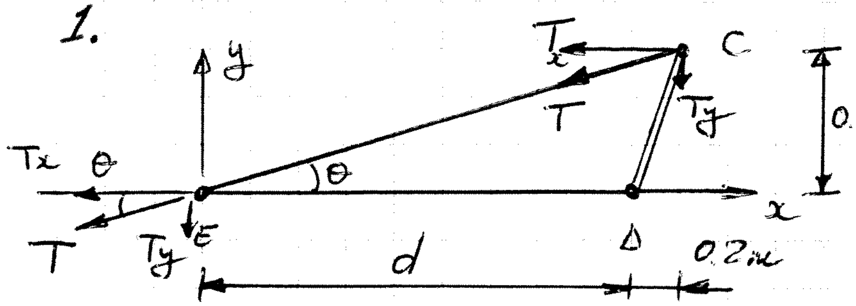
CIV 100F: SOLUTIONS TO ASSIGNMENT No. 2

1.



Force $\vec{T}$, i.e. the tension in cable CE, can be resolved into its rectangular components, $\vec{T_x}$ and $\vec{T_y}$, and can also be applied at E since $\vec{T}$ is a sliding vector. Hence, the moment of $\vec{T}$ about point D will be given by only the y component of $\vec{T}$ since the line of action of $\vec{T_x}$ passes through point D, i.e. the moment arm is zero.

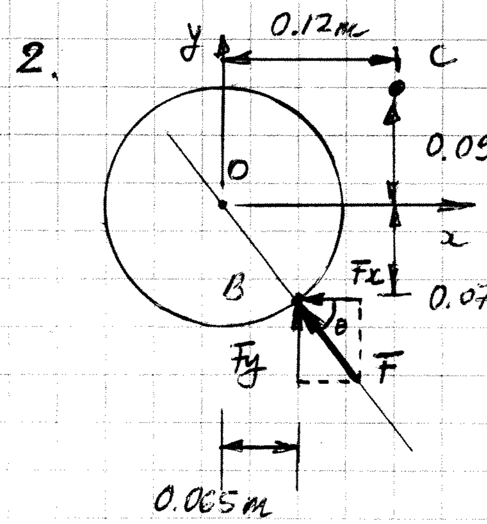
$$\therefore M_D = (T \sin\theta)d, \text{ where } \sin\theta = \frac{0.875}{\sqrt{(d+0.2)^2+(0.875)^2}}.$$

But
$$\underbrace{960}_{\text{Req'd moment to rotate the post about D}} = (d)\underbrace{(2400)}_{\text{Force in the cable}}\frac{0.875}{\sqrt{(d+0.2)^2+(0.875)^2}}$$

$$\therefore \sqrt{(d+0.2)^2+(0.875)^2} = 2.1875d, \text{ or } d^2+0.4d+0.04+0.7656 = 4.785d^2$$

$$\therefore 3.7852d^2 - 0.4d - 0.8056 = 0 \qquad \therefore \underline{d_{min} = 0.517\ m}$$

2.



The line of action of $\vec{F}$ is line OB, and $\vec{F}$ can also be resolved into its rectangular components.

$$d_{OB} = \sqrt{(65)^2+(72)^2} = 97\ mm, \quad \cos\theta = \frac{65}{97} \text{ and } \sin\theta = \frac{72}{97}.$$

$$\therefore F_x = F\cos\theta = (485)\frac{65}{97} = 325N$$

$$F_y = F\sin\theta = (485)\frac{72}{97} = 360N$$

Now, $M_C = -F_x d_y - F_y d_x =$

$$= -(325)(0.072+0.09) - (360)(0.12-0.065) = -(325)(0.162) - (360)(0.055) =$$

$$= -72.45\ Nm \qquad \therefore \underline{M_C = 72.5\ Nm} \circlearrowright$$

3. $d_{AB} = \sqrt{(24-0)^2+(16-(-8))^2+(-18-10)^2} = 44\ m$

Assume an imaginary force $\vec{F}$ acting at point A and directed from A to B. Then,

$$\vec{F} = F\vec{\lambda}_{AB} = \frac{F}{44}\left(24\vec{i} + 24\vec{j} - 28\vec{k}\right) = \frac{F}{11}\left(6\vec{i} + 6\vec{j} - 7\vec{k}\right).$$

By definition, the moment of $\vec{F}$ about C is $\vec{M_C} = \vec{r}_{CA} \times \vec{F}$, and its magnitude: $M_C = |\vec{r}_{CA} \times \vec{F}| = Fd$, where

$$\vec{r}_{CA} = 3\vec{i} - 10\vec{j} + (10-a)\vec{k}$$

$$\therefore \vec{M_C} = \frac{F}{11}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -10 & (10-a) \\ 6 & 6 & -7 \end{vmatrix} = \frac{F}{11}\Big\{[70-6(10-a)]\vec{i} + [6(10-a)+21]\vec{j} +$$

$$+ [18+60]\vec{k}\Big\} = \frac{F}{11}\left\{[10+6a]\vec{i} + [81-6a]\vec{j} + 78\vec{k}\right\}.$$

$$\therefore M_C^2 = \left(\frac{F}{11}\right)^2\left[(10+6a)^2 + (81-6a)^2 + 78^2\right] = (Fd)^2,\ \text{hence}$$

$$d^2 = \frac{1}{121}\left[(10+6a)^2 + (81-6a)^2 + (78)^2\right]$$

To calculate d_{min}, find the derivative of d^2 and equal it to 0:

$$\frac{d(d^2)}{da} = \frac{1}{121}\left[2(10+6a)(6) + 2(81-6a)(-6)\right] = 0,\ \text{or}$$

$$(10+6a) - (81-6a) = 0 \qquad \underline{\therefore a = 5.92\ m}$$

---

4. $BA = \sqrt{(-3)^2+(3)^2+(-1.5)^2} = 4.5\ m$ and

$BD = \sqrt{(-0.08)^2+(0.38)^2+(0.16)^2} = 0.42\ m$

(a) $$\vec{T}_{BA} = \frac{T_{BA}}{4.5}\left(-3\vec{i} + 3\vec{j} - 1.5\vec{k}\right) = \frac{T_{BA}}{3}\left(-2\vec{i} + 2\vec{j} - \vec{k}\right)$$ (From: $\vec{T}_{BA} = \vec{\lambda}_{BA} \cdot T_{BA} = \frac{\vec{BA}}{BA} T_{BA}$.

$$\vec{\lambda}_{BD} = \frac{\vec{BD}}{BD} = \frac{1}{0.42}\left(-0.08\vec{i} + 0.38\vec{j} + 0.16\vec{k}\right) = \frac{1}{21}\left(-4\vec{i} + 19\vec{j} + 8\vec{k}\right)$$

$$\cos\theta = \frac{\vec{T}_{BA} \cdot \vec{\lambda}_{BD}}{T_{BA} \cdot 1} = \frac{(-2)(-4) + (2)(19) + (-1)(8)}{3(21)} = 0.60317$$

$$\underline{\therefore \theta = 52.9°}$$

(b) $(T_{BA})_{BD} = \vec{T}_{BA} \cdot \vec{\lambda}_{BD} = T_{BA}(1)\cos\theta =$

$= (540)(0.60317)\ N \qquad \underline{\therefore (T_{BA})_{BD} = 326 N}$

5. $\vec{M}_O = \vec{r}_{OA} \times \vec{F}$, $\quad \vec{r}_{OA} = \left[(-0.1)\vec{i} + (0.275)\vec{j} - (0.675)\vec{k}\right]$ m

Also, $\vec{F} = F\left[(\cos\theta)(\cos\phi)\vec{i} - \sin\theta\,\vec{j} + (\cos\theta)(\sin\phi)\vec{k}\right]$ N

Hence, $$M_O = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -0.1 & 0.275 & -0.675 \\ (\cos\theta)(\cos\phi) & -\sin\theta & (\cos\theta)(\sin\phi) \end{vmatrix} =$$

$$= F\Big[\big(0.275(\cos\theta)(\sin\phi) - 0.675(\sin\theta)\big)\vec{i} + \big(-0.675(\cos\theta)(\cos\phi) + 0.1(\cos\theta)(\sin\phi)\big)\vec{j} + \big(0.1(\sin\theta) - 0.275(\cos\theta)(\cos\phi)\big)\vec{k}\Big] \text{ N m}$$

So that:
$$M_x = F\left[0.275(\cos\theta)(\sin\phi) - 0.675(\sin\theta)\right] \quad (1)$$
$$M_y = F\left[-0.675(\cos\theta)(\cos\phi) + 0.1(\cos\theta)(\sin\phi)\right] \quad (2)$$
$$M_z = F\left[0.1(\sin\theta) - 0.275(\cos\theta)(\cos\phi)\right] \quad (3)$$

From (1): $$(\cos\theta)(\sin\phi) = \frac{1}{0.275}\left(\frac{M_x}{F} + 0.675\sin\theta\right) \quad (4)$$

From (3): $$(\cos\theta)(\cos\phi) = \frac{1}{0.275}\left(0.1\sin\theta - \frac{M_z}{F}\right) \quad (5)$$

Substituting (4) and (5) into (2):

$$M_y = F\left\{-0.675\left[\frac{1}{0.275}\left(0.1\sin\theta - \frac{M_z}{F}\right)\right] + 0.1\left[\frac{1}{0.275}\left(\frac{M_x}{F} + 0.675(\sin\right.\right.\right.$$

$$M_y = \cancel{F}\left\{\left[\cancel{\frac{-0.0675}{0.275}(\sin\theta)} + \frac{0.675}{0.275}\frac{M_z}{\cancel{F}}\right] + \left[\frac{0.1}{0.275}\frac{M_x}{\cancel{F}} + \cancel{\frac{0.0675}{0.275}(\sin\theta)}\right]\right\}$$

$$= \frac{1}{0.275}\left(0.675\,M_z + 0.1\,M_x\right) = \frac{1}{0.275}\left[(0.675)(-81) + (0.1)(-77\right.$$

$$\therefore\ \underline{M_y = -227 \text{ N}\cdot\text{m}}$$