Breakeven, Sensitivity and Risk Analysis

Breakeven Analysis

- Performed when we are completely uncertain of the possible values a parameter can take on
- This method identifies the set of values for which an investment decision is justified economically

Sensitivity Analysis

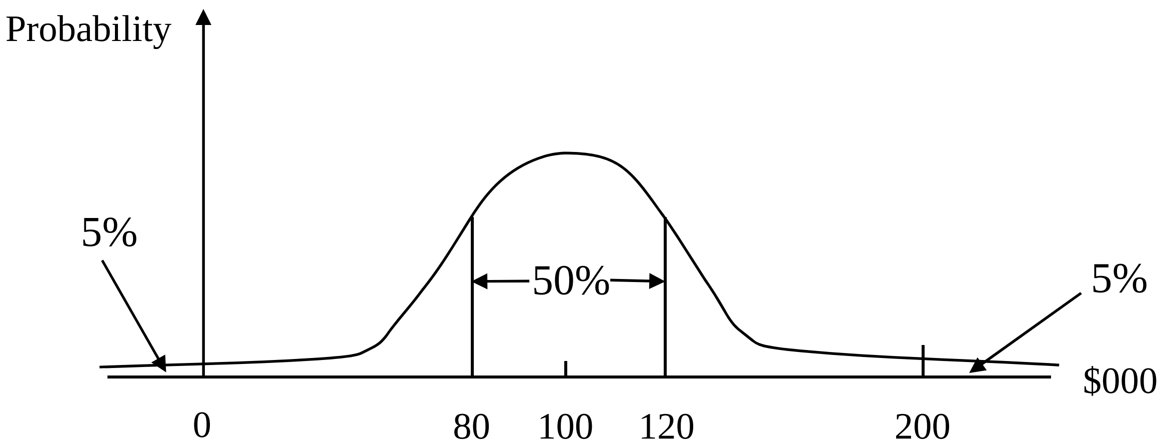
- Performed when we are reasonably sure of the possible values a parameter can take on, but uncertain of their chances of occurrence.
- This method identifies the sensitivity of the measure of merit to changes in, or errors in estimating, parameter values.

Risk Analysis

- Performed when probabilities can be assigned to the values of various parameters.
- This method aggregates the probability distributions of the input parameters to obtain the probability distribution for the measure of merit.
- Aggregation usually done through simulation.

Investment Decisions Under Risk

- Any business forecast, including the estimation of future cash flows, is subject to uncertainty



- Single value “best” or “most likely” estimate of $100,000
- Risk is associated with project variability - the more variable the expected future returns, the riskier the investment.
- Investors are generally risk-averse and will require a risk premium to enter into a risky investment.
- The firm’s cost of capital is directly related to the risk of the firm’s investment.
- Must evaluate projects in light of the market’s trade-off for risk and return.
- Must attempt to quantify the forecaster’s subjective beliefs about the future.
- Express predictions in terms of probabilities

Expected Value Methodology

Project A - oil well investment

Project B - real estate investment

PROJECT A			**PROJECT B**		
Outcome	Probability	NCF	Outcome	Probability	NCF
1	0.2	0	1	0.2	9,000
2	0.6	10,000	2	0.6	10,000
3	0.2	20,000	3	0.2	11,000

What is the expected value of each project?

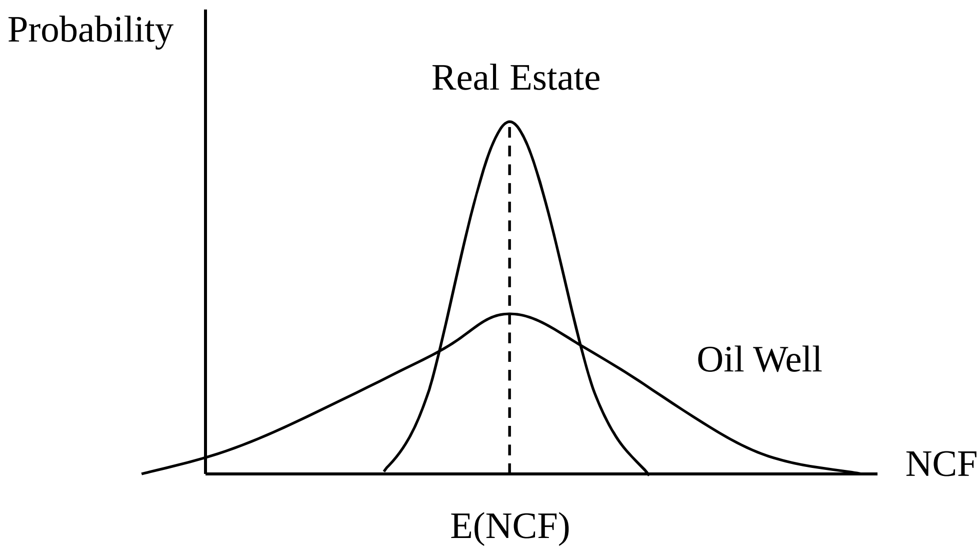
$$E(NCF) = P_1\ NCF_1 + P_2\ NCF_2 + \ldots + P_N\ NCF_N$$

PROJECT	N	P_N	NCF_N	$P_N\ NCF_N$
A	1	0.2	0	0
	2	0.6	10,000	6,000
	3	0.2	20,000	4,000
			$E(NCF_A)$ =	10,000
B	1	0.2	9,000	1,800
	2	0.6	10,000	6,000
	3	0.2	11,000	2,200
			$E(NCF_B)$ =	10,000

- The expected value of each project is the same.

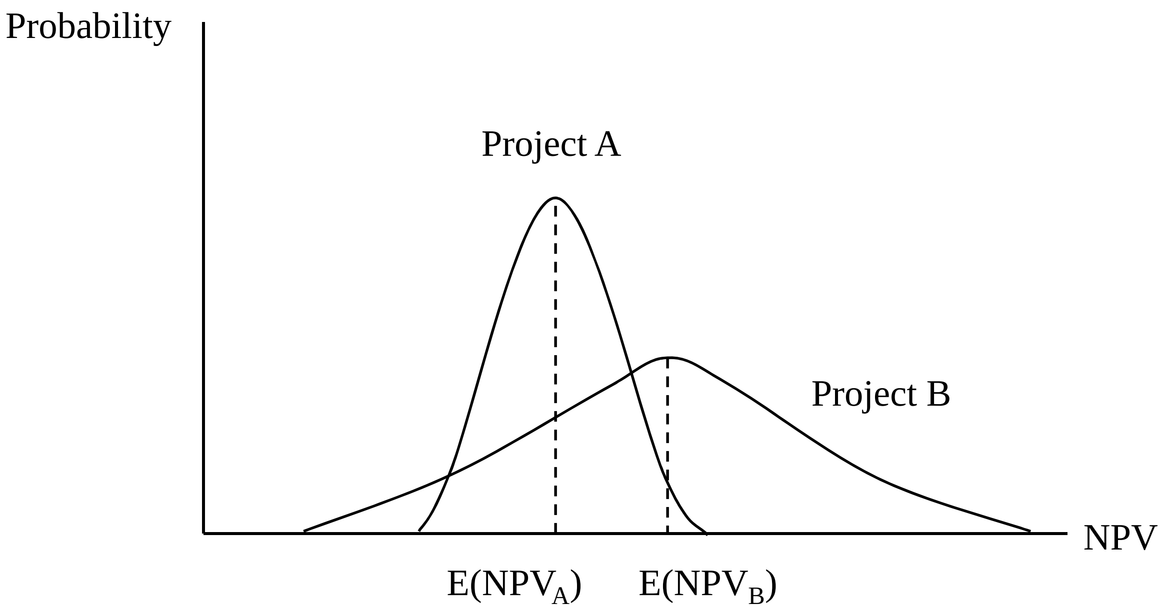
Risk and Project Cash Flow Variability

- Given that the oil well investment and the real estate investment have the same expected value, is one preferable over the other?



- Risk-averse investors would choose the real estate project over the oil well project.
- While the expected values are equal, the variability of returns around the expected value is much greater in the case of the oil well.
- This increased variability in the NCF represents greater risk to the investor.
- The oil well investment offers no greater compensation for undertaking the additional risk.

Evaluation Criteria



Risk and Return on Two Projects

- Once risk has been assessed, how does one rank projects which differ in both risk and expected returns?
- Can no longer say Project B will be preferred over Project A just because it has the higher NPV.
- Ranking of projects must include risk element.
- The additional value of Project B may not be sufficient to compensate for the increased risks associated with the larger variability in returns.

Risk-Adjusted Discount Rates

- Vary the discount rate applied to a project's expected cash flows on the basis of perceived risk.
- Attempt to take into account shareholders' attitudes regarding the risk-return relationship.

Example:

	Project Class	Applicable Discount Rate
I	Low risk, cash flows highly predictable	12%
II	Normal business risk	15%
III	Speculative	18%

Consider a project requiring an initial investment of \$100,000. Cash inflows of \$30,000 per year are expected for the next five years.

Discount Rate Applied	NPV
12%	\$8,150
15%	\$ 605
18%	-\$6,190

This investment would be rejected if it had higher than normal business risk.

Breakeven Analysis

- The point where two alternatives are equal is called the breakeven point.
- When alternative project cash flows are at the point of <u>equivalence</u>, we are <u>indifferent</u> as to the choice of one alternative over the other.
- The choice between the two then rests on a judgement as to which side of the breakeven point the element will likely fall.
- The analysis will focus on the point at which operations just break even

  i.e., neither making nor losing money

Let x = a variable quantity
(e.g. units of production)

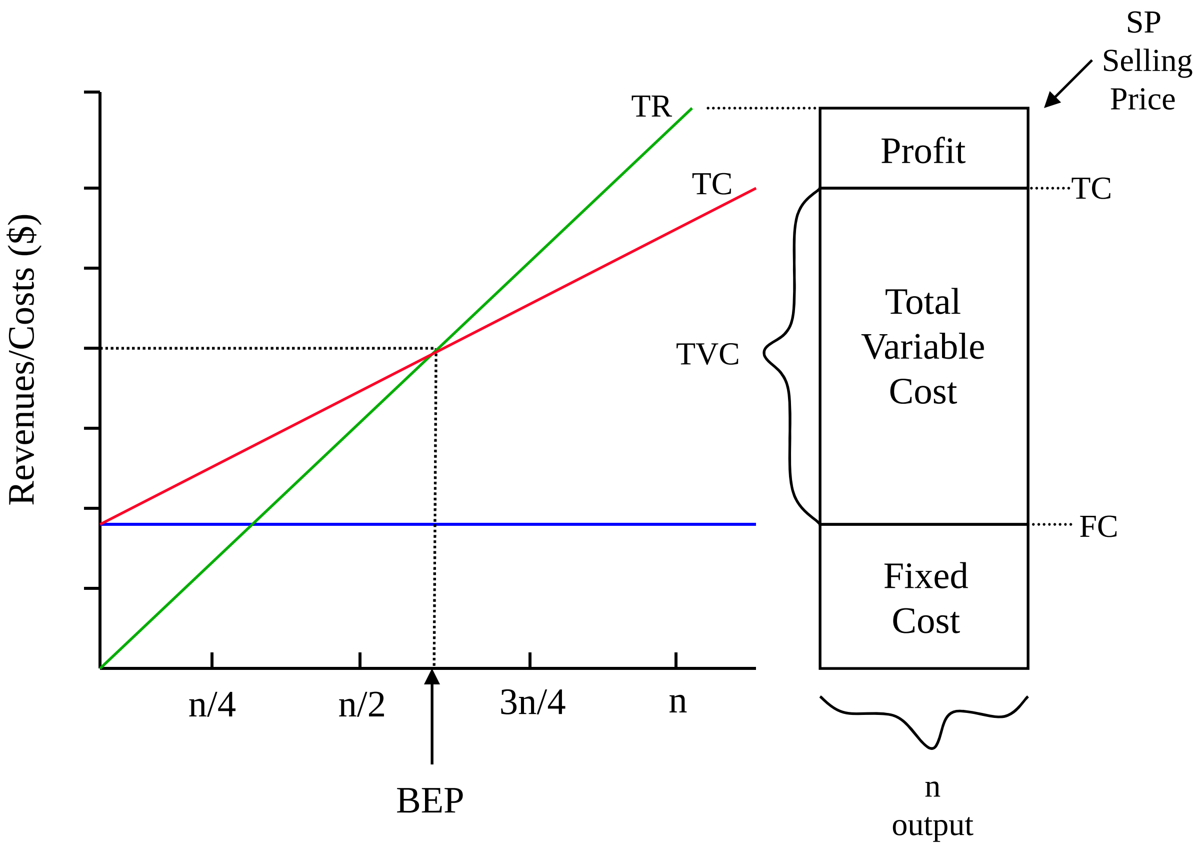
Then, if $Y = F(x)$ and $Y = G(x)$,

the value of x (call it x^*),

when $F(x^*) = G(x^*)$

is termed the **breakeven value**.

Breakeven Charts



Revenue: TR = nSP

Cost: TC = nVC + FC

Gross Profit: Z = TR - TC = n(SP-VC) - FC

Break-even Point: $\Rightarrow$ Z=0

$$\therefore \quad n\Big|_{BEP} = \frac{FC}{SP - VC}$$

Contribution = SP - VC $\xrightarrow{n<BEP}$ Fixed Costs; $\xrightarrow{n>BEP}$ Profit

Breakeven-Point Alternatives

- Objective: lower breakeven point
- Sales level well above BEP is a sign of business success.
- Profitability less affected in economic downturns.
- Profit is achieved when TR(n) > TC(n).
- The break-even point can be lowered by:
    1. increasing the slope of the TR(n), i.e., increase the unit selling price of $SP.
    2. decreasing the value of FC, the fixed costs

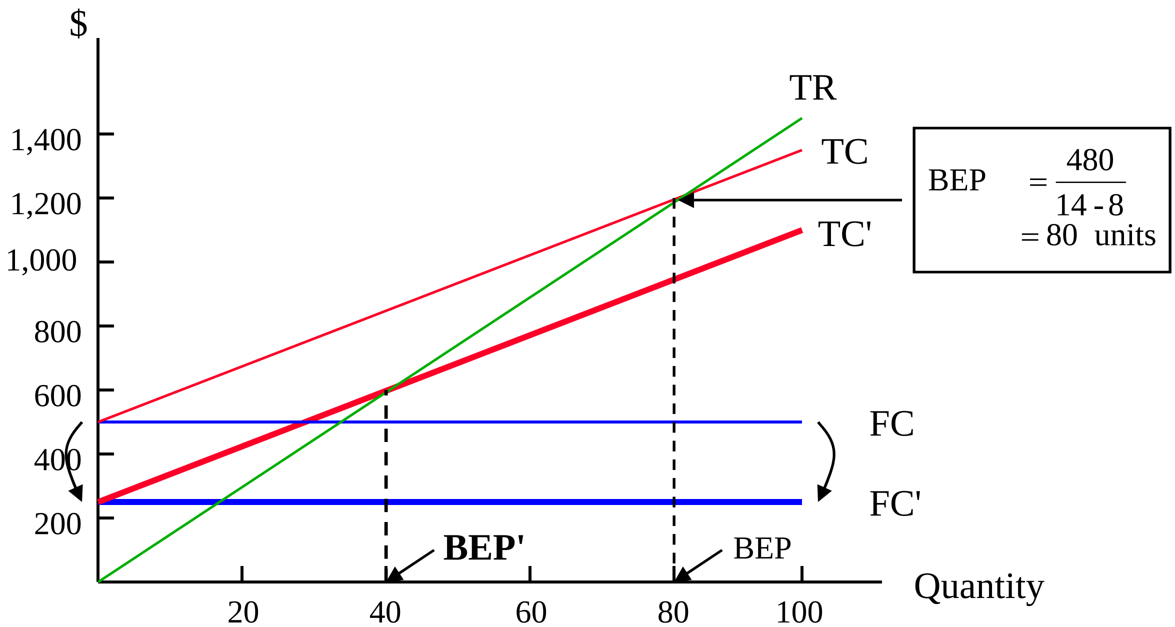
$$n\Big|_{BEP} = \frac{FC}{SP - VC}$$

3. decreasing the slope of TC(n), i.e., decrease the unit variable cost VC(n).

TC(n) = VC(n) + FC

Lowering the Breakeven Point

- Fixed Cost Reduction



VC = \$8.00/unit

SP = \$14.00/unit

TR(n=100) = nSP = 100 × \$14 = \$1,400

TC(n=100) = VC + FC = 100 × \$8 + \$480 = \$1,280

FC = \$480

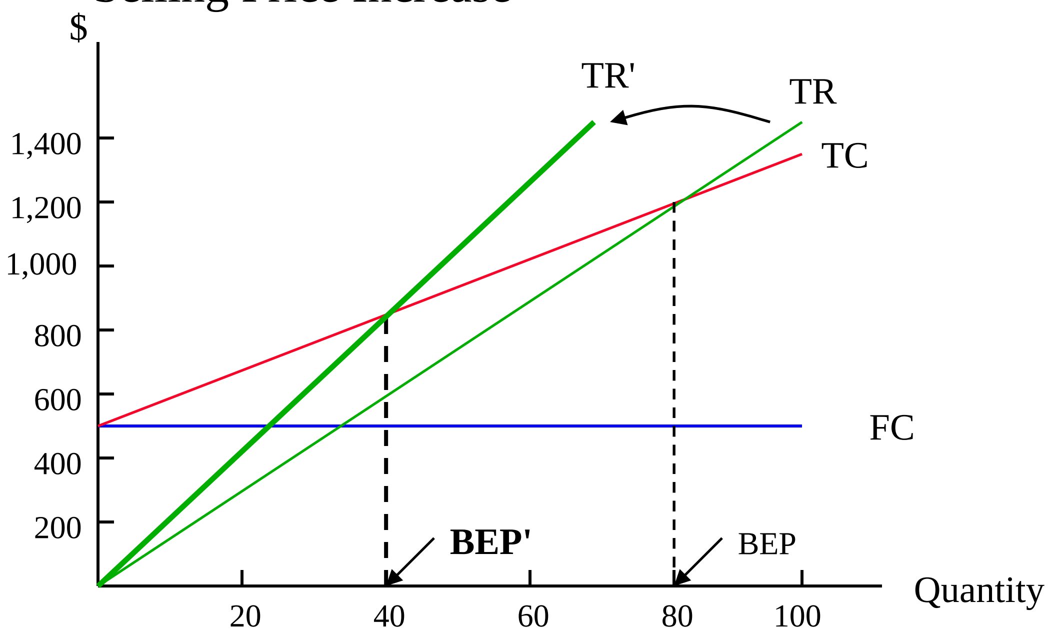
BEP = 80 units

<u>Reduce FC by Half</u>

$$FC' = \frac{480}{2} = \$240 \Rightarrow BEP' = \frac{240}{14-8} = 40 \text{ units}$$

Lowering the Breakeven Point

- Selling Price Increase



VC = $8.00 per unit	SP = $14.00/unit
TR = $1,400	TC = $1,280
FC = $480	BEP = 80

- Increasing the price SP changes the slope of TR and augments the contribution per unit.

$$\text{Contribution'} \quad = \quad \frac{FC}{BEP'} = \frac{480}{40} = \$12.00\,/\,\text{unit}$$

$$SP' \quad = \quad VC + \text{contribution'}$$

$$= \quad \$8.00 + \$12.00 = \$20.00 \ \text{ per unit}$$

Break Even Analysis

- Determine the value of X that will yield a breakeven situation

Investment	\$50,000	k=10%
Annual Fixed Costs	\$10,000	
Annual Variable Costs	\$2.00 per unit	
Revenue	\$4.50 per unit	
Planning Horizon	5 years	

At what level of output does the NPV = 0?

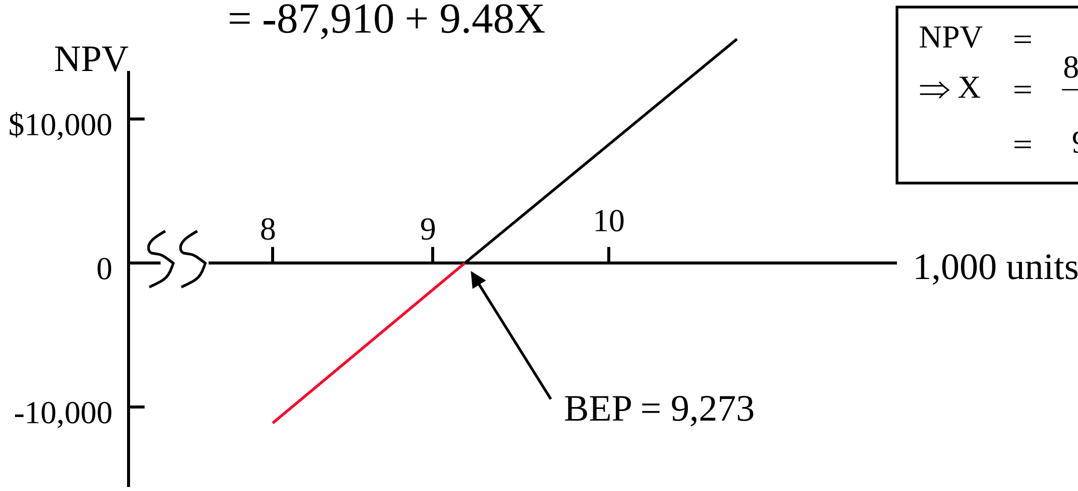
Let X = the annual quantity

Then NPV = -50,000 - 10,000(P|A 10,5) - 2x(P|A 10,5) + 4.5 x (P|A 10,5)

= -50,000 - 10,000(3.791) + (4.5-2)(3.791)X

= -87,910 + 9.48X

$$\text{NPV} = 0 \quad \Rightarrow X = \frac{87{,}910}{9.48} = 9{,}273$$



Electric Motor Efficiency

- Two 10 kW motors are under consideration
- Motor A - sells for $400
  - efficiency rating 85%
- Motor B - sells for $300
  - efficiency rating 78%
- Cost of electricity - $0.042/kW-hour
- Five-year planning period
- Equal salvage values
- MARR of 20% is to be used
- Assume motor is loaded to capacity

What is the range of values for hours used per year for each motor to be preferred?

i.e. For what range of usage are the costlier enhancements to Motor A to gain its efficiency justified on an economic basis?

Electric Motor Efficiency

Let x = annual usage in hours

Electricity cost/year for 100% efficient motor:

$$\frac{10-\text{KW}}{\text{motor}}\ \frac{\$0.042}{\text{KW-hour}}\ \frac{x\ \text{hours}}{\text{year}} = \frac{\$0.42x}{\text{year}}\quad \text{per motor}$$

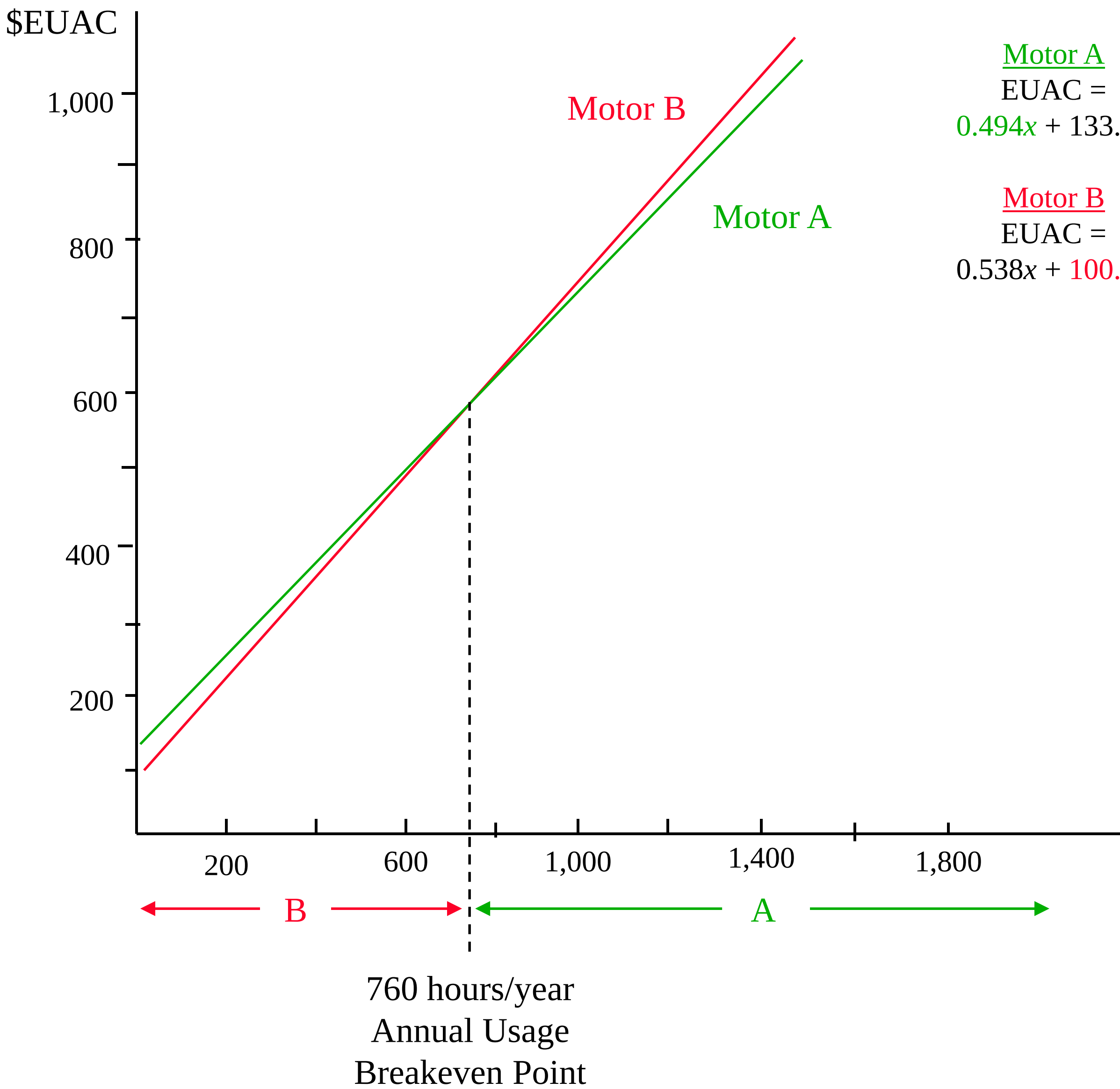
Equivalent uniform annual cost:

$$\begin{aligned}
\text{EUAC}_A(20\%) &= \frac{\$0.42x}{0.85} + \$400\,(\text{A}\,|\,\text{P } 20{,}5) \\
&= \$0.494x + \$133.76 \\
\text{EUAC}_B(20\%) &= \frac{\$0.42x}{0.78} + \$300\,(\text{A}\,|\,\text{P } 20{,}5) \\
&= \$0.538x + \$100.32
\end{aligned}$$

Let $\text{EUAC}_A(20\%) = \text{EUAC}_B(20\%)$
and solve for x

$$x = \frac{\$33.44\,/\,\text{year}}{\$0.044\,/\,\text{hour}} = 760\ \text{hours/year}$$

Electric Motor Efficiency



Motor A
EUAC =
$0.494x + 133.76$

Motor B
EUAC =
$0.538x + 100.32$

Cost Reduction Study

- What level of annual savings need to be realized in order to justify the study?
- The study will cost \$20,000 to perform.
- The planning period is five years.
- MARR = 10%

$$NPV = -\$20{,}000 + A\underbrace{(P|A\ 10,5)}_{3.7908}$$
$$= -\$20{,}000 + 3.7908A$$

Setting NPV = 0 and solving for the breakeven value of A yields

$$3.7908\,A = 20{,}000$$
$$\Rightarrow A = \$5{,}276$$

Therefore, if the annual savings are greater than \$5,276, then the study is justified.

Sensitivity Analysis

- Sensitivity analyses are performed when the range of the possible values A parameter can take on is known.
- However, there is uncertainty as to their chances of occurrence.
- The sensitivity of the measure of merit to changes, or errors in estimating, parameter values is shown.
- For what values of the uncertain parameters does the project become unprofitable?

<u>Cost Reduction Study</u>

- The cost study will cost $20,000 to perform
- Annual savings are estimated at $7,000
- MARR = 10%, five year planning period
- How sensitive is the economic viability of the study to errors in estimating the initial cost and the annual savings?
- Conduct a sensitivity analysis
- Let x = % error in estimating first cost
  y = % error in estimating savings

Cost Reduction Study

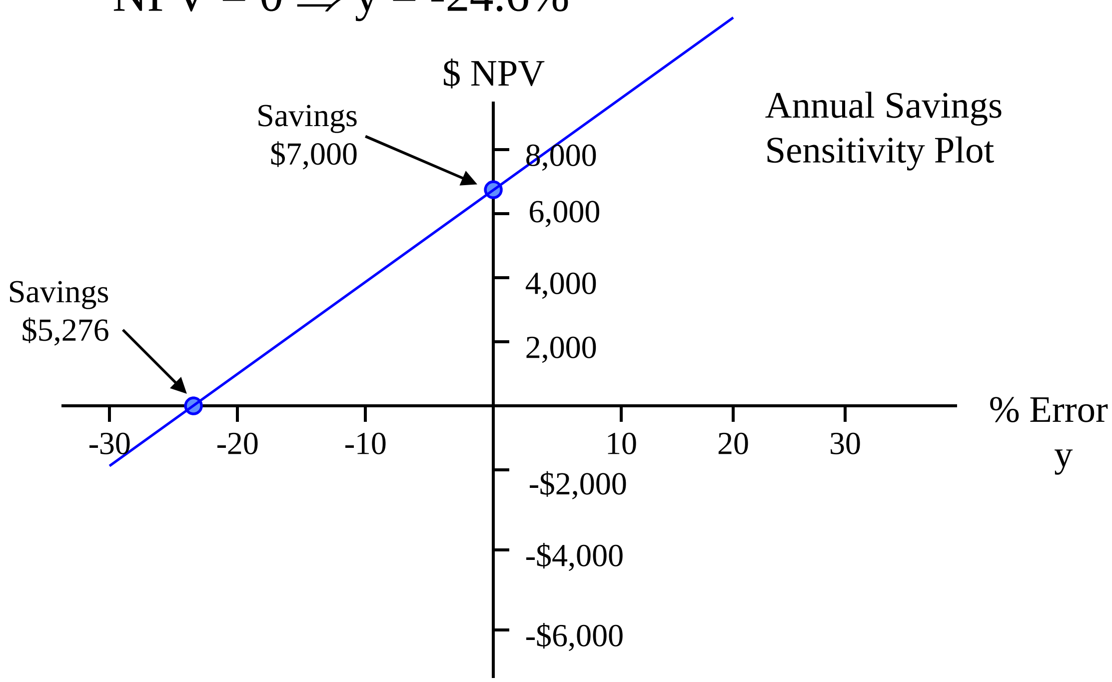
- First, consider errors in estimating the annual savings only

  First Cost = \$ 20,000

  Annual Savings = \$ 7,000 (1+y)

- The study should be done if NPV $\geq$ 0

  $$NPV = -20{,}000 + 7{,}000(1+y)\ (P|A\ 10{,}5)$$
  $$= 6{,}535.6 + 26{,}535.6y$$
  $$y = 0 \Rightarrow NPV = 6{,}535.6$$
  $$NPV = 0 \Rightarrow y = -24.6\%$$



Cost Reduction Study

- Second, consider errors in estimating the first cost only

  First Cost = $ 20,000(1+x)

  Annual Savings = $ 7,000

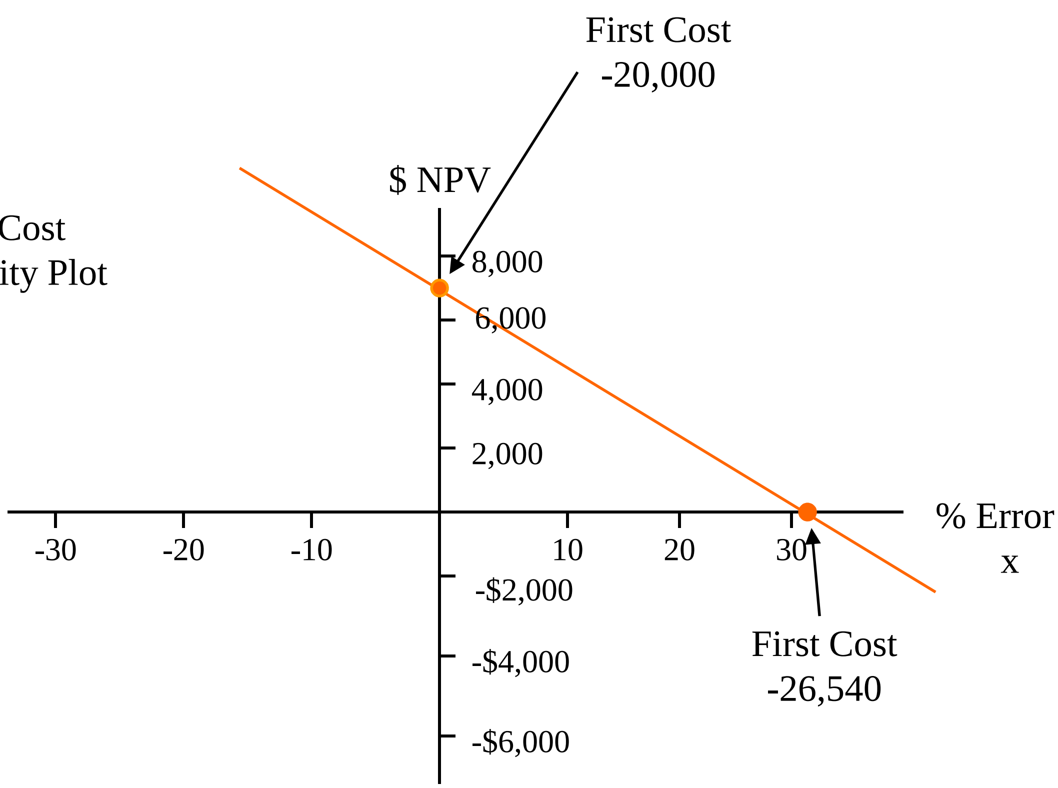
- NPV = -20,000(1+x) + 7,000 (P|A 10,5)

  = -20,000x + 6535.6

  x = 0 $\Rightarrow$ NPV = 6,535.6

  NPV = 0 $\Rightarrow$ x = 32.7%

First Cost
Sensitivity Plot



Cost Reduction Study

Then First Cost = \$20,000(1+x)

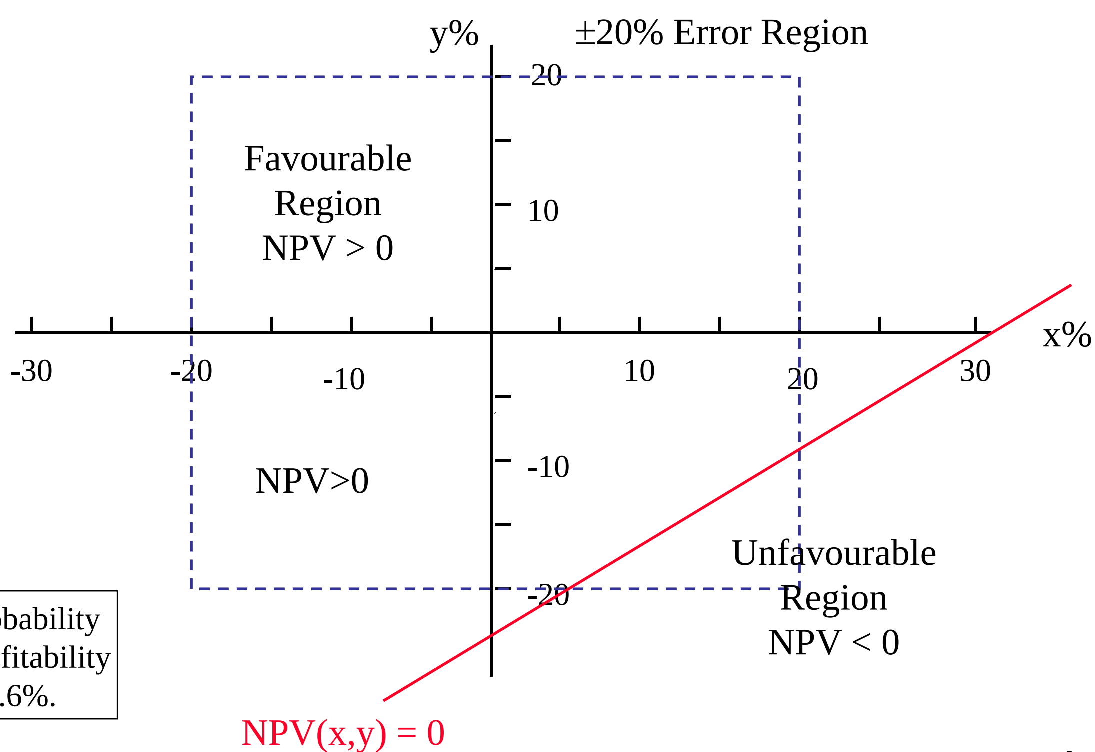
Annual Savings = \$ 7,000(1+y)

The project is justified economically when:

$$NPV(x,y) = -20{,}000(1+x) + 7{,}000(1+y)(P|A\ 10,5) \geq 0$$
$$= 6{,}535.6 - 20{,}000x + 26535.6y \geq 0$$

Solving for y

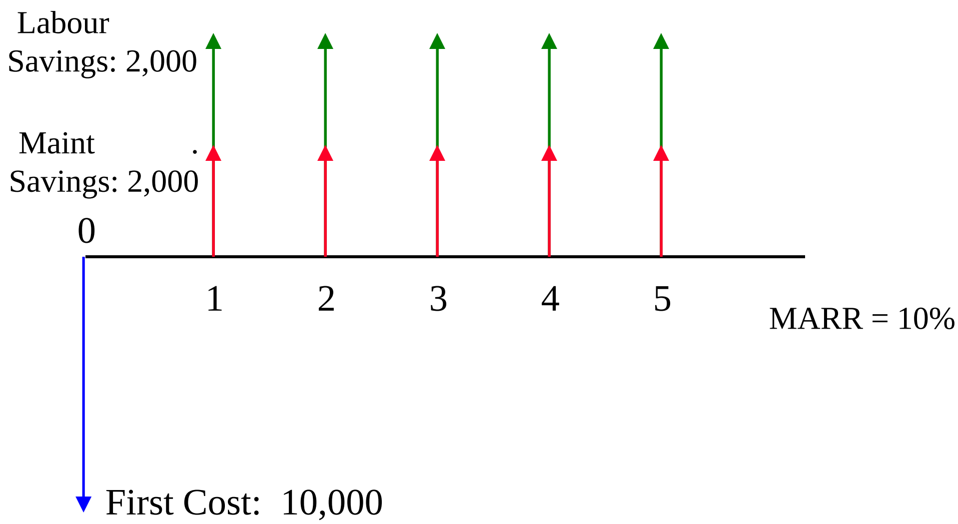
$$y \geq -0.246 + 0.754x$$



The probability of unprofitability is 4.6%.

Sensitivity Analysis

- What are the effects of making errors in estimating cash flows?
- Consider a project where two of the cash flow estimates are subject to error



- labour savings - % error = x
- initial cost - % error = y

$$\text{NPV} = -10{,}000(1+y) + 2{,}000(1+x)(\text{P}\,|\,\text{A }10{,}5)+(\text{P}\,|\,\text{A }10{,}5)2{,}000$$
$$= 5{,}164 - 10{,}000\text{y} + 7{,}582\text{x}$$

For what values of x and y is the NPV = 0?

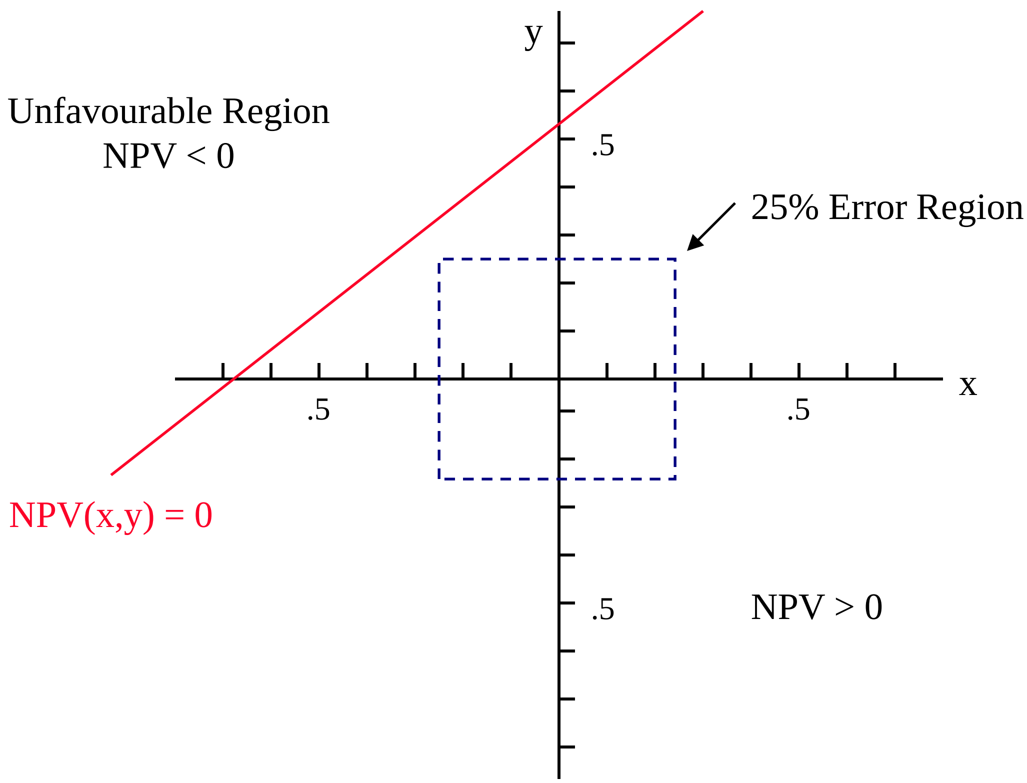
$$0 = 5{,}164 - 10{,}000\text{y} + 7{,}582\text{x}$$

Sensitivity Analysis

x = % error in labour savings

y = % error in initial cost

$$0 = 5{,}164 - 10{,}000y + 7{,}582x \Rightarrow y = .52 + .76x$$



- If we are certain that the estimates for labour savings and first cost are correct within plus or minus 25%, then the sensitivity analysis indicates that the NPV will always be greater than zero.