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On efficiency, savings, wealth transfers and risk-aversion in electricity markets with uncertain supply*

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Abstract This paper studies the wealth transfers which arise from implementing a stochastic program in a single-settlement energy-only pool market and establishes when the SP mechanism leads to lower clearing prices, lower consumer costs and higher generator costs, or vice versa. By implementing the SP mechanism in the New Zealand Electricity Market, we uncover computational evidence that the wealth transfer from consumers to generators can be an order of magnitude larger than the cumulative system savings. We also consider participants who are risk averse and model their risk aversion using law-invariant coherent risk measures. We uncover a closed form characterization of the optimal pre-commitment behaviour for a given real-time dispatch policy, with arbitrary risk aversion. When participants cannot trade risk, a risked equilibrium exists which provides less pre-commitment than when participants are risk-neutral. Alternatively, when participants trade a rich set of financial instruments, a second risked equilibrium exists which provides more pre-commitment than when generators are risk-neutral. Consequently, introducing an auxiliary risk market alters the energy-only market by increasing pre-commitment, depressing nodal prices and providing a wealth transfer from generators to consumers.

Keywords Wind energy · Stochastic programming · Spot market · Coherent risk measure · Risky equilibria · Risk trading.

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1 Introduction

Renewable power generation is an increasingly attractive investment option for participants in electricity pool markets, as it does not emit carbon and has a marginal cost of zero. Furthermore, investment in intermittent renewable generation is attractive from a regulatory standpoint, as wind and solar generators reduce the expected dispatch cost and do not emit carbon. However, renewable investment increases supply-side uncertainty. This creates difficulties for independent system operators (ISOs) when clearing electricity pool markets, as inflexible coal and nuclear generators may require several hours or more to implement a dispatch and intermittent power output is unknowable this far in advance.

When intermittent renewable generators supply a small proportion of electricity, the ISO can efficiently manage deviations from forecasts by procuring suitable amounts of frequency-keeping and reserve generation. However, when intermittent generators supply a larger proportion of electricity, procuring reserves as a back-up generation source becomes expensive and more efficient grid management strategies are required. Consequently, some pool markets employ a two-market structure, where: (1) a pre-commitment market is cleared by assuming that renewable generation takes on its forecast value, and (2) a real-time market is cleared at the commencement of each trade period to balance deviations between renewables' forecast and realized generation output. This two-market structure allows inflexible generators to implement a dispatch, by providing them with a pre-commitment setpoint. However, the pre-commitment and real-time nodal prices might fail to converge in expectation, as the expected adjustment cost is not priced when solving for the pre-commitment setpoint. Indeed, the recent paper [48] claims this price distortion is a market design flaw which may lead to systematic arbitrage opportunities.

More sophisticated uncertainty management strategies comprise modelling intermittent renewable generation as a random variable, and solving for a pre-commitment setpoint by determining the optimal solution to a two-stage stochastic optimization problem. The stochastic program has the objective of minimizing the expected cost of generation plus adjustment, and the real-time market has the objective of minimizing the cost of generating electricity plus adjusting to manage fluctuations from forecast renewable generation. This strategy for clearing an electricity pool market is known as stochastic dispatch, and has been studied by authors including [8], [9], [45], [36], [29], [48], [15], [7], and [47]. Stochastic dispatch induces efficiency savings because it explicitly prices deviations between the pre-commitment setpoint and the real-time market in the first stage; thereby considering both flexibility and production costs when selecting the generators who balance deviations in intermittent output.

The cumulative expected payoff to generators, consumers and the ISO from implementing stochastic dispatch in a system cleared by deterministic dispatch is non-negative (see, e.g., [6]). However, the existence of cumulative savings does not imply that all market participants benefit from implementing stochastic dispatch. Indeed, implementing stochastic dispatch could decrease generator profits by two orders of magnitude more than the cumulative payoff and decrease consumer costs by almost as much, leaving generators dissatisfied. Alternatively, implementing stochastic dispatch could lead to dissatisfied consumers if it increased generator profits by two orders of magnitude more than the cumulative payoff, and consumer costs increased by nearly as much. The possibility of, and indeed-as we will see later-the existence of, one-sided wealth transfers leads to the following question: *Do generators or consumers benefit from the introduction of a stochastic dispatch mechanism, and under what conditions?*

1.1 Contributions and structure

- In Section 2, we formally introduce the Stochastic Dispatch Mechanism, or SDM, as described by Zakeri et al. in [47]. For benchmarking purposes, we also introduce a myopic Expected Value Dispatch Mechanism, or EVDM. We provide two interpretations of SDM: one which relates each generator's marginal deviation cost to their risk-aversion and SDM to a risk-averse dispatch mechanism, and one which relates each generator's marginal deviation cost to a regularization term and SDM to a robust dispatch mechanism.
- In Section 3, we revisit and elaborate upon the main results obtained by Zakeri et al. in [47], and motivate our subsequent analysis.
- In Section 4, we implement SDM and EVDM in the New Zealand Electricity Market for all 35040 trade periods which occurred in $2014 - 2015$, and evaluate their relative effectiveness's. We show that while SDM provides net efficiency savings on the order of 0.1%, it also provides significantly larger wealth transfers between the participants: a wealth transfer from consumers to generators which is two orders of magnitude larger in trade periods where SDM requires generators to ramp up more than EVDM, and a wealth transfer from generators to consumers which is two orders of magnitude larger in trade periods where SDM requires generators to ramp down more than EVDM. Overall, in $2014-2015$, switching the NZEM from EVDM to SDM is equivalent to a one-sided wealth transfer from consumers to generators, where the wealth transferred is an order of magnitude greater than the efficiency savings.
- In Section 5, we study SDM in a risk-averse context. When generators are endowed with law-invariant coherent risk measures and cannot trade risk, we demonstrate that the resultant risk-averse competitive equilibrium admits a solution, provided nodal prices are capped. We obtain a closed-form relationship between each generator's pre-commitment and their real-time dispatch, and demonstrate that generator risk aversion without financial instruments results in pre-commitment supply shortfalls. Alternatively, when generators can trade Arrow-Debreu securities, a second risked equilibrium exists which corresponds to a risk-averse social plan. Moreover, we obtain a second closed-form relationship between each generator's pre-commitment setpoint and their dispatch, and demonstrate that risk-aversion with a risk market causes excess pre-commitment. Consequently, introducing a risk market is favourable for consumers, and becomes more favourable as generators become more risk-averse. We also develop strictly positive lower bounds on the expected profits of risk-averse generators, and demonstrate that these bounds reduce to 0 upon introducing an auxiliary risk market.

- In Section 6, we illustrate the impact of risk-aversion on pre-commitment, when participants can trade risk, by endowing a system optimizer with the CVaR risk measure in two settings: a six-node toy network, and the NZEM.

2 Deterministic and stochastic dispatch mechanisms

In this section, we introduce our notation and two separate dispatch mechanisms. The first dispatch mechanism is a deterministic mechanism called the Expected Value Dispatch Mechanism, or EVDM, which fixes wind supply at its expected value when determining the pre-commitment setpoint and takes a recourse action when renewable generation output is revealed. The second dispatch mechanism is called the Stochastic Dispatch Mechanism, or SDM, which explicitly models uncertainty in wind supply using a probability distribution when determining the pre-commitment setpoint and takes a recourse action after renewable generation output is revealed. SDM is a relaxation of the dispatch mechanism considered by [36], where non-physical constraints on the first stage are relaxed (see [7] for a justification of relaxing non-physical constraints). We follow [36] in assuming that all generator offers are submitted before the pre-commitment setpoint is determined, and remain intact in real time.

Both dispatch models are coupled with a single-settlement payment scheme, wherein charges to participants under different realizations of renewable generation are known ex-ante and incurred ex-post.

2.1 Our notation

We use lower case Roman lettering for constants, upper case Roman lettering for random variables, and lower case Greek lettering to refer to dual prices. Our notation closely follows that of [36], [15] and [47].

We follow [6] and [43] in letting uncertainty be represented by the scenario $\omega \in \Omega$, which occurs with probability $\mathbb{P}(\omega)$ and prescribes all uncertainty caused by intermittency in our stochastic dispatch models. We denote a random variable by $Z : \Omega \mapsto \mathbb{R}$, its realized value in scenario ω by $Z(\omega)$ and the space of all measurable functions by $\mathcal{Z}$. We assume the sample space Ω is finite, which is an approximation obtained by sampling from the true distribution of uncertain outcomes. The triple $(\Omega, \mathcal{F}, \mathbb{P})$ then defines a discrete probability space on which the uncertain outcomes occur. Consequently, the dispatch models considered here are Sample Average Approximations for which one can derive asymptotic convergence results as the sample size increases (see [43]). We note that any computational dispatch model must work with a finite Ω and therefore restrict our attention to this case.

The sets and indices used throughout this paper are defined as follows:

- $\mathcal{F}$ is the set of flows which obey thermal limits, line capacities and the DC load-flow constraints imposed by Kirchhoff's laws. We assume that $\mathcal{F}$ is closed, convex and non-empty, meaning that $0 \in \mathcal{F}$.
- i is the index of a generation unit. We assume perfect competition, meaning the ownership of each generation unit is irrelevant and each generation tranche can be thought of as operated by a separate generator.
- $j(i)$ is the index of the node j where generator i is located.

- $\mathcal{N}$ is the set of all nodes in the network.
- $\mathcal{T}(n)$ is the set of all generators located at node n.

The problem data used throughout this paper are defined as follows:

- c_i is generator i's marginal generation cost.
- $D_n(\omega)$ is the inelastic demand at node n in scenario ω.
- $G_i(\omega)$ is generator i's production capacity in scenario ω.
- $r_{u,i}$ is generator i's marginal cost of upward deviation.
- $r_{v,i}$ is generator i's marginal cost of downward deviation.

To avoid out-of-merit-order dispatches we follow [47] in requiring that $r_{u,i}$ and $r_{v,i}$ are equal for all tranches offered by a generation unit (see Section 2.1 of [47] for an in-depth discussion of this matter). We also require that $r_{u,i}, r_{v,i} > 0$ for some generator i, as otherwise all generation units are infinitely flexible and we can implement a wait-and-see policy wherein we dispatch all generation units after uncertainty is realised.

The variables used throughout this paper are defined as follows:

- x_i is generator i's pre-commitment setpoint, the amount generator i prepares to produce before uncertainty is realized.
- $X_i(\omega)$ is generator i's real-time dispatch in scenario ω.
- $U_i(\omega)$ is generator i's upward deviation from its setpoint in scenario ω. It is equal to $\max(X_i(\omega) - x_i, 0)$.
- $V_i(\omega)$ is generator i's downward deviation from its setpoint in scenario ω. It is equal to $\max(x_i - X_i(\omega), 0)$.
- $F(\omega)$ is the vector of branch flows through the network in scenario ω.
- $\tau_n(F(\omega))$ is the net amount of energy injected from the grid into node n in scenario ω.

2.2 The Expected Value Dispatch Mechanism

In the deterministic Expected Value Dispatch Mechanism, or EVDM, wind supply is assumed to be equal to its expected value in the first stage, and a recourse action is taken once wind generation output takes on a realization in the second stage. EVDM does not consider uncertainty when determining the pre-commitment setpoint, and therefore does not price the expected cost of deviating to a real-time dispatch. Consequently, it has a higher expected system cost than SDM, although it may outperform SDM under some realisations of renewable generation output (see, e.g., [6]).

EVDM's pre-commitment setpoint, x^*, is found by solving the following deterministic linear optimization problem:

$$\begin{aligned}
\min \quad & c^\top x \\
\text{s.t.} \quad & \sum_{i \in \mathcal{T}(n)} x_i + \tau_n(f) \geq \bar{D_n}, \ \forall n \in \mathcal{N}, \\
& f \in \mathcal{F}, \\
& 0 \leq x \leq G,
\end{aligned}$$

where $\bar{D_n} := \sum_\omega \mathbb{P}(\omega) D_n(\omega)$ is the expected demand at node n, and the supply demand balance is enforced via a $\geq$ constraint which allows load to be shed, in order that EVDM satisfies a constraint qualification such as Slater's; in this regard, see Section 5.2.3 of [10].

After the pre-commitment setpoint x^* is found, the scenario $\hat{\omega}$ is realized, and the ISO solves the following recourse problem for the real-time dispatch $X(\hat{\omega})$:

$$\begin{aligned}
\min \quad & c^\top X(\hat{\omega}) + r_u^\top U(\hat{\omega}) + r_v^\top V(\hat{\omega}) & \\
\text{s.t.} \quad & \sum_{i \in \mathcal{T}(n)} X_i(\hat{\omega}) + \tau_n(F(\hat{\omega})) \geq D_n(\hat{\omega}), \ \forall n \in \mathcal{N}, & [\lambda_n(\hat{\omega})], \\
& x^* + U(\hat{\omega}) - V(\hat{\omega}) = X(\hat{\omega}), & \\
& F(\hat{\omega}) \in \mathcal{F}, & \\
& 0 \leq X(\hat{\omega}) \leq G(\hat{\omega}), & \\
& U(\hat{\omega}), V(\hat{\omega}) \geq 0, &
\end{aligned}$$

where $\lambda_n(\hat{\omega})$ is the marginal cost of supplying one additional unit of electricity at node n in scenario $\hat{\omega}$, which is matched with its corresponding constraint via square brackets. Because the supply-demand balance constraint is an inequality constraint, we have that $\lambda_n(\hat{\omega}) \geq 0$.

After solving both stages, the ISO pays $\lambda_{j(i)}(\hat{\omega}) X_i(\hat{\omega})$ to generator i and charges $\lambda_n(\hat{\omega}) D_n(\hat{\omega})$ to consumer n. The ISO does not incur a penalty for deviating from f to $F(\hat{\omega})$, and is therefore guaranteed to not be out of pocket (see [47]). Generators however, incur a deviation cost which is not priced in the first stage, and therefore may not even recover their costs in expectation.

2.3 The Stochastic Dispatch Mechanism

In the Stochastic Dispatch Model, or SDM, renewable generation output is modelled by a set of samples from a continuous distribution, which constitute an ensemble forecast of future uncertainty. Consequently, SDM is a Sample Average Approximation which yields pre-commitment setpoints that asymptotically converge to the optimal setpoint as the number of scenarios considered increases [43]. SDM's pre-commitment setpoint is determined by solving the following stochastic linear optimization problem for x^*, as defined by Zakeri et al. in [47]:

$$\begin{aligned}
\text{SLP:} \ \min \quad & \mathbb{E}_\omega[c^\top X(\omega) + r_u^\top U(\omega) + r_v^\top V(\omega)] & \\
\text{s.t.} \quad & \sum_{i \in \mathcal{T}(n)} X_i(\omega) + \tau_n(F(\omega)) \geq D_n(\omega), & \forall \omega \in \Omega, \ \forall n \in \mathcal{N}, \\
& x + U(\omega) - V(\omega) = X(\omega), & \forall \omega \in \Omega, \\
& F(\omega) \in \mathcal{F}, & \forall \omega \in \Omega, \\
& 0 \leq X(\omega) \leq G(\omega), & \forall \omega \in \Omega, \\
& U(\omega), \ V(\omega), \ x \geq 0, & \forall \omega \in \Omega.
\end{aligned}$$

After the auctioneer solves SLP and determines the setpoint x^* which clears the pre-commitment market, uncertainty is realized and the auctioneer solves the same recourse LP as EVDM in order to clear the real-time market and adjust generators at minimum cost. Generators are compensated using the same pricing mechanism as under EVDM, which provides expected fuel and deviation cost recovery under SDM, since both quantities are priced when clearing the first stage ([47], [15]).

Remark 1. A risk-averse interpretation of SDM
As noted by Ogryczak and Ruszczyński in [32], the optimal solution to SLP is also the optimal solution to the following risk-averse stochastic linear program:

$$\begin{aligned}
\min \quad & \mathbb{E}_\omega[(c-r_u)^\top X(\omega)] + r_v^\top \mathrm{CVaR}_{\eta_i}[X_i(\omega)] & \\
\text{s.t.} \quad & \sum_{i\in\mathcal{T}(n)} X_i(\omega) + \tau_n(F(\omega)) \geq D_n(\omega), & \forall\omega\in\Omega,\ \forall n\in\mathcal{N}, \\
& x + U(\omega) - V(\omega) = X(\omega), & \forall\omega\in\Omega, \\
& F(\omega)\in\mathcal{F}, & \forall\omega\in\Omega, \\
& 0 \leq X(\omega) \leq G(\omega), & \forall\omega\in\Omega, \\
& U(\omega),\ V(\omega),\ x \geq 0, & \forall\omega\in\Omega,
\end{aligned}$$

where $\eta_i := \frac{r_{u,i}}{r_{u,i}+r_{v,i}}$ may be interpreted as a measure of each generator's risk-aversion. In fact, we will see that $r_{u,i}$ and $r_{v,i}$ fully prescribe each generator's risk-aversion, in the coherent sense of Artzner et al. [5]. We defer the proof of this result to Proposition 17.

Remark 2. A robust interpretation of SDM
In this paper, we treat the deviation costs incurred by the generators as purely physical costs. However, *when the deviation costs are symmetric*, we can also use the commonly known relation $||Y||_1 = \max_{||\eta||_\infty \leq 1} \eta^\top Y$ to rewrite SDM as the following robust expected-value linear program:

$$\begin{aligned}
\text{ROSLP:} \ \min \max_{||\eta(\omega)||_\infty\leq 1} \quad & \mathbb{E}_\omega[c^\top X(\omega) + \Delta c(\omega)^\top(x - X(\omega))] & \\
\text{s.t.} \quad & \Delta c(\omega) = R\ \eta(\omega), & \forall\omega\in\Omega, \\
& \sum_{i\in\mathcal{T}(n)} X_i(\omega) + \tau_n(F(\omega)) \geq D_n(\omega), & \forall\omega\in\Omega,\ \forall n\in\mathcal{N}, \\
& x + U(\omega) - V(\omega) = X(\omega), & \forall\omega\in\Omega, \\
& F(\omega)\in\mathcal{F}, & \forall\omega\in\Omega, \\
& 0 \leq X(\omega) \leq G(\omega), & \forall\omega\in\Omega, \\
& U(\omega),\ V(\omega),\ x \geq 0, & \forall\omega\in\Omega,
\end{aligned}$$

where R is a diagonal matrix which encodes the marginal deviation costs incurred by the system.

This observation illustrates that SDM can also be thought of as a regularized stochastic linear program which minimizes the expected real-time dispatch cost subject to generating a pre-commitment policy which the real-time market does not excessively deviate from.

3 Risk-neutral theoretical results

In this section, we revisit the main results derived by Zakeri et al. in [47], in order to motivate our subsequent analysis. They are laid out as follows:

Proposition 3. *Let all generation agents be risk-neutral price-takers who are dispatched under SDM; then for each generator* i, x_i^* *is a* $\frac{r_{u,i}}{r_{u,i}+r_{v,i}}$ *quantile of the probability distribution of* $X_i^*(\omega)$, *i.e., a* $\frac{r_{u,i}}{r_{u,i}+r_{v,i}}$-VaR *of* $X_i^*(\omega)$.

Proof. See [43] for the result, and [48], [47] for applications of the result to SDM. □

Proposition 4. *Let agent* i *be a risk-neutral price-taking generation agent who offers the deterministic quantity* G_i *in each scenario, incurs the marginal costs* c_i, $r_{u,i}$, $r_{v,i} > 0$, *makes a pre-commitment decision* $x_i^* > 0$, *and is dispatched under SDM; then* $M_i(x_i^*, \omega)$, *generator* i*'s profit margin in scenario* ω, *is defined as the following function, which is non-decreasing in* $X_i(\omega)$:

$$M_i(x_i^*, \omega) = \begin{cases} -r_{v,i}x_i^*, & \text{if } 0 \leq X_i^*(\omega) < x_i^*, \\ (\lambda_{j(i)}(\omega) - c_i)x_i^*, & \text{if } X_i^*(\omega) = x_i^*, \\ r_{u,i}x_i^*, & \text{if } x_i^* < X_i^*(\omega) < G_i, \\ \big(\lambda_{j(i)}(\omega) - c_i - r_{u,i}\big)G_i + r_{u,i}x_i^* & \text{if } X_i^*(\omega) = G_i. \end{cases}$$

Proof. Rearrange Equation (3) in [47]. □

Corollary 5. *The relationship between agent* i*'s optimal real-time dispatch and the marginal price at node* $j(i)$ *is described by the following expression:*

$$\lambda_{j(i)}(\omega) \begin{cases} \leq c_i - r_{v,i}, & \text{if } X_i^*(\omega) = 0, \\ = c_i - r_{v,i}, & \text{if } 0 < X_i^*(\omega) < x_i^*, \\ \in [c_i - r_{v,i}, c_i + r_{u,i}] & \text{if } X_i^*(\omega) = x_i^*, \\ = c_i + r_{u,i}, & \text{if } x_i^* < X_i^*(\omega) < G_i, \\ \geq c_i + r_{u,i}, & \text{if } X_i^*(\omega) = G_i. \end{cases}$$

Proof. Re-arrange Proposition 2 in [47]. □

The above results suggest a relationship between the relative magnitudes of the payoffs from stochastic and deterministic dispatch: If generation units have a lower pre-commitment setpoint under SDM than EVDM, then expected nodal prices will be higher under SDM than EVDM, leading to higher cumulative generator profits and higher cumulative consumer costs. Alternatively, if generation units have a lower pre-commitment setpoint under SDM than EVDM, then expected nodal prices will be lower under SDM than EVDM, leading to lower cumulative generator profits and lower cumulative consumer costs. However, we are unable to conclusively show that this relationship persists in all systems.

Consequently, we implement SDM and EVDM in the New Zealand Electricity Market (NZEM), utilize them for the 35040 trade periods which occurred in the NZEM between 1 January 2014 and 31 December 2015, and study the savings provided to the market participants. We proceed to introduce the NZEM, the software used to clear it, and our experimental setup. Our methodology closely follows that of [25], although we consider the full NZEM rather than aggregating generation units into 18 nodes. Our methodology is also identical to that of [47], although we consider welfare transfers and risk-aversion rather than cost-recovery. We repeat the discussion of the methodology here, so that the paper is self-containing.

4 Risk-neutral numerical results

4.1 Vectorised Scheduling Pricing and Dispatch

The New Zealand Electricity Market (NZEM) is an energy-only pool market where generators and retailers submit bids to buy and sell electricity in each half hour trade period. Bids consist of up to five tranches (marginal-price generation-quantity pairs) per participant, and are permitted between 36 and 2 hours before each trade period commences. Transpower, the system operator then clears the market using software called Scheduling, Pricing and Dispatch (SPD, see [4]).

Vectorised Scheduling Pricing and Dispatch (vSPD) is a publicly available replica of SPD, which is written in GAMS and allows each trade period from 2004 onwards to be independently replicated (see [31]). We implement SDM and EVDM in the NZEM by modifying vSPD to clear two-hour-ahead pre-commitment markets using a nonanticipative forecast, and clear the real-time markets using realized wind.

4.2 Generating wind scenarios via quantile regression

To generate nonanticipative ensemble forecasts, we model the geographically adjacent Tararua, Te Apati and Te Rere Hau wind farms in the Central North Island as one wind farm with a maximum output capacity of 300 MW, the geographically adjacent West Wind and Mill Creek wind farms as a second wind farm with a maximum output capacity of 200 MW, assume the two wind farms are conditionally independent[1], and treat the remaining wind farms as deterministic negative demand. Our approach models 500 MW of the 690 MW of wind production in New Zealand at an affordable computational cost.

We generate splines corresponding to the 1st, 5th, 15th, 25th, ..., 95th, 99th quantiles of the conditional distribution of future wind at both locations using techniques developed by Pritchard in [35] and implemented in the R package Quantreg (see [26]). Our modelling approach is a form of importance sampling, which places emphasis on extreme amounts of wind as a surrogate for the higher adjustment costs incurred in these scenarios. The spline coefficients are determined using historical wind farm data from $2011-2013$ (see [31]). Wind scenarios in SDM correspond to the outer product of the two sets of splines, and each occur with probability $\frac{1}{144}$. Consequently, our approach indicates how the savings from SDM are distributed after taking account of out-of-sample effects.

[1] This assumption is valid, as the average correlation between the two sets of wind farms was 0.079 from 1 January 2011 to 31 December 2013.

4.3 Modelling the marginal deviation costs

The generator offer stacks in vSPD do not provide the marginal deviation costs. Therefore, we estimate them in the same manner as [25] and assume they are of the form

$$\mathrm{r}_{\mathrm{u,i}} = \frac{\mathrm{K}}{\text{Ramp up rate}},\ \mathrm{r}_{\mathrm{v,i}} = \frac{\mathrm{K}}{\text{Ramp down rate}},$$

where the ramp rates are provided by the parameter i_TradePeriodOfferParameter within vSPD's GDX input files, and K is a constant which depends on underlying market conditions such as generator flexibility and the degree to which hydro generators are risk-averse to dry winters.

To determine K, observe that in the NZEM in $2014-2015$, reserve prices are on the order of \$2 per MW, ramp rates for hydro plants are on the order of 100 MW per hour and ramp rates for thermal plants are on the order of 5 MW per hour (see [31]). Consequently, $\mathrm{K} = 10$ provides marginal deviation costs of \$0.1 per MW for hydro generators and \$2 per MW for thermal generators; a set of costs which bound reserve prices from below. Similarly, $\mathrm{K} = 100$ gives marginal deviation costs of \$1 per MW for hydro generators and \$20 per MW for thermal generators; a set of costs which bound reserve prices from above. Therefore, using $\mathrm{K} = 10$ and $\mathrm{K} = 100$ in two separate experiments provides bounds on the cumulative savings which arise from stochastic dispatch.

4.4 Solving stochastic optimization problems in CPLEX

The market clearing problems are modelled using GAMS version 24.4.3 (see [11]) and a modified version of vSPD version 2.0.4 [31], and solved using CPLEX 12.6.1 on an Intel Xeon 2.3 GHZ processor with 32 GB of RAM and 4 cores. The required computational time for each deterministic LP is negligible. However, the deterministic equivalents of the stochastic LPs arising from SDM have $400-800$ thousand rows, $2-5$ million columns and $4-8$ million non-zero entries after CPLEX performs a presolve operation, and require $1000-3000$ seconds per trade period to solve to optimality with default CPLEX parameters. Consequently, each experiment requires around 2 years to conduct with default CPLEX parameters.

Table 1 depicts the parameters used to decrease the required computational time for $\mathrm{K} = 10$ by an order of magnitude, to around 2.5 months. The required computational time for $\mathrm{K} = 100$ was decreased even further, by crashing the basis from the $\mathrm{K} = 10$ experiment which corresponds to the same trade period using the Dual Simplex Method (see [23] for a discussion of parameter tuning in CPLEX or [40] for a discussion of reusing an existing basis in GAMS).

Table 1 CPLEX parameters used to solve SLP for K = 10 (see [23]).

Parameter	**Value**	**Meaning**
lpmethod	4	Solve using Log-Barrier
barcolnz	50	Manage columns with 50+ non-zero entries separately
barorder	3	Use Nested Dissection when ordering rows
scaind	1	Aggressively scale the problem matrix
objllim	0	Bound the objective from below with 0

4.5 Efficiency savings from SDM

The cumulative savings to market participants are measured by computing the difference between the real-time market clearing costs under EVDM and SDM. The total available savings are measured by computing the difference between the real-time market clearing cost under EVDM and the real-time market clearing cost of an anticipative dispatch which makes its pre-commitment decision after uncertainty is revealed. In the parlance of stochastic optimization, the cumulative and available savings are called the Value of Stochastic Solution and the Value of Perfect Information (see [6]).

Tables 2 and 3 depict the savings to the NZEM participants under several distinct market conditions, and indicate that the net savings to the participants are between $\$63,700$ and $\$408,300$ per year and the cost of uncertainty is between $\$221,700$ and $\$1,269,700$ per year. The notation "$0-2.5\%$ Wind" indicates the savings in a year of NZEM trade periods with fewer than 2.5% of generation supplied by wind. Around 5% more hydro generation occurs in our payoff numbers than in the NZEM in $2014-2015$, because we have removed constraints on ramping in vSPD to ensure that the market considered is consistent with SLP and more generally the energy-only dispatch models considered in the literature. One could account for the excess water usage by inflating the hydro offer prices, but we are studying general patterns in the efficiency savings, and therefore relaxing the ramping constraints is sufficiently accurate for our purposes.

Table 2 Efficiency savings and available efficiency savings 1 Jan 2014 to 31 Dec 2015 K = 10.

		95% CI Savings per Year		95% CI Available Savings per Year	
Kind of trade period	No. TPs	Lower Limit	Upper Limit	Lower Limit	Upper Limit
Overall	35,040	$63,671.97	$71,118.11	$221,700.25	$231,918.73
0 − 2.5% Wind	11,280	$69,316.39	$76,522.04	$214,362.64	$224,076.91
2.5 − 5% Wind	12,258	$49,446.38	$56,672.47	$211,567.84	$221,313.34
5 − 100% Wind	11,502	$73,321.71	$81,198.44	$239,764.34	$250,896.04
0 − 30% Thermal	8,553	$38,570.72	$44,224.84	$177,825.31	$186,865.56
30 − 35% Thermal	13,161	$56,610.52	$63,630.66	$197,969.71	$207,634.71
35 − 100% Thermal	13,326	$86,906.06	$95,604.60	$273,552.19	$284,771.98
0 − 60% Hydro	12,290	$91,555.62	$100,429.07	$280,106.55	$291,699.57
60 − 65% Hydro	10,348	$57,082.71	$64,165.46	$211,632.71	$221,479.08
65 − 100% Hydro	12,402	$41,721.38	$47,685.45	$172,574.70	$181,275.92

Table 3 Efficiency savings and available efficiency savings 1 Jan 2014 to 31 Dec 2015 K = 100.

Kind of trade period	No. TPs	95% CI Savings Per Year Lower Limit	95% CI Savings Per Year Upper Limit	95% CI Available Savings Per Year Lower Limit	95% CI Available Savings Per Year Upper Limit
Overall	35,040	\$370,625.76	\$408,309.35	\$1,220,905.53	\$1,269,739.82
0% − 2.5% Wind	11,280	\$357,759.78	\$391,674.00	\$1,193,985.52	\$1,240,515.34
2.5% − 5% Wind	12,258	\$291,387.78	\$327,908.03	\$1,169,549.75	\$1,217,000.63
5% + Wind	11,502	\$468,261.33	\$510,250.19	\$1,302,423.46	\$1,354,711.07
0% − 30% Thermal	8,553	\$262,063.47	\$291,570.45	\$1,047,412.32	\$1,088,460.10
30% − 35% Thermal	13,161	\$336,704.46	\$371,440.59	\$1,123,905.71	\$1,168,940.50
35% + Thermal	13,326	\$474,630.32	\$518,965.18	\$1,429,213.90	\$1,485,087.06
0% − 60% Hydro	12,290	\$519,906.59	\$566,029.24	\$1,463,710.82	\$1,521,181.03
60% − 65% Hydro	10,348	\$339,214.04	\$375,537.97	\$1,180,457.86	\$1,227,586.96
65% + Hydro	12,402	\$250,583.89	\$278,329.44	\$1,015,953.47	\$1,054,951.72

Tables 2-3 illustrate that the hour to hour wind fluctuations do not have a significant impact on the savings from stochastic dispatch, at least with current wind supply levels. Instead, whether hydro-thermal generators can readily backup intermittent wind generators accurately predicts the savings from stochastic dispatch. This is unsurprising: if hydro-thermal generation is readily available then generators with marginal deviation costs of O(\$0.1-\$1) backup the intermittent generation. Otherwise, inflexible thermal generators with marginal costs of deviation of O(\$2-\$20) backup the intermittent generation. Consequently, SDM provides more significant savings in dry winters where water is scarce.

Tables 4 and 5 summarize the payoffs to generators, consumers and the ISO from implementing stochastic dispatch, against the sizes of the pre-commitment magnitudes from SDM and EVDM. The notation "SDM > EVDM" indicates the savings in a year of trade periods conditioned on the pre-commitment magnitude from SDM being larger, and the notation "SDM < EVDM" depicts the converse. The payoff numbers do not sum to the cumulative payoff: there is a discrepancy of around \$5000 per year for K = 10. This discrepancy arises because cumulative savings are measured from objective function values and do not account for any of: vSPD rounding nodal prices to two decimal places, differences in penalty violation cost magnitudes, or differences in the real-time network flow losses.

Table 4 Participant payoffs by first stage dispatch 1 Jan 2014 to 31 Dec 2015 K = 10.

Participant	95% CI Savings per Year SDM > EVDM (17,429 TPs) Lower Limit	95% CI Savings per Year SDM > EVDM (17,429 TPs) Upper Limit	95% CI Savings per Year SDM < EVDM (17,558 TPs) Lower Limit	95% CI Savings per Year SDM < EVDM (17,558 TPs) Upper Limit
Generator	−\$11,506,439.68	−\$10,216,958.72	\$15,462,554.60	\$16,664,920.60
Consumer	\$10,514,038.10	\$11,822,209.90	−\$16,938,656.24	−\$15,711,265.36
ISO	−\$303,396.25	−\$205,734.95	\$304,031.16	\$355,772.04
Net	\$32,416.12	\$59,739.08	\$65,996.04	\$93,085.56

Table 5 Participant payoffs by first stage dispatch 1 Jan 2014 to 31 Dec 2015 K = 100.

	95% CI Savings per Year SDM > EVDM (17,269 TPs)		95% CI Savings per Year SDM < EVDM (17,755 TPs)	
Participant	Lower Limit	Upper Limit	Lower Limit	Upper Limit
Generator	−\$51,998,269.26	−\$46,565,409.18	\$101,386,692.20	\$106,597,425.85
Consumer	\$47,935,816.68	\$53,486,681.48	−\$108,032,738.40	−\$102,716,346.70
ISO	−\$1,462,547.78	−\$1,057,539.69	\$1,668,855.14	\$1,932,540.52
Net	\$70,945.62	\$267,786.64	\$340,262.58	\$496,166.03

The expected payoff numbers illustrate that whether the pre-commitment setpoint from SDM or EVDM is larger has a significant impact on the payoffs to generators and consumers, with consumers standing to benefit heavily from more pre-commitment and generators standing to benefit heavily from less pre-commitment. Consequently, consumers have a vested interest in the pre-commitment setpoint magnitude under SDM being greater than expected demand, and generators have a vested interest in the converse.

Table 6 depicts the payoff numbers for the 7 gentailers who own generation units in the NZEM, broken down by whether the pre-commitment setpoint magnitude is larger under SDM or EVDM. We do not compute the payoff numbers for the experiment with K = 100, because the pre-commitment setpoints are quite similar, meaning the real-time payoffs follow the same pattern.

Table 6 Payoffs by gentailer 1 Jan 2014 to 31 Dec 2015 K = 10.

	95% CI Savings per Year SDM > EVDM (17,429 TPs)		95% CI Savings per Year SDM < EVDM (17,558 TPs)	
Generator	Lower Limit	Upper Limit	Lower Limit	Upper Limit
Contact Energy	−\$2,250,872.73	−\$1,997,026.47	\$3,163,499.63	\$3,419,114.77
Genesis Energy	−\$2,558,880.89	−\$2,255,615.11	\$3,469,396.43	\$3,739,733.17
Meridian Energy	−\$3,584,718.82	−\$3,176,249.18	\$4,360,564.41	\$4,721,803.59
MRP	−\$1,932,786.06	−\$1,707,169.14	\$2,822,589.90	\$3,038,200.50
Norske Skog	−\$117,469.65	−\$103,282.35	\$172,856.08	\$186,654.32
Todd Energy	−\$199,639.02	−\$172,836.18	\$227,688.83	\$253,059.97
Trustpower	−\$877,572.93	−\$778,417.47	\$1,216,686.33	\$1,309,347.27

5 Risk-averse theoretical results

To date, we have considered energy-only markets where the participants are risk-neutral price-takers. This approach is justified when participants are dispatched under SLP for a large number of independent trade periods, as then the Law of Large Numbers implies they are concerned with their expected payoff, irrespective of fluctuations in particular trade periods. However, Proposition 4 illustrates the potential shortcomings of this approach. Suppose that generator i makes the pre-commitment decision $x_i^* > 0$. Then it incurs the loss $-r_{v,i}x_i^*$ when its real-time dispatch is less than x_i^*, has a payoff between $-r_{v,i}x_i^*$ and $r_{u,i}x_i^*$ if $x_i^* =$

$X_i^*(\omega)$, and otherwise has a strictly positive payoff. That is, generator i receives a variable amount of revenue and does not consider mitigation strategies to reduce its incurred risk. We believe that this is not always reasonable behaviour, since generator i might be risk-averse and act to decrease the variability in its payoff, particularly if, for instance, generator i is a peaker unit who usually does not participate in SLP. Therefore, in this section, we discuss stochastic dispatch in a risk-averse context.

We restrict our attention to law-invariant coherent risk measures because of their natural dual interpretation and their representability via Kusuoka's Theorem (see [28]). Our analysis resembles the analysis of risk-trading conducted by [37] and the analysis of risk-trading in hydro-thermal scheduling systems conducted by [33], although we restrict our attention to energy-only markets. This restriction allows us to exploit the properties of risk-averse newsvendors established by authors including [18], [41], [13], [12] and particularly [14].

Our analysis also resembles that of the working paper [16], which extends the analysis of [37] to multiple stages and situations where Arrow-Debreu securities can only be traded in a proper subset of $\mathbb{R}^{|\Omega|}$. However, the authors of [16] take the first stage to correspond to a capacity investment decision rather than a forward market, and do not explicitly consider the relationship between risk-aversion and pre-commitment.

We proceed to remind the reader of the definitions of (1) a coherent risk measure and (2) the risk-neutral competitive equilibrium which corresponds to the social plan implemented by SLP. These definitions allow us to easily translate the risk-neutral equilibrium defined by SLP's Lagrangian into the following two risk-averse situations: (1) a risk-averse competitive equilibrium without risk instruments, (2) a risk-averse competitive equilibrium with a complete risk market. Comparing both situations then uncovers evidence that SLP provides incentives for risk-neutral participants to behave in a risk-averse manner, but these incentives disappear upon introducing forward contracts; a situation similar to the risk-neutral price-making oligopolistic network studied by [3], wherein introducing forward contracts provides incentives for participants to behave in a social-welfare maximizing manner.

Definition 6. A coherent risk measure $\rho : \mathcal{Z} \mapsto \mathbb{R}$ is a function which measures the risk-adjusted disbenefit of a random variable Z in a manner satisfying the four axioms defined by Artzner et al. in [5] (see [43] for a general theory).

Each coherent risk measure possesses a dual representation, which is given by:

$$\rho(Z) = \max_{\mu \in \mathcal{D}} \mathbb{E}_\mu[Z],$$

where $\mathcal{D}$ is a convex subset of probability measures [5]. We follow [33] in referring to $\mathcal{D}$ as a risk set.
By Kusuoka's Theorem (see [28], [14], or [42]) each coherent risk measure can be represented in the following mean-risk form for some risk coefficient κ and some

risk set $\mathcal{D}$:

$$\begin{aligned}\rho(Z) &= -\mathbb{E}[Z] + \kappa \sup_{\mu\in D}\int_{\beta=0}^{\beta=1}\frac{1}{\beta}r_\beta[Z]\mu d\beta,\\ &= -\kappa \sup_{\mu\in D}\int_{\beta=0}^{\beta=1}\mathrm{CVaR}_\beta[Z]\mu d\beta,\end{aligned}$$

where

$$r_\beta[Z] = \min_{\eta\in\mathbb{R}}\mathbb{E}[\max((1-\beta)(\eta-Z),\beta(Z-\eta))]$$

is the weighted mean-deviation from the βth quantile,

$$\mathrm{CVaR}_\beta[Z] = -\max_{t\in\mathbb{R}}\{t-\frac{1}{\beta}\mathbb{E}[t-Z]_+\}$$

is the conditional value at risk, or average value at risk, κ is a constant which prices risk by balancing the desirability of maximizing $\mathbb{E}[Z]$ with the undesirability of fluctuations towards the left tail of Z, and $\mathcal{D}$ is a convex subset of probability measures. These definitions can be found in [38], [41] or [43]. We note that while the set $\mathcal{D}$ is uniquely defined for spectral [2] risk measures such as CVaR, it is not uniquely defined in general. Nevertheless, we can define $\mathcal{D}$ to be the minimal generating set for a given risk measure ρ; in this regard, see Remark 34 of [43].

As observed by Philpott et al. in [33], whenever the sample space Ω is finite, at least one of the worst-case probability measures in $\mathcal{D}$ is an extreme point of $\mathcal{D}$. Consequently, in a Sample Average Approximation setting, $\rho(Z)$ is equal to the optimal objective value of the following linear program:

$$\begin{aligned}\rho(Z) = \min \quad & \theta\\ \text{s.t.} \quad & \theta \geq \sum_{\omega}\mathbb{P}_{m\omega}Z_\omega,\ \forall m,\end{aligned}$$

where $\mathbb{P}_m$ is the measure which corresponds to the mth extreme point of $\mathcal{D}$.

5.1 Risk-averse SDM without risk trading: A risked equilibrium

We begin by defining the risk-neutral competitive equilibrium which corresponds to SLP, as it can easily be elicited from SLP's Lagrangian. This serves to introduce notation and allows for a straightforward conversion to a risk-averse competitive equilibrium.

We consider a market of risk-neutral agents, each of whom maximize their expected profit under the probability measure $\mathbb{P}(\omega)$. Throughout this section, we assume that SLP satisfies a constraint qualification, such as Slater's constraint qualification; in this regard, see Section 5.2.3 of [10]. We also assume that each scenario occurs with a strictly positive probability. Indeed, since Ω contains a finite number of scenarios and SLP provides complete recourse, we can ignore all other scenarios without loss of optimality.

Taking the Lagrangian of SLP (see [15]) and decoupling by generation agent allows us to elicit the problem RNP(i) solved by each generation agent i, the problem PISO(ω) solved by the ISO in each scenario ω, and the market clearing condition. We formalize this observation in the following definitions:

Definition 7. Given prices $\lambda_{j(i)}(\omega)$ in each scenario ω, each risk-neutral generation agent i determines its own dispatch $(x^*, X^*(\omega), U^*(\omega), V^*(\omega))$ by solving the following stochastic optimization problem in order to maximize its expected profit:

$$\begin{aligned}\text{RNP}(i): \ \max \sum_{\omega} \mathbb{P}(\omega)\Big((\lambda_{j(i)}(\omega) - c_i)X_i(\omega) - r_{u,i}U_i(\omega) - r_{v,i}V_i(\omega)\Big)\\ \text{s.t. } x_i + U_i(\omega) - V_i(\omega) = X_i(\omega),\ \forall \omega \in \Omega,\\ 0 \le X_i(\omega) \le G_i(\omega),\ U_i(\omega), V_i(\omega), x_i \ge 0.\end{aligned}$$

As observed by [47], each generation agent i recovers its costs in the long-run, since it has the choice of not participating by choosing the action $(x, X, U, V) = (0, 0, 0, 0)$, which yields a payoff of 0 in each scenario. Therefore, agent i only participates when the prices $\lambda_{j(i)}(\omega)$ provide it with a non-negative expected profit.

Definition 8. Given prices $\lambda_n(\omega)$ in each scenario ω, the ISO determines the optimal distribution of branch flows throughout the network, $F^*(\omega)$, by solving the following optimization problem in order to maximize its cummulative rental, as defined by Philpott and Pritchard in [34]:

$$\begin{aligned}\text{PISO}(\omega): \ \max \sum_{n} \lambda_n(\omega)\tau_n(F(\omega))\\ \text{s.t. } F(\omega) \in \mathcal{F},\end{aligned}$$

where we write PISO(ω) rather than PISO to emphasise that the ISO's problem decouples by scenario, meaning the ISO is a wait-and-see agent which chooses the same action under any risk measure. As observed by [34], [47], the ISO recovers its costs in each scenario (i.e. achieves revenue adequacy), since it is a wait-and-see agent which can shed load by choosing the action $F(\omega) = 0,\ \forall \omega \in \Omega$ and earning a certain payoff of 0.

Definition 9. Given participant actions $(x, X(\omega), U(\omega), V(\omega), F(\omega))$ and prices $\lambda_n(\omega)$ in each scenario ω, determining whether the actions and prices constitute a risk-neutral competitive equilibrium is equivalent to determining whether $(x, X(\omega), U(\omega), V(\omega)) \in \arg\max \text{RNP}$, $F(\omega) \in \arg\max \text{PISO}(\omega)$, and whether the participants collective choices of actions and dual prices satisfy the following market clearing condition:

$$0 \le \sum_i X_i(\omega) + \tau_n(F(\omega)) - D_n(\omega) \perp \lambda_n(\omega) \ge 0,\ \forall n \in \mathcal{N},\ \forall \omega \in \Omega. \tag{1}$$

The Condition (1) requires that for each node n and each scenario ω, either supply equals demand or the price of electricity is zero.

To simplify the subsequent analysis, we follow [27] in replacing Condition (1) with a market-clearing agent. The equivalent market clearing agent's problem is defined as follows:

Definition 10. Given participant actions $(x, X(\omega), U(\omega), V(\omega), F(\omega))$, the market-clearing agent in scenario ω solves the following optimization problem to determine the optimal choice of nodal prices:

$$\begin{aligned} \mathrm{MC}(\omega):\ \max -\sum_{n} \lambda_n(\omega)\big(&\sum_{i\in T(n)} X_i(\omega) + \tau_n(F(\omega)) - D_n(\omega)\big) \\ \text{s.t. }\ &\lambda_n(\omega) \geq 0,\ \forall n \in \mathcal{N}, \end{aligned}$$

where we write $\mathrm{MC}(\omega)$ rather than MC to emphasise that the market clearing agent is a wait-and-see agent.

Observe that if supply exceeds demand at node n in scenario ω then the market clearing agent's optimal choice of nodal price is $\lambda_n(\omega) = 0$. Alternatively, if supply equals demand then the agent is free to choose any $\lambda_n(\omega) \geq 0$. Consequently, the market-clearing agent earns a payoff of 0 in each scenario.

We label the MOPEC problem defined by the collection of $\mathrm{RNP}(i)$, $\mathrm{PISO}(\omega)$ and $MC(\omega)$ as RNEQ. This definition leads us to the following result:

Proposition 11. *Let SLP satisfy a constraint qualification. If $(x, X(\omega), U(\omega), V(\omega), F(\omega))$ solves SLP then there exists a probability measure $\mathbb{P}(\omega)$ and a set of prices $\lambda_n(\omega)$, $\forall n \in \mathcal{N}$, $\forall \omega \in \Omega$ such that $(x, X(\omega), U(\omega), V(\omega), F(\omega), \mathbb{P}(\omega), \lambda_n(\omega))$ solves RNEQ.*

Proof. See Theorem 3 of [33]. □

We now consider the equivalent risk-averse equilibrium, wherein each generation agent i is endowed with a coherent risk measure ρ_i, with a view to show that the risk-averse equilibrium always admits at least one solution. Observe that since the ISO and the market clearing agent do not have first-stage actions, they perform the same action under any coherent risk measure. Therefore, we continue using the problems $\mathrm{MC}(\omega)$ and $\mathrm{PISO}(\omega)$ without loss of generality.

The risk-averse generation problem which each agent i solves is defined as follows:

Definition 12. Given prices $\lambda_i(\omega)$ in each scenario ω, generation agent i maximizes its risk-adjusted expected profit by determining the actions $(x, X(\omega), U(\omega), V(\omega))$ which solve the following stochastic optimization problem:

$$\begin{aligned} \mathrm{RAP}(i):\max\quad & \rho_i\Big((\lambda_{j(i)}(\omega) - c_i)X_i(\omega) - r_{u,i}U_i(\omega) - r_{v,i}V_i(\omega)\Big) \\ \text{s.t.}\quad & x_i + U_i(\omega) - V_i(\omega) = X_i(\omega), && \forall \omega \in \Omega, \\ & 0 \leq X_i(\omega) \leq G_i(\omega), && \forall \omega \in \Omega, \\ & U_i(\omega), V_i(\omega), x_i \geq 0, && \forall \omega \in \Omega, \end{aligned}$$

where ρ_i is a coherent risk measure. Observe that generation agent i recovers its risk-adjusted costs in the long-run, since it has the choice of not participating by choosing the action $(x, X, U, V) = (0, 0, 0, 0)$ and earning a payoff of 0 in each scenario. Therefore, agent i only chooses an alternate action when the prices $\lambda_{j(i)}(\omega)$ incentivise it to do so.

The collection of the problems PISO(ω), RAP(i) and $MC(\omega)$ then defines a risk-averse competitive equilibrium, which we refer to as RAEQ. The problem RAEQ cannot be solved directly by commercial solvers such as CPLEX or Gurobi, but it can be solved by more specialized codes such as the Extended Mathematical Programming framework; in this regard, see Appendix A of [33]. The properties of RAEQ are the focus of our study in Section 5.1. Our subsequent analysis assumes that RAEQ admits a solution, and therefore requires an existence result. To obtain this existence result, we require the following intermediate lemma:

Lemma 13. *Let generation agent i be a risk-averse price-taking generation agent endowed with the coherent risk measure ρ_i. Then agent i's optimization problem, RAP(i), has a closed, convex and bounded strategy set.*

Proof. The constraint $0 \leq X_i(\omega) \leq G_i(\omega)$ implies the optimal choice of x_i, x_i^*, satisfies the inequality $0 \leq x_i^* \leq \max_\omega\{G_i(\omega)\}$, by Proposition 3. Therefore, we can introduce the constraint $0 \leq x_i^* \leq \max_\omega\{G_i(\omega)\}$ into the problem RAP(i), without loss of optimality. The restrictions on X and x then imply that $0 \leq U_i(\omega), V_i(\omega) \leq G_i(\omega)$, since $U_i(\omega) = \max(X_i(\omega) - x_i, 0)$ and $V_i(\omega) = \max(x_i - X_i(\omega), 0)$. Therefore, the strategy space RAP(i) is bounded.

The strategy space is also closed and convex, because it is defined by the intersection of a set of linear inequality constraints. □

We also require the following assumption:

Assumption 14. The optimal choice of dual price $\lambda_n(\omega)$ is bounded from above by the Value of Lost Load, or VOLL, for all nodes n and all scenarios ω.

The above assumption is common in power system applications; for instance, the NZEM has a price cap of VOLL=\$20,000 per MWh, meaning consumers are willing to curtail their load at a marginal price of \$20,000 per MWh in the short-run. Indeed, there is a broad theory of the impact of Assumption 14 on power systems; in this regard, we refer the reader to the book [44].

We combine the above lemma and assumption to yield the following result, which is similar to a result demonstrating that a risk-averse electricity transmission expansion problem admits an equilibrium in the working paper [27]:

Theorem 15. *Suppose Assumption 14. Then RAEQ admits a solution.*

Proof. To show this result, we follow the steps of Rosen's theorem [39] in arguing that the following three conditions hold: (1) each participant's strategy set is non-empty, (2) each participant's strategy set is a closed, convex and bounded set, and (3) each participant's payoff function is concave in her strategy space, and continuous in all other participants' strategy spaces.

The Condition (1) holds, as each participant can choose the feasible action of setting all their decision variables to 0, and therefore all participants have non-empty strategy sets.

To show that the Condition (2) holds, we consider each class of agent separately. First, the problem PISO(ω)'s strategy space is the closed, convex and bounded set $\mathcal{F}$, which implies that the condition (2) holds for PISO(ω). Second, by Lemma 13, each generation agent i's strategy set is closed, convex and bounded. Finally, by Assumption 1, the market clearing agent's strategy space is a bounded set which can be seen to be closed and convex by inspection. Therefore, Condition (2) holds for all participants in RAEQ.

Condition (3) holds for all generation agents, as their decision variables are continuous and they are endowed with coherent risk measures, i.e., convex risk measures where positive homogeneity also holds (see [41]). Consequently, their payoff functions are concave with respect to maximization. Similarly, Condition (3) holds for the ISO and market-clearing agents, as they solve wait-and-see optimization problems by choosing continuous decision variables from convex strategy sets.

Therefore, it follows from Rosen's Theorem (see [39]) that RAEQ admits a solution. □

We believe that the approach of demonstrating that a competitive equilibrium exists by replacing market clearing conditions with market-clearing agents has broader applications than the setting studied in this paper. For instance, it can be used to show that the MOPEC formed by a set of risk-averse hydroelectric generation agents who do not trade risk, as studied in [33], admits at least one solution whenever the dual variables are bounded by VOLL (although there may be multiple equilibria, and these equilibria may be functions of VOLL). We elaborate on potential multiplicity of solutions to RAEQ in the following remark:

Remark 16. Existence does not imply uniqueness.
Theorem 15 shows that, with a price cap of VOLL, there exists a set of prices which clear the market when the participants are risk-averse. However, this theorem does not necessarily imply that there exists a unique set of prices which clear the market. Indeed, Abada et al. have shown that generation capacity investment models may admit multiple equilibria [1] and Gerard et al. have shown that there exists an energy-only pool market with risk averse generators which admits multiple equilibria [20]. Consequently, we cannot guarantee that RAEQ admits a unique solution. Indeed, due to the counterexample in [20], such a guarantee does not exist, at least in general.

Having shown that RAEQ admits a solution, we consider the problem RAP(i)'s first-order optimality condition, with a view to obtain insight into the relationship between x^* and $X^*(\omega)$. To do so, we need to change perspective and assume that Ω represents the true distribution of uncertainty, in order to exploit the Kusuoka representation of coherent risk measures. This change of perspective implies that the below results hold for the true distribution of uncertainty, while solutions to RAEQ constitute Sample Average Approximation (SAA), estimators of the solution for the true distribution. However, SAA estimators for variational inequalities are known to converge exponentially towards the true solution as we increase the sample size (see [46]). Therefore, the below results also hold for RAEQ, provided that our sample of the underlying distribution is sufficiently rich.

After assuming that Ω fully describes the distribution of uncertain outcomes, we also need to change our understanding of the dispatch and remuneration process. To see why this is the case, consider the auctioneer's price-setting problem MC(ω) in scenario ω, and assume that the optimal choice of dual price is $VOLL > \lambda_n^*(\omega) > 0$ for some node n. Then it is not too hard to see that the corresponding DC-load-flow constraint must be met exactly, because otherwise the unique optimal choice of nodal price would be VOLL. Therefore, we have that any $\lambda_n(\omega) \in [0, VOLL]$ is an optimal choice of dual price at this node, with all such choices providing the auctioneer with a payoff of 0. That is, the most natural remuneration scheme suggested by RAEQ provides highly redundant dual prices.

Consequently, we require a more reasonable remuneration scheme, which we obtain by assuming that participants are dispatched and remunerated in the same manner as SLP, although they may be risk-averse when making their pre-commitment decision. In this context, each generation agent i solves a risk-averse newsvendor problem to determine their pre-commitment behaviour. This observation allows us to characterize the impact of a generator's risk-aversion on their pre-commitment behaviour in the following results:

Proposition 17. *Let generator i be risk-averse and endowed with the law invariant coherent risk measure $\rho : \mathcal{Z} \mapsto \mathbb{R}$ with the following Kusuoka representation:*

$$\rho[Z] = -\mathbb{E}[Z] + \kappa \sup_{\mu \in D} \int_{\beta=0}^{\beta=1} \frac{1}{\beta} r_\beta[Z] \mu d\beta.$$

Then, ***for a given set of second stage dispatches*** *$X_i^*(\omega)$, generator i makes the pre-commitment decision x_i^*, where:*

$$x_i^* = F^{-1}_{X_i(\omega)}\Big(\frac{r_{u,i}}{(r_{u,i} + r_{v,i})(1 + \kappa(1 - \bar{\beta}))}\Big),$$
$$\bar{\beta} = \int_0^1 \mu^{RN} \beta d\beta;\ \kappa \in [0, \frac{1}{\bar{\beta}}].$$

Proof. See Appendix A.1. □

Proposition 17 indicates that as the risk coefficient $\kappa(1 - \bar{\beta})$ increases from the risk-neutral level $\kappa(1 - \bar{\beta}) = 1(1 - 1) = 0$, the pre-commitment setpoint decreases. We formalize this observation in the following corollaries to Proposition 17:

Corollary 18. *Let generator i be endowed with the coherent risk measure ρ_i. Then, for a given set of second stage dispatches $X_i^*(\omega)$, generator i makes the pre-commitment decision x_i^*, which is bounded from above by the following expression:*

$$\begin{aligned} x_i^* &= F^{-1}_{X_i(\omega)}\Big(\frac{r_{u,i}}{(r_{u,i} + r_{v,i})(1 + \kappa(1 - \bar{\beta}))}\Big), \\ &\leq F^{-1}_{X_i(\omega)}\Big(\frac{r_{u,i}}{(r_{u,i} + r_{v,i})(1 + 1(1 - 1))}\Big) = x_{i,RN}, \end{aligned}$$

where $x_{i,RN}$ is generator i's risk-neutral choice of pre-commitment decision for the set of second-stage dispatches $X_i^*(\omega)$.

Corollary 19. *Let generator i be endowed with the β-*CVaR *risk measure. Then, for a given set of second stage dispatches $X_i^*(\omega)$, we have that $\kappa = \frac{1}{\beta}$, $\bar{\beta} = \beta$, and generator i makes the pre-commitment decision to produce x_i^*, where:*

$$x_i^* = F^{-1}_{X_i(\omega)}\Big(\frac{r_{u,i}}{(r_{u,i} + r_{v,i})(1 + \frac{1}{\beta}(1 - \bar{\beta}))}\Big) = F^{-1}_{X_i(\omega)}\Big(\frac{\beta r_{u,i}}{r_{u,i} + r_{v,i}}\Big).$$

Note that this result was previously obtained in Section 2.2 *of the appendix to [12].*

The analysis in the previous sections indicates that modifying the total pre-commitment magnitude impacts the payoffs to the market participants. Consequently, a pertinent question is "what is the impact of generator risk-aversion on the generator's expected payoff?". We provide a lower bound on this quantity in the following proposition:

Proposition 20. *Let generator i's risk-aversion be represented by the risk measure ρ_i, which has a Kusuoka representation such that $\bar{\beta}_i := \int_0^1 \mu^{RN} \beta_i d\beta_i$, $\kappa_i \in [0, \frac{1}{\bar{\beta}_i}]$, and combine these quantities by defining $\alpha_i := \frac{1}{1+\kappa_i(1-\bar{\beta}_i)}$. Then generator i's expected profit is at least $(1-\alpha_i) r_{u,i} x_i^*$. This quantity is 0 if generator i is risk-neutral and positive otherwise.*

Proof. Proposition 17 shows that the quantity $\alpha_i := \frac{1}{\kappa_i - \kappa_i \bar{\beta}_i}$ summarizes the relationship between generator i's pre-commitment and its production, since x_i^* is a $\frac{\alpha_i r_{u,i}}{r_{u,i}+r_{v,i}}$ quantile of the distribution of $X_i^*(\omega)$. Therefore, $X_i(\omega)^* \le x_i^*$ with probability $\frac{\alpha_i r_{u,i}}{r_{u,i}+r_{v,i}}$. Applying Proposition 4 then reveals that generator i receives a payoff of at least $-r_{v,i} x_i^*$ with probability $\frac{\alpha_i r_{u,i}}{r_{u,i}+r_{v,i}}$, and at least $r_{u,i} x_i^*$ with probability $\frac{(1-\alpha_i) r_{u,i} + r_{v,i}}{r_{u,i}+r_{v,i}}$. Computing the expected payoff then yields the result. □

The above results suggest that generator risk-aversion results in a lower pre-commitment magnitude, moving the setpoint from the optimal risk-neutral setpoint and thereby decreasing the cumulative system welfare. Moreover, Proposition 20 indicates that generators have an incentive to behave in a risk-averse manner, as doing so increases their expected profit. One might question whether this situation can arise in a dispatch mechanism such as SDM, or whether the supply shortfalls which arise from risk aversion are confined to risk-averse competitive partial equilibria. Proposition 17 provides an answer to this query: generators can express their risk-aversion by inflating the relative magnitude of $r_{v,i}$, their marginal cost of deviating downward, in order that the auctioneer treats them as risk-averse. Much like the risk-averse partial equilibria, the dispatch policy generated by considering offers from risk-averse participants who manipulate their offers provides lower expected social welfare than the dispatch policy generated by considering risk-neutral participants.

A recent numerical study by Kazempour and Pinson [24] confirms our findings that risk-aversion negatively impacts market outcomes, by demonstrating that in a two-market stochastic equilibrium where generators are endowed with the CVaR risk criterion, generators prefer to pre-commit less generation when they are more risk-averse[2]. On a similar note, Gülpinar and Oliveira [22] recently studied stochastic dispatch models in an oligopolistic setting, and showed that risk-aversion can cause supply shortfalls in the Iberian electricity market. Moreover, they remarked that it was impossible to distinguish between legitimate risk-aversion and market collusion in this setting. This observation suggests that risk-aversion and the ensuing supply shortfalls are a real concern in electricity markets with high wind penetration.

Indeed, Proposition 20 raises questions about the viability of a stochastic single-settlement market where a small number of participants are scheduled to deviate in different scenarios, and suggests that if the participants are risk-averse then it may be preferable to continue using EVDM to avoid supply shortfalls.

[2] In this model, the total pre-commitment remains constant as generator risk-aversion increases, because the model's dispatch mechanism requires that the total pre-commitment volume equals expected demand. However, the model's forward market clearing price increases with risk-aversion (even as the expected real-time clearing price decreases). This phenomenon indicates that generators need to be provided with stronger incentives to pre-commit the same amount of generation, and therefore the generators prefer to pre-commit less generation.

Fortunately, a recent paper by Ralph and Smeers [37] provides a means to extend SDM in order to cope with risk-aversion; namely increasing the efficiency of the wholesale market by introducing an auxiliary financial market wherein generators and the ISO can trade risk[3]. If generators and the ISO are endowed with intersecting risk sets then trading Arrow-Debreu securities on an exchange causes each participant's effective risk-aversion to decrease to the least risk-averse participant's risk-aversion, leaving only the residual risk commonly called system or market risk. In this case, each generator's pre-commitment decision is equivalent to the decision made by a risk-averse system optimizer who uses the *least* risk averse agent's risk set as its own [37]. We remark that the intersecting risk set assumption is necessary in this setting; if it is violated then participants will sense nonexistent arbitrage opportunities and bet infinitely against each other, causing the corresponding risk-averse system optimization problem to be unbounded and implying that no set of prices can clear the risk market.

One might object that assuming a perfectly complete market for risk is unrealistic, because wind supply is drawn from a continuous and unknown distribution, while trading Arrow-Debreu securities requires that uncertainty is drawn from a discrete probability space. This objection is partially accurate, because there is no easy way to translate Arrow-Debreu securities into a continuous setting. However, a recent numerical study by de Maere d'Aertrycke et al. [17] demonstrates that, in a capacity expansion setting, both fully and partially liquid risk markets yield similar market outcomes to those which arise when participants trade Arrow-Debreu securities. Moreover, a subsequent numerical study [27] corroborates this finding, by demonstrating that introducing a limited number of Contracts for Difference (CFDs) into a transmission expansion model results in outcomes similar to those obtained by introducing Arrow-Debreu securities. Therefore, our consideration of a complete risk market counterfactual is justified, albeit informally, by the tendency of such counterfactuals to emulate energy-only markets wherein financial instruments such as CFDs are traded on an exchange.

5.2 Risk-averse SDM with risk trading: A risk-averse social planner

In this section, we extend our analysis from the previous section to consider a stochastic energy-only market where participants can trade Arrow-Debreu securities on an exchange. We begin by defining the market clearing problem solved to dispatch all generators at minimum risk-adjusted expected cost. Our formulation is similar to that laid out by [33], and requires the following definition and notation:

Definition 21. An Arrow-Debreu security is a contract which charges the price π_ω to receive a payoff of 1 in scenario ω. We let $W_{i\omega}$ denote the bundle of Arrow-Debreu securities held by agent i (see [37] for a general theory).

- θ_i is generator i's risk-adjusted payoff.

[3] In practise, allowing the ISO to trade risk is problematic. However, to make the following analysis tractable, we require that all market participants can trade risk, and therefore we assume that allowing the ISO to trade risk is a reasonable first approximation to an identical situation where the ISO cannot trade risk. This counterfactual is a reasonable proxy: if the generation agents can trade risk among themselves but not the ISO then a risked equilibrium exists, by Section 6.3 of [16].

- θ_k is the ISO's risk-adjusted payoff.
- $W_{i\omega}$ is the quantity of Arrow-Debreu securities purchased by generator i in scenario ω.
- $W_{k\omega}$ is the quantity of Arrow-Debreu securities purchased by the ISO in scenario ω.
- $\mathbb{P}_{im\omega}$ is the probability measure corresponding to the mth extreme point of generator i's risk set.
- $\mathbb{P}_{km\omega}$ is the probability measure corresponding to the mth extreme point of the ISO's risk set.

Assume that each generator submits the same offers as in SDM, that all generators and the ISO submit their risk sets before the market is cleared, and that the intersection of the participants' risk sets is non-empty. Then clearing the market is equivalent to minimizing cumulative risk-adjusted disutility [37], which we achieve by solving the following risk-averse stochastic linear program:

$$
\begin{aligned}
\text{RASLP:}\quad & \min \sum_i \theta_i + \theta_k \\
\text{s.t.}\quad & \theta_i \geq \sum_\omega \mathbb{P}_{im\omega}(c_i X_i(\omega) + r_{u,i}U_i(\omega) + r_{v,i}V_{i,\omega} - W_{i\omega}), && \forall i, \forall m, \\
& \theta_k + \sum_\omega \mathbb{P}_{km\omega} W_{k\omega} \geq 0, && \forall m, \\
& \sum_{i \in \mathcal{T}(n)} X_i(\omega) + \tau_n(F(\omega)) \geq D_n(\omega), && \forall \omega \in \Omega, \\
& \sum_i W_{i\omega} + W_{k\omega} = 0, && \forall \omega \in \Omega, \quad [\pi_\omega], \\
& x + U(\omega) - V(\omega) = X(\omega), && \forall \omega \in \Omega, \\
& F(\omega) \in \mathcal{F}, && \forall \omega \in \Omega, \\
& 0 \leq X(\omega) \leq G(\omega),\ U(\omega), V(\omega), x \geq 0, && \forall \omega \in \Omega,
\end{aligned}
$$

where we follow [33] in enumerating the extreme points of each generator's risk set in order to express the market-clearing problem as a single linear optimization problem, participants are remunerated for their dispatch in the same manner as SDM, and participants are remunerated with the term $W_{i\hat{\omega}} - \sum_\omega \pi_\omega W_{i\omega}$ in scenario $\hat{\omega}$ for their financial instruments as per [37]. Note that we could equivalently follow [13] and [14] in writing the market clearing problem as a mean-deviation from quantile model where we use the finiteness of Ω to write the Kusuoka representation of ρ over a (potentially exponentially many but) finite number of constraints. We remind the reader that since setting $W = 0$ is a feasible choice in RASLP, the expected cummulative social welfare in RASLP will never be worse than that in RAEQ [37]. Moreover, since SLP maximizes expected cummulative social welfare, RASLP never outperforms SLP in this regard [37].

As noted by Ralph and Smeers in [37], the dual prices of the Arrow-Debreu securities, π_ω, are equal to the system optimizer's risk-adjusted probability measure. That is, the participants submit bids and offers for Arrow-Debreu securities, and the auctioneer uses these bids and offers to infer the appropriate amount of probability mass to assign to each scenario, in order to optimally hedge against the residual market risk. This hedging procedure is a marked improvement over a central planner deciding the amount of probability mass to assign to each scenario, as risk markets provide incentives for participants to (1) take on risk by purchasing Arrow-Debreu securities and (2) invest in order to improve the quality of the ensemble forecast, while central-plan based systems provide neither of these incentives. Moreover, the equivalence between the dual prices and the worst-case probability measure allows us to rewrite the system optimization objective function in the following form, where $\mathcal{D}$ is the intersection of the risk sets of the generators and the ISO:

$$\min_{x,u,v} \max_{\mu \in \mathcal{D}} \mathbb{E}_\mu[c^\top X(\omega) + r_u^\top U(\omega) + r_v^\top V(\omega)],$$

which is equivalent to the following objective:

$$\min_{x,u,v} \rho(c^\top X(\omega) + r_u^\top U(\omega) + r_v^\top V(\omega)),$$

where ρ is the coherent risk measure coupled with its dual risk set $\mathcal{D}$: a set which corresponds to the residual risk which no generation agent is willing to take on and the system cannot hedge against. If $\mathcal{D} = \{\mathbb{P}(\omega)\}$ then (1) there exists a risk-neutral agent which absorbs all risk in the market, (2) the Arrow-Debreu securities are priced at $\mathbb{P}(\omega)$, and (3) there is no residual system risk (see [37] or [33]).

Our main interest in this paper is determining the impact of the existence of financial instruments on the pre-commitment setpoint. Consequently, to exploit the Kusuoka representation of a coherent risk measure, we change our perspective and assume that Ω represents the true distribution of uncertainty. Strictly speaking, the optimal solution to RASLP constitutes a Sample Average Approximation, or SAA, estimator of the optimal solution for the true distribution, and therefore the reader may wonder whether the following theorems apply to SAA estimators. However, SAA estimators are known to converge almost surely to the optimal solution for the underlying distribution (see [43]). Therefore, the below results hold almost surely true for solutions to RASLP, wherever the sample of the underlying distribution is sufficiently rich.

We proceed to consider the system optimization's first-order optimality condition with respect to each generator i. As we are considering a system optimization problem rather than an individual generator's problem, our objective is risk-adjusted expected fuel cost minimization rather than risk-adjusted expected profit maximization, and we are therefore risk-averse to scenarios with high fuel plus deviation costs rather than scenarios with low nodal prices. This observation leads to the following proposition, which is similar to Proposition 17:

Proposition 22. *Suppose that the system is risk-averse and endowed with the law invariant coherent risk measure $\rho : \mathcal{Z} \mapsto \mathbb{R}$, which has the Kusuoka representation:*

$$\rho[Z] = -\mathbb{E}[Z] + \kappa \sup_{\mu \in D} \int_{\beta=0}^{\beta=1} \frac{1}{\beta} r_\beta[Z] \mu d\beta.$$

Then, for a given dispatch policy $X^*(\omega)$, each generator makes the pre-commitment decision to produce x_i^*, where:

$$x_i^* = F^{-1}_{X_i(\omega)}\Big(\frac{r_{u,i} + (r_{u,i} + r_{v,i})(\kappa - \kappa\bar{\beta})}{(r_{u,i} + r_{v,i})(1 + \kappa(1 - \bar{\beta}))}\Big),$$
$$\bar{\beta} = \int_0^1 \mu^{RN}\beta d\beta,\ \kappa \in [0, \frac{1}{\bar{\beta}}].$$

Proof. See Appendix A.2. □

We remind the reader that the dispatch policy $X^*(\omega)$ obtained from RASLP and used in Proposition 22 is not necessarily the same dispatch policy as that obtained from RAEQ and used in Proposition 17. In particular, both dispatch policies are functions of (1) the problem data and (2) their (respective and possibly different) pre-commitment setpoints. Consequently, when the market participants can trade Arrow-Debreu securities we can readily construct a competitive equilibrium by solving RASLP, while when the participants cannot trade Arrow-Debreu securities we must instead formulate and solve a Multiple Optimization Problem with Equilibrium Constraints, or MOPEC, using more specialized codes such as EMP; in this regard, see Appendix A of [33].

Proposition 22 indicates that although generator risk-aversion causes pre-commitment shortfalls in the absence of financial instruments, it also results in excess pre-commitment with a complete market for risk. Moreover, the amount of excess pre-commitment is dictated by the residual system risk coefficient $(\kappa - \kappa\bar{\beta})$. We formalize this observation in the following corollaries to Proposition 22:

Corollary 23. *Let the system be endowed with the coherent risk measure ρ. Then, for a given second stage dispatch policy $X(\omega)$, each generator makes the pre-commitment decision x_i^*, which is bounded from below by the following expression:*

$$\begin{aligned} x_i^* &= F^{-1}_{X_i(\omega)}\Big(\frac{r_{u,i} + (r_{u,i} + r_{v,i})(\kappa - \kappa\bar{\beta})}{(r_{u,i} + r_{v,i})(1 + \kappa(1 - \bar{\beta}))}\Big), \\ &\geq F^{-1}_{X_i(\omega)}\Big(\frac{r_{u,i} + (r_{u,i} + r_{v,i})(1(1-1))}{(r_{u,i} + r_{v,i})(1 + 1(1-1))}\Big) = x_{i,RN}, \end{aligned}$$

where $x_{i,RN}$ is generator i's risk-neutral pre-commitment decision.

Corollary 24. *Let the system be endowed with the β-*CVaR *risk measure. Then, for a given second stage dispatch policy $X(\omega)$, we have that $\kappa = \dfrac{1}{\beta}$, $\bar{\beta} = \beta$, and each agent makes the pre-commitment decision x_i^*, where:*

$$x_i^* = F^{-1}_{X_i(\omega)}\Big(\frac{r_{u,i} + (r_{u,i} + r_{v,i})(\frac{1}{\bar{\beta}} - \kappa\bar{\beta})}{(r_{u,i} + r_{v,i})(1 + \frac{1}{\bar{\beta}}(1 - \bar{\beta}))}\Big) = F^{-1}_{X_i(\omega)}\Big(\frac{r_{u,i} + r_{v,i}(1 - \beta)}{r_{u,i} + r_{v,i}}\Big).$$

This result is similar to Corollary 19, except greater residual system risk results in more, not less, pre-commitment.

We remind the reader that the second-stage dispatches from SLP and RASLP are distinct in general, meaning we cannot make a direct comparison between x_i^* and $x_{i,RN}$. Corollaries 23-24 do however apply to situations where the least risk-averse participant's behaviour is sufficiently close to risk-neutrality that the second-stage dispatches under SLP and RASLP are identical. Consequently, the above corollaries can be thought of as risk-averse sensitivity analysis results.

The analysis in the previous sections suggests that increasing the total amount of pre-commitment decreases expected nodal prices. Consequently, a pertinent question is "does an auxiliary risk market remove the positive relationship between a generator's risk-aversion and its expected payoff, as laid out in Proposition 20?". The answer is in the affirmative, as we demonstrate in the following proposition:

Proposition 25. *Let the system's risk-aversion be represented by the risk measure ρ, which has a Kusuoka representation such that $\bar{\beta} := \int_0^1 \mu^{RN} \beta d\beta$, $\kappa \in [0, \frac{1}{\bar{\beta}}]$, and combine these two quantities by defining $\alpha := \frac{1}{1+\kappa(1-\bar{\beta})}$. Then generator i's expected profit is at least $-(1-\alpha) r_{v,i} x_i^*$. This quantity is 0 when the least risk-averse agent is risk-neutral and is negative otherwise.*

Proof. Proposition 22 shows that the quantity $\alpha_i := \frac{1}{\kappa_i - \kappa_i \beta_i}$ summarizes the relationship between generation agent i's pre-commitment and its production, since x_i^* is a $\frac{r_{u,i}+(1-\alpha_i) r_{u,i}}{r_{u,i}+r_{v,i}}$ quantile of the distribution of $X_i^*(\omega)$. Therefore, $X_i(\omega)^* \leq x_i^*$ with probability $\frac{r_{u,i}+(1-\alpha_i) r_{u,i}}{r_{u,i}+r_{v,i}}$. Applying Proposition 4 then reveals that each generation agent i receives a payoff of at least $-r_{v,i} x_i^*$ with probability $\frac{r_{u,i}+(1-\alpha) r_{v,i}}{r_{u,i}+r_{v,i}}$ and receives a payoff of at least $r_{u,i} x_i^*$ with probability $\frac{\alpha r_{v,i}}{r_{u,i}+r_{v,i}}$. Computing the expected payoff then yields the result. □

The above analysis might appear to suggest that expected cost-recovery is not guaranteed in RASLP. However, this is not the case, as the above analysis does not include payoffs from the auxiliary risk market. Moreover, by comparison with the feasible choice of non-participation in both markets, which has a certain payoff of 0 under any coherent risk measure, we can show that risk-averse generators recover their risk-adjusted costs *in expectation.* Therefore, by the standard minimax inequality, risk-averse generation agents must also recover their risk-neutral costs *in expectation.* However, profits from the auxiliary risk market are derived by assuming risk, unlike the situation described in Proposition 20. Consequently, the wealth transfers from consumers to generators observed in Section 4 may be mitigated by completing a market for risk, particularly when generators are risk-averse. Moreover, it may be possible to distinguish between *genuine* risk-aversion and gaming in the presence of financial instruments.

We proceed to compare the risk-hedging strategy studied in this paper with an alternative risk-hedging strategy recently proposed by the authors in [15] in the following remark:

Remark 26. A second way to hedge risk: A Price of Information
The payment mechanism studied in this paper comprises paying each generator i the term $\lambda_{j(i)}(\omega) X_i(\omega)$ in scenario ω. A second payment mechanism, as suggested by the authors in [15], comprises paying each generator i the term $\lambda_{j(i)}(\omega) X_i(\omega) - \rho_i(\omega) x_i$ in scenario ω, where $\rho_i(\omega)$ is the dual multiplier on generator i's nonanticipativity constraint and the term $x_i \rho_i(\omega)$ constitutes an ex-post information payment (charge) in scenarios where agent i incurs a loss (makes

a profit)[4]. This payment mechanism provides (1) cost recovery in every scenario, and (2) expected revenue equivalence with the payment mechanism studied here [15]. Moreover, it can be shown that compensating participants according to this payment mechanism results in agent i receiving a certain payoff of 0 unless $X_i(\omega) = G_i$, in which case agent i earns a non-negative payoff. It is not too hard to see that compensating participants in this manner mitigates the impact of risk-aversion on the dispatch, since generators do not improve their worst-case payoff by deviating from their optimal risk-neutral pre-commitment decision. Moreover, this payment mechanism translates naturally to a Sample Average Approximation setting, while Arrow-Debreu securities do not (although we have argued in this paper that forward contracts can perform similar roles). However, charging an ex-post information rent is equivalent to a central planner managing risk on behalf of the participants, while managing risk aversion via a risk market is equivalent to providing an exchange mechanism through which the market participants can hedge risk among themselves. This issue is of significance, because markets are thought to provide better long-run incentives than central planners (see, e.g., [44]). Market designers therefore need to choose between mechanisms which provide expedient risk-hedging and mechanisms which provide more complex risk-hedging but also provide perhaps better long-run incentives.

6 Risk-averse numerical results

The theoretical results in the previous section indicate that, for a given set of real-time dispatches, if generators can trade risk via contracts then their pre-commitment setpoint depends on the market risk and becomes higher as the market risk increases. One might be tempted to infer that as the market risk increases, each generator's pre-commitment also increases. However, this is not necessarily the case. Indeed, a more risk-averse system might procure generation from more flexible but more expensive sources in order to hedge against low wind scenarios, in which case cheaper and less flexible generation units may pre-commit less generation. In this section, we demonstrate that this situation arises in the six-node network introduced by Pritchard et al. [36] and studied by [48], [15]. We use the problem data described in [15] but reproduce it here in order that this paper is self-containing.

6.1 A six node example under the CVaR criterion

Consider a transmission network with the topology depicted in Figure 1, which comprises two inflexible thermal generators who cannot ramp up or down in the second stage, two flexible hydro generators who can ramp up or down at marginal costs of 35 and 20, and two intermittent wind generators who can ramp up or down without incurring a deviation cost. The output capacity of each generator and their marginal cost of generation are indicated by the notation "X@$Y". The

[4] In the paper [15] the payment mechanism is stated differently, because the marginal cost term c_i is assigned to the first-stage decision in [15] and to the second-stage decision here. However, by repeating the analysis in [15] with the cost term assigned to the second-stage decision, we can show that the two payment mechanisms are equivalent.

two wind generators independently draw an output capacity from the sample space $\{30, 50, 60, 70, 90\}$ with each realization occurring with equal probability, resulting in 25 scenarios each having probability 0.04. There is a single deterministic and inflexible consumer who requires 264 units of generation in each scenario, and a transmission constraint dictates that up to 150 units can be transmitted from node A to node B or vice versa. The lines are assumed to obey DC load-flow constraints imposed by Kirchhoff's voltage laws and have equal reactances, meaning $\frac{1}{6}$ of the power generated by the hydro generators flows via the constrained line, and $\frac{1}{3}$ of the power generated by Wind 2 flows via the constrained line. We prevent dual degeneracy by imposing quadratic losses on all transmission lines, with a loss coefficient of 10^{-8}. The market is cleared by minimizing the risk-adjusted fuel plus deviation cost, where the system optimizer is endowed with the β-CVaR risk measure for several different values of β.

Fig. 1 The six node example studied in [36], [48] and [15].

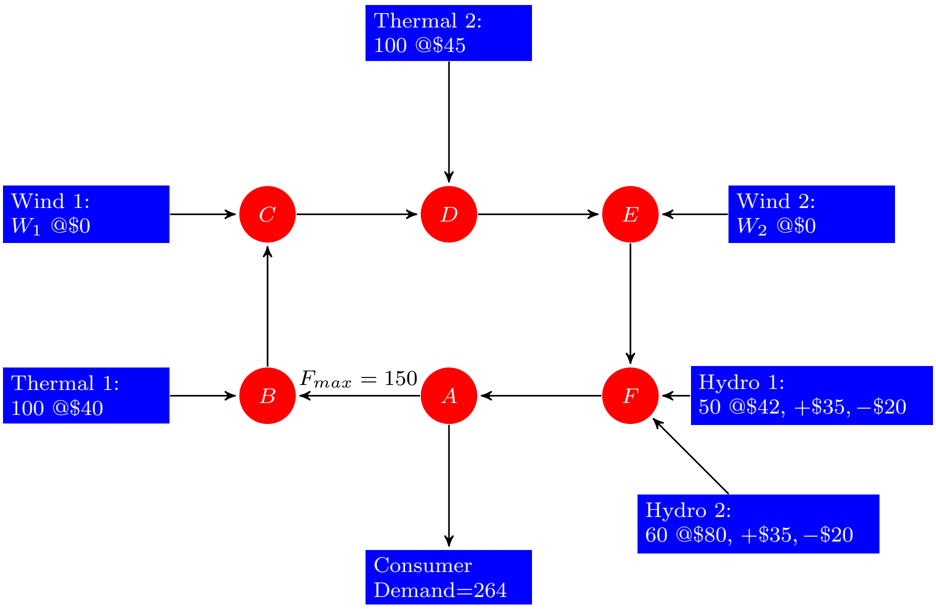

Table 7 depicts the pre-commitment behaviour of each deterministic generator as the market becomes more risk-averse, which corresponds to the least risk-averse generation agent in the market becoming more risk-averse. Observe that the system never cumulatively pre-commits less generation with higher residual market risk. However, the distribution of second-stage dispatches changes with market risk, causing Hydro 1 to pre-commit less generation at moderate risk-aversion levels than when the system is risk-neutral. This observation justifies our previous remark that increased system risk does not cause each generator's pre-commitment to increase.

Table 7 The pre-commitment behaviour of each deterministic generator in Figure 1 by risk.

Gen	β coefficient in CVaR ($\beta = 0$ is risk-neutral)									
(MW)	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
Thermal 1	79	94	84	79	79	79	74	74	76.5	74
Thermal 2	75	40	50	55	55	45	50	50	45	40
Hydro 1	50	50	50	40	40	42.5	40	40	37.5	40
Hydro 2	0	0	0	0	0	0	0	0	0	0
Cumulative	204	184	184	174	174	166.5	164	164	159	154

The previous section demonstrates that increased pre-commitment decreases expected nodal prices. Combining this finding with Table 7 suggests that forward contracts induce lower expected nodal prices in the presence of generator risk-aversion. The reader may appreciate that this result is similar to the seminal result of Allaz and Vila in [3], who show that forward contracts lower nodal prices in an oligopolistic setting where risk-neutral participants exercise market power.

6.2 Risk-averse SDM in the NZEM

To see how a fully liquid risk market impacts generator pre-commitment, we modify SDM's objective function to a β-CVaR risk measure for ten different values of β (by an abuse of notation we refer to a worst-case risk measure as $\beta = 1$, although we implement it via a minimax LP) and solve for the pre-commitment policies for the first 7 days of 2014.

Table 8 depicts the additional pre-commitment procured for ten distinct values of β. For the 335 trade periods considered, enlarging the system risk set never results in a smaller amount of generation being procured, which is consistent with our findings in Proposition 22 and Corollary 24. Consequently, by our numerical results, residual market risk decreases expected real-time nodal prices and benefits consumers at the expense of generators in the NZEM. This is the opposite situation to when no risk-trading instruments exists. When computing Table 8, we omit trade period 34 of 2 January, because line congestion makes the problem infeasible in some scenarios, making the worst-case optimal penalty violation cost non-zero and meaning the equality $x + U - V = X$ is violated when $\beta = 1$.

Table 8 Additional pre-commitment procured by a risk-averse planner, 1-7 Jan 2014, K = 10.

Gen	β coefficient in CVaR ($\beta = 1$ is a worst-case risk measure)									
(MW)	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
Max	397.01	335.43	234.36	187.11	136.09	101.41	78.62	60.04	39.68	21.77
3rd Qu.	175.22	125.82	96.25	77.92	62.50	49.54	37.86	29.51	19.00	9.78
Mean	137.28	103.37	78.76	63.13	49.54	38.22	29.48	21.80	14.57	7.20
Median	108.15	77.77	59.60	47.38	38.67	30.03	23.98	17.93	11.89	6.05
1st Qu.	78.35	59.16	45.11	35.31	28.61	22.48	17.81	13.30	9.18	4.37
Min	18.13	3.08	1.86	0.00	0.00	0.00	0.00	0.00	0.00	0.00

To determine the impact of system risk-aversion on real-time nodal prices, we back-test the risk-averse pre-commitment policies on realized wind generation, in the same manner as our risk-neutral experiments in Section 4.

Tables 9 and 10 depict the real-time nodal prices at Haywards (HAY2201) and Benmore (BEN2201); the two reference nodes on either end of the High Voltage Direct Current, or HVDC, cable which connects New Zealand's two main islands. Observe that although additional market risk does not depress nodal prices in each trade period, it does depress expected nodal prices.

Table 9 Haywards nodal prices with a risk-averse planner, 1-7 Jan 2014, K = 10.

Price	β coefficient in CVaR ($\beta = 1$ is a worst-case risk measure)									
($ per MWh)	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
Max	71.77	71.77	71.77	71.77	71.77	71.77	71.77	71.77	71.77	71.77
3rd Qu.	42.45	42.45	42.44	42.45	42.44	42.470	42.49	42.50	42.50	42.50
Mean	29.46	29.47	29.51	29.56	29.59	29.66	29.74	29.84	29.88	29.94
Median	35.06	35.06	34.76	35.06	34.76	36.03	36.01	36.03	36.01	36.19
1st Qu.	4.47	4.47	4.47	4.47	4.47	4.47	4.47	4.48	4.60	4.60
Min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 10 Benmore nodal prices with a risk-averse planner, 1-7 Jan 2014, K = 10.

Price	β coefficient in CVaR ($\beta = 1$ is a worst-case risk measure)									
($ per MWh)	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
Max	66.02	66.02	66.02	66.02	66.02	66.02	66.02	66.03	66.02	66.02
3rd Qu.	40.04	40.03	40.03	40.04	40.03	40.06	40.07	40.07	40.07	40.07
Mean	27.52	27.56	27.58	27.62	27.66	27.72	27.79	27.89	27.93	27.98
Median	33.10	32.71	32.71	33.10	32.71	34.02	33.99	34.02	33.99	34.17
1st Qu.	4.11	4.11	4.11	4.11	4.11	4.11	4.11	4.12	4.33	4.32
Min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

7 Conclusions

Our results demonstrate that generator risk-aversion tends to result in less pre-commitment and higher expected nodal prices; an outcome which causes higher consumer costs and higher generator profits. Alternatively, introducing a complete set of forward contracts results in more pre-commitment and depressed expected nodal prices; a result which causes better consumer outcomes, worse generator outcomes and improved system efficiency in the presence of generator risk-aversion. Taken together, these results indicate that forward contracts perform similar roles in energy-only markets with risk-averse price-takers and the oligopolistic network studied by Allaz and Vila in [3]. We believe that similar results can be obtained with minor extensions to our model, such as multiple pre-commitment markets or quadratic deviation costs. However, we caution that forward contracts may not depress expected nodal prices with, for instance, the nonconvexities induced by unit commitment. Indeed, Murphy and Smeers have demonstrated in [30] that forward contracts do not depress nodal prices in the oligopolistic market of [3] when convexity is violated by introducing an investment capacity stage. Consequently, generator risk-trading may not be favourable for consumers in the presence of unit commitment.

Our risk-averse analysis can be applied to two-stage capacity investment problems, such as those studied by authors including [19], [1], [17] and [16]. Indeed, capacity investment is a special case of pre-commitment where the marginal cost of deviating upwards is infinite, and results pertaining to risk-averse newsvendors who cannot backorder can therefore be applied to yield capacity-investment analogues of the results derived in this paper; in this regard we refer the reader to [14].

Our results also suggest that wind power futures may be necessary to account for the market imperfections introduced by intermittent generation, particularly in the presence of risk aversion. This observation was previously made by Gersema and Wozabal in [21], who arrived at a similar conclusion by considering a two-agent equilibrium model with intermittency. Moreover, as noted by [21], the European Energy Exchange (EEX) recently introduced wind power futures to mitigate the market imperfections introduced by intermittency. This action provides further evidence that wind futures may be necessary to mitigate market imperfections.

Appendix A Proofs of Propositions

A.1 Proof of Proposition 17

To show this result, we model an arbitrary generator as a risk-averse newsvendor by using a combination of the notation in Section 2.2 of the appendix to [12] and the notation in [14], and we convert to the notation used in the main body of this paper ex-post. This choice maintains consistency with the newsvendor literature, because conventional newsvendor models assume the cost of stocking a product is incurred in the first stage, whilst we assume that the cost of stocking a product is incurred in the second stage, and modify our deviation costs accordingly. Our approach can be viewed as a generalization of that taken in Section 5 of [14], as we include the possibility that newsvendors might back-order in the second stage and incur an additional cost for doing so (i.e. ramp up their plant's production at a marginal cost of $c_i + r_{u,i}$), while the analysis conducted by Choi et al. in [14] precludes the possibility that $X_i^*(\omega) > x_i^*$.

We require the following terms:

- e is the marginal emergency order cost.
- s is the marginal salvage value.
- p is the marginal sale price.
- c is the marginal ordering cost.
- x is the initial order quantity.
- D is the stochastic demand.
- $y_+ = \max(y, 0)$ is the positive component of y.
- $\Pi(x, D)$ is the newsvendor's profit with initial stock x and demand D.
- ρ is a law-invariant coherent risk measure.

We follow [12] in defining the newsvendor's profit function as:

$$\begin{aligned}\Pi(x, D) &= pD - cx + s(x-D)_+ - e(D-x)_+,\\ &= (p-e)D + (e-c)x - (e-s)(x-D)_+,\\ &= (e-c)x + Z_+.\end{aligned}$$

Using Definition 6, we invoke Kusuoka's Theorem (see [28]) to represent the law-invariant coherent risk measure ρ via the following expression:

$$\rho[Z] = -\mathbb{E}[Z] + \kappa \sup_{\mu \in D} \int_{\beta=0}^{\beta=1} \frac{1}{\beta} r_\beta[Z] \mu d\beta,$$

where $r_\beta[Z] = \min_{\eta \in \mathbb{R}} \mathbb{E}[\max((1-\beta)(\eta - Z), \beta(Z-\eta))] = \beta(\mathrm{CVaR}_\beta[Z] + \mathbb{E}[Z])$.

The above expression permits a representation of the newsvendor's risk-adjusted profit via the following function:

$$\begin{aligned}\rho(\Pi(x, D)) &= -\mathbb{E}[\Pi(x, D)] + \kappa \sup_{\mu \in D} \int_{\beta=0}^{\beta=1} \frac{1}{\beta} r_\beta[\Pi(x, D)] \mu d\beta,\\ &= -(e-c)x - \mathbb{E}[Z_+] + \kappa \sup_{\mu \in D} \int_{\beta=0}^{\beta=1} \frac{1}{\beta} r_\beta[Z_+] \mu d\beta,\end{aligned}$$

as $(e-c)x$ is invariant and $r_\beta[Z+a] = r_\beta[Z]$ for nonrandom a. Moving $\mathbb{E}[Z_+]$ within the integral and using the substitution
$r_\beta[Z] = \beta(\mathrm{CVaR}_\beta[Z] + \mathbb{E}[Z])$, provides the following expression:

$$\rho(\Pi(x,D)) = -(e-c)x + \sup_{\mu \in D} \int_{\beta=0}^{\beta=1} \Big(\mathbb{E}[Z_+](\kappa\beta - 1) + \kappa\beta \mathrm{CVaR}_\beta[Z_+]\Big)\mu d\beta.$$

Observe that the β quantile of Z_+ must be lower than in the risk-neutral case. In the risk-neutral case, the optimal choice of x is the $\frac{c-s}{e-s}$ quantile of D (see Proposition 3 or [18]), which corresponds to equality between the βth quantile of Z_+ and $(p-s)D$. In the risk-averse case, the βth quantile of Z_+ is (equal or) lower and is therefore equal to $(p-s)D-(e-s)x$ for some x and some D. This observation allows us to define the partial derivatives of the expectation and CVaR terms within $\rho(\Pi(x,D)$ as follows:

$$\begin{aligned}
\frac{\partial \mathbb{E}[Z_+]}{\partial x} &= -(e-s)\mathbb{P}(x > D), \\
\frac{\partial \mathrm{CVaR}_\beta[Z_+]}{\partial x} &= -\frac{\partial}{\partial x}\Big\{(p-s)D - (e-s)x - \frac{1}{\beta}\mathbb{E}[(p-s)D - (e-s)x - Z_+]\Big\}, \\
&= (e-s) - \frac{1}{\beta}(e-s) + \frac{1}{\beta}(e-s)\mathbb{P}(x > D), \\
&= (e-s)(1-\frac{1}{\beta}) + \mathbb{P}(x > D)(e-s)\frac{1}{\beta}.
\end{aligned}$$

Now, assume that the supremum over the risk set D is uniquely attained at the measure $\hat{\mu}$; then we have the following first-order optimality condition:

$$\begin{aligned}
\frac{\partial \rho(\Pi(x,D))}{\partial x} &= -(e-c) \\
&+ \int_{\beta=0}^{\beta=1} \Big((e-s)(1+\kappa-\kappa\beta)\mathbb{P}(x > D) - \kappa(e-s)(1-\beta)\Big)\hat{\mu} d\beta, \\
&= -(e-c) + \mathbb{P}(x > D)(e-s)\Big(1 + \kappa - \kappa\big(\int_{\beta=0}^{\beta=1} \beta\hat{\mu} d\beta\big)\Big) \\
&- \kappa(e-s)(1 - (\int_{\beta=0}^{\beta=1} \beta\hat{\mu} d\beta)).
\end{aligned}$$

Setting this condition to 0 and re-arranging for $\mathbb{P}(x > D)$ yields:

$$\mathbb{P}(x > D) = \frac{(e-c) + \kappa(e-s)(1-\bar{\beta})}{(e-s)(1+\kappa-\kappa\bar{\beta})},$$

where $\bar{\beta} = \int_0^1 \mu^{RN}\beta d\beta$ is the expected value of the risk-averse probabilities with respect to the risk-neutral probabilities.

Netting against 1 to find $\mathbb{P}(x \leq D)$ then yields:

$$\mathbb{P}(x \leq D) = \frac{(c-s)}{(e-s)(1+\kappa-\kappa\bar{\beta})}.$$

To convert to our notation, observe that $r_u = c-s$ and $r_v = e-c$, giving
$r_u + r_v = e - s$. Therefore, we have that:

$$\mathbb{P}(x \leq D) = \frac{r_u}{(r_u + r_v)(1+\kappa-\kappa\bar{\beta})},$$

as required. Note that the corresponding quantile is not necessarily unique. □

A.2 Proof of Proposition 22

To show this result, we invoke the observation made by [47] that for a given set of second-stage dispatches $X^*(\omega)$, the system solves a newsvendor problem in order to determine the pre-commitment setpoint which minimizes the term

$$\mathbb{E}[r_{u,i}U_i(\omega) + r_{v,i}V_i(\omega)]$$

for each generator i. Consequently, we use the same notation as in the proof of Proposition 19. We require $p = 0$, as we are considering a system optimization problem and any revenue accrued by a generator is provided by the ISO. Therefore, the system's residual cost with respect to a particular generator's pre-commitment decision, $\Pi(x, D)$, is defined by the following expression:

$$\begin{aligned}
\Pi(x, D) &= -cx + s(x - D)_+ - e(D - x)_+,\\
&= -eD + (e - c)x - (e - s)(x - D)_+,\\
&= (e - c)x + Z_+.
\end{aligned}$$

By following the steps outlined in Appendix A.1, we obtain the following risk-adjusted profit function:

$$\rho(\Pi(x, D)) = -(e - c)x - \mathbb{E}[Z_+] + \kappa \sup_{\mu \in D} \int_{\beta=0}^{\beta=1} \frac{1}{\beta} r_\beta[Z_+]\mu d\beta.$$

Now observe that since $p = 0$, the risk-neutral critical fractile becomes $-eD$. Since $s(x-D)-ex$ gives a lower system cost than $-eD$ we therefore have that the critical fractile within each CVaR term becomes equal to $-eD$. This situation is similar to Section 5.3 of [14], although we include emergency holding costs. Consequently, the partial derivatives of the terms which constitute ρ become:

$$\begin{aligned}
\frac{\partial \mathbb{E}[Z_+]}{\partial x} &= -(e - s)\mathbb{P}(x > D),\\
\frac{\partial \mathrm{CVaR}_\beta[Z_+]}{\partial x} &= -\frac{\partial}{\partial x}\Big\{ - eD - \frac{1}{\beta}\mathbb{E}[-eD - Z_+]\Big\} = \frac{1}{\beta}(e - s)\mathbb{P}(x > D).
\end{aligned}$$

Now assume that $\mu = \hat{\mu}$ is the unique optimal pdf. Then, we have the following first-order condition:

$$\begin{aligned}
\frac{\partial \rho(\Pi(x, D))}{\partial x} &= -(e - c) + \int_{\beta=0}^{\beta=1} \Big((e - s)(1 + \kappa - \kappa\beta)\mathbb{P}(x > D)\Big)\hat{\mu} d\beta,\\
&= -(e - c) + \mathbb{P}(x > D)(e - s)\Big(1 + \kappa - \kappa \int_{\beta=0}^{\beta=1} \beta\hat{\mu} d\beta\Big).
\end{aligned}$$

Setting this condition to 0 and re-arranging for $\mathbb{P}(x > D)$ yields:

$$\mathbb{P}(x > D) = \frac{(e - c)}{(e - s)(1 + \kappa - \kappa\bar{\beta})},$$

where $\bar{\beta} = \int_0^1 \mu^{RN}\beta d\beta$ is the expected value of the risk-averse probabilities with respect to the risk-neutral probabilities.
Netting against 1 to find $\mathbb{P}(x \leq D)$ then yields:

$$\mathbb{P}(x \leq D) = \frac{(c - s) + (e - s)(\kappa - \kappa\bar{\beta})}{(e - s)(1 + \kappa - \kappa\bar{\beta})}.$$

To convert to our notation, observe that $r_u = c - s$ and $r_v = e - c$, giving $r_u + r_v = e - s$. Therefore, we have that:

$$\mathbb{P}(x \leq D) = \frac{r_u + (r_u + r_v)(\kappa - \kappa\bar{\beta})}{(r_u + r_v)(1 + \kappa - \kappa\bar{\beta})},$$

as required. Note that the corresponding quantile is not necessarily unique. □

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