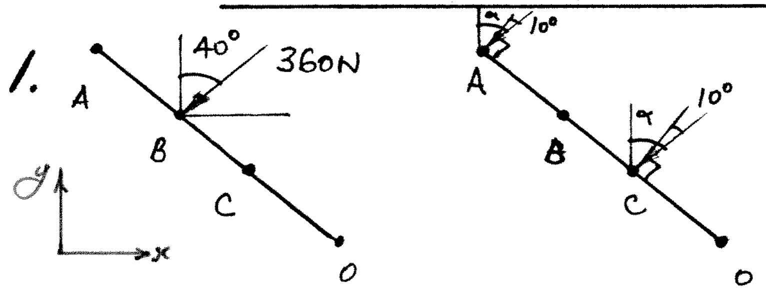
SOLUTIONS TO ASSIGNMENT No. 3

1.



For equivalence:

$\Sigma F = \Sigma F'$ or $\Sigma F_x = \Sigma F_x'$ (1), $\Sigma F_y = \Sigma F_y'$ (2)

and

$\Sigma M = \Sigma M'$ (3)

(1) $\Sigma F_x: \; -360 \sin 40° = -F_A \sin\alpha - F_C \sin\alpha$

(2) $\Sigma F_y: \; -360 \cos 40° = -F_A \cos\alpha - F_C \cos\alpha$

Divide (1) by (2):

$$\frac{\sin 40°}{\cos 40°} = \frac{-(F_A+F_C)\sin\alpha}{-(F_A+F_C)\cos\alpha}$$

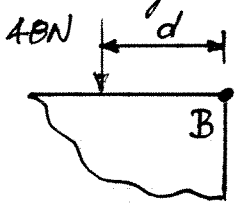
$\therefore \underline{\alpha = 40°}$ – which was actually expected.

(3) $\Sigma M_B: \quad 0 = (0.4) F_A \cos 10° - (0.35) F_C \cos 10°$ or $F_A = \frac{7}{8} F_C$.

But $F_A + F_C = 360N = \frac{7}{8}F_C + F_C$. Hence: $\underline{F_A = 168N}$ and $\underline{F_C = 192N}$

---

2. The given system has to be replaced with the following:



(a) The new force $F = 48N$, since the couple's resultant is zero.

From the given system: $\Sigma M_B = (0.4)(15)\cos 40° + (0.24)(15)\sin 40° = 6.9103\ Nm$

Also, for equivalence, on the new system:

$\Sigma M_B = F \cdot d$ or $6.9103 = 48 \cdot d \quad \therefore \underline{d} = \frac{6.9103}{48} = \underline{0.144\ m}$

(b) Similarly, by knowing $d = 0.1m$ and solving for α:

$\Sigma M_B = (0.4)(15)\cos\alpha + (0.24)(15)\sin\alpha = (0.1)(48)$

OR $\quad 5\cos\alpha + 3\sin\alpha = 4, \quad 25\cos^2\alpha = (4 - 3\sin\alpha)^2$

But $\quad \cos^2\alpha = 1 - \sin^2\alpha$, hence $25(1-\sin^2\alpha) = 16 - 24\sin\alpha + 9\sin^2\alpha$

and $\quad 34\sin^2\alpha - 24\sin\alpha - 9 = 0$

Then $\quad \sin\alpha = \frac{24 \pm \sqrt{(24)^2 - 4(34)(-9)}}{2(34)}$

$\sin_1\alpha = 0.97686 \quad \therefore \underline{\alpha_1 = 77.7°}$

~~$\sin_2\alpha = -0.27098 \quad \therefore \alpha_2 = -15.72°$~~

---

3. $d_{AJ} = \sqrt{(18)^2 + (-14)^2 + (-3)^2} = 23\ mm$, hence

$\vec{F} = \frac{46}{23}(18\vec{i} - 14\vec{j} - 3\vec{k}) = (36)\vec{i} - (28)\vec{j} - (6)\vec{k}\ N$

Also, $d_{AC} = \sqrt{(-45)^2 + (0)^2 + (-28)^2} = 53\ mm$, and

$$\therefore \vec{M} = \frac{2120}{53}(-45\vec{i} - 28\vec{k}) = -1800\vec{i} - 1120\vec{k} \text{ Nmm}$$

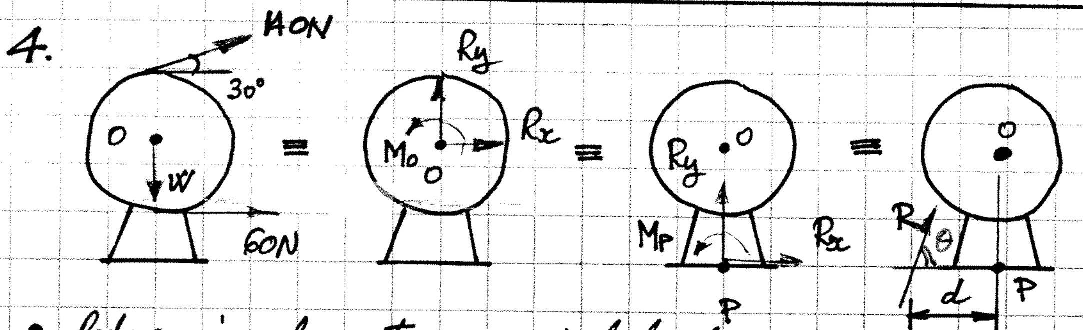
Now, $\vec{M}'_{@H} = \vec{M} + \vec{r}_{HA} \times \vec{F}$, where $\vec{r}_{HA} = 45\vec{i} + 14\vec{j}$ mm , then

$$\vec{M}' = (-1800\vec{i} - 1120\vec{k}) + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 45 & 14 & 0 \\ 36 & -28 & -6 \end{vmatrix} = (-1800\vec{i} - 1120\vec{k}) + \{[(14)(-6)]\vec{i} +$$

$$+ [(45)(-6)]\vec{j} + [(45)(-28) - (14)(36)]\vec{k}\} \text{ Nmm}$$

$$\therefore \vec{M}' = (-1.884\vec{i} + 0.27\vec{j} - 2.88\vec{k}) \text{ Nm} \quad \text{and}$$
$$\vec{F}' = (36\vec{i} - 28\vec{j} - 6\vec{k}) \text{ N}$$
} Force-Couple System @ H.

4.



$W = mg = (3.26)(9.81)$
$= 32N$

- Reduce given forces to an equivalent force-couple system at O:

ΣF_x: $140\cos 30° + 60 = R_x$ or $R_x = 181.2\text{ N} \rightarrow$

ΣF_y: $140\sin 30° - 32 = R_y$ or $R_y = 38\text{N} \uparrow$

ΣM_o: $-(140)(40) + (60)(40) = M_o$ or $M_o = -3200\text{ Nmm} = 3200\text{ Nmm}$

- Move the equivalent force-couple to P: $R_x = 181.2\text{ N}$, $R_y = 38\text{N}$ and

ΣM_P: $-3200 - (181.2)(80) = M_P$ $\therefore M_P = -17{,}696\text{ Nmm} = 17{,}696\text{ Nmm}$

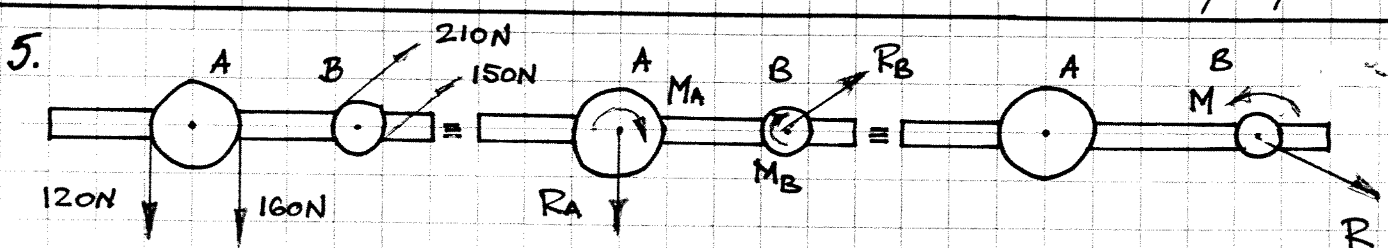
- Calculate the resultant force: $R = \sqrt{(181.2)^2 + (38)^2} = \underline{185.2N}$

and $\theta = \tan^{-1}\left(\frac{38}{181.2}\right) = \underline{11.84°}$

- And, finally, calculate the position of the resultant force:

ΣM_P : $-17{,}696 = (38)(d)$ $\therefore$ $\underline{d = 466\text{ mm}}$ to the left of P.

5.



- Reduce the forces acting on each pulley to two equivalent force-couples acting at the centre of each pulley A and B:

* Pulley A: ΣF_y: $-120 - 160 = R_A$ or $R_A = -280\text{ N} = 280\text{N} \downarrow$

ΣM_A: $(120)(20) - (160)(20) = M_A$ or $M_A = -800\text{Nmm} = 800\text{Nmm}$

* Pulley B: $\Sigma F: 210 + 150 = R_B \quad \therefore R_B = 360N \nearrow$

$\Sigma M_B: (150)(15) - (210)(15) = M_B \quad \therefore M_B = -900 Nmm = 900 Nmm \circlearrowright$

- Combine $\vec{R}_A$ and $\vec{M}_A$ and $\vec{R}_B$ and $\vec{M}_B$ into an equivalent force-couple system at B:

$\Sigma F_x: 360 \cos 25° = R_x \quad \therefore R_x = 326.3N \rightarrow$

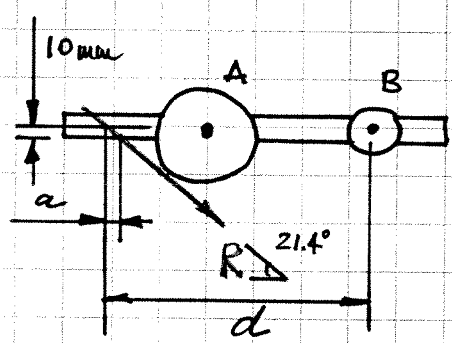
$\Sigma F_y: -280 + 360 \sin 25° = R_y \quad \therefore R_y = -127.9N = 127.9N \downarrow$

$\Sigma M_B: -800 + (280)(60) - 900 = M \quad \therefore M = 15,100 \; Nmm \circlearrowleft$

- Finally, replace this system with the single equivalent force:

$R = \sqrt{(326.3)^2 + (-127.9)^2} = \underline{350 N}; \quad \theta = \tan^{-1}\left(\frac{127.9}{326.3}\right) = \underline{21.4°}$

- Also ...



$\Sigma M_B = 15,100 \; Nmm = (127.9)(d)$

$\therefore d = 118.1 \; mm$

$a = \frac{10}{\tan 21.4°} = 25.52 \; mm$, and the line of action of $\vec{R}$ intersects the lower edge of the bracket $118.1 - 25.52 = \underline{92.6 \; mm}$ to the left of the centre of pulley B.