Set Theory

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1 Preliminaries

Let x, y, z stand for sets. Let $x \in y$ denote x is an element of y. Let $x \notin y$ denote x is not an element of y.

Axiom 1 $(x \in y) \Rightarrow x - < - y$.

Theorem 1 $x \notin x$.

Proof by induction on x. **qed.**

Signature 1 *The* empty set *is the set that has no elements. Let* $\emptyset$ *denote the empty set.*

Definition 1 x *is* nonempty *iff* x *has an element.*

Definition 2 *A subset of* y *is a set* x *such that every element of* x *is an element of* y*. Let* $x \subseteq y$ *stand for* x *is a subset of* y*. Let* $y \supseteq x$ *stand for* x *is a subset of* y*.*

Definition 3 *A* proper subset *of* y *is a subset* x *of* y *such that there is an element of* y *that is not an element of* x*.*

Proposition 1 $x \subseteq x$.

Proposition 2 *If* $x \subseteq y \subseteq x$ *then* $x = y$.

Definition 4 $x \cup y = \{u \mid u \in x \text{ or } u \in y\}$.

Definition 5 $\{x\} = \{\text{sets } y \mid y = x\}$.

2 Ordinal Numbers

[ordinal/-s]

Definition 6 *x is* transitive *iff every element of x is a subset of x. Let* Trans(x) *stand for x is transitive.*

Definition 7 *An* ordinal *is a set x such that x is transitive and every element of x is transitive. Let* Ord(x) *stand for x is an ordinal. Let α, β, γ stand for ordinals.*

Theorem 2 *$\emptyset$ is an ordinal.*

Definition 8 $x + 1 = x \cup \{x\}$.

Theorem 3 *$\alpha + 1$ is an ordinal.*

Theorem 4 *If* $\mathrm{Ord}(\beta)$ *and $\alpha \in \beta$ then* $\mathrm{Ord}(\alpha)$.

Theorem 5 $(\alpha \in \beta) \wedge (\beta \in \gamma) \Rightarrow (\alpha \in \gamma)$.