TEN RULES FOR COMPUTING CONVOLUTIONS

1. $h(t) * [ax(t) + by(t)) = a[h(t) * x(t)] + b[h(t) * y(t)]$
   Break up convolutions using linear combinations.
2. If $h(t) * x(t) = y(t)$ then $h(t-a) * x(t+b) = y(t-a+b)$
   This is *very* useful if the given functions have delays.
3. $x(t) * \delta(t-a) = x(t-a)$ and $x(t) * u(t) = \int_{-\infty}^{t} x(\tau)d\tau$
   $\frac{dh}{dt} * x(t) = h(t) * \frac{dx}{dt} = \frac{dy}{dt}$; similarly for integrals.
   #1-#3 greatly simplifies many convolutions in EECS 216.
4. If both $h(t)$ and $x(t)$ are causal, then
   $y(t) = h(t) * x(t) = [\int_0^t h(\tau)x(t-\tau)d\tau]u(t)$ is also causal.
5. If $x(t)$ is a more complicated function than $h(t)$, use
   $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$ so you don't substitute $t-\tau$.
6. If $h(t)$=0 outside $0 < t < L$ and $x(t)$=0 outside $0 < t < M$,
   then $y(t) = h(t) * x(t) = 0$ outside $0 < t < (L+M)$. And use #2!
7. Even*even=even; Even*odd=odd; Odd*odd=even functions.
   Use #2 to shift symmetric and antisymmetric functions.
8. $\delta(t)*\delta(t) = \delta(t)$ and u(t)*u(t)=r(t)=tu(t) and rect(t)*rect(t)=triangle(t).
9. If $y(t) = h(t) * x(t)$ then $\int y(t)dt = [\int h(t)dt][\int x(t)dt]$. Good check.
10. Discretize $h(t)$ and $x(t)$ and use Matlab's `conv` to check form.
    You still need to set the scale factor properly (use #9).

EX #1: Compute $y(t) = e^{-t}u(t) * [u(t) - u(t-a)]$.

Sol'n: Using #1-#5, $y(t) = e^{-t}u(t) * u(t) - e^{-t}u(t) * u(t-a)$

1^{st} term$=\int_0^t e^{-\tau}d\tau\, u(t) = [1-e^{-t}]u(t)$. 2^{nd} term$=[1-e^{-(t-a)}]u(t-a)$.

$y(t) = [1-e^{-t}]u(t) - [1-e^{-(t-a)}]u(t-a)$=RC circuit pulse response.

EX #2: Compute $y(t) = e^{-t}u(t) * \frac{t}{T}[u(t) - u(t-T)]$.

Hint: Note $\frac{t}{T}[u(t) - u(t-T)] = \frac{1}{T}\int_0^t [u(t) - u(t-T) - T\delta(t-T)]dt$.

Sol'n: Abusing notation for clarity and using #3 and Ex #1 above, we have

$$y(t) = \int_0^t [e^{-t}u(t) * \tfrac{1}{T}[u(t) - u(t-T) - T\delta(t-T)]]dt$$

$$y(t) = \tfrac{1}{T}\int_0^t [1-e^{-t}]dt\, u(t) - \tfrac{1}{T}\int_T^t [1-e^{-(t-T)}]dt\, u(t-T) - \int_T^t e^{-(t-T)}dt\, u(t-T)$$

$$y(t) = \tfrac{1}{T}[t+e^{-t}-1]u(t) - \tfrac{1}{T}[(t-T)+e^{-(t-T)}-1]u(t-T) + [e^{-(t-T)}-1]u(t-T)$$

Compare to Soliman and Srinath p. 60–yes, they DO agree (try it!).