1. We'll use the terms option A to refer to not taking the degree and option B to taking the degree. Let N be the number of years remaining in Bob's career (65-45=20), i the discount rate, i^* the IRR, j_A, j_B the differing salary growth rates, and S_{At}, S_{Bt} the salary (thousands) in year t under each option. $S_{B0} = S_{B1} = 0$ and $S_{B2} = 100$.

   (a) Since Alice and Bob have investments, they could cash those in to pay tuition. The cost would be foregone returns, similar to the question earlier in the semester about selling equity in a company versus borrowing to buy a property. The investment returns are not guaranteed, but neither is the return on the degree. Their mortgage is also coming due for renewal and they have a lot of equity in the home. It would be possible to add the amount on to the mortgage and thereby benefit from a low cost of borrowing. Borrowing would be cheapest.

   (b) The direct costs are due to tuition. We assume living expenses remain the same, so they are irrelevant. There are also opportunity costs since by taking a leave Bob is passing up the opportunity to earn his salary.

   (c) This question is meant to test your understanding of the concept of discount rate. There is no unique correct answer, but there are several considerations that would make some answers clearly invalid. First, the discount rate should exceed the government bond yields since those are "risk-free". Second, they are able to borrow at 4.0% on their mortgage, so that's their cost of debt and a *minimum* discount rate. That amount is certain for the next five years. For a riskier investment, the rate should be higher to reflect the uncertainty. Depending on whether they are more confident of the investment returns of the returns from studying, the rate would be higher or lower than their investment rate. Let's set it (subjectively) at 6% to suggest that Bob is a little more confident about the MBA than his financial investments.

   Also, it's worth noting that qualitatively the higher the discount rate the less favourable school looks because the future salary increases are weighted less than the current cost.

   (d) This year's salary is paid regardless, so ignore it. Option A has Bob's salary continuing from a year from now (remember we assume EOY salary payment) into retirement, so the PV of future salary from today is $S_{A0}(P|A, i, j_A, N)$. If on the other hand he opts to go back to school, the PV is $-25 - \frac{25}{1+i}$ for tuition and $(P|F, i, 2)(P|A, i, j_B, N-2)S_{B2}$ for future salary. The difference at the discount rate i is the NPV of that decision. It is calculated as -16570 at 6% in an attached spreadsheet. Not looking good for back-to-school because the opportunity cost is very high. It may be better for Bob to pay more for an "executive" MBA so he can keep working or take less time away from work.

   (e) IRR is independent of any discount rate chosen (depends only on cash flows) so you should come up with the same value of 5.3% using either trial and error with the NPV formulas above or using Excel. That's a fairly low rate so unless Bob is very certain of the return, the *economic* analysis says no-go.

   (f) As nice as it is, receiving an inheritance does not change the economics. Bob still has the same returns available: pay down the mortgage (a guaranteed return of 4.0% for the next five years, and variable until the balance is fully paid); invest in retirement account (an expected but not guaranteed 6.25% return); or pay tuition (expecting to receive the IRR 5.3% over the balance of his career). Even though the $50,000 seems to be "easy money", one should still consider time value when making economic choices.

2. This question is a classic instance of differing lifetimes. Assuming you will need a car indefinitely, you can calculate any one of: cost for a common multiple lifetime (7,4,3)=84 years, perpetual, or equivalent uniform cost (exactly EUAC, but monthly). They will give equivalent results if the same discount rate is used. Equivalent monthly is used below since it offers a clean interpretation, and is easy to calculate for the lease option. We choose 6% as a discount rate since it's the return we think we can achieve on invested cash (assuming you can easily carry the payments out of income so there's no risk of losing the car).

   To calculate the salvage values, take $P_0 \cdot 0.85^t$ for the used car and $P_0 \cdot 0.6 \cdot 0.85^{t-1}$ for the new car. We own nothing at the end of the lease, so its salvage value is 0.

   Costs are compared in the table below:

Option	Lifetime (m)	Time 0 cost	Monthly cost	Yearly cost	Salvage
Buy new	84	3000	550 (60m)	0	6788.7
Lease	48	0	400	0	0
Used	60	15000	0	1500+j%/yr	6655.6

   The equivalent monthly cash flow for the lease e_{lease} is easy: 400. There's a trick for the new car, though: we pay it off over 5 years but replace after 7 so can't just add 550 per month to its monthly equivalent. First find PV, then convert to equivalent monthly:

$$e_{new} = (A|P, i^{(12)}, 84) \left[P_{new} + m_{new}(P|A, i^{(12)}, 60) - S_{new,7}(P|F, i, 7) \right] \tag{1}$$

   Note above that $(A|P, i, N)(P|A, k, M)$ above would cancel out only if $i = k$ and $M = N$. Likewise for the used car, we need to include all that maintenance expense:

$$e_{used} = (A|P, i^{(12)}, 60) \left[P_{used} + a_{used}(P|A, i, j, 5) - S_{used,5}(P|F, i, 5) \right] \tag{2}$$

   As evaluated in the attached spreadsheet, the equivalent monthly costs are 436.60 to buy new, 400 to lease, and 370.44 to buy used.

i	6	P|A,i12,12	11.629
i12	1.004867551	A|P,i12,12	0.086
i1	1.06		
		P|F,i,1	0.943396226
Depreciation model			
Year 1	0.6		
Year 2+	0.85		
New car		Lease	
Purchase price	30000		
Down payment	-3000		0
Periodic payment	-550		-400
N	60		48
(P|A,i12,M) monthly	51.92		42.71
PV (pmt)	-31558.10		-17085.09
Gradient payment			
Growth rate j			
(P|A,i,j,N) annually			
PV (Maint)			
Interval (yr)	7		4
Salvage value	6788.69		0.00
NPV	**-30043.23**		**-17085.09**
(A|P,i,N) annual	0.18		0.29
EUAC	**-5381.80**		**-4930.61**
(A|P,i12,M) monthly	0.01		0.02
Monthly uniform cost	**-436.60**		**-400.00**

3. (a) For payback, just add cash flows until they're positive. For discounted, discount the cash flows using $(P|F, i, n)$ first then accumulate until it's positive (just like PP). For NPV, add up all the discounted cash flows. IRR is calculated in the usual way (for the volume of calculation here, Excel is recommended; it has an IRR() function). Note an IRR, PP, and DPP do not exist for projects with non-positive cash flows (C1,C2). All the calculations are done on an attached spreadsheet. The columns for each project are: CF (cash flow), DCF (discounted cash flow; the PV of that year's cash flow today), CCF (cumulative cash flow, the sum of undiscounted cash flows 0..t), and NPV(0..t) (the sum of DCF in years 0..t).

   (b) Since C1 or C2 is mandatory, our option set is reduced: we must do C1 so do that and reduce the capital budget to $70m with a new option set: upgrading C1 to C2, A1/A2, and B. B's IRR is less than MARR so it is definitely undesirable under any circumstances and can be removed from the option set. Note that A2 and C2 are incompatible since their capital cost (70+30) exceeds the budget (90). So, enumerate the possibilities and look at NPV: C1/A1 (-36.66), C2/A1 (-30.81), C1/A2 (-35.7), C2/A2 infeasible. Clearly C2/A1 is the best NPV given the capital constraint. The mandatory project has a large negative return, which we offset partially by the profits from A1.

   (c) We can now spend up to $50m more. Since the maximum projects being considered total *at most* $100m (A2+C2), the capital budget is effectively unlimited so we can evaluate each project in isolation. Do the mandatory projects and any projects where IRR exceeds MARR. C1 is mandatory and we've already established that A1 is desirable (IRR exceeds MARR). The option set is: upgrade to C2 or not, and upgrade to A2 or not. So, we need to check the IRR of the incremental projects C2-C1 and A2-A1. From spreadsheet, both have an IRR exceeding MARR (equivalently a positive NPV at MARR), so we do C2 and A2 for total NPV -29.87. It's not good news for the company that the NPV comes out negative, but this is the least negative value so we go for it (do nothing is not an option because C1/C2 mandatory). Note that adding the non-mandatory project B always decreases NPV at the MARR so we would not choose to do so.

10% MARR

Year	P|F i N		A1 CF	DCF	CCF	NPV(0..t)	A2 CF	DCF	CCF	NPV(0..t)
	0	1.00	-60	-60.00	-60	-60.00	-70	-70.00	-70	-70.00
	1	0.91	20	18.18	-40	-41.82	20	18.18	-50	-51.82
	2	0.83	25	20.66	-15	-21.16	25	20.66	-25	-31.16
	3	0.75	30	22.54	15	1.38	40	30.05	15	-1.10
	4	0.68	20	13.66	35	15.04	25	17.08	40	15.97
IRR			20.9%				19.5%			
NPV @10%			15.04				15.97			
PP			3				3			
DPP			3				4			

Year	P|F i N		C1 CF	DCF	CCF	NPV(0..t)	C2 CF	DCF	CCF	NPV(0..t)
	0	1.00	-20	-20.00	-20	-20.00	-30	-30.00	-30	-30.00
	1	0.91	-10	-9.09	-30	-29.09	-5	-4.55	-35	-34.55
	2	0.83	-10	-8.26	-40	-37.36	-5	-4.13	-40	-38.68
	3	0.75	-10	-7.51	-50	-44.87	-5	-3.76	-45	-42.43
	4	0.68	-10	-6.83	-60	-51.70	-5	-3.42	-50	-45.85
IRR			n/a				n/a			
NPV @10%			-51.70				-45.85			
PP			n/a				n/a			
DPP			n/a				n/a			

Year	P|F i N		A2-A1 CF	DCF	CCF	NPV(0..t)	C2-C1 CF	DCF	CCF	NPV(0..t)
	0	1.00	-10	-10.00	-10	-10.00	-10	-10.00	-10	-10.00
	1	0.91	0	0.00	-10	-10.00	5	4.55	-5	-5.45