THE OUTPUT RESISTANCE OF A POWER SUPPLY AN INTRODUCTORY RESISTANCE EXERCISE

Karys Littlejohns
Student#
karys.littlejohns@mail.utoronto.ca

Peter Leong
Student# 1010892955
peter.leong@mail.utoronto.ca

1 INTRODUCTION

Sources of electricity produce a voltage potential across their terminals called an electromotive force (*emf*) or an open circuit voltage, V_∞. In practice, when a closed circuit is made, a current I will be drawn and the voltage at the terminals, V called *terminal voltage* will fall below V_∞. A plot of what the terminal voltage V vs. current I may look like is shown in 1.

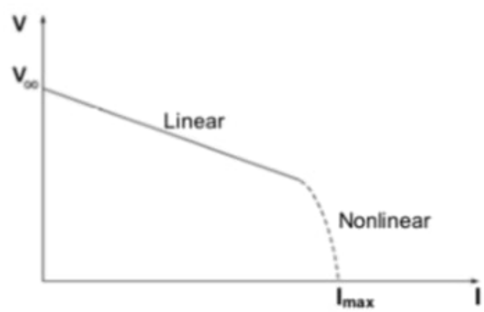

Figure 1: Terminal voltage vs. current

For most cases, many powers sources will exhibit a linear variation of R for small current values, with nonlinear behaviour at higher currents. The linear part of the curve can be described by:

$$V = V_\infty - RI$$

where R is the *output resistance of the powers source*. In this linear regime, according to Thevenin's theorem, the power source is completely represented by the equivalent circuit shown below.

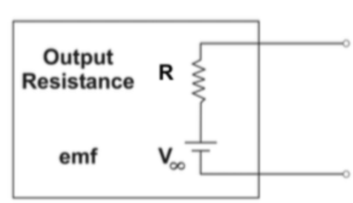

Figure 2: Equivalent circuit of an electric power source.

The output resistance (R) can be determined by attaching different external resistances of the load (R_l) to the power source, and measuring the current and voltage with a multimeter. 3 shows two possible ways of doing this. Both would be equivalent **if** the multimeter were ideal. However, in this exercuse we will measure with real, not ideal, multimeters.

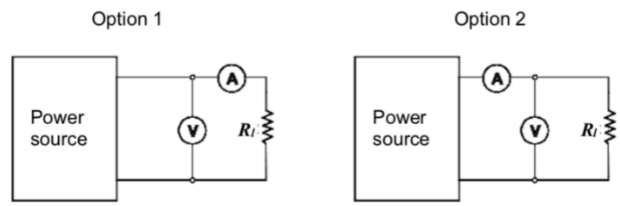

Figure 3: Possible circuits for determining the output resistance of a power source

2 Pre-Lab Exercises

Without connecting circuits and making measurements, we can expect the the voltmeter and ammeter readings to differ between the two options due to current and voltage leakage. More specifically, in option 1, we can expect the current to differ as the voltmeter allows some current to pass through. The amount of leakage will depend on the load resistance. Similarly, in option 2, we can expect that the voltage will differ as the ammeter will draw some voltage. Its leakage will too depend on the load resistance.

To calculate the internal resistances of the voltmeter and the ammeter we can use basic current and voltage division. For the voltmeter, we can calculate the current that we expect to pass through the ammeter and solve for the voltmeter resistance.

2.1 Ammeter Internal Resistance Derivation

Starting from Ohm's law: $V = I(R_A + R_L)$ Solve for R_A:

$$\frac{V}{I} = R_A + R_L \quad \Rightarrow \quad R_A = \frac{V}{I} - R_L$$

2.2 Voltmeter Internal Resistance Derivation

Starting from Ohm's Law: $V = I\left(\frac{R_V R_L}{R_V + R_L}\right)$ Solve for R_V:

$$\frac{V}{I} = \frac{R_V R_L}{R_V + R_L} \quad \Rightarrow \quad \frac{V}{I}(R_V + R_L) = R_V R_L \quad \Rightarrow \quad \frac{V}{I}R_V + \frac{V}{I}R_L = R_V R_L$$

$$\frac{V}{I}R_L = R_V\left(R_L - \frac{V}{I}\right) \quad \Rightarrow \quad R_V = \frac{\frac{V}{I}R_L}{R_L - \frac{V}{I}} = \frac{VR_L}{IR_L - V}$$

3 The Experiment

We began by measuring the resistance values of the provided resistors. For the subsequent circuit experiments, we selected the two highest and two lowest resistance values to represent the load resistance conditions for the voltmeter and ammeter configurations, respectively. The measured values with their associated uncertainties are presented below.

Table 1: Resistor Values and Uncertainties

	R_{l1}	R_{l2}	R_{l3}	R_{l4}	R_{l5}	R_{l6}
Value (Ω)	100.32	219.91	461.3	2.6760 k	26.814 k	101.57 k
Uncertainty (Ω)	$\pm$0.25	$\pm$0.49	$\pm$0.1.42	$\pm$.0005 k	$\pm$0.006 k	$\pm$0.25 k

3.1 Circuit Option 1

INCLUDE DIAGRAM/SKETCH OF CIRCUIT OPTION 1

From this data, we applied a linear fit to determine the slope m_1 and its uncertainty. We obtained a slope value of $-0.046 \pm 0.004\ \Omega$. The residuals plot and χ^2 analysis suggested potential underestimation of uncertainties.

Table 2: Circuit 1 Readings & Results

Resistance R_{li} (Ω)	Uncertainty ΔR_{li} (Ω)	Voltage V (V)	Uncertainty ΔV (V)	Current I (mA)	Uncertainty ΔI (mA)	Ammeter Resistance R_A (Ω)	Uncertainty ΔR_A (Ω)
100.32	$\pm$0.25	6.499	$\pm$0.005	63.67	$\pm$0.18	1.78	$\pm$0.39
219.91	$\pm$0.49	6.500	$\pm$0.005	29.315	$\pm$0.064	1.85	$\pm$0.71
26.814 k	$\pm$60	6.501	$\pm$0.005	0.241	$\pm$0.051	161.10	$\pm$5708.77
101.57 k	$\pm$250	6.501	$\pm$0.005	0.063	$\pm$0.005	1620.48	$\pm$8193.92
					Average:	446.3	$\pm$3475.9

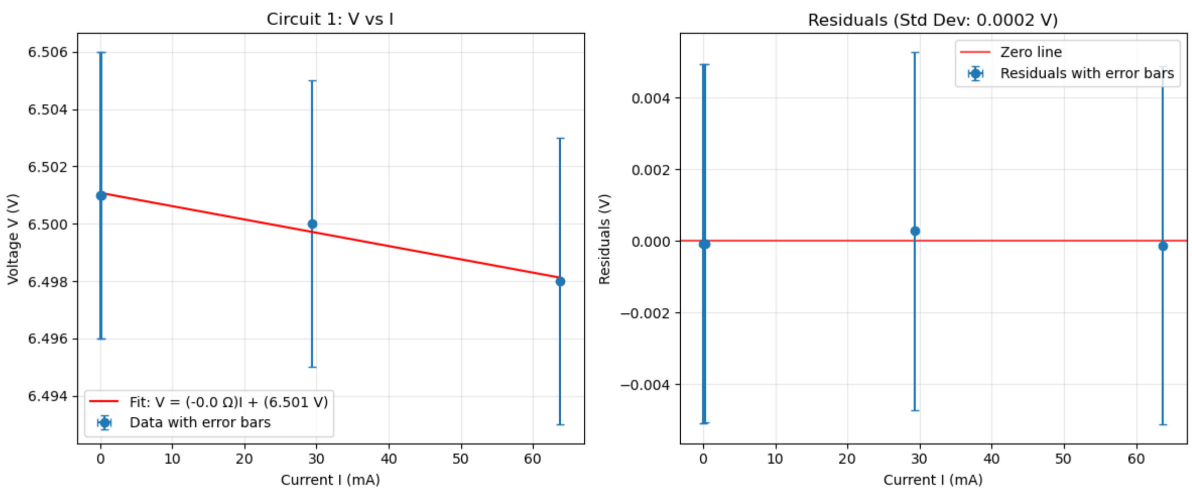

Figure 4: Linear regression of voltage versus current measurements showing an ideal voltage source behavior. The slope of $-1.813 \pm 0.003\ \Omega$ represents the effective circuit resistance, while the intercept of $6.501 \pm 0.000\ V$ corresponds to the source voltage. The excellent fit quality ($\chi^2 = 0.003$, $\chi^2/\nu = 0.001$, $p = 0.9987$) indicates the data are consistent with the linear model within measurement uncertainties. Reading uncertainity was used for both V and I since they were greater than statistical uncertainties.

3.2 Circuit Option 2

INCLUDE DIAGRAM/SKETCH OF CIRCUIT OPTION 2

Table 3: Circuit 2 Readings & Results

Resistance R_{li} (Ω)	Uncertainty ΔR_{li} (Ω)	Voltage V (V)	Uncertainty ΔV (V)	Current I (mA)	Uncertainty ΔI (mA)	Voltmeter Resistance R_V (Ω)	Uncertainty ΔR_V (Ω)
100.32	$\pm$0.25	6.386	$\pm$0.005	63.60	$\pm$0.18	-1.13×10^5	$\pm 4.94 \times 10^5$
219.91	$\pm$0.49	6.448	$\pm$0.005	29.322	$\pm$0.064	7.05×10^6	$\pm 7.27 \times 10^8$
26.814 k	$\pm$59	6.501	$\pm$0.005	0.243	$\pm$0.051	1.18×10^7	$\pm 1.09 \times 10^9$
101.57 k	$\pm$250	6.501	$\pm$0.005	0.065	$\pm$0.005	6.53×10^6	$\pm 3.29 \times 10^7$
					Average:	6.31×10^6	$\pm 4.62 \times 10^8$

From this data, we applied a linear fit to determine the slope m_2 and its uncertainty. We obtained a slope value of $-0.046 \pm 0.004\ \Omega$. The residuals plot and χ^2 analysis suggested potential underestimation of uncertainties.

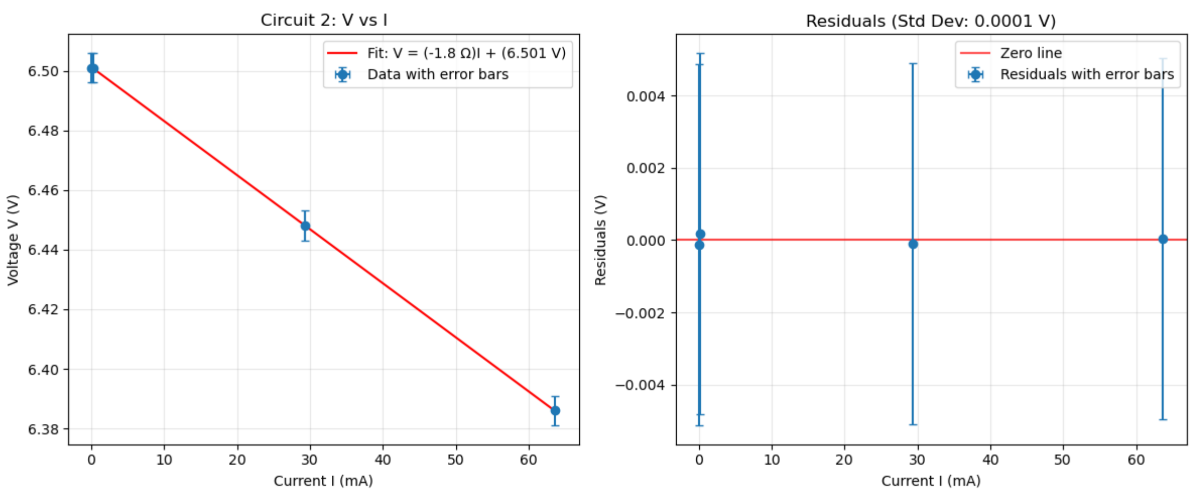

Figure 5: Linear regression of voltage versus current measurements for the voltmeter circuit configuration. The slope of $-0.046 \pm 0.004\ \Omega$ indicates minimal effective resistance, while the intercept of $6.501 \pm 0.000\ V$ represents the source voltage. The fit quality ($\chi^2 = 0.003$, $\chi^2/\nu = 0.001$, $p = 0.9987$) suggests potential underestimation of measurement uncertainties given the small residuals relative to error bars. Reading uncertainity was used for both V and I since they were greater than statistical uncertainties.

4 Analysis

5 Conclusion

References