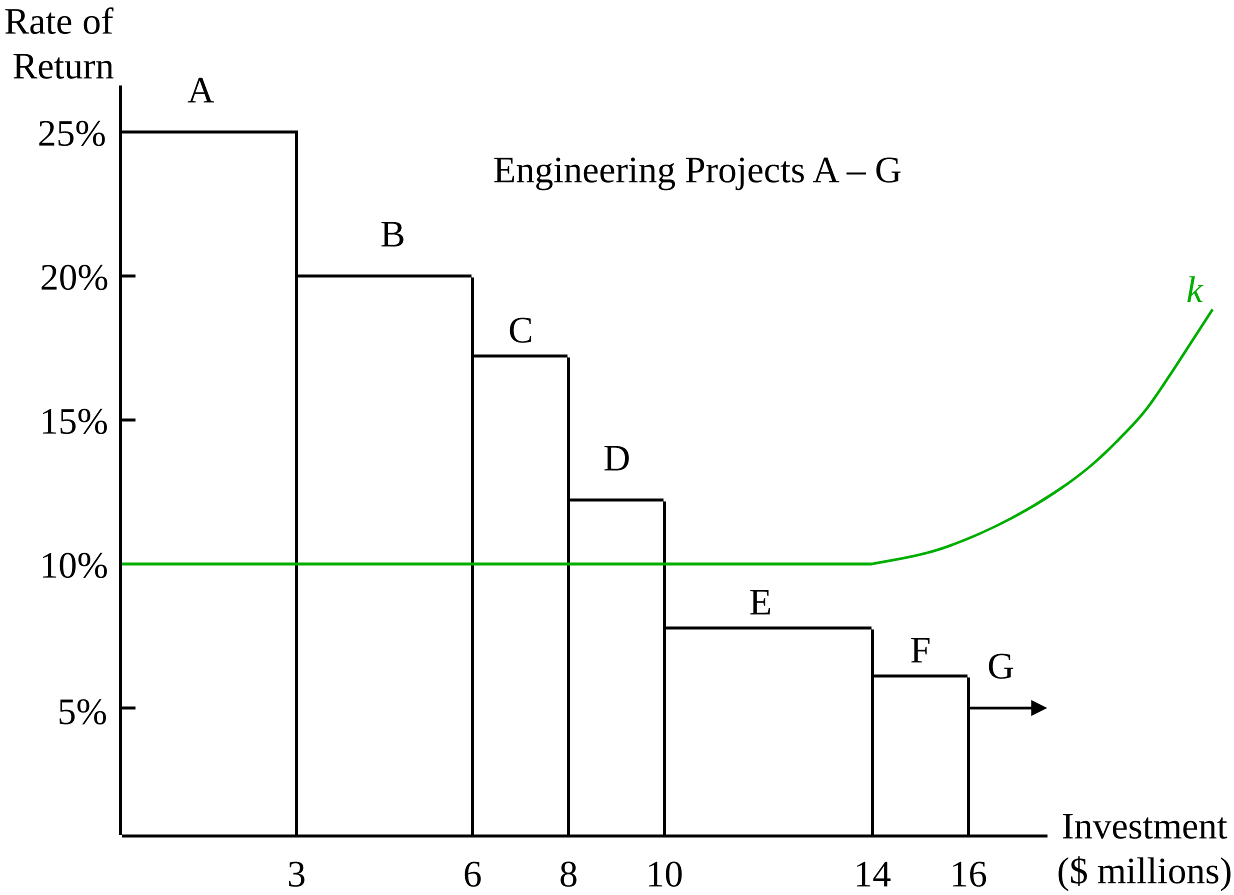
Capital Budgeting

- Initially assume no taxes and perfect certainty
- Process of planning expenditures whose returns are expected to extend beyond one year
- Significance of capital budgeting
  - long-term effects
  - loss of flexibility
  - over-investment – unnecessary extra costs
  - under-investment – loss of market share
  - timing – capital assets must come on stream at the right time
- Optimal capital budget
  - level of investment that maximizes the present value or worth of the firm
- Goals: (1) the firm's management should make decisions that the shareholders would
  (2) management does not have to know the shareholders
- Level of investment is determined by supply and demand under conditions of uncertainty

  supply – cost of capital schedule

  demand – available investment opportunities

Capital Budgeting



<u>Cost of Capital</u>

$$k = \frac{D}{V} k_D + \frac{E}{V} k_E$$

<u>Economic Theory of the Firm</u>

Marginal Revenue = Marginal Cost

Corporate Finance
Sources of Long-Term Financing

What is the cost of capital for the firm?

This is the "hurdle rate" applied to projects in the capital-budgeting process (MARR – Minimum Attractive Rate of Return).

<u>Assets</u>	<u>Liabilities & Owner's Equity</u>	
current assets	short-term liabilities	working capital
long-term assets	long-term debt equity (common stock) preferred shares retained earnings	long-term sources (capital structure)

Example: \$5 000 000 investment – 50% debt & 50% equity

Before:	A	L & OE	
CA	2	STL	1
LTA	5	LTD	2
		E	4
	7		7

After:	A	L & OE	
CA	2	STL	1
LTA	10	LTD	4.5
		E	6.5
	12		12

Note the intermediate step when CA = 7 just after the sale of the debt & equity (an additional \$5 million of cash).

Investment Proposals

Replacement

- replacement decisions are the simplest to make
- cost savings generally well-known
- high degree of confidence

Expansion: additional capacity in existing product lines

- greater uncertainty
- must accurately estimate future demand
- can use previous production and sales experience to reduce uncertainty

Expansion: new product lines

- greatest uncertainty
- these strategic proposals are often key to the competitive position of the firm
- crude estimates are better than none

Ranking Investment Proposals

- Make investment decisions in engineering projects that will maximize the value of the firm

  1) which of several mutually exclusive projects should be selected?
  2) which projects, and how many in total, should be accepted?

Comparison of Alternatives
Eight-Step Methodology

1. Generate the set of feasible, mutually exclusive projects
   – generally there is more than one way to meet an objective
2. Define the planning period to be used in the economic study
   – estimate the economic life of the asset
3. Define the cash flow profiles for each alternative
4. Specify the time value of money
5. Specify the method of ranking the alternatives
   – Payback Period
   – Net Present Value
   – Internal Rate of Return
6. Rank the alternatives
7. Perform supplementary analyses
8. Select the preferred alternative

Defining Mutually Exclusive Alternatives

Step 1

- Which investment proposals should be selected for further consideration?
- Those selected become the investment alternatives

Alternative	X_1	X_2	X_3	
1	0	0	0	do nothing
2	0	0	1	accept proposal 3 only
3	0	1	0	accept proposal 2 only
4	0	1	1	accept proposals 2 & 3 only
5	1	0	0	accept proposal 1 only
6	1	0	1	accept proposals 1 & 3 only
7	1	1	0	accept proposals 1 & 2 only
8	1	1	1	accept all three proposals

- What are the constraints?
- Which alternatives are feasible?
- Budget limitations **Reject**
  - insufficient funds to accept all 3 **8**
- Mutually exclusive proposals
  - 1 and 2 are alternative computer designs **7**
- Contingent proposals
  - 3 is contingent on 2 **2 & 6**
- Therefore, consider Alternatives 1, 3, 4 & 5 for further study

Defining the Planning Period

<u>Step 2</u>

- Period of time over which the alternatives are compared
- Defines the width of the “window” that is used to view the cash flows generated by an alternative
- Use the economic, not physical, life of the asset

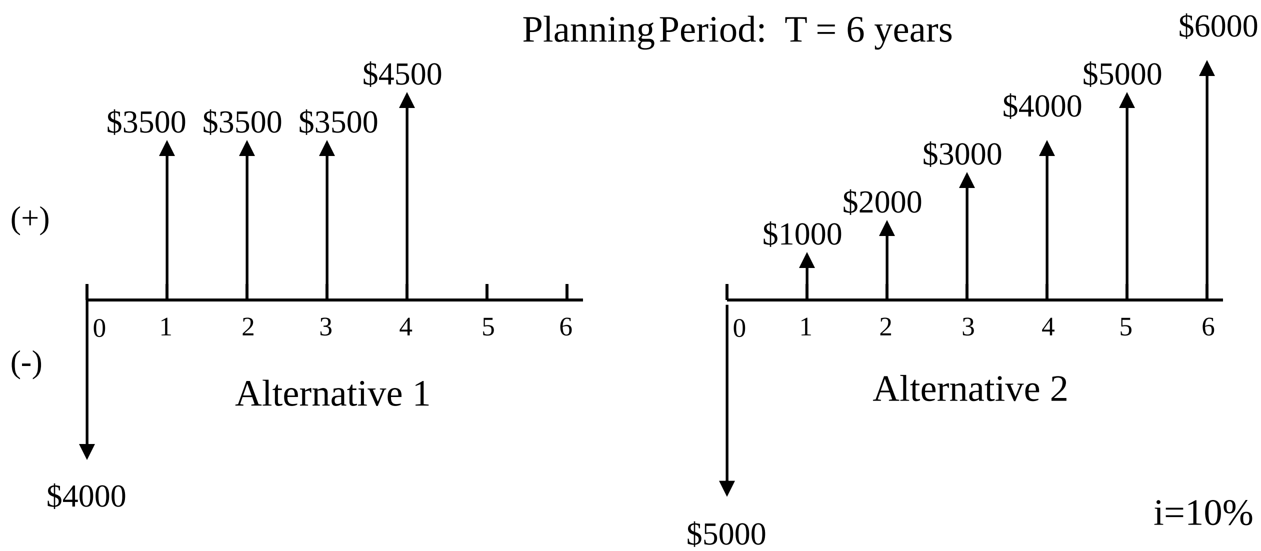
1. Least common multiple of lives (T_{LCM})
   - $T_1 = 5$, $T_2 = 7 \Rightarrow T_{LCM} = 35$ years

2. Shortest project life (T_S)
   - Requires salvage value of the project with the longer life

3. Longest project life (T_L)
   - Difficult to specify cash flows for project with shortest life between T_S and T_L (reinvest in asset with shorter life?)

4. Standard Planning Period
   - Same planning period for all economic alternatives
   - Wrap up all projects and determine salvage value using the same project life
   - Must be careful to consider projects whose major benefits are late in the project life

Defining the Planning Period



1. "One-shot" Mutually Exclusive Investments

FW_1 = 4 500 (F|P 10,2) + 3 500 (P|A 10,3) (F|P 10,6)

– 4 000 (F|P 10,6)

= $13 779

FW_2 = 1 000 (F|A 10,6) + 1 000 (A|G 10,6) (F|A 10,6)

– 5 000 (F|P 10,6)

= $16 014

2. Mandatory Service Requiring Reinvestment in Alternative 1

FW_1 = 13 779 – 4 000 (F|P 10,2) + 3 500 (F|P 10,1) + 3 500 + 1 000

= $17 289

Assumption: As the equipment purchased at EOY 4 still has two years of economic life, it will have a salvage value of $1 000 at EOY 6.

Developing Cash Flow Profiles

Step 3

- Estimate all cash flows that will occur during the planning period for each alternative
- Costs and revenues that will be the same regardless of the alternative can be omitted
- Beware of allocated, non-cash and average costs

$$\begin{aligned}\text{Net cash flow} &= \text{benefits} - \text{costs}\\ &= \text{revenues} - \text{expenses}\end{aligned}$$

	Plan A			Plan B		
Year	R	E	NCF	R	E	NCF
1	800	650	150	500	300	200
2	800	600	200	600	300	300
3	800	550	250	700	350	350

Incremental cash flows Plan B versus Plan A

$$\Delta NCF = B - A$$

Year	NCF_A	NCF_B	ΔNCF
1	150	200	50
2	200	300	100
3	250	350	100

Specify the Time Value of Money

Step 4

- use the cost of capital for the firm as the basis for the time value of money
- opportunity cost – even if funds are raised entirely from internal sources (retained earnings)
- minimum attractive rate of return (MARR)
- if the Net Present Value of the project cash flows discounted at the MARR is negative, then the project does not add value to the company and should be rejected
- the cost of capital is often adjusted to compensate for the perceived risk of the project under evaluation

	Project Class	MARR
I	Low risk, cash flows – highly predictable	8%
II	Normal business risk	12%
III	Speculative	18%

Specify the Measure of Merit

Step 5

- on what basis does one evaluate alternatives?

1) Payback Period Method

2) Cash Flow Value Method

    - Present Value (Discounted Cash Flow Analysis)
    - Annual Worth
    - Future Value

3) Internal Rate of Return Method

4) Savings/Investment Ratio Method (Chapter 6)

5) External Rate of Return Method

The evaluation process should select those projects that add the greatest value to the firm.

Payback Period Method

- accept all project proposals where the original investment is recovered from the net cash flows in less than the payback period
- easy to calculate, easy to understand

$1 000 investment – three-year payback period required

Year	A	B	
1	$500	$100	
2	400	200	Accept A
3	300	300	Reject B
4	100	400	
5		500	
6		600	

- Biased against long-term investments – the ones that are often key to a firm's long-run success

Year	X	Y	
1	$600	$200	Accept
2	400	400	Either
3	200	600	X or Y

- fails to take into account the time value of money
- discounted payback period – can be used as a risk indicator

Net Present Value Method

- discounted cash flow technique
- recognizes the time value of money
- convert all net cash flows to their equivalent value at the beginning of the planning horizon

$$\text{NPV} = \left[\frac{F_1}{1+k} + \frac{F_2}{(1+k)^2} + \cdots + \frac{F_n}{(1+k)^n} \right] - I$$

F_n – net cash flow in Year n

K – cost of capital

I – initial cost of the project

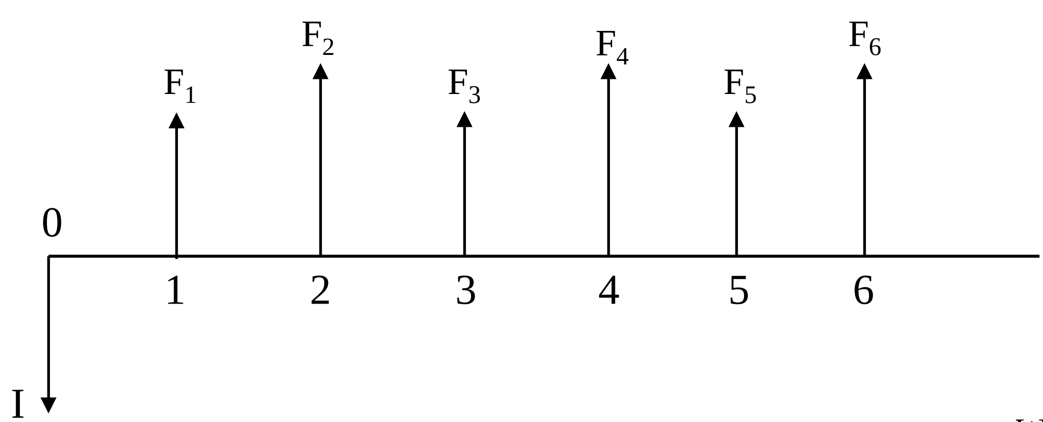
n – planning period

- for independent projects

  NPV > 0 – accept

  NPV < 0 – reject

- for mutually exclusive projects, accept the one with the highest NPV



Net Present Value Method

	Plan A			Plan B		
Year	Rev	Exp	NCF	Rev	Exp	NCF
0	0	300	–300	0	400	–400
1	800	650	150	500	300	200
2	800	600	200	600	300	300
3	800	550	250	700	350	350

- What is the NPV of both projects?
- Cost of capital (MARR) = 10%

Year	NCF	(P|F k,n)	PV(NCF)	NCF	(P|F k,n)	PV(NCF)
0	–300	1.0000	–300	–400	1.0000	–400
1	150	0.9091	136	200	0.9091	182
2	200	0.8264	165	300	0.8264	248
3	250	0.7513	188	350	0.7513	263
		PV_A =	189		PV_B =	293

$PV_B > PV_A$ and $PV_B > 0$

Therefore, choose Plan B.

What if Plan A had an annual depreciation charge of $100 and Plan B $150?

Incremental Net Present Value Method

- Can also use an incremental analysis
- Is Plan B better than Plan A?
- Plan A is the reference

$$\Delta NCF = B - A$$

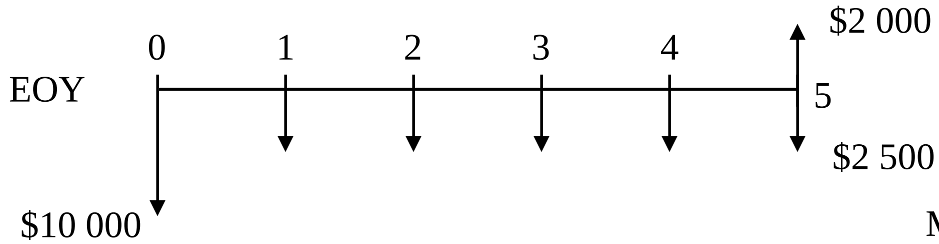
Year	NCF_A	NCF_B	ΔNCF
0	–300	–400	–100
1	150	200	50
2	200	300	100
3	250	350	100

Year	ΔNCF	(P|F i,n)	PV(NCF)
0	–100	l.0000	–100
1	50	0.9091	45
2	100	0.8264	83
3	100	0.7513	75
		Δ PV =	103

- Plan B is better than Plan A
- The incremental cash flows have a positive NPV
- Does this mean that Plan B should necessarily be undertaken?

Discounted Cash Flow Approach

Alternative 1



MARR = 10%

- Revenues are the same in each alternative
- Select the alternative that minimizes the cost
- Can use the PW, FW or AW approach

Present Worth $\boxed{- \$18\ 235}$

$PW_1 = -10000 - 2\ 500(P|A\ 10,5) + 2\ 000(P|F\ 10,5)$

Future Worth $\boxed{- \$29\ 368}$

$FW_1 = -10\ 000(F|P\ 10,5) - 2\ 500(F|A\ 10,5) + 2\ 000$

Annual Worth $\boxed{- \$4\ 810}$

$AW_1 = -10\ 000(A|P\ 10,5) - 2\ 500 + 2\ 000(A|F\ 10,5)$

- Each approach will select the correct alternative

Note: $FW_1 = PW_1\ (F|P\ 10,5)$

$= -18\ 235\ (1.6105)$

$= -29\ 368$

Internal Rate of Return Method

- IRR is that interest rate which equates the present value of the expected future cash flows to the initial cost outlay

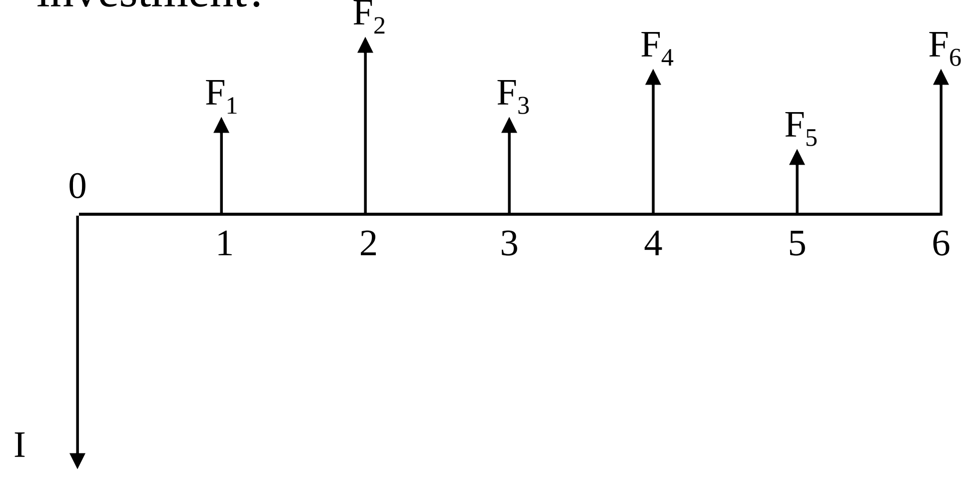
$$0 = \frac{F_1}{(1+i^*)} + \frac{F_2}{(1+i^*)^2} + \ldots + \frac{F_n}{(1+i^*)^n} - I$$

$$i^* = \text{IRR}$$

- Can say if i* > k, then NPV > 0

IRR and NPV will always give the same
accept-reject decisions.

- However, IRR and NPV can rank projects differently
- This is a problem if capital is limited or if projects are mutually exclusive
- Always use NPV if there is a conflict because the project with the highest NPV contributes more to the value of the firm.
- Can we claim that the IRR is the rate of return on investment?



Internal Rate of Return Method

- Evaluate Plan A and Plan B using the IRR method
- MARR = 10%

Year	**Plan A**	**Plan B**
0	–300	–400
1	150	200
2	200	300
3	250	350

Plan A:
$$0 = -300 + \frac{150}{1+i^*} + \frac{200}{(1+i^*)^2} + \frac{250}{(1+i^*)^3}$$

$$i_A^* = 40\%$$

Plan B:
$$0 = -400 + \frac{200}{1+i^*} + \frac{300}{(1+i^*)^2} + \frac{350}{(1+i^*)^3}$$

$$i_B^* = 44\%$$

$i^*_B > i^*_A$ Therefore, choose Plan B over Plan A.

$i^*_B > 10\%$ Therefore, accept Plan B.

Note that despite being chosen by the IRR method,
Plan B may be inferior to Plan A!

Internal Rate of Return

Let

A_{jt} = net cash flow for alternative j in time period t

then

i^*_j = IRR for Alternative j

when

$$0 = \sum_{t=0}^{n} A_{jt}(1 + i_j^*)^{n-t}$$

(Future Value approach)

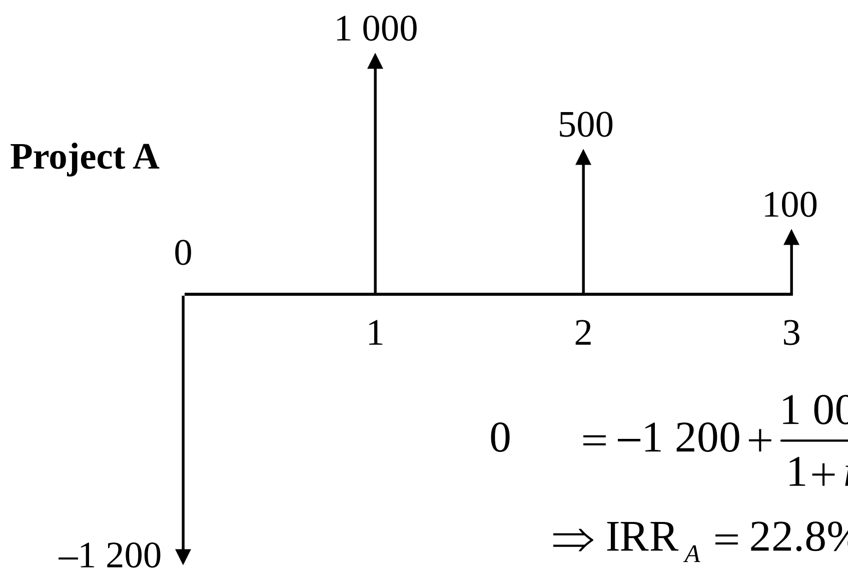
or

$$0 = \sum_{t=0}^{n} A_{jt}(1 + i_j^*)^{-t}$$

(Present Value approach)

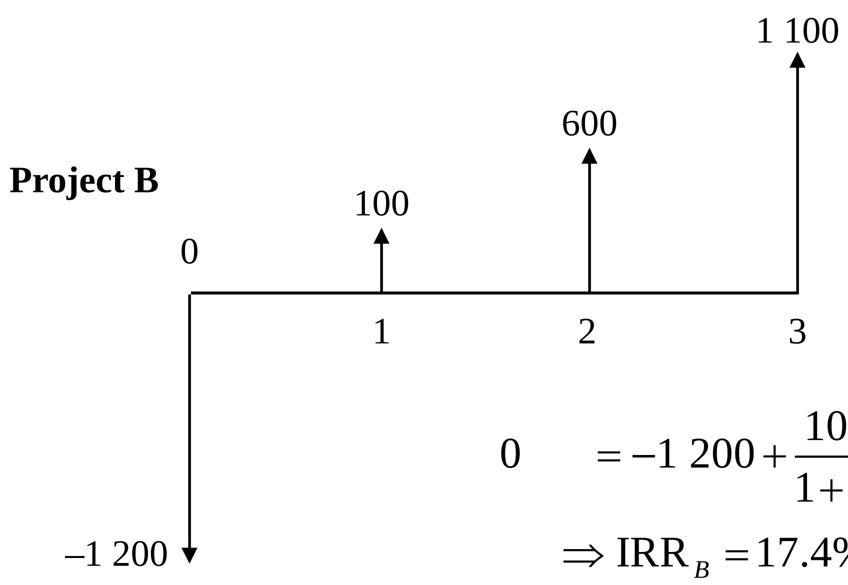
IRR Project Evaluation

Rising versus Declining Cash Flow Profiles



$$0 = -1\,200 + \frac{1\,000}{1+i^*} + \frac{500}{(1+i^*)^2} + \frac{100}{(1+i^*)^3}$$

$$\Rightarrow \text{IRR}_A = 22.8\% = i_A^*$$



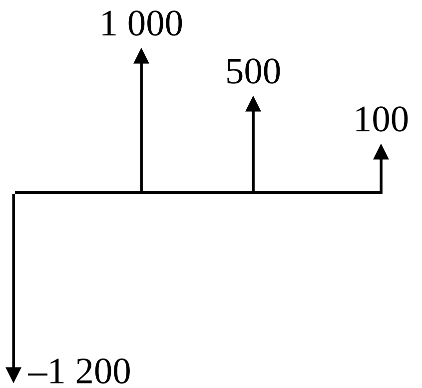
$$0 = -1\,200 + \frac{100}{1+i^*} + \frac{600}{(1+i^*)^2} + \frac{1\,100}{(1+i^*)^3}$$

$$\Rightarrow \text{IRR}_B = 17.4\% = i_B^*$$

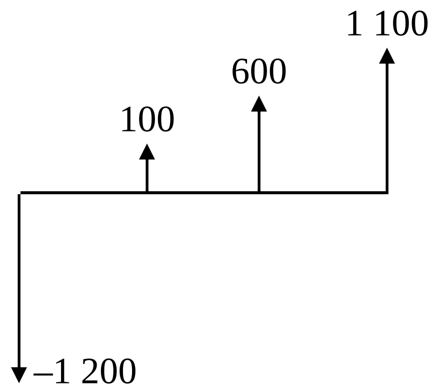
Is **Project A** always preferred over **Project B** ?

NPV Versus IRR

Project A



Project B

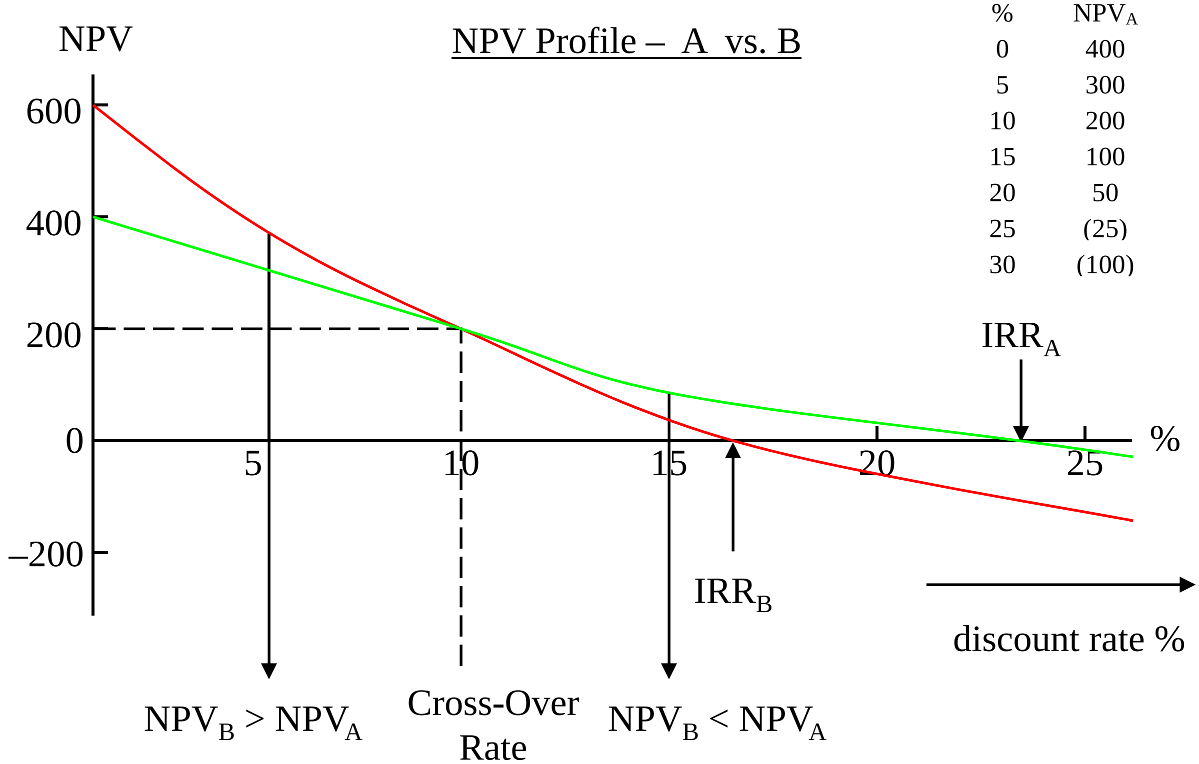


$$NPV_A = -1\,200 + \frac{1\,000}{1+i} + \frac{500}{(1+i)^2} + \frac{100}{(1+i)^3}$$

$$NPV_B = -1\,200 + \frac{100}{1+i} + \frac{600}{(1+i)^2} + \frac{1\,100}{(1+i)^3}$$

NPV Profile – A vs. B

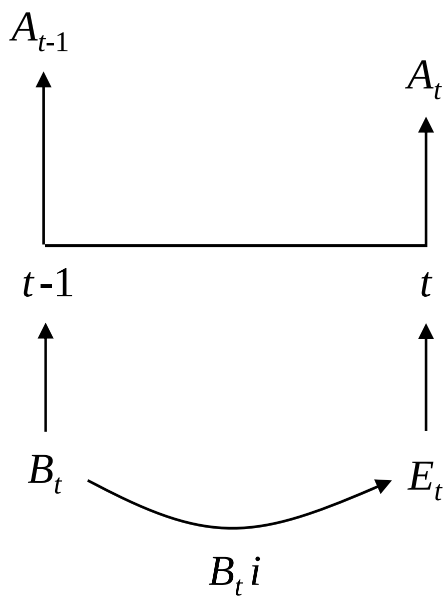
%	NPV_A	NPV_B
0	400	600
5	300	400
10	200	200
15	100	50
20	50	(85)
25	(25)	(175)
30	(100)	(250)



Therefore, a conflict between NPV & IRR exists if the cost of capital (MARR) is below the cross-over rate.

IRR Reinvestment Rate

What interest rate for the reinvestment of cash flows is implicitly assumed by the Internal Rate of Return methodology?
Is the rate assumed by IRR always achievable?



Let A_t = net cash flow for Period t

B_t = unrecovered balance at the <u>beginning</u> of Period t

E_t = unrecovered balance at the <u>end</u> of Period t

I_t = interest on the unrecovered balance during Period t

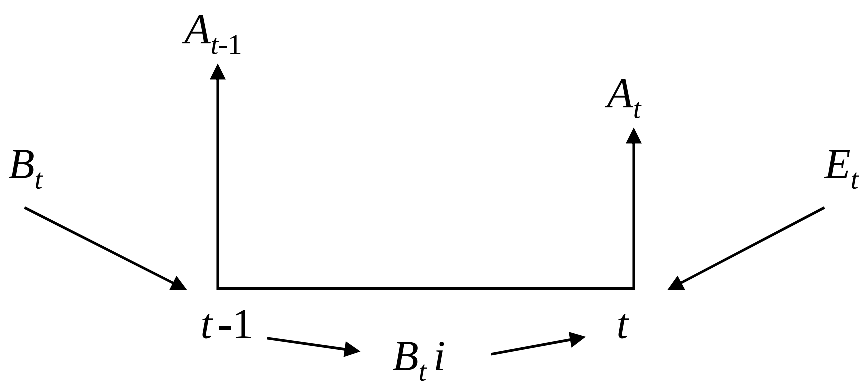
Then

$E_0 = A_0$

$B_t = E_{t-1} \qquad t = 1,\ldots,n$

$E_t = A_t + B_t + I_t \qquad t = 1,\ldots,n$

IRR Reinvestment Rate



Interest earned on unrecovered balance:

$$I_t = B_t\, i \qquad\qquad t = 1,...,\text{n}$$

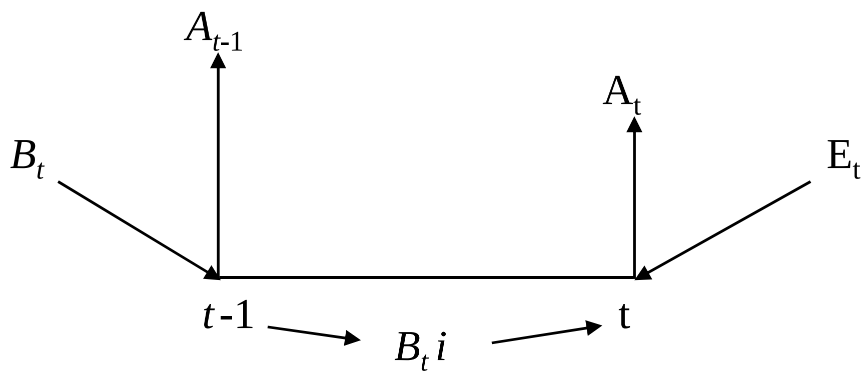
What is the rate of interest earned implicit in the IRR?

$$\begin{aligned} E_n &= A_n + B_n + I_n \\ &= A_n + B_n\,(1 + i) && (I_n = B_n i) \\ &= A_n + E_{n\text{-}1}\,(1 + i) && (B_n = E_{n\text{-}1}) \end{aligned}$$

but

$$E_{n\text{-}1} = A_{n\text{-}1} + E_{n\text{-}2}\;\;(1 + i)$$

IRR Reinvestment Rate



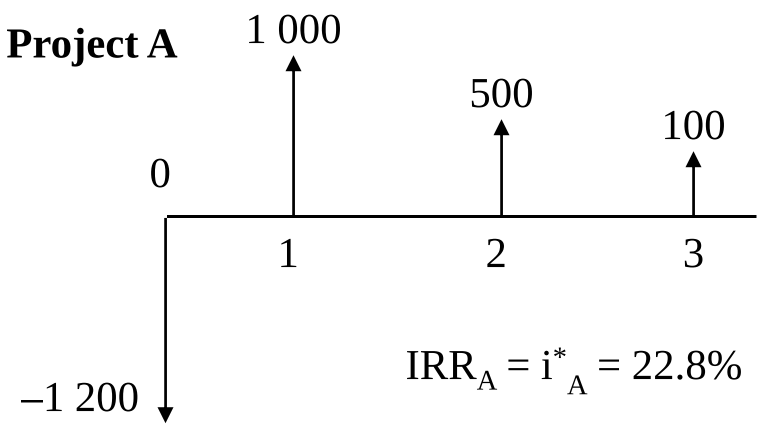
$$\begin{aligned}
E_n &= A_n + E_{n-1}(1+i) \\
&= A_n + A_{n-1}(1+i) + E_{n-2}(1+i)^2 \\
&= A_n + A_{n-1}(1+i) + A_{n-2}(1+i)^2 + \ldots + A_1(1+i)^{n-1} + A_0(1+i)^n \\
&= \sum_{t=0}^{n} A_t(1+i)^{n-t}
\end{aligned}$$

(Recursive Relationship)

$$E_n = 0 = \sum_{t=0}^{n} A_t(1+i^*)^{n-t} \Rightarrow i = i^*$$

$$\boxed{\therefore \quad I_t = B_t i^* \qquad\qquad t = 1, \ldots, n}$$

IRR: Reinvestment of Cash Flows



$IRR_A = i^*_A = 22.8\%$

t	A_t	B_t	I_t	E_t
0	–1 200			–1 200
1	1 000	–1 200	–274	–474
2	500	–474	–108	–82
3	100	–82	–19	0

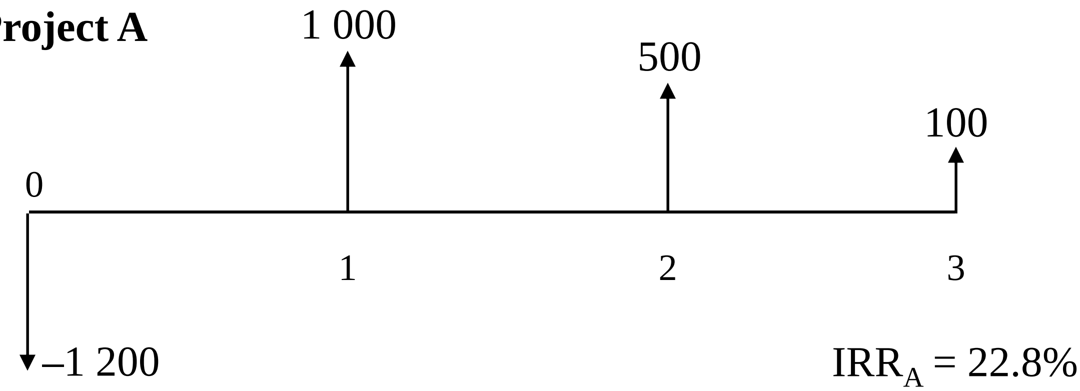
For **Project A** to be better than **Project B**, the IRR method assumes that cash flows can be reinvested at the IRR rate of $i^*_A = 22.8\%$.

$1 200 Investment	1 200 (F|P 22.8,3)	= 2 222
Cash Flows	1 000 (F|P 22.8,2)	= 1 508
Reinvested at	500 (F|P 22.8,1)	= 614
22.8%	100 (F|P 22.8,0)	= 100
	Σ Reinvested Cash Flows	2 222

Are there any 22.8% investments available for reinvestment of the project cash flows in Year 1 and Year 2?

IRR: Reinvestment of Cash Flows

Project A



If Project A has a 22.8% rate of return on investment, then the future value of the $1 200 investment should be

$$1\,200\,(1 + 0.228)^3 = \$2\,222$$

The project's value comes from the three cash flows that must be reinvested at 22.8% to generate a FV of $2 222.

$$1\,000\,(1 + 0.228)^2 + 500\,(1 + 0.228) + 100 = \$2\,222$$

But it is unlikely that a company with a MARR of 10% will have projects as good as this one available in Years 1 and 2. It is safer to assume reinvestment of cash flows at the MARR.

$$1\,000\,(1 + 0.1)^2 + 500\,(1 + 0.1) + 100 = \$1\,860$$

This implies a revised rate of return (i_R) on the $1 200 investment of

$$1\,200\,(1 + i_R)^3 = 1\,860 \Rightarrow \boxed{i_R = 15.7\%}$$