Modeling Municipal Solid Waste Collection Systems Using Derived Probability Distributions. I: Model Development

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Abstract: This paper presents a derived probability model that can be used to estimate vehicle and labor requirements for municipal solid waste collection systems. The model consists of closed-form solutions for important performance parameters such as the mean and variance of the total collection time for a route of given size. The results of the model agree well with the results of Monte Carlo–simulation models of curbside collection, but the derived probability model has several advantages over simulation techniques. The analytical nature of the model allows it to be easily and directly implemented on a spreadsheet. The proposed model will allow solid waste managers to examine a wide range of collection alternatives without the time and expense of simulation modeling.

INTRODUCTION

The collection of residential solid waste has become increasingly complex over the past decade. In addition to weekly refuse collection, which they have provided for many years, a growing number of municipalities now collect recyclable materials at the curbside. For example, there were an estimated 9,349 curbside recycling collection programs in the United States in 1998, up from 1,042 in 1988 (Glenn 1999). Although these programs are very important in terms of waste diversion, they have undoubtedly added to total waste collection costs. The cost of managing residential solid waste in the United States in 1990 has been estimated to be on the order of U.S. $19.5 billion annually, with collection accounting for approximately 50% of this total (Tchobanoglous et al. 1993). Because this estimate predates most of the expansion in collection systems noted above, the current cost of collecting residential solid waste in the United States likely exceeds U.S. $10 billion annually.

As collection costs increase, new waste collection programs may strain public works budgets. Some municipalities are experimenting with alternative collection systems such as ''wet/dry'' collection systems and ''co-collection'' of waste and recyclables to reduce costs. However, municipal solid waste managers often find that they have very little objective information on the expected cost of these alternatives. They also often find that the techniques available to evaluate and compare different collection scenarios are very limited and insufficient for their needs.

The purpose of this paper is to present an analytical approach to modeling municipal waste collection systems that could be applied to refuse collection, curbside recycling, yard waste collection, or co-collection. The specific objectives of this paper are

- To review the literature on residential waste collection efficiency and modeling
- To present the development of a derived probability model (DPM) of residential waste collection that generates estimates of vehicle and labor requirements for a given collection route
- To compare the results of the DPM to existing approaches

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BACKGROUND

The primary purpose of modeling municipal solid waste collection systems is to estimate the number of vehicles and the amount of labor required to provide collection service to a specific area. There have, to date, been two primary methods of modeling solid waste collection systems: deterministic models based on average collection rates and simulation models (Wilson 2001).

Tchobanoglous et al. (1993) described a deterministic model that is widely used for determining vehicle and labor requirements for refuse collection programs. The model requires estimates of the portion of the working day available for collection activity and of the average service time per household. Route time is determined by multiplying the number of households on the route by the average service time per household and then adding all unproductive time such as that spent traveling to and from a disposal site. Vehicle requirements are determined by dividing the total time on the route by the time in a working day and rounding up to the next integer value.

The deterministic model works well under many circumstances, especially when average productivity data from an existing neighborhood can be used to estimate the cost of expanding a collection service. However, problems with the method appear when it is used to model service changes. For example, average productivity data from a garbage collection program generally cannot be used to design a curbside recycling program. The two programs would have different average service times because they involve different activities at the curb and the number of residents setting out material for collection may differ significantly between the two programs.

The other approach to the problem of modeling solid waste collection systems has been to employ simulation techniques. Quon et al. (1965) were the first to apply simulation techniques to the solid waste collection problem. Truitt et al. (1969) proposed a simulation model that examined the impact of transfer stations on collection costs. These models have fallen out of use, in part, because simpler deterministic procedures provided acceptable results for most refuse collection systems in operation prior to the 1990s.

Simulation of curbside collection stayed out of favor until it was revived by Everett and Shahi (1996a,b, 1997) to examine the efficiency of curbside yard waste collection. A similar model was developed for curbside collection of recyclable materials (Everett and Riley 1997; Everett et al. 1998a,b). The Everett simulation models explicitly considered the effect of set-out rate on collection efficiency and estimated route time by separating total route time into travel time, collection time,

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and delays such as waiting time at traffic signals. Travel time was estimated based on an empirical relationship between observed average truck velocities and distance between stops. Collection time was estimated from a linear relationship between time and the number of containers set out for collection. To determine the variability in the response, simulations were done for various set-out rates.

Simulation models are an improvement on deterministic procedures because they address the variability in the collection problem caused by variable set-out rates. However, there are problems inherent in the use of simulation. The major problem is that the models cannot be used to determine optimal conditions directly and they require many runs with one-at-a-time parameter variation. As a result, simulation of complicated systems, such as co-collection systems, would be very complex. The Everett models are also influenced by the use of an empirical travel time equation. For example, both Everett and Shahi (1996a) and Everett and Riley (1997) noted the probable importance of vehicle acceleration on travel time, but this parameter does not appear directly in either model.

This previous work has resulted in a general understanding that a number of variables are important in determining the efficiency of waste collection operations. These variables include the distance between collection stops, amount of waste generated at each stop, set-out rate, loading time per stop, distance or travel time to the disposal or transfer facility, any nonproductive time such as breaks or unloading time, and acceleration characteristics of individual collection vehicles. Many of these variables describe the physical characteristics of the collection process. These state variables are beyond the control of the analyst and many are stochastic in nature. They may change not only from city to city but also from route to route within any individual city. For example, the service density (i.e., the number of houses per unit distance) is generally higher in the urban core and lower in the suburban fringe of any city. This means that the travel time from stop to stop and the total number of stops that can be served in a day will be different in these two types of areas.

There are also a number of important decision variables facing a solid waste program operator. These include the type of collection service provided, frequency of the service, size of the collection crew, capacity of the collection vehicle, size of the collection route, and specific work rules such as the length of the working day and specific policies on paying overtime. These decision variables are under the direct control of the program operator and can be adjusted, although usually only over the medium to long term. Unfortunately, previous research has not provided a good understanding of the fundamental relationships among these decision variables, state variables listed above, and overall cost of the resulting collection system. The remainder of this paper outlines an analytical DPM that relates all of the state and decision variables listed above (with the exception of waste generation rates) and provides estimates of the labor and capital requirements of the resulting collection system.

DERIVED PROBABILITY COLLECTION MODEL

The approach taken here will be to estimate the total time required to service a route of N residences from first principles, using vehicle dynamics and probability theory. This analytical approach postulates a probability distribution for the number of households along a route that set out material on any given collection day (the set-out rate). From this basic assumption, the probability distributions of a number of important collection efficiency parameters can be derived.

In concept, this analytical model is very similar to simulation models. The goal is to identify the distance that a vehicle must travel until it reaches a residence that has set material out for collection. Once this distance is known, a travel time can be estimated and summed over all residences. This total travel time is then added to the total collection time for all residences that require service, to determine a total route time. Knowing the total route time, the equations of Tchobanoglous et al. (1993) can then be used to determine overall labor and vehicle requirements and the resulting system costs.

The development of a derived probability collection model is presented below. The model assumes that only one side of the street is collected at a single pass and that, at most, only one set-out is collected at each stop. This single-side, single-stop (or S4) stop rule is the simplest to model (Everett et al. 1998b). It also reflects many municipal collection operations, especially those involving a right-hand-drive vehicle operated by a single driver/collector. Several more complex stop rules are possible (Everett et al. 1998b) and could be modeled in a similar fashion. The model presented here also assumes that stops are equally spaced and that the capacity of the collection vehicle is essentially unlimited. These restrictions on the model will be addressed in future research.

Set-Out Distribution

Everett and Shahi (1996a) used simulation modeling to show that travel time is a function of set-out rate and that the frequency and distribution of set-outs along a route have a direct effect on total travel time. Set-out rates vary dramatically among collection programs. Reported set-out rates for recycling programs range from a low of 16% of households to >70% (Wilson 2001). Set-out rates for refuse collection are typically close to 100%. Such major variations in set-out rates are likely to have a dramatic impact on the efficiency of a collection program.

To investigate this effect analytically, consider a portion of a route, as shown in Fig. 1. Assume that, at each residence on a route, there is a probability of θ that material will be set out at the curb for collection and a probability of $(1 - \theta)$ that no material will be set out at that residence. Furthermore, assume that this probability is the same for all houses. Then, the process for determining the total number of stops setting out material X on a route with N potential stops is equivalent to conducting N independent Bernoullian trials of an event with a probability of θ in each trial. The variable X thus has a binomial distribution and the probability of seeing $X = j$ set-outs on a route with N houses is

$$P(X = j) = \binom{N}{j} \theta^{j}(1 - \theta)^{N-j} \tag{1}$$

An observer counting the number of houses setting material out for collection will calculate a set-out rate of $(X/N) \times 100\%$ for that route for that particular collection period. From (1), the expected value of X is $N\theta$ (Blake 1979). Therefore, the expected value of the set-out rate for any route is $E[X/N] = (N\theta)/N$. That is, the long-term average set-out rate for the route is θ. The variance of X is $N\theta(1 - \theta)$, so the variance in the observed set-out rate (X/N) for a given route will be

$$\mathrm{var}\left[\frac{X}{N}\right] = \frac{1}{N^2}\,\mathrm{var}[X] = \frac{\theta(1 - \theta)}{N} \tag{2}$$

Stop-to-Stop Travel Distance

The distance that a truck must travel between stops will vary depending on the set-out distribution along the route. Consider a collection vehicle stopped in front of a residence, as shown in Fig. 1. The problem is to determine the number of houses that the vehicle will pass before it reaches the next house that has set material out for collection. The probability that the very next house on the route has set material out for collection is

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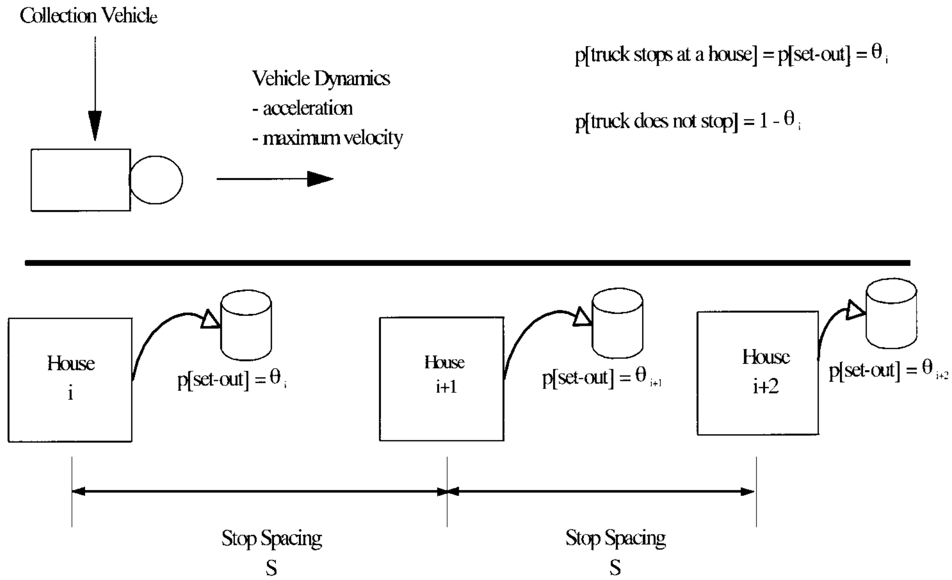

FIG. 1. Concept Diagram for DPM

θ, whereas the probability that the next house has not set out material is $(1 - \theta)$. Therefore, the probability that the truck must stop at the first house is θ. Similarly, the probability that the truck must stop at the second house (given that the first house did not set out material for collection) is $\theta(1 - \theta)$. In general, if Y is the number of houses until the next stop with material to be collected, the probability that the truck will stop at the kth house is given by

$$P(Y = k) = \theta(1 - \theta)^{k-1}, \quad k \geq 1 \tag{3}$$

which is a geometric distribution with mean $1/\theta$ and variance $(1 - \theta)/\theta^2$ (Blake 1979). Because the distance between adjacent houses is S, the distance between stops D is simply

$$D = S \times Y \tag{4}$$

Eqs. (3) and (4) can be used to determine the probability density function of D. The distribution of D is a linear function of the probability of stopping at the next house. Therefore, the distribution of D is also geometric and is given by

$$P(D = Sk) = P(Y = k) = \theta(1 - \theta)^{k-1} \tag{5}$$

The expected value of D, the mean distance that a truck must travel between stops, is S/θ; and the variance of the distance between stops is $S^2(1 - \theta)/\theta^2$.

Stop-to-Stop Travel Time Equation

Knowing the probability distribution of the distance between stops, it is now possible to determine the probability distribution of travel times between stops by assuming a relationship between travel distance and travel time. Tchobanoglous et al. (1993) suggested a linear empirical relationship between travel time and travel distance, but this relationship is only valid for longer travel distances such as travel to and from a transfer station or disposal site. Everett and Shahi (1996a) proposed the following nonlinear, empirical relationship, which is more representative of the short stop-to-stop distances encountered in curbside collection:

$$T = \frac{D}{V_{\max}[1 - \exp(-0.003D)]} \tag{6}$$

where T = stop-to-stop travel time; and $V_{\max}$ = maximum velocity (m/s).

The methodology developed here begins with the observation that the velocity profile of a waste collection vehicle is similar to that of a bus or other transit vehicle. The vehicle accelerates from a stop (perhaps reaching a maximum velocity), decelerates to come to a standstill at the next stop, and then spends some time providing service at a stop. The movement of transit vehicles has been well studied [e.g., Vuchic (1981) and Victor and Santhakumar (1986)]. The major difference between waste collection vehicles and transit vehicles is that waste collection stops are very closely spaced.

If vehicle acceleration and deceleration rates are constant and equal, two distinct velocity profiles are possible (Fig. 2). In the first case [Fig. 2(a)], the vehicle accelerates for half the distance to the next stop and then immediately decelerates until it comes to rest in front of that stop. In the second case [Fig. 2(b)], the vehicle accelerates to a maximum velocity $V_{\max}$, travels at constant speed for a period of time, and then decelerates to a stop.

In general, the acceleration rate will not equal the deceleration rate and these rates and the maximum velocity will not be constant. However, results from simulation models of transit systems suggest that reasonable results can be obtained by assuming the velocity profiles presented in Fig. 2. For example, Victor and Santhakumar (1986) reported that a 15% increase in acceleration and deceleration rates resulted in a reduction in total route time of <2%. Similarly, a 15% increase in maximum velocity resulted in a reduction in total route time of 3.8%.

Proceeding on the assumption of the velocity profiles presented in Fig. 2, the time T required to travel a distance D is given by

$$T = 2\sqrt{\frac{D}{a}}, \quad D \leq \frac{V_{\max}^2}{a} \tag{7a}$$

$$T = \frac{V_{\max}}{a} + \frac{D}{V_{\max}}, \quad D > \frac{V_{\max}^2}{a} \tag{7b}$$

where a = average acceleration (and deceleration) rate. The restriction on D is necessary to determine whether or not the vehicle has traveled far enough to reach a maximum velocity.

Eqs. (6) and (7) provide significantly different estimates of the stop-to-stop travel time for closely spaced stops, although both methods provide similar estimates of the travel time for large stop spacings. Eq. (7) will be used in this paper because it has been used successfully to model the movement of transit

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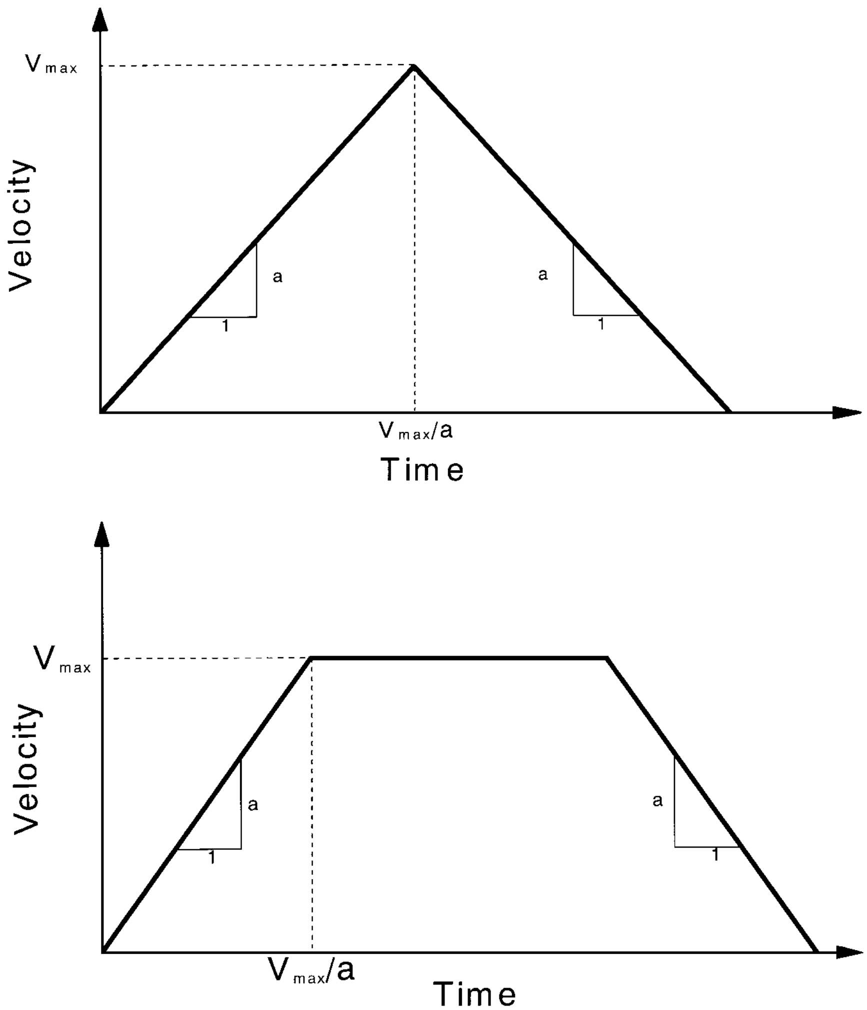

FIG. 2. Assumed Velocity Profiles for Collection Vehicles ($S < V_{max}^2/a$)

vehicles that have similar operating characteristics to waste collection vehicles (Victor and Santhakumar 1986). Eq. (7) also has the advantage, unlike (6), of predicting zero travel time for a stop-to-stop travel distance of zero.

Consider first the case where stops are spaced far enough apart to ensure that the collection vehicle always reaches a maximum velocity. From (7), T is then linear in D, and from (4), D is linear in Y. As a result, T is a linear function of the geometrically distributed variable Y. It is therefore possible to show that the stop-to-stop travel time T has the mean

$$E[T] = E\left[\frac{V_{max}}{a} + \frac{SY}{V_{max}}\right] = \frac{V_{max}}{a} + \frac{S}{V_{max}}E[Y] = \frac{V_{max}}{a} + \frac{S}{\theta V_{max}},$$
$$S > \frac{V_{max}^2}{a} \tag{8}$$

and, assuming V_{max}, S, and a to be constant, a variance given by

$$\mathrm{var}[T] = \mathrm{var}\left[\frac{V_{max}}{a} + \frac{SY}{V_{max}}\right] = \left(\frac{S}{V_{max}}\right)^2 \mathrm{var}[Y]$$
$$= \left(\frac{S}{V_{max}}\right)^2 \frac{(1-\theta)}{\theta^2}, \quad S > \frac{V_{max}^2}{a} \tag{9}$$

It is important to note that (8) and (9) are only valid for houses that are spaced relatively far apart. For example, if V_{max} is only 5.6 m/s (20 km/h) and a is 1 m/s^2, S must be at least 30.9 m for (8) and (9) to apply. Generally this will only be the case on some suburban or rural collection routes.

In cases where $S < V_{max}^2/a$, the relationship between D and T is not always linear and the derivation of $E[T]$ and var[T] are not as straightforward. However, it is still possible to derive analytical expressions for $E[T]$ and var[T] for stop spacings over specific ranges. For example, if $S < V_{max}^2/a < 2S$, there is a probability θ that waste will be set out at the next house and the vehicle will only travel a distance S before stopping. Therefore, there is a probability of θ that the truck will operate according to the velocity profile in Fig. 2(a). If that house does not set out waste, the distance to the next stop will be at least $2S$ and the vehicle will operate according to the velocity profile in Fig. 2(b). This occurs with a probability of $(1 - \theta)$. Thus, for this stop spacing, it is possible to show that the expected travel time is given by

$$E[T] = 2\theta\sqrt{\frac{S}{a}} + (1-\theta)\left[\frac{V_{max}}{a}\right] + \left[\frac{S(1-\theta^2)}{V_{max}\theta}\right],$$
$$S < \frac{V_{max}^2}{a} < 2S \tag{10}$$

Similar expressions have been derived for smaller stop spacings. Table 1 presents analytical expressions for $E[T]$ for stop spacings of practical interest. Derivations of these expressions are not presented here but are available from the writers upon request. Analytical expressions for var[T] for various stop spacings are also possible, but (9) provides a reasonable approximation in most circumstances.

The need for a number of travel time equations that depend on the stop spacing is a drawback to this approach. However,

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TABLE 1. Expected Value of Stop-to-Stop Travel Time for Various Stop Spacings

Stop spacing	Mean stop-to-stop travel time
$S > V_{max}^2/a$	$E[T] = (V_{max}/a) + (1/\theta)(S/V_{max})$
$S < V_{max}^2/a < 2S$	$E(T) = 2\theta\sqrt{S/a} + (1-\theta)[V_{max}/a] + \{[S(1-\theta^2)]/(V_{max}\theta)\}$
$2S < V_{max}^2/a < 3S$	$E(T) = 2\theta\sqrt{S/a} + 2\theta(1-\theta)\sqrt{2(S/a)} + (1-\theta)^2[V_{max}/a] + \{[S(1-\theta)(1-2\theta)]/(V_{max}\theta)\}$
$S \ll V_{max}^2/a$	$E[T] \approx 2\sqrt{S/a}\{[7\theta^{-(1/2)} + \theta^{1/2}]/8\}$

in practical terms, it is not problematic for two reasons. First, drivers seldom operate their vehicles according to the velocity profile shown in Fig. 2(a) because the constant acceleration/deceleration pattern is very uncomfortable over a full working day. Faced with closely spaced stops, most drivers will simply operate their vehicle according to the velocity profile shown in Fig. 2(b) by reducing their maximum speed. Second, for most reasonable values of θ, the chances of traveling past more than a few houses until coming to a stop is quite small. For example, if $\theta = 0.6$, the probability of traveling past more than three houses before stopping can be determined from (3) to be only 6.4%. Therefore, the equations presented in Table 1 are sufficient for most practical purposes.

Total Travel Time

Over any route of N residences with a set-out rate of θ, the total travel time TT can be determined by multiplying the expected stop-to-stop travel time [given by (8), (10), or the expressions in Table 1] by the expected number of stops $N\theta$. Because stop-to-stop travel time depends on stop spacing, so does the total travel time. If $S > V_{max}^2/a$, the mean stop-to-stop travel time is given by (8) and the mean total travel time is

$$E[TT] = N\theta E[T] = \frac{N\theta V_{max}}{a} + \frac{SN}{V_{max}}, \quad S > \frac{V_{max}^2}{a} \tag{11}$$

Expressions for $E[TT]$ for stop spacings of $<V_{max}^2/a$ can be obtained by multiplying the appropriate expression for $E[T]$ given in Table 1 by $N\theta$.

Examining (11) reveals that total travel time consists of two components: SN/V_{max}, which is simply the time required to travel past N residences spaced a distance S apart at a velocity of V_{max}, and a penalty of V_{max}/a for each stop, which represents the time lost bringing the vehicle to a stop ($V_{max}/2a$) and time lost accelerating back up to top speed ($V_{max}/2a$). An alternative interpretation of (11) is that TT is a linear function of the binomial variate X, the number of stops made on a route on any given day. Using this interpretation, it is possible to determine that the variance in total travel time, assuming S, N, and V_{max} to be constant, is

$$\mathrm{var}[TT] = \mathrm{var}\left[\frac{XV_{max}}{a} + \frac{SN}{V_{max}}\right] = \frac{V_{max}^2}{a^2}\mathrm{var}[X] = \frac{N\theta(1-\theta)V_{max}^2}{a^2}, \quad S > \frac{V_{max}^2}{a} \tag{12}$$

Eq. (12) suggests that the variance in total travel time is primarily due to the variance in the number of stops made on any given day. Again, analytical expressions for var[TT] for various stop spacings are possible, but (12) provides a reasonable approximation.

It is very useful to know the mean and the variance of the total travel time, but it is also useful to know the underlying distribution of TT. Wilson (2001) showed that, for any reasonably sized collection route, total travel time will be approximately normally distributed with a mean given by (11) and a variance given by (12). Knowing the time required to travel a route, it is now necessary to examine the time required to load materials set out along the route into the collection vehicle.

Loading Time Equation

The amount of time spent loading materials placed at the curbside into the collection vehicle can only be derived from time and motion studies in the field. Pfeffer (1992) provided a discussion of such time and motion studies. Everett and Shahi (1996a) and Everett and Riley (1997) provided estimates of loading times for curbside collection of yard waste and recyclables, respectively. Everett et al. (1998b) estimated loading time using the distance walked by collectors and the mass of material collected at each set-out.

From the perspective of estimating total loading time for a given route, it is sufficient to note that a collection crew will serve a large number of residences on any given route. Therefore, the total loading time will be the sum of a large number of random variables, and, according to the central-limit theorem, this sum will be approximately normally distributed for a large number of stops. In particular, if the collection vehicle makes X stops, then the total loading time will be the sum of X individual loading times or

$$LT = \sum_{i=1}^{X} lt_i \tag{13}$$

where LT = total loading time; and lt_i = loading time for stop i. Because the expected value of X is $N\theta$, the expected total loading time is

$$E[LT] = N\theta\mu_{LT} \tag{14}$$

and the variance in loading time is

$$\mathrm{var}[LT] = N\theta(1-\theta)\mu_{LT}^2 + N\theta\sigma_{LT}^2 \tag{15}$$

where μ_{LT} = mean loading time per residence; and σ_{LT}^2 = variance of the loading time per residence. Note that the variance in total loading time is due not only to the variance in individual loading times but also to the variance in the number of stops made. This is because of a positive correlation between the number of stops made and the total loading time.

Stop Signs, Stoplights, and Other Delays

Collection vehicles encounter various delays along a route that are not related to travel or loading time, such as delays at stop signs and traffic lights. The sum of these delays can be expressed

$$DT = \sum_{i=1}^{N_{delay}} d_i \tag{16}$$

where DT = total delay; N_{delay} = number of delays on a route; and d_i = duration of the ith delay. These delays can be included in a DPM by again depending on the central-limit theorem. Given even a moderate number of such delays, the sum of the delay time will be approximately normally distributed with mean $N_{delay}\mu_{delay}$ and variance $N_{delay}^2\sigma_{delay}^2$, where μ_{delay} is the mean time per delay and σ_{delay}^2 is the variance in the delay.

Estimating Route Time

It is now possible to estimate total route time by adding total travel time, total loading time, and total delays. For example, consider the case where $S > V_{max}^2/a$. The total route time is then

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$$RT = \frac{SN}{V_{\max}} + \frac{XV_{\max}}{a} + \sum_{i=1}^{X} lt_i + \sum_{i=1}^{N_{\text{delay}}} d_i, \quad S > \frac{V_{\max}^2}{a} \quad (17)$$

Eq. (17) shows that the time required to complete a route is the sum of the time required to drive past all houses at a constant velocity plus a time penalty at each stop, due to deceleration and acceleration, plus the total loading time plus the total delay. The mean total route time $E[RT]$ can be determined by taking the expected value of (17), which yields

$$E[RT] = \frac{SN}{V_{\max}} + \frac{N\theta V_{\max}}{a} + \theta N\mu_{LT} + N_{\text{delay}}\mu_{\text{delay}}, \quad S > \frac{V_{\max}^2}{a} \quad (18)$$

The variance in route time, var[RT] is

$$\text{var}[RT] = N\theta(1 - \theta)\left[\frac{V_{\max}^2}{a^2} + \frac{2V_{\max}\mu_{LT}}{a} + \mu_{LT}^2\right] + N\theta\sigma_{LT}^2 + N_{\text{delay}}^2\sigma_{\text{delay}}^2, \quad S > \frac{V_{\max}^2}{a} \quad (19)$$

The variance in total route time is the sum of the variances in travel time, loading time, and delays, plus a term that arises because of the covariance between the number of stops and the loading time per stop. The presence of this covariance term is logical, because a loading time is only added if a stop is made. If the loading time per stop is constant, X and lt_i will be perfectly correlated.

Having obtained analytical expressions for total route time, vehicle labor requirements can now be obtained by dividing total route time by the time available per truck per collection period. Tchobanoglous et al. (1993) provided the following expression for the time available per truck per collection period:

$$3{,}600C_p^w[H(1 - B) - (t_1 + t_2) - N_d(s + h)] \quad (20)$$

where C_p^w = number of working days per collection period; H = length of the working day; B = percentage of H spent on nonproductive activities; t_1 = time spent at the beginning of the day, prior to the first collection stop of the day; t_2 = time spent at the end of the day after collection is complete; s = time spent at the unloading site; h = round-trip haul time from the route to the unloading facility; and N_d = number of trips each vehicle completes each working day.

The number of vehicles required is obtained by dividing (17) by (20), or

$$NOV = \frac{\dfrac{SN}{V_{\max}} + \dfrac{XV_{\max}}{a} + \displaystyle\sum_{i=1}^{X} lt_i + \sum_{j=1}^{N_{\text{delay}}} d_j}{3{,}600C_p^w[H(1 - B) - (t_1 + t_2) - N_d(s + h)]}, \quad S > \frac{V_{\max}^2}{a} \quad (21)$$

where NOV = number of vehicles required. Note that, in contrast to the deterministic procedure outlined by Tchobanoglous et al. (1993), NOV is now a random variable. Of course, municipalities cannot operate a random number of vehicles and must select a specific integer valued fleet size $NOVI$. However, rather than simply selecting one fixed fleet size, (21) suggests that a municipality should consider the implications of various fleet sizes.

Having selected a fleet size, the total time per collection period required to complete all collection activities from N residences is

$$TTT = \frac{\left(\dfrac{SN}{V_{\max}} + X\left(\dfrac{V_{\max}}{a} + \mu_{LT}\right) + \displaystyle\sum_{j=1}^{N_{\text{delay}}} d_j\right)}{3{,}600(1 - B)} + NOVIC_p^w\left[\frac{t_1 + t_2 + N_d(s + h)}{(1 - B)}\right], \quad S > \frac{V_{\max}^2}{a} \quad (22)$$

Since TTT is a function of X, it is an approximately normal random variable with mean

$$E[TTT] = \frac{\left(\dfrac{SN}{V_{\max}} + N\theta\left(\dfrac{V_{\max}}{a} + \mu_{LT}\right) + N_{\text{delay}}\mu_{\text{delay}}\right)}{3{,}600(1 - B)} + NOVIC_p^w\left[\frac{t_1 + t_2 + N_d(s + h)}{(1 - B)}\right], \quad S > \frac{V_{\max}^2}{a} \quad (23)$$

and variance

$$\text{var}[TTT] = \frac{N\theta(1 - \theta)\left(\dfrac{V_{\max}}{a} + \mu_{LT}\right)^2}{[3{,}600(1 - B)]^2} + N_{\text{delay}}^2\sigma_{\text{delay}}^2, \quad S > \frac{V_{\max}^2}{a} \quad (24)$$

Eqs. (21) and (22) form the basis of a DPM for municipal curbside waste collection. They provide estimates of the number of vehicles and the total time required to collect waste from a route of N residences as functions of the state and decision variables that describe a solid waste collection operation. From these two equations, many other parameters of interest can be determined. For example, total labor requirements per collection period can be determined by multiplying (22) by the number of employees (drivers and collectors) per vehicle. Labor costs can then be determined by multiplying further by an hourly labor rate. For stop spacings of $<V_{\max}^2/a$, expressions similar to (21) and (22) can be derived from the equations presented in Table 1.

SENSITIVITY ANALYSIS

The closed-form nature of the DPM presented above allows for direct sensitivity analysis. Each of the equations can be differentiated with respect to any parameter of interest. For example, evaluation of the derivatives of (23) over the ranges for typical parameter values expected in the field suggests that the expected total collection time is most sensitive to the number of residences on the route N, set-out rate θ, and mean loading time per stop μ_{LT}. The expected total collection time is relatively insensitive to average stop spacing S, maximum velocity $V_{\max}$, acceleration rate a, and delays. This suggests that future data collection efforts should focus on measuring set-out rates and loading times for various collection alternatives. The ability to perform this type of analytical sensitivity analysis is an important feature of the DPM that is not possible with simulation models. It allows the analyst to directly estimate the impact that a change to any parameter might have on the overall response of the collection system.

MODEL VERIFICATION

The DPM was verified primarily through comparison to the results of Monte Carlo–simulation modeling and limited field testing (Wilson 2001). Wilson (2001) also examined the validity of the assumptions used in the development of the DPM. This work shows that the DPM is consistently capable of accurately predicting the results of Monte Carlo simulations of residential curbside waste collection, including simulations that have, in turn, been verified with field data. Although this does not directly validate the DPM, it does confirm that the DPM can replicate the results of the best available method for modeling municipal solid waste collection operations with considerably less time and effort. An example of the comparison of the DPM to simulation modeling is presented in the following section.

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COMPARISON WITH SIMULATION MODELING

The DPM presented above was compared to the results of the DPM with results generated by a discrete event Monte Carlo–simulation program developed by the writers. Specifically, (7)–(24) were compared to means and variances calculated by a computer simulation model of curbside collection, written in C. The program, CURBSIM, is based on the methodology presented in Everett and Shahi (1996a), with the exception of the calculation of stop-to-stop travel time. CURBSIM calculates travel time according to both (6), presented by Everett and Shahi (1996a), and (7) to allow for comparison between the two methods. A large number of CURBSIM runs were made at various set-out rates and stop spacings.

In all cases, the travel times predicted using the equations in Table 1 agreed very well with the results from the Monte Carlo simulations calculated using (7). However, there was some disagreement between these results and the simulation results calculated using (6). The magnitude of the difference varied with stop spacing and set-out rate. These differences were due only to the equations used to calculate travel times, because both simulations calculated loading times and delays identically. These differences were expected because the DPM is based on (7).

This disagreement was examined further through the consideration of a hypothetical case study, similar to a hypothetical curbside recycling example presented in Everett and Riley (1997). The study area is characterized by relatively large distances between residences (S = 30 m) and relatively large loading times at the curb (μ_{LT} = 45 s) and may represent a suburban or rural collection program with considerable sorting of recyclables at the curb. Typical input parameters were taken from several sources (Vuchic 1981; Tchobanoglous et al. 1993; Everett and Shahi 1996a; Everett and Riley 1997) and are listed in the notes of Table 2.

TABLE 2. Vehicle and Labor Requirements for Hypothetical Urban Area of 10,000 Residences (Case 2)

		Labor Requirement $E[TTT]$ (h/week)		
Set-out rate θ (%)	Number of vehicles *NOVI*	Derived probability estimate	Simulation results from Eq. (7)	Simulation results from Eq. (6)
0	1	15.8	15.5	15.5
10	1	22.1	21.9	21.2
20	1	28.5	28.2	27.8
30	1	34.8	34.5	34.7
40	2	49.3	49	50.1
50	2	55.5	55.3	57.3
60	2	61.8	61.5	64.5
70	2	68	67.7	71.7
80	2	74.1	73.9	79
90	3	88.5	88.3	94.5
100	3	94.7	94.4	101.8

Note: N = 10,000, S = 10 m, $V_{\max}$ = 4.5 m/s, a = 1.0 m/s/s, μ_{LT} = 15 s, N_{ss} = 50, μ_{ss} = 10 s, N_{tl} = 10, μ_{tl} = 30 s, C_p^w = 5 days, H = 8 h, B = 0.15, t_1 = 0.2 h, t_2 = 0.2 h, s = 0.25 h, h = 0.25 h, N_d = 2 trips.

Table 2 compares the expected labor requirements predicted by the DPM to the average labor requirements calculated by CURBSIM for each of the two travel time equations. The results are shown graphically in Fig. 3. There is very good agreement between the predicted route time and the simulated route time calculated using (7). There is also generally good agreement between the analytical model and the results simulated using (6).

This hypothetical case study demonstrates that the DPM developed in this paper can reproduce results generated using existing simulation models with relative ease compared to the effort required in simulation modeling of the same system. The example also shows that the modeling results are dependent on the travel time equation used.

CONCLUSIONS

This paper presents a DPM for municipal solid waste collection systems. The model agrees well with the results of Monte Carlo simulations; however, the model has distinct advantages over simulation models of curbside collection. First, the analytical nature of the model means that it can be easily and directly implemented on a spreadsheet. Second, the approach allows for direct consideration of the stochastic aspects of the municipal solid waste collection problem. Third, the model explicitly incorporates all of the parameters known to be of importance in curbside collection efficiency, with the exception of waste generation rates and vehicle capacity. Finally, the sensitivity of the model to all input parameters can be investigated directly.

The proposed model could be used by solid waste managers to examine a wide range of collection alternatives for a specific municipality, without the time and expense of simulation modeling or conducting pilot projects. In addition, the DPM could be used to examine the efficiency of curbside collection programs in general. For example, the model could provide general solutions to several practical problems of interest to solid waste managers, such as the economics of using overtime to avoid the purchase of an additional vehicle, optimal size of routes and vehicles, and impact of changes in collection frequency. Future research will focus on applying the model to such general problems, with a view to drawing conclusions and making recommendations that are applicable to a wide range of curbside collection programs. Plans are also under way for further validation of the model using data collected from electronic tachometers mounted on municipal waste collection vehicles.

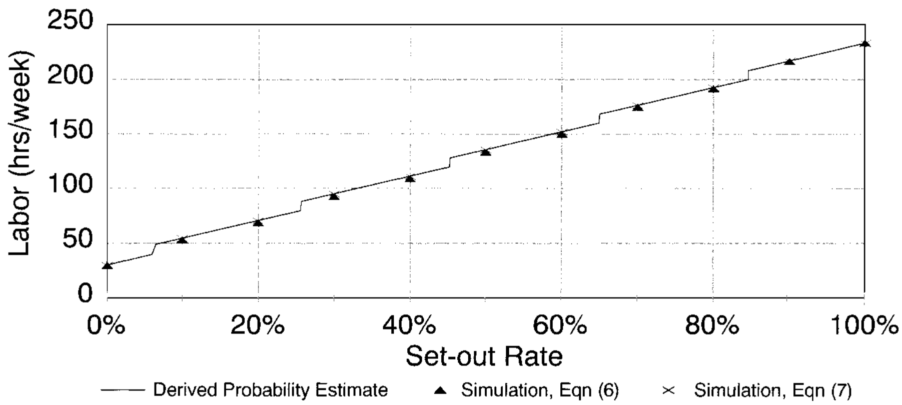

FIG. 3. Labor Requirements for Hypothetical Case Study

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NOTATION

The following symbols are used in this paper:

a = average acceleration (and deceleration) rate (m/s^2);
C_p^w = number of working days in each collection period;
D = mean distance that truck must travel between stops (m);
DT = total delay (s);
d_i = duration of ith delay (s);
$E[\ ]$ = expectation operator;
H = length of working day (h/day);
h = round-trip haul time from end of route to unloading facility (h/trip);
LT = total loading time (s);
lt_i = loading time for stop i (s);
N = number of residences;
NOV = minimum number of vehicles required;
$NOVI$ = integer number of vehicles required (i.e., smallest integer larger than NOV);
N_d = number of routes each vehicle completes each working day;
N_{delay} = number of delays on route;
RT = total route time (s);
S = distance between adjacent houses (m);
s = time spent at unloading site (h/vehicle/trip/day);
T = stop-to-stop travel time (s);
TT = total travel time on route (s);
TTT = total time required to complete all activities in collection area of N residences (h/week);
t_1 = time spent prior to first collection stop of day (h/vehicle/day);
t_2 = time spent after last collection stop of day (h/vehicle/day);
$V_{\max}$ = maximum collection vehicle velocity (m/s);
$\text{var}[\ ]$ = variance operator;
W = off-route time factor, percentage of H;
X = number of households setting out material for collection on any given day;
Y = number of houses to next stop;
z = standard normal variate;
θ = set-out rate (percentage of households setting out material for collection on given day);
θ_i = probability that household i sets out material on given collection day;
μ_{LT} = mean collection time per residence (s);
μ_{ss} = mean delay per stop sign (s);
μ_{tl} = mean delay per traffic light (s);
σ^2_{LT} = variance in collection time per residence (s^2);
σ^2_{delay} = variance in delays on route (s^2); and
$\Phi(\ )$ = cumulative distribution function of standard normal distribution.

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