_1}vz - x_1vz_1 = \cancel{z}vx - zvz_2 - \cancel{z_2}vx + z_2vz_2 - \cancel{x}vz + xvz_2 + \cancel{x_2}vz - x_2vz_2$$

$$(vz_1 - vz_2)x + (vx_2 - vx_1)z + (z_2 - z_1)vx + (x_1 - x_2)vz = z_2vx_2 - x_2vz_2 + x_1vz_1 - z_1vx_1$$

$$(x - x_1)(vy - vy_1) - (y - y_1)(vx - vx_1) = (x - x_2)(vy - vy_2) - (y - y_2)(vx - vx_2)$$

$$\cancel{x}vy - xvz_1 - \cancel{x_1}vy + x_1vz_1 - \cancel{y}vx + yvx_1 + \cancel{y_1}vx - y_1vx_1 = \cancel{x}vy - xvz_2 - \cancel{x_2}vy + x_2vz_2 - \cancel{y}vx + yvx_2 + \cancel{y_2}vx - y_2vx_2$$

$$(vy_2 - vy_1)x + (vx_1 - vx_2)y + (y_1 - y_2)vx + (x_2 - x_1)vy = x_2vy_2 - y_2vx_2 + y_1vx_1 - x_1vy_1$$