

Satz 1. *The set of prime numbers is infinite.*

Beweis. Let A be a finite set of prime numbers. Take a function P and a number r such that $A = \{P_1, \dots, P_r\}$. Take $n = P_1 \cdots P_r + 1$. Take a prime divisor p of n . Let us show that p is not an element of A . Assume the contrary. Take i such that $(1 \leq i \leq r \text{ and } p = P_i)$. $\{P_1, \dots, P_r\} \subseteq \text{Dom } P$ and $\text{Ran } P \subseteq \mathbb{N}$. P_i divides $P_1 \cdots P_r$ (by Mult-Prod). Then p divides 1 (by DivMin). Contradiction. qed. Hence A is not the set of prime numbers. \square