Applications of Probability Model to Analyze the Effects of Electric Vehicle Chargers on Distribution Transformers

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Abstract—Society's increased concern over green house gas emission and the reduced cost of electric vehicle technologies has increased the number of electric vehicles (EV) and plug-in hybrid vehicles on the road. Previous studies into the effects of electric vehicles on the electric system have focused on transmission, generation, and the loss of life of distribution transformers. This paper focuses specifically on identifying distribution transformers that are most susceptible to excessive loading due to the implementation of electric vehicles. The authors use a binomial probability model to calculate the probability that a specific distribution transformer will experience excessive loading. Variables to the function include the existing peak transformer demand, number of customers connected to the transformer, and the most common EV charger demand. Also included in the paper is an optimization approach that utilizes the results from the binomial function to determine the optimal replacement strategy to minimize replacement costs. An extension of the approach is also utilized to explore the effectiveness of EV targeted demand side management programs. The authors apply the described algorithms to 75 000 distributions transformers within a distribution system located in Denver, Colorado.

Index Terms—Binomial distribution, demand side management, distribution planning, distribution transformers, electric vehicles.

I. INTRODUCTION

THE increased concern about reducing green house gas emissions has led to the continued increase in the number of pure electric vehicles and plug-in hybrid electric vehicles on the road. These vehicles, which the authors will refer to as electric vehicles (EVs), can increase mileage by 60% given the same amount of primary energy [1]. The number of electric vehicles will continue to grow over the next 20 years, at which time EVs may comprise as much as 50% of new vehicle purchases [2].

Studies have been performed over the last decade to determine the effects of EV chargers on the electric power system. EV effects on generation and transmission were studied in [3]

and [4]. The effects on electric distribution systems were analyzed in [1], with major emphasis placed on distribution transformers, cable systems, and switches. The increased burden due to EV chargers causing faster loss of life in distribution transformers is addressed in [5]. While the reduced loss of life of a distribution transformer due to EV chargers is a long-term effect, in the short term, EV chargers may result in a high number of blown transformer fuses due to the excessive loading that may occur during peak periods. This can result in an increased number of customer outages.

The North American Standard SAEJ1772 categorizes EV chargers into three different operating groups, commonly referred to as Level 1, Level 2, and Level 3 [6]. The power drawn from these chargers can range between 1 kW and 240 kW for periods between 30 min and 7 h, although most Level 1 and 2 chargers will not exceed a peak demand of 10 kW [6]. Studies have shown the peak charging period of EV chargers, if not controlled by the utility company, will likely occur after 5 p.m. and may coincide with existing customer load peaks, thereby significantly increasing the maximum power draw of the distribution transformer [1], [5], [7]. For transformers that have a peak demand that is near or above its rated capacity, the addition of several chargers can push the total demand above its fuse ratings. These ratings typically range between 180% and 300% depending upon transformer size, available fuse size, and protection scheme [8]. Therefore identifying transformers that are susceptible to excessive loading due to EV can have an impact on preventing future customer outages.

In the late 1990s, Xcel Energy experienced a high number of distribution transformer failures caused from unexpected loading that was due to widespread implementation of air conditioning units. Prior to this period most homes were not equipped with central air conditioning units and relied upon low energy demand swamp coolers. During this AC implementation period, the number of failed transformers due to excessive loading averaged approximately 600 (1% of all transformers) annually, which also contributed to approximately 84 000 customer outage minutes annually.

Considering that transformers can serve as many as 20 households, the probability of a transformer being overloaded or resulting in an unplanned outage due to fuse-blowing during peak hours can be significant if the utility does not incorporate demand side management programs to control charging times. Another approach utilities can employ to avoid excessive distribution transformer loading due to EV penetration is to identify the transformers that have the highest probability of overload based

on existing peak demand, number of connected customers, and the most common EV charger size.

In this paper, the authors describe a novel load forecasting approach that utilizes a binomial distribution to identify transformers that have a high probability of being overloaded with the addition of one or several EV chargers. Variables to the function include the peak transformer demand, the number of customers served by the transformer, and the most common size of an EV charger. Existing load growth forecasting approaches along with their validity for application to EV demand growth are discussed in Section II. The binomial model is described in detail in Section III. Section IV illustrates the application of the binomial function to determine the total number of susceptible distribution transformers located in Denver, Colorado. This section also illustrates the use of a binomial function to optimize replacement strategy and evaluate demand side management (DSM) programs.

II. EXISTING APPROACHES TO FORECASTING

Evaluating the impact of EVs on the existing distribution infrastructure will require planning engineers to forecast the load growth associated with EV charger demand. As always, the three most important questions that planning engineers must answer are [9]:

1) "Where" will the system be impacted by the increased load demand?
2) "When" will the impact occur?
3) "What are the costs" associated with the impact?

To answer these questions, planning engineers must rely upon load growth forecasting techniques. Such prediction techniques can consist of short-term forecasting, which extends between 1 and 6 years, and long-term forecasting, which can span over 20 years [9], [10]. Because the EV penetration growth paradigm is being developed, it is unknown if evaluating the impact of EVs requires long-term or short-term forecasting. Variables such as customer acceptance and available capacity of the existing system infrastructure will dictate whether short- or long-term forecasting is required for a specific geographical location. However, because there is so little historical data regarding a continuously increasing EV penetration, existing techniques for forecasting, long-term or short-term, may not be applicable.

Trending is the most common approach to short-term load growth forecasting [9]. Utilizing a spatial approach, which divides larger service areas into smaller and discreet areas based on customer class or type, city, subdivision, or geographical location, distribution planners match historical load growth information to growth curves that predict future growth [10]–[12]. The approach typically utilizes multiple regression curve-fitting techniques that depend on historical data just to initiate. Because so little information is known about EV penetration growth, even if these approaches are applied, the results would be subject to inaccuracies. Another problem with applying these trending approaches is that they can be significantly inaccurate at high resolution spatial forecasting, which hinders the evaluation of the impact of small distribution areas such as secondary service conductors and distribution transformers, which may only consist of several customers [9].

The most accurate means to forecast load growth is through simulations-based methods that attempt to predict the size of the increased demand along with the causes of the increased demand that include economic growth, historical data, and customer type. Because of the accuracy associated with this approach if applied correctly and the intensive effort required to apply, the simulation-based approach is typically applied to long-term planning. Unfortunately the simulation approach heavily relies upon historical data and information regarding the causes of load growth, such the methods described in [13], which in the case of the paper would be increased EV penetration. Aspects that affect EV penetration include customer acceptance, purchase costs, and government subsidies, all of which are difficult to predict for long-term planning on small spatial levels [2], [9]. Because of the lack of information regarding the attributes that drive EV penetration growth, the load demanded by future EV chargers can be considered "causal" event loads in that more information regarding the driving factors are required to make accurate long-term forecasts for specific areas.

The approach described in the paper, which would be considered a high resolution spatial forecasting analysis, is sufficient for a stand alone analysis or as an input to the simulation-based approach method, where historical growth information is unknown or nonexistent. Instead of trying to predict growth based on historical growth data, the authors have developed an approach that calculates the probability of impact on utility devices based on their existing peak demand, number of connected customers, which accounts for the possibility of growth, and a given EV penetration rate, which if applied to future EV growth predictions can determine "when" a device is impacted. Once the probability of impact is derived through the binomial distribution function described in the following section, an economic impact study is performed to the answer the question of "how much" will the load growth cost. However, before evaluations are performed to answer "when" and "how much," distribution engineers must first determine "where" will the impacts occur.

III. PROBABILISTIC MODEL

For a distribution planner to answer the question of "where" will the system be impacted, they must first determine if specific devices are susceptible to overloading due to the increased demand. Although there are many devices on the distribution system that may be impacted by EV penetration growth, the authors are focusing on evaluating the susceptibility of distribution transformers, or in layman's terms, which distribution transformers on the existing system are susceptible to overloading based on their existing available capacity and number of residential customers, which represents growth potential. Load growth is ignored in the calculations because this occurrence is typically the result of new subdivision developments and/or new business, which require the installation of new distribution transformers. The authors have chosen to identify these susceptible transformers by utilizing a binomial distribution function.

The probabilistic model considered for this paper is founded in binomial distribution theory. The model describes the probability of a distribution transformer overloading due to connec-

tion of new EV. The authors define overload as the limit in which the loading on a transformer exceeds acceptable levels, which could be well above the nameplate rating. After a brief introduction to the binomial random variable, the specific methods needed to use the distribution are explored. Finally, the assumptions taken to use this distribution are enumerated.

A. Mathematical Introduction to the Binomial Distribution

A binomial random variable describes the number of "successes" expected from a series of trials. An experiment such as "What are the chances of seeing three heads when flipping five fair coins?" is described using a binomially distributed random variable [8], [14].

The probability mass function of a binomial random variable X is as follows:

$$P[X = k] = \frac{n!}{k!(n-k)!}p^k(1-p)^{n-k} \qquad (1)$$

where $P[X = k]$ is the probability of k successes in n trials, each with a probability of p [14].

B. Creation of Transformer Overloading Model

To describe the probability of several EV being connected to a transformer, a binomial random variable as in (1) is used. The number of trials n is the total number of households connected to a given transformer. The probability of a single customer choosing to connect an EV to the transformer is p. P then describes the probability that k electric vehicles are on the transformer.

However, interest is not in the probability of k electric vehicles being on the transformer, but the probability that the transformer experiences excessive loading due to electric vehicles. To find this, the parameters of the distribution function need to be adjusted so that the number of electric vehicles that can be placed on the transformer before it overloads can be found. This number is a function of S_P, the peak demand of the transformer; S_R, the acceptable limit to which a transformer can be loaded; and S_C, the peak demand of an electric vehicle charger. Let the capacity for the number of EV that can be connected without overloading the transformer be n_{cap}:

$$n_{cap} = \left\lfloor \frac{S_R - S_P}{S_C} \right\rfloor \qquad (2)$$

where

n_{cap}	maximum number of EVs that can be connected to a transformer before acceptable loading limit is exceeded;
S_R	acceptable peak demand on transformer;
S_P	existing peak demand on transformer;
S_C	average peak demand of an EV charger.

The peak acceptable demand on a distribution transformer is an arbitrary value that represents the loading on a transformer that "may" lead to blown fuses, distribution transformers with shortened lives, and customer outages if not addressed. Although the loading of EV chargers may only result in adverse transformer loading during a short period of time, such as in the summer, which may not significantly reduce the age of

the transformer, a peak demand that blows protection fuses will result in added operational cost that a utility company will typically mitigate by upgrading the transformer to prevent future costs and prevent customer outages [9], [15]–[19]. Many utility companies utilize a predetermined loading limit with and without a duration threshold to determine if a transformer requires upgrading due to load [20]. Although these limits may not be able to identify that a specific transformer will have a shorten life due to loading above threshold limits, for a large population size they have proven to minimize the customer impact to failed transformers and outages due to repeated blown fuses.

An additional option for establishing the peak demand is based on the protection scheme of distribution transformer. Transformer fusing is generally based on two principles. The first is to protect the transformer from experiencing loading that can significantly reduce the life of the transformer. Fuses rated for this type of protection typically ranges between 150%–250% of nameplate [21]. In the last 15 years, many utility companies have incorporated a second type of protection philosophy which is more focused on protecting the entire distribution system from transformer failures instead of preventing the transformer from failing [20]. These fuses are typically sized between 180%–300% depending upon available sizes. The one thing that both of these approaches have in common is that they result in unplanned customers outages and the replacement of distribution transformers if loading limits are exceeded [7].

From a distribution system planning perspective, either one of these limits would be sufficient for application to the acceptable peak loading of transformer. Because the probability density function is utilized to predict which transformers are likely to experience adverse loading that impact customer reliability and operational costs, employing the existing company guidelines for identifying overloaded transformers is paramount. Although these limits may not necessarily reflect the actual aging of the distribution transformer, they will reflect the loading limits that typically impact customers and operational costs.

The only exception would be in cases where fuse sizes are near 300% of the nameplate rating. Although the fuses will not operate until loading reaches these high levels, liquid immersed transformers began to experience oil degradation at much lower loads, which can ultimately reduce the operating capacity of the transformer and make them more susceptible to failure due to EV charger loading at lower loading levels [21]–[23]. The reduced capacity can result in failures although loading never reached 300%. Therefore utilizing this as an upper threshold would lead to a forecast that identifies fewer transformers than actual. The level in which oil degradation occurs is a function of the oil type, loading, and environmental conditions, therefore the acceptable peak loading that reflects this oil characteristics must be location dependant.

The demand of the most commonly utilized EV chargers has yet to be determined due to the current low implementation rate. As mentioned previously, EV chargers can demand anywhere between 1 kW and 240 kW depending upon the level of charger [5]. When considering driving distance and the required home electric upgrades for Level 3 chargers, the current consensus is

that the most commonly utilized chargers will be a Level 2 and draw any between 2 kW and 7 kW.

The value for n_{cap} is a worst-case number, as it assumes no load diversity within the collection of EV chargers as well as the existing coincident load. It also assumes that all EVs will be charging at rated demand during peak loading.

Let n_{EV} be the number of electric vehicles that are connected to the transformer, i.e., $n_{EV} = 0, 1, \ldots, n_{max}$ where n_{max} is the total possible number of electric vehicles that can be connected to the transformer. For simplicity, the authors assume that only one EV is connected per customer household, which is currently the common perception of implementation. Therefore n_{max} is equivalent to the number of homes connected to a transformer. For this scenario, the function is expanded by setting n_{max} equal to the number of homes connected to the transformer multiplied by the average number of EVs per residence. As penetration of EVs grows, it may be necessary to incorporate more than one vehicle per customer.

Let F be a binomial random variable with n_{max} trials with the probability p of a single household connecting an EV, i.e., $F \sim bin(n_{max}, p)$. As such, the probability that a transformer is at its maximum capacity of EVs is the probability n_{cap} vehicles are on a transformer, described by (3):

$$P[F = n_{cap}] = \frac{n_{max}!}{n_{cap}!(n_{max} - n_{cap})!}p^{n_{cap}}(1 - p)^{n_{max} - n_{cap}}$$

(3)

where

$P[F = n_{cap}]$	probability mass function;
n_{cap}	maximum number of EVs that can connect to transformer before acceptable loading limit is exceeded;
n_{max}	maximum possible number of EVs per transformer;
p	the probability that single residential customer will charge an EV during peak loading.

However, interest is not in if a transformer is at capacity, but if it is overloaded. As such, any n_{EV} ranging from the minimum number of EVs that will cause an overload, $n_{cap} + 1$, up to n_{max} must be considered. As such, the cumulative distribution function for a binomially distributed random variable is used, which describes $P[F \le n_{cap}]$. The cumulative distribution is the sum of all $P[F = n_{EV}]$ for $n_{EV} = \{0, 1, \ldots, n_{cap}\}$. This shows the probability of a transformer being at or below full loading. The complement probability is used to find the value of interest $P[F > n_{cap}]$.

As such, the probability of a transformer overloading due to electric vehicle charging is given by (4):

$$P[F > n_{cap}] = 1$$
$$- \sum_{n_{EV}=0}^{n_{cap}} \frac{n_{max}!}{n_{EV}!(n_{max} - n_{EV})!}p^{n_{EV}}(1 - p)^{n_{max} - n_{EV}}$$

(4)

where $P[F > n_{cap}]$ is the complement of the cumulative distribution.

Once the transformers with the highest probability of failure have been determined based on p, planning engineers must esti-
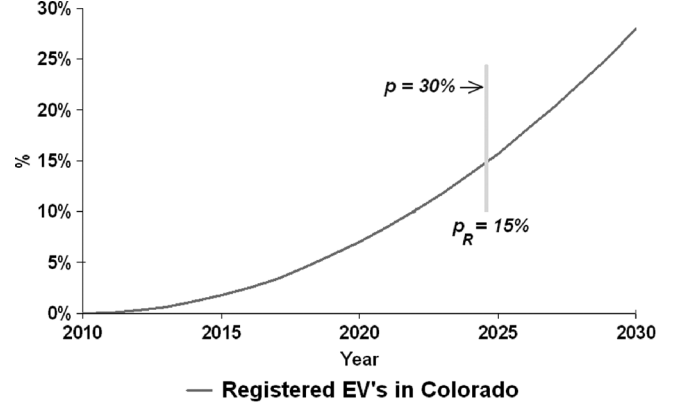

Fig. 1. Illustration of the predicted moderate growth rate of EV penetration in Colorado as a percentage of the total passenger vehicles registered.

mate "when" the results will come to fruition. The authors define p as the probability that a single customer will connect an EV to his or her transformer. Utilizing simple probability, p can also be considered the EV penetration for a given customer population size (i.e., 10%, 20%, and 30%) [14]. Currently, little is known about the relationship between utility customer and EV ownership. Therefore, it is wise to analyze the binomial functions utilizing multiple penetration levels. To determine the year in which the results of each analysis will occur, a utility can apply predictions for future registered vehicles in a given area to approximate the year. As EV penetration increases and more is known about the EV customer demographic, it may be possible to customize p for specific neighborhoods or subdivisions.

Several entities have performed studies to predict the sale and registration of EVs throughout the U.S. [6]. Several studies include aggressive, moderate, and mild penetration scenarios for the next 50 years. To determine the year in which the analysis results for a given p will occur, p is converted from EV penetration per total customers (p) to EV penetration per total vehicles registered in the given area (p_R). Fig. 1 illustrates the EV penetration rate per vehicles registered in Colorado up to 2030. For example, in Xcel Energy's Colorado operating area a penetration of $p = 30\%$ is equivalent to approximately 300 000 EVs. Dividing this value by the total registered vehicles in Xcel's Colorado territory equates to p_R equal to 15%, which is predicted to occur in 2024.

System planners can also utilize p as an indicator to extend or move forward the forecasted impact date. As more information becomes available regarding customer acceptance and overall implementation, more detailed penetration growth curves that are geographically dependant will be developed. Although the penetration rate growth forecasts may change from what is expected today, the results of the binomial distribution function will remain the same with only the expected impact date changing.

C. Assumptions Needed for Binomial Model

To use a binomial distribution, several assumptions must be considered. Mathematically, the events in the sample space must be independent and have consistent probability, meaning that the outcome of one event does not affect the uniform proba-

bility of other events occurring in the sample space. In the context of the problem at hand, this assumes that the probability of the event of one customer connecting an electric vehicle to the grid is independent of his neighbor's choice to do so. While this may not be the case because of phenomena like "keeping up with Joneses," this can be dealt with macroscopically (utilizing different p's for different subdivisions and areas as described in the previous section).

Secondly, all events must be binary, meaning they can only succeed or fail and the probability of a success is the complement of the probability of a failure. Subtly contained within this assumption is that all EV utilize identical charging technology. This is because the event being studied is "does a customer have an EV?" For a multiple-charging technology scenario, the outcomes for the event become "No," "Yes, a type A charger," and "Yes, a type B charger." These outcomes are mutually exclusive and cover the full sample space; as such this event would not meet the binary event criterion.

This assumption stands in contradiction to the literature, such as [6], which indicates several charging options are likely to come to market. However, to simplify analysis the assumption of identical charging technologies must be made. For scenarios in which multiple charging options need to be considered, a weighted average of their demand based off their penetration percentages were used in the analysis, resulting in a loss of fidelity. A more rigorous treatment would be to do a nested binomial analysis, as is done with the EV DSM. However, a weighted average is realistic enough to give a ballpark estimate of the effects of a mixed charger scenario.

Likewise, the possibility of different charging times and state-of-charge raise the possibility of the event being non-binary due to the chargers being non-coincident. Assuming the probability of this occurring could be formulated, the fact that this causes the random variable to be non-binary can worked around in an identical fashion to how demand side management is handled in Section IV-C, as the goal of DSM is to create such a diverse charging condition. Because such diversity is not the primary way in which EV charging manifests—that is to say the EV charging clumps around peak loading [1], [5], [7]—the binary RV assumption holds, with the caveats discussed above which could be addressed to further improve the model.

IV. APPLICATIONS OF THE MODEL

With a model which is capable of describing the overload probability of a transformer, several analyses can be performed to determine the monetary impacts of high EV implementation as a function of EV penetration rates. The costs associated with EV penetration include determining the expected transformer replacement rate, the effectiveness of proactive replacement program, and the effectiveness of a DSM program.

These analyses were performed on residential customers in the Denver area served by Xcel Energy. The sample included nearly 75 000 transformers and 550 000 customers across 400 feeders. The goal was determine how many transformers will be overloaded at multiple EV penetration rates and if a proactive replacement program or DSM program would minimize the overall impact. Future work will include extending this analysis from the assessment of distribution transformers' loading to the
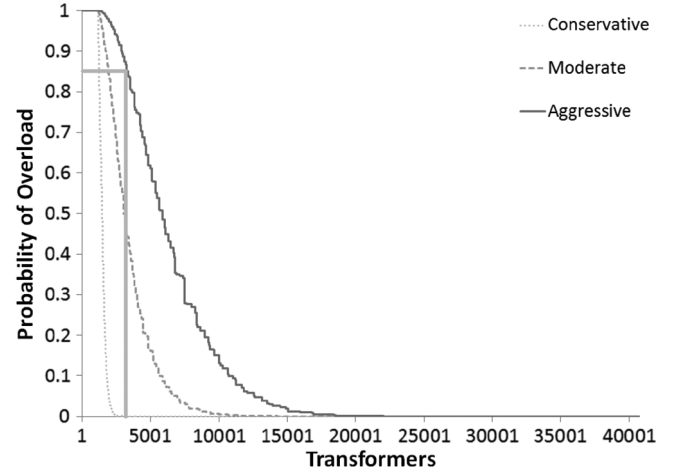
Fig. 2. Probability of overload for the nearly 45 000 25-kVA transformers in Xcel Energy service area in Denver, CO for conservative, moderate, and aggressive EV adoption scenarios. The horizontal line indicated the $P[F_{n_p}]$ threshold and the vertical line indicate the n_p versus n_r boundary for the aggressive model at a proactive replacement value (C_p) that is 15% of the reactive replacement cost (C_r).

assessment of fuse sizing, lateral loading, secondary loading, and substation bank issues.

A. Expected Number of Transformer Replacements

In order to gain an idea of the possible best and worst case scenarios, several factors were varied. One factor was the class of chargers used, which included Level 1 (1.2 kW), a low Level 2 charger (3.8 kW) [6], and a mixture of both classes using the weighted average method described previously. Also varied were the EV penetration percentages, which spanned between 10% to 30%. A total of 15 scenarios were considered for the various transformer sizes in Xcel Energy's fleet to gain an idea of the possible extreams that EV penetration effects could have. However, such a large number of scenarios proved cumbersome to work with, so three representative scenarios—conservative (Level 1 chargers at 10% penetration), moderate (25%/75% Level 1/Level 2 charger mix at 20% penetration), and aggressive (level 2 chargers at 30% penetration)—were chosen. The acceptable load limit was 180%, which coincides with protection fuse size. If a transformer is loaded beyond 180% it needs to be replaced to accommodate increased EV demand. Fig. 2 shows the conservative, moderate, and aggressive scenarios for the 25-kVA transformer class.

The expected transformer replacement rates for a scenario is simply the area under the curve. For the three generalized scenarios, they are 1485, 3497, and 6441 transformers or 3.64%, 8.58%, and 15.8% for conservative, moderate, and aggressive, respectively.

B. Optimal Proactive and Reactive Replacement

A natural use of this statistical model is to explore if a proactive transformer replacement program should be implemented. In a proactive replacement scenario, transformers with a probability of excessive loading over a certain threshold would be replaced proactively, hoping to gain some sort of salvage value and reduce the cost due to overloads and outages. The question

then becomes what is this probability of overload threshold at which to proactively replace equipment?

Consider a population of five transformers which have an overload probability, as calculated by (4), to be 90%, 70%, 50%, 30%, and 10%. Assume the (potentially sub-optimal) replacement strategy is to replace transformers with at least a 70% chance of overload; two transformers (the 90% and 70% ones) would be replaced proactively and we would expect to replace 0.9 transformers reactively (the expected number of failures for the three remaining transformers, i.e., $0.5 + 0.3 + 0.1$). In this way, the number of transformers to replace proactively and reactively can be found. Several of the results for various thresholds need to be calculated in order to create a foundation on which to find the optimal replacement strategy.

To find the optimal replacement strategy, an optimization must be formulated to minimize the costs of proactive and reactive replacements considering the number of proactive replacements with special deference for the expected value of reactive replacements. Mathematically, this is formed as (5):

$$\min \quad C_t = n_p(C_r - C_p) + n_r C_r$$

$$s.t.$$

$$n_r = \sum_{i=n_p+1}^{n_t} P[F_i] \qquad (5)$$

where

C_t total costs of replacement for entire system $P[F_i]]$ expected value for the ith transformer's random variable, per (4);

n_t number of transformers in population;

n_r number of reactive replacements;

n_p number of proactive replacements;

C_r reactive cost to replace a transformer;

C_p value realized from a proactive replacement (planned and reduced outage time, salvage or redeployment value, etc.).

For a given overload probability (y-axis of Fig. 2), the corresponding transformer ordinal (x-axis of Fig. 2) provides the value of n_p. To find n_r, the curve is "integrated" (i.e., summed) on the bounds n_p to n_t. In order to avoid a curve fit of Fig. 2 and the nonlinearities that would result, the authors have opted to bin in 1% increments the overload probability, find the corresponding n_p's and n_r's, and solve the optimization numerically. Costs have also been per-unitized to where $C_r = 1$ and C_p is varied between 5% and 15% of C_r. As such, any total replacement value less than 1.0 is beneficial, though possibly sub-optimal. The results are shown in Fig. 3. Table I shows the replacement rates for various C_p as a percentage of replacement costs and the per-unitized costs experienced at $P[F_{n_p}]$—the optimal replacement threshold. As can be seen, for even the most aggressive scenario with a large proactive replacement value, it is still optimal to let most transformers fail and replace them, retroactively.

C. Electric Vehicle Targeted Demand Side Management

Because of the long charging time of EVs and the temporal overlap between daily commutes and load peaks, it is likely
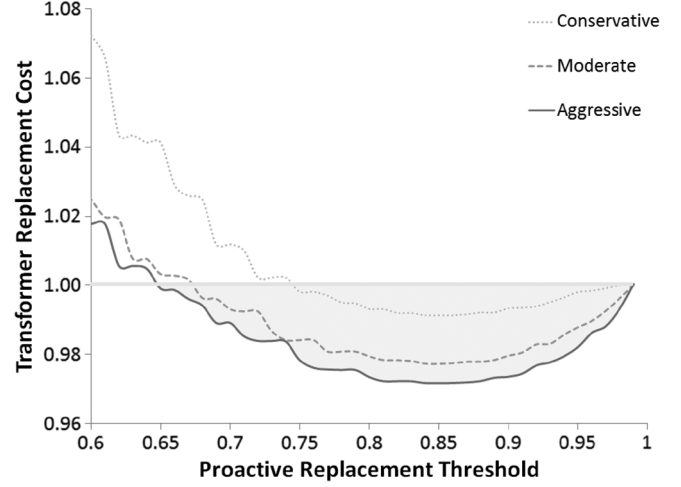

Fig. 3. Nominal transformer replacement cost as a function of proactive replacement rate for the 15% salvage value (C_s) scenario. The optimal proactive rate is the value which minimizes this function. The shaded region indicates the beneficial region for the aggressive scenario. Replacement of transformers with overload probabilities of more than 0.65 is beneficial, but 0.85 is optimal.

TABLE I
OPTIMAL PROBABILITY THRESHOLD AND TOTAL REPLACEMENT COST

Scenario	C_p as a percentage of C_r					
	5%		10%		15%	
	$P[F_{n_p}]$	C_t	$P[F_{n_p}]$	C_t	$P[F_{n_p}]$	C_t
Conservative	95%	0.999	90%	0.997	84%	0.991
Moderate	95%	0.998	90%	0.990	85%	0.997
Aggressive	95%	0.997	90%	0.987	85%	0.972

that EVs will be coincident with the peak loading. However, if a DSM program is implemented so that EV owners are incentivized to not charge their vehicle during peak loading most of the capacity issues related to EVs could be minimized. To help establish what effects DSM programs might have, the baseline model has been further refined to reflect DSM participation.

Recall that for each transformer, a binomial probability is calculated to determine if enough homes will get EVs to cause an overload. We now introduce a second, nested series of binomial experiments to see, for households which have an EV, if they participate in a DSM program. This model can then be used to see to what degree curtailment from DSM will avert an overload.

Take for example a transformer with five homes on it which also only has capacity for one electric vehicle, as described in Table II. Assume an EV penetration of 15% and a DSM participation rate of 10%. To analyze the effectiveness of DSM, a second series of four binomial probabilities must now be formulated, one for each event which causes an overload (i.e., for the events that 2 (event A), 3 (B), 4 (C), and 5 (D) households connect an EV):

$$P[OK_D|A] = P[X_A \geq 1], \qquad X_A \sim bin(2, 0.1)$$
$$P[OK_D|B] = P[X_B \geq 2], \qquad X_B \sim bin(3, 0.1)$$
$$P[OK_D|C] = P[X_C \geq 3], \qquad X_C \sim bin(4, 0.1)$$
$$P[OK_D|D] = P[X_D \geq 4], \qquad X_D \sim bin(5, 0.1).$$

TABLE II
EXAMPLE OF DSM OUTCOMES FOR A TRANSFORMER WITH FIVE HOUSEHOLDS AND CAPACITY FOR ONE EV

Event	#1	#2	#3	#4	#5	Prob †	DSM Outcomes					k [a]	n [b]	Prob ‡
D	X	X	X	X	X	0.01%	XXXXX XXXXD XXXDD XXDDD XDDDD DDDDD					4	5	0.05%
C	X	X	X	X	O	0.22%	XXXXO XXXDO XXDDO XDDDO DDDDO					3	4	0.37%
B	X	X	X	O	O	2.44%	XXXOO XXDOO XDDOO DDDOO					2	3	2.80%
A	X	X	O	O	O	13.82%	XXOOO XDOOO DDOOO					1	2	19.00%
	X	O	O	O	O	39.15%								
	O	O	O	O	O	44.37%								

X = Home has an EV Connected, O = Home has no EV, D = Home has EV with DSM.
† Probability of specific event, i.e., $P[A]$, $P[B]$, etc. The sum of non-overloading events (i.e., the unlabeled events) is the probability of no overload because of sufficient capacity, i.e., $P[OK_C] = 0.3915 + 0.4437 = 0.8352$.
‡ Probability of any DSM event occurring which averts an overload, i.e., $P[OK_D|A]$, $P[OK_D|B]$, etc.
[a] k = # of EVs needed to participate in DSM to avert an overload
[b] n = # of EVs that may choose to participate in DSM.

A binomial random variable is created for each event. The probability that enough people participate in EV DSM to cause the transformer to not overload is the conditional probability that EV DSM causes the transformer to not overload (event OK_D) given that a number of EVs are connected which would otherwise cause the transformer to overload (events A, B, C, and D).

The law of total probability can now be applied to find the probability that DSM averts an overload using the conditional probabilities:

$$P[OK_D] = P[OK_D|A]P[A] + P[OK_D|B]P[B]$$
$$+ P[OK_D|C]P[C] + P[OK_D|D]P[D] \quad (6)$$
$$= (0.19)(.1382) + (0.028)(0.0244)$$
$$+ (0.0037)(0.0022) + (0.0005)(0.0001)$$
$$= 0.0269. \quad (7)$$

Taking this probability with that calculated from the initial model (of no overload because of enough capacity on the transformer, event OK_C), the total probability of no overload is found. The results for the five customer example is also given:

$$P[OK] = P[OK_C] + P[OK_D]$$
$$= 0.8352 + 0.0269 = 0.8622. \quad (8)$$

The general form of this process can be formulated as follows. Let $X \sim bin(k; n, p)$ with k successes in n trials with probability p. Then let

$$f_X[k, n, p] = P[X = k] \quad (9)$$
$$F_X[k, n, p] = P[X \leq k] \quad (10)$$

where $f_X[k, n, p]$ is the probability mass function (PMF) and $F_X[k, n, p]$ is the cumulative density function (CDF).

With these definitions, the probabilities of averted overloads due to capacity (event OK_C) and DSM (event OK_D) along with overall overload (event OL) are

$$P[OK_C] = F_X[n_{cap}, n_{max}, p_{EV}] \quad (11)$$
$$P[OK_D] = \sum_{n=n_{cap}+1}^{n_{max}} (1 - F_X[(n - n_{cap} - 1), n, p_{DSM}])$$
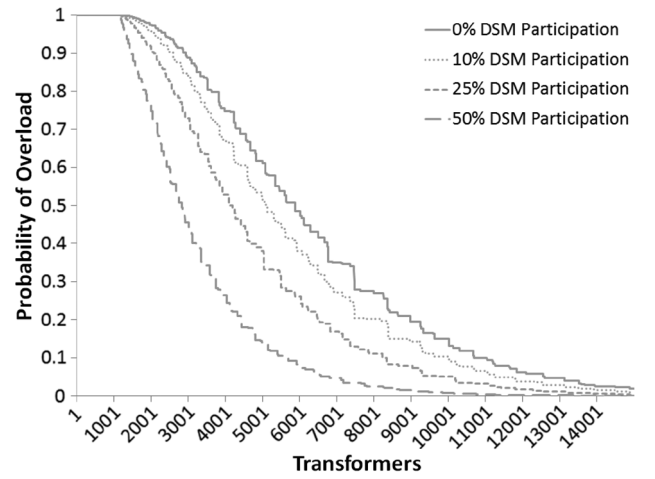$$* f_X[n, n_{max}, p_{EV}] \quad (12)$$

Fig. 4. Effectiveness of DSM programs targeted at EV for the aggressive scenario.

TABLE III
NUMBER OF TRANSFORMERS REQUIRING REPLACEMENT FOR VARIOUS EV PENETRATIONS AND DSM PARTICIPATION RATES

Scenario	No DSM	10% DSM	25% DSM	50% DSM
Conservative	1485	1454	1407	1330
Moderate	3497	3201	2783	2159
Aggressive	6441	5741	4747	3272

$$P[OL] = 1 - P[OK] = 1 - (P[OK_C] + P[OK_D]). \quad (13)$$

This analysis was performed on the Denver data for the three EV penetration scenarios (conservative, moderate, and aggressive) and assumed DSM participation rates (0%, 10%, 20%, and 50% participation). As Fig. 4 shows, EV targeted DSM produce substantial reductions in transformer replacements for the aggressive EV penetration scenario. However, at the lower EV penetration levels DSM is unlikely to provide much benefit. The summary of these results are enumerated in Table III.

V. CONCLUSION

The increased penetration of EVs may have a significant effect on distribution transformer loading. Utility companies must develop procedures or algorithms to identify these overload-susceptible transformers before they result in a high number

of customer outages. The authors have presented a method that utilizes a binomial distribution to predict the probability that a distribution transformer will be loaded above a specific level based on existing peaking demand, the number of customers served by said transformer, and average size of EV charger. Also presented in the paper is an optimization approach that utilizes data from the binomial distribution to determine the replacement strategy in regards to economic feasibility. The authors finally present a DSM strategy that utilizes information from binomial function.

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