ECE 472F - PROBLEM SET #3 (suggested completion date: October 30th, 2015)

1. The parents of a small child have decided to create an education fund by making equal annual deposits into a mutual fund of government and corporate bonds. They believe that the historical real return for the fund of 2% annually is a good estimate for the returns from the fund over their planning horizon. The first deposit will be made on the child's fifth birthday and the last deposit on the fifteenth birthday. Then starting on the eighteenth birthday, the following annual withdrawals to cover the actual cost of college will be made. The parents have assumed a constant rate of inflation over the planning horizon and that education costs will increase at the same rate as the general rate of inflation. What equal annual deposits must the parents make?

Birthday	Withdrawal
18	$18 100
19	$18 825
20	$19 575
21	$20 360

2. The ABC Credit Union offers two different savings accounts. The "Savings Advantage" account has an annual interest rate of 2.4 per cent compounded quarterly. Interest is calculated on the minimum quarterly balance in the account. The "Money Multiplier" account offers an annual rate of 2 per cent compounded quarterly. However, interest is earned from the time of deposit.

   An investor plans to deposit $10 000 on January 1st and $6 000 on February 1st into one of these accounts. Which account should the investor choose to maximize accumulated interest at the end of the first quarter (March 31)?

   (a) How much additional interest (in dollars and cents) will the investor earn versus the other account?
   (b) If the number of accounts is not a constraint, what is the optimal savings strategy and how much further ahead is the investor?

   (Assume: (1) that the balance in an account is determined by the daily closing value;
   (2) that all months are of equal length; i.e., a monthly interest rate may be used.)

3. Maintenance costs for the new engineering building are expected to be $100 000 each year for the first five years; $125 000 for each year 6 through 10 inclusive; and $175 000 each year after that. Assume that the building will have an indefinite service life. The donated funds will earn 6% per year in the University's endowment investment pool.

   (a) What amount of donations should the Dean of Engineering seek now in order create a fund to cover all the expected future maintenance costs for the building?
   (b) How much less will the Dean need to raise if the service life of the building is assumed to be 35 years?

4. During the time that Nortel was attempting to make a financial comeback, Nortel went to the debt market to raise funds by issuing convertible bonds. Consider an investor at that time evaluating a new issue of a 7-year Nortel convertible bond with a face value of $100 000 and a bond rate of 5% per annum payable semi-annually. The bond is currently selling for $103 000. She plans on owning the bond for five full years (receiving the interest payment at the end of Year 5) at which time she will sell it. Her expected annual rate of return is 6%. In light of current interest rates, discuss why the bond is selling at a premium above its face value. What must she sell the Nortel bond for (or convert it into Nortel stock) at the end of Year 5 to realize her expected rate of return? Assume that she can reinvest her interest payments at 2% compounded semi-annually.

   A convertible bond may be exchanged for a company's equity shares at a stated price at the option of the bondholder. Convertible bonds, which act as stock-bond hybrids, have become increasingly popular recently as Nortel, Lucent and a host of other blue-chip companies have run into tough times. Lenders prefer convertible bonds because if the company and its stock stage a comeback, the financier can convert the bonds to stock and score a big win. If no comeback occurs, the lender still has a guaranteed return.

   Would this have been a good investment if the bond had been purchased in 2005?

5. Given the three cash flow profiles shown below, determine the values of X and Y so that all three cash flow profiles are equivalent at an annual interest rate of 5%. Show all your work and use an approach that minimizes the number of calculations.

End of Year	Cash Flow A	Cash Flow B	Cash Flow C
0	-10 000	-2 000	0
1	2 000	2X	2Y
2	4 000	1.75X	2Y
3	6 000	0	2Y
4	8 000	1.25X	Y
5	10 000	X	Y

6. A young couple shopping for a new home are trying to decide how "much house" they can afford to buy. They can afford $3 000 per month in mortgage payments and do not want to finance more than 75% of the purchase price. Keeping below the 75% limit means that they do not have to pay costly mortgage insurance. They will obtain a three-year mortgage with monthly payments at 6% compounded monthly. The amortization period will be 25 years.

   (a) What is the most expensive house that they can buy and what down payment do they need?
   (b) How much interest will they pay over the three-year term?
   (c) Assuming their house increases in value by 3% per year, what percentage of the house do they own at the end of the three-year term?

7. A civil engineering firm is considering a BOOT (Build-Own-Operate-Transfer) project to build a toll road. As the name implies, the firm builds the project, which they then own and operate for a term. After 40 years from the start of the project, ownership is transferred to the government without any further compensation.
The project is funded by a mix of bank loans, long-term bonds, and equity from investors (the firm, life insurers, and pension funds[1]) with a required real rate of return of 12%. Inflation is projected to be 2%.
Construction is estimated to take 5 years, with beginning-of-year constant-dollar costs as follows: $50m, $110m, $120m, $130m, and $140m.
Operation starts at the beginning of year 6. Annual operating & maintenance costs are expected to be $50m nominal in the first year, increasing with inflation.

   (a) What is the annual capital recovery charge in today's constant dollars during the operating period?
   (b) What must be the present worth of the revenue in today's dollars to make the project worthwhile?
   (c) Assume that revenue also increases with inflation during the operating term. How much (nominal dollars) must the toll road take in in its first operating year to meet profit objectives?
   (d) What if revenue increases 4% annually?

8. Government of Canada (GoC) real-return bonds (RRBs)[2] are a useful financial instrument. For investors who are concerned about inflation (e.g. indexed pensions, conservative individuals) they provide a means of hedging inflation risk by linking returns to inflation. They also provide a means of "trading" inflation so that we can infer market expectations of inflation.
An RRB is similar to a normal (*nominal*) bond like the ones discussed in class. It pays a coupon that is a fixed percentage of the face value semi-annually (coupon Fc/2 for annual coupon rate c). However, unlike a nominal bond, its face value changes in response to the CPI (Consumer Price Index). At issue, the bond is associated with the value of the CPI at the time of issue, called the *base CPI (CPI0)*. The face value Ft at time t is then F0(CPIt/CPI0).
The cash flows from an RRB are Ftc/2 = F0CPIt/2CPI0 semi-annually, and Fm=F0CPIm/CPI0 at maturity.

   (a) Assuming constant annual inflation, write out the equation for real (constant-dollar) yield on a regular par bond with coupon rate cn. What about a par RRB with coupon rate cr? Use these answers to explain why RRBs are are called real-return.
   (b) Suppose a 30-year GoC par RRB has a coupon rate of 5% while a 30-year "normal" par GoC bond has a rate of 8%. Based on this, what constant <u>effective annual</u> inflation rate does the market anticipate over the next 30 years?
   (c) If you can buy a GoC RRB with 5% coupon and 15 years to maturity for 95-1/8 while a nominal GoC with the same maturity yields 6.2%, what are the <u>effective annual</u> nominal, real, and expected inflation rates for the 15-year term implied by the market prices?
   (d) All else being equal, if inflation expectations increase, what happens to the bond yields and prices?

[1] Since they have very, very long-term (i.e., human lifetimes, many decades long) liabilities, life insurers and pension funds generally search for very long-term stable assets to match them. Consequently, life insurers and pension funds are active investors in utilities and real estate.

[2] http://www.finiki.org/wiki/Real_Return_Bonds

9. Loans with a flexible payment schedule are generally priced with a *floating rate* of interest, which changes over time in response to fixed-income market conditions. Canadian banks generally price consumer products in terms of the *prime rate* P, which is the rate they offer their best ("prime") customers[3]. The prime rate in turn varies up and down depending on the Bank of Canada's target overnight rate, set by the Bank of Canada to keep inflation near 2%.

   Many students use government loans (OSAP) to finance part of their university education. The loans are interest-free while the student remains in school.

   Following graduation, there is a 6-month grace period where no payments are required, no interest is charged on the provincial part, and interest accrues (is added to the principal balance) on the federal part. You have up to 15 years to repay the loan, though the standard plan is 9.5 years. OSAP loans generally contain two components: a Federal component, which charges interest at P+2.5%, and a provincial component at P+1%, both compounded monthly.

   When you make a payment, the principal component of the payment is split between the federal and provincial parts in proportion to the outstanding balance. Let P be the prime rate, Bi the balance at the beginning of the i-th month after graduation, and f be the percentage of the balance that is federal.

   (a) Show that the federal and provincial components will be paid off at the same time due to the proportional split of payments. You may assume that the payment exceeds the interest (otherwise it would never be paid off!)
   (b) Assume you have a balance B7=10,000 at the end of the interest-free window. Compare the total interest cost to pay off the loan in the following four years versus the standard schedule of nine-and-a-half years. Assume the prime rate stays constant at its current value of 3% (note 5-6% is more typical and it exceeded 10% in the 1980s) and the loan is 50% federal, 50% provincial.
   (c) Suppose you receive some money from family as a graduation gift, and decide to use part of it to either invest or repay loans. For the next six months, interest accrues only on the federal portion of your loan. What must be your after-tax effective annual rate of return to justify investing instead of repaying your loan? *[Hint: answer will be in terms of variables P and f]* Calculate an example with the federal balance as 50% of total and a 3% prime rate
   (d) A tax-savvy friend points out that you will receive a tax credit of 15% of all interest paid/accrued on student loans. How does that change the answer above?
   (e) After the six-month grace period, assume you can make the minimum monthly payments. What is your optimal strategy now for any extra money you have to repay/invest? *[Answer in terms of P, f, and your after-tax rate of return i]*
   (f) Create an amortization schedule to pay off a loan of $6000 that is 1/3 federal and 2/3 provincial by the end of month 30. Use equal monthly payments starting at the end of month 6, assuming a prime rate of 3%. Show the federal & provincial components of balance outstanding, interest, and principal payments. *[Hint: Excel is your friend here.]*

[3] This has historically been true but in recent years with a very competitive mortgage market, customers have sometimes been offered discounts as low as ~P-0.9%. However, that is generally not done for any other products.