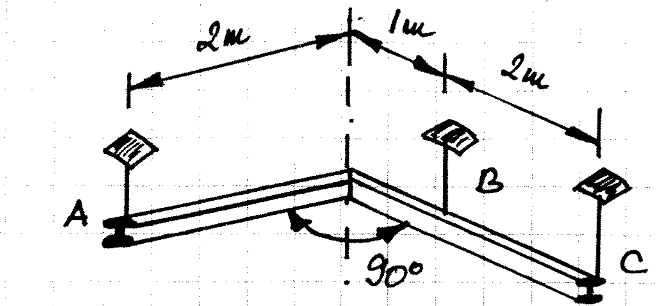
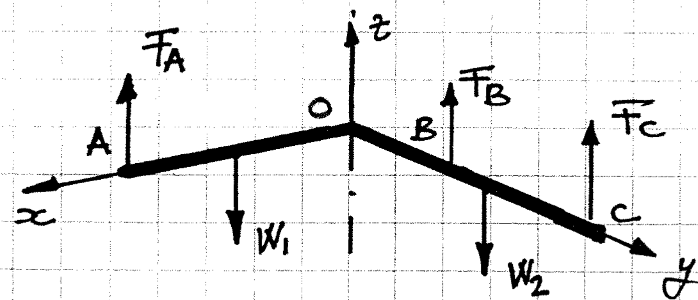
EXAMPLE:

The two steel I beams, each with a mass of 40 kg per metre length are welded together at right angles and lifted by the vertical cables so that the beams remain in a horizontal plane. Compute the tension in each of the cables A, B and C.





Free Body Diagram

As for 2D problems, a free body diagram must always be drawn

$W_1 = 40 \times 2 \times 9.81 = 784.8\,N$

$W_2 = 40 \times 3 \times 9.81 = 1177.2\,N$

Parallel forces $\rightarrow$ 3 equations of equilibrium:

$\vec{M}_c = 0 \Rightarrow$

(i) $\sum M_{y_c} = 0 \qquad -2F_A + 1 \times 784.8 = 0 \qquad \therefore \underline{F_A = 392\,N}$

(ii) $\sum M_{x_c} = 0 \qquad -392 \times 3 + 784.8 \times 3 - F_B \times 2 + 1177.2 \times 1.5 = 0$

$\therefore \underline{F_B = 1472\,N}$

98.0 N

(iii) $\sum F_z = 0 \qquad -784.8 - 1177.2 + 392 + 1472 + F_c = 0 \;\therefore \underline{F_c = 98.0\,N}$

- Alternatively, we can solve the problem by using vector notation.

$\underline{\sum M_c = 0}$

$-(\overrightarrow{\text{Moment about } C \text{ due to } W_2}) = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -1.5 & 0 \\ 0 & 0 & -1177.2 \end{vmatrix} = 1765.8\,\vec{i}\ Nm$

$-(\overrightarrow{\text{Moment about } C \text{ due to } F_B}) = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -2 & 0 \\ 0 & 0 & +F_B \end{vmatrix} = -2F_B\,\vec{i}\ Nm$

– (Moment about C due to $\vec{W}_1$) $= \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ +1 & -3 & 0 \\ 0 & 0 & -784.8 \end{vmatrix} = 2354.4\vec{i} + 784.8\vec{j}$ Nm.

– (Moment about C due to $\vec{F}_A$) $= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ +2 & -3 & 0 \\ 0 & 0 & F_A \end{vmatrix} = -3F_A\vec{i} - 2F_A\vec{j}$ Nm

– (Moment about C due to $\vec{F}_C$) $= 0$.

$\therefore \vec{M}_c$ = Total moment about C $= (4120.2\vec{i} - 2F_B\vec{i} - 3F_A\vec{i}) + (784.8\vec{j} - 2F_A\vec{j}) + 0\vec{k}$ Nm.

For $\vec{M}_c$ to be zero : $\begin{Bmatrix} 4120.2 - 2F_B - 3F_A = 0 \\ 784.8 - 2F_A = 0 \end{Bmatrix}$

Note: We are effectively using three equations of equilibrium here by saying that $M_{cx}=0$, $M_{cy}=0$ and $M_{cz}=0$.

Hence, $\underline{F_A = 392N}$ and

$$\underline{F_B} = \frac{4120.2 - 3(392.4)}{2} = \underline{1472\ N}$$

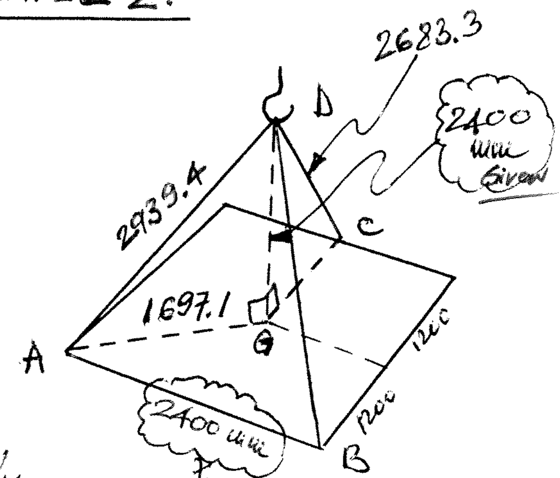
Also, $\Sigma F = 0$

$\Rightarrow \quad F_A\vec{k} + F_B\vec{k} + F_C\vec{k} - W_1\vec{k} - W_2\vec{k} = 0$

$\therefore \quad F_c = 784.8 + 1177.2 - 392 - 1472 \quad N$

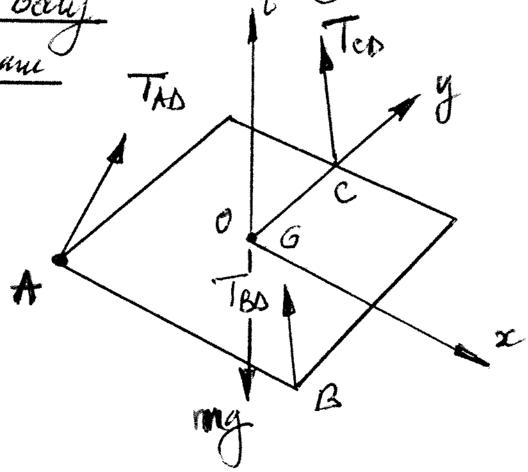
$\therefore \quad \underline{F_c = 98\ N}$

EXAMPLE 2:



- The square plate of mass 1800 kg is lifted by 3 cables, keeping the plate horizontal.
- Calculate the Tension in the 3 cables

Free Body Diagram



- $\Sigma M_{x_A} = 0$

$$\Rightarrow \underbrace{T_{CD}\left(\frac{2400}{2683.3}\right)}_{T_{CD_z}}(2.4\,m) - mg\,(1.2\,m) = 0$$

$$\therefore T_{CD} = \frac{(1800 \times 9.81)(1.2)}{2.147} =$$

$$= 9{,}871\ N$$

$$\therefore \underline{T_{CD} = 9.87\ kN}$$

- $\Sigma F_x = 0 \Rightarrow T_{AD_x} = T_{BD_x}$, because T_{CD} has no x component

Then, $\therefore \underline{T_{AD} = T_{BD}}$

- $$\Sigma F_z = 0 \Rightarrow \underbrace{9.87\left(\frac{2400}{2683.3}\right)}_{T_{CD_z}} + \underbrace{2T_{AD}\left(\frac{2400}{2939.4}\right)}_{T_{AD_z} \text{ or } T_{BD_z}} - \underbrace{(1.800)(9.81)}_{[kN]} = 0$$

$$\therefore 8.83\,kN + 1.633\,T_{AD} - 17.66 = 0$$

$$\therefore \underline{T_{AD} = T_{BD} = 5.41\,kN}$$