SOLUTIONS TO ASSIGNMENT

1/ $A(0,0,0)$; $B(6,0,0)$; $C(0,-2,1.5)$; $D(9,0,0)$; $E(12,0,0)$; $F(0,0,-5)$

$$\vec{F}_{BC} = F_{BC}\vec{\lambda}_{BC} = F_{BC}\frac{-6\vec{i}-2\vec{j}+1.5\vec{k}}{\sqrt{(-6)^2+(-2)^2+(1.5)^2}} = F_{BC}\left(-\tfrac{12}{13}\vec{i}-\tfrac{4}{13}\vec{j}+\tfrac{3}{13}\vec{k}\right);\ \vec{r}_{AB}=6\vec{i}$$

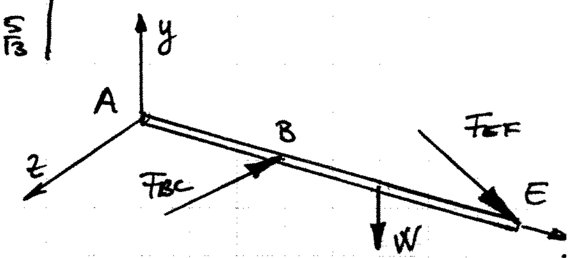
$$\vec{F}_{EF} = F_{EF}\vec{\lambda}_{EF} = F_{EF}\frac{-12\vec{i}+0\vec{j}-5\vec{k}}{\sqrt{(-12)^2+(0)^2+(-5)^2}} = F_{EF}\left(-\tfrac{12}{13}\vec{i}+0\vec{j}-\tfrac{5}{13}\vec{k}\right);\ \vec{r}_{AE}=12\vec{i}$$

$\vec{W} = -58.86\vec{j}$ KN; $\vec{r}_{AD} = 9\vec{i}$. Now, sum the moments about A, $\Sigma M_A = 0$

$$\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 9 & 0 & 0\\ 0 & -58.86 & 0\end{vmatrix} + \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 6 & 0 & 0\\ -\frac{12}{13} & -\frac{4}{13} & \frac{3}{13}\end{vmatrix}F_{BC} + \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 12 & 0 & 0\\ -\frac{12}{13} & 0 & -\frac{5}{13}\end{vmatrix}F_{EF} = 0$$

Hence,

$$\begin{cases}-\frac{24}{13}F_{BC} - 529740 = 0\\ -\frac{18}{13}F_{BC} + \frac{60}{13}F_{EF} = 0\end{cases} \qquad \begin{cases}\underline{\underline{F_{BC} = 287\text{ KN (C)}}}\\ \underline{\underline{F_{EF} = 86.1\text{ KN (C)}}}\end{cases}$$



2/ $A(0,-1.5,2.5)$; $B(0,1.5,2.5)$; $C(0,0,0)$; $D(0,0,-1)$

By writing $\Sigma M_c = 0$ one can solve for three unknowns, i.e. T_1, T_2, D_y.

$$\vec{T}_1 = T_1\frac{-4\vec{i}-1.5\vec{j}+2.5\vec{k}}{\sqrt{(-4)^2+(-1.5)^2+(2.5)^2}} = \left[-0.808\vec{i}-0.303\vec{j}+0.505\vec{k}\right]T_1;\ \vec{r}_1 = 4\vec{i}+\vec{k}$$

$$\vec{T}_2 = T_2\frac{-2.5\vec{i}+1.5\vec{j}+2.5\vec{k}}{\sqrt{(-2.5)^2+(1.5)^2+(2.5)^2}} = \left[-0.651\vec{i}+0.391\vec{j}+0.651\vec{k}\right]T_2;\ \vec{r}_2 = 2.5\vec{i}+\vec{k}$$

$\vec{W} = -(100)(9.81)\vec{k} = -981\vec{k}$; $\vec{r}_W = 2\vec{i}$

$\vec{D}_y = D_y\vec{j}$; $\vec{r}_D = 1\vec{k}$, Hence

$$\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 4 & 0 & 1\\ -0.808 & -0.303 & 0.505\end{vmatrix}T_1 + \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 2.5 & 0 & 1\\ -0.651 & 0.391 & 0.651\end{vmatrix}T_2 + \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 2 & 0 & 0\\ 0 & 0 & -981\end{vmatrix} + \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 0 & 0 & 1\\ 0 & 1 & 0\end{vmatrix}D_y = 0$$

For equilibrium, the terms in $\vec{i}, \vec{j}, \vec{k}$ resulting after expanding the above determinants must each be zero. A system of 3 equations with 3 unknowns results, and hence : $\underline{\underline{T_1 = 347N}}$, $\underline{\underline{T_2 = 431N}}$ and $\underline{\underline{D_y = -63.1\text{ N}}}$. To solve for the force at C write $\Sigma F = 0$, hence:

$$[-0.808\vec{i} - 0.303\vec{j} + 0.505\vec{k}](347) + [-0.651\vec{i} + 0.391\vec{j} + 0.651\vec{k}](431) + 63.1\vec{j} + (-981)\vec{k} + C_x\vec{i} + C_y\vec{j} + C_z\vec{k} = 0.$$

The terms in $\vec{i}$, $\vec{j}$ and $\vec{k}$ must be equal to zero, hence $C_x = 560.96$ N, $C_y = 0.00$ N, $C_z = 525.18$ N

$$\therefore C = \sqrt{(560.96)^2 + (525.18)^2} = \underline{\underline{768\text{ N}}}$$

3./ A moment summation equation about point D will solve for T, C_y, C_z: $\Sigma M_D = 0$. Then, a force summation equation will solve for D_x, D_y, D_z: $\Sigma F = 0$

- The 50N force: $50\,[0.707\,\vec{j} + 0.707\,\vec{k}] = 35.35\,\vec{j} + 35.35\,\vec{k}$. Also:

$\vec{r}_{50} = 0.2\,\vec{i} - 0.1061\,\vec{j} + 0.1061\,\vec{k}$; $\vec{r}_T = 0.2\,\vec{i} - 0.15\,\vec{k}$; $\vec{r}_{65} = 0.45\,\vec{i} - 0.08\,\vec{j}$

$\vec{r}_{80} = 0.45\,\vec{i} + 0.08\,\vec{j}$ and $\vec{r}_c = 0.75\,\vec{i}$

$\therefore \Sigma \vec{M}_D = 0$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.2 & -0.1061 & 0.1061 \\ 0 & 35.35 & 35.35 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.2 & 0 & -0.15 \\ 0 & T & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.45 & -0.08 & 0 \\ 0 & 0 & -65 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.45 & 0.08 & 0 \\ 0 & 0 & -80 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.75 & 0 & 0 \\ 0 & C_y & C_z \end{vmatrix} = 0$$

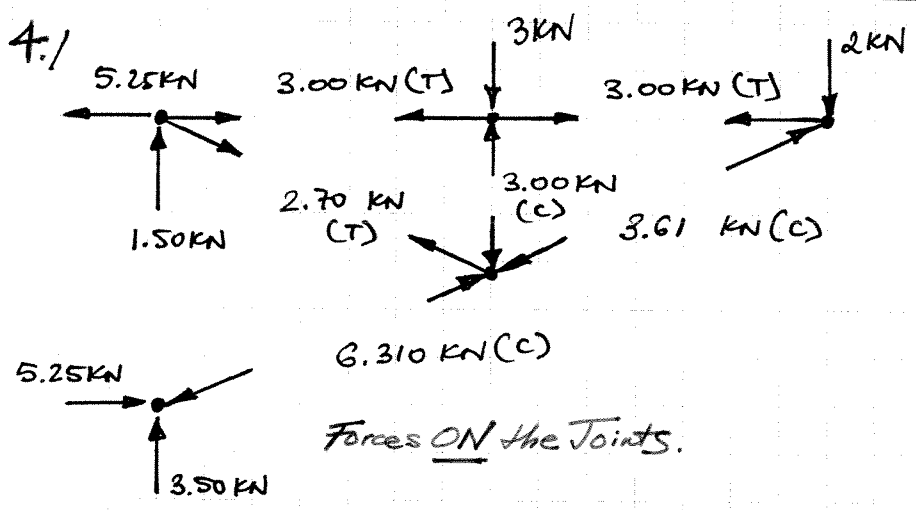
Again, for equilibrium, the terms in $\vec{i}, \vec{j}, \vec{k}$ must be equal to zero, hence:

$\underline{\underline{T = 58\,N}}$, $\underline{\underline{C_y = -24.9\,N}}$ and $\underline{\underline{C_z = 77.6\,N}}$

$\therefore \Sigma \vec{F} = 0 \quad 35.35\,\vec{j} + 35.35\,\vec{k} + 58\,\vec{j} - 65\,\vec{k} - 80\,\vec{k} - 24.9\,\vec{j} + 77.6\,\vec{k} + D_x\,\vec{i} + D_y\,\vec{j} + D_z\,\vec{k} = 0$

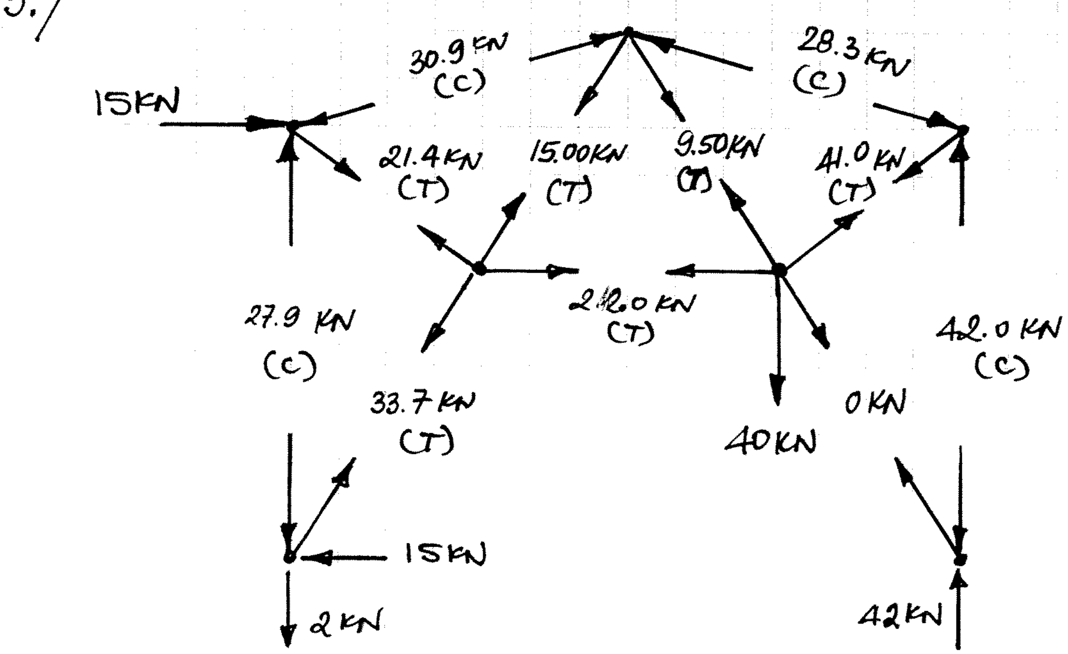
and $\underline{\underline{D_x = 0\,N}}$, $\underline{\underline{D_y = -68.5\,N}}$ and $\underline{\underline{D_z = 32.0\,N}}$

4./



One can solve this truss using the method of joints by either calculating the reactions first an then analysing joint by joint or by starting at the C node, analysing the joints one by one and finally computing the reaction forces at the supports.

5./



Although this truss is geometrically symmetric the applied loads are not symmetric, therefore one must analyse equilibrium of all joints throughout the structure.