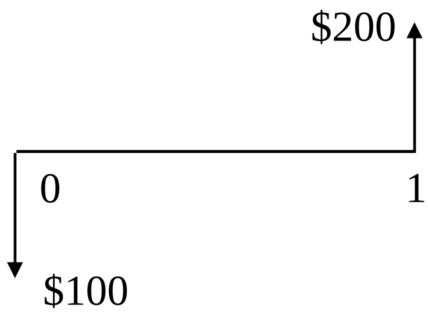
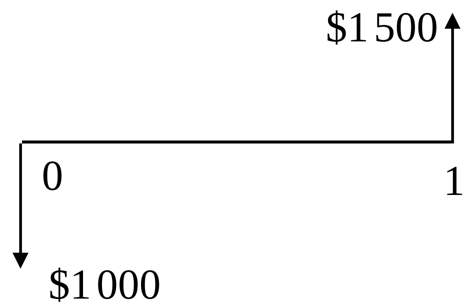
IRR vs. NPV: Mutually Exclusive Projects

A



B



<u>IRR METHOD</u>

$$\begin{aligned} 0 &= -100 + 200\,(P|F\ i,1) \\ &= -100 + \frac{200}{1+i} \\ \Rightarrow i_A^* &= 100\% \end{aligned}$$

$$\begin{aligned} 0 &= -1\,000 + 1\,500\,(P|F\ i,1) \\ &= -1\,000 + \frac{1\,500}{1+i} \\ \Rightarrow i_B^* &= 50\% \end{aligned}$$

<u>NPV METHOD @ MARR = 10%</u>

$NPV_A = -100 + 200\ (P|F\ 10,1)$ $NPV_B = -1\,000 + 1\,500(P|F10,1)$

$= \$82$ $= \$364$

The IRR & NPV methods rank the projects differently:

IRR – A is better

NPV – B is better

IRR vs. NPV : Mutually Exclusive Projects

- Which Project Is Really Better?
  - Look at the rate of return (IRR) on the incremental investment

IRR:

$$
\begin{aligned}
0 &= -900 + 1\,300\,(P|F\ i,1) \\
&= -900 + \frac{1\,300}{1+i} \\
\Rightarrow i^*_{INC} &= 45\%
\end{aligned}
$$

The IRR on incremental investment of $900 is 45%.

As this is greater than the MARR of 10%, the incremental investment of $900 in B is worthwhile.

$\therefore$ Confirms the NPV result.
Project B is better.

N.B. Section 4.7.2 - Incremental Approaches

IRR versus NPV - Different Rankings

Alternative	Value		Ranking		
	NPV	IRR	NPV	IRR	
A	$100K	15%	1	3	
B	$ 70K	18%	2	1	NPV>0 IRR > MARR
C	$ 40K	17%	3	2	
					MARR=10 %
D	–$ 20K	3%	4	5	
E	–$ 45K	5%	5	4	NPV<0 IRR < MARR

Note:

1. Same accept/reject recommendations
2. Different ranking of alternatives

Consider the effect on:

1. Independent projects
2. Mutually exclusive projects
3. Budget constraints

IRR: Multiple Roots

$$i^* = \text{IRR} \Rightarrow 0 = \sum_{t=0}^{n} A_{jt}(1+i_j^*)^{n-t}$$

n^{th} Degree Polynomial

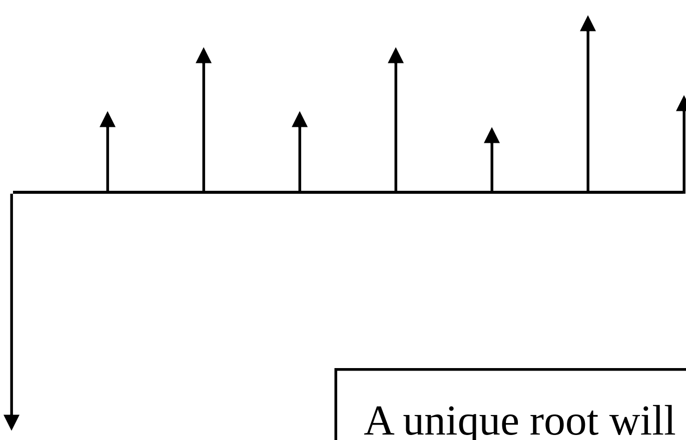
$$0 = A_0x^n + A_1x^{n-1} + \ldots + A_{n-1}x + A_n$$

where $x = (1 + i^*)$

General Case: n distinct roots exist. However,

$$\#\text{Roots} \le \begin{matrix}\#\text{ of changes of sign in}\\ < A_0, A_1, \ldots, A_n >\end{matrix}$$

$\therefore$ For a "Normal" Cash Flow Profile



A unique root will exist.

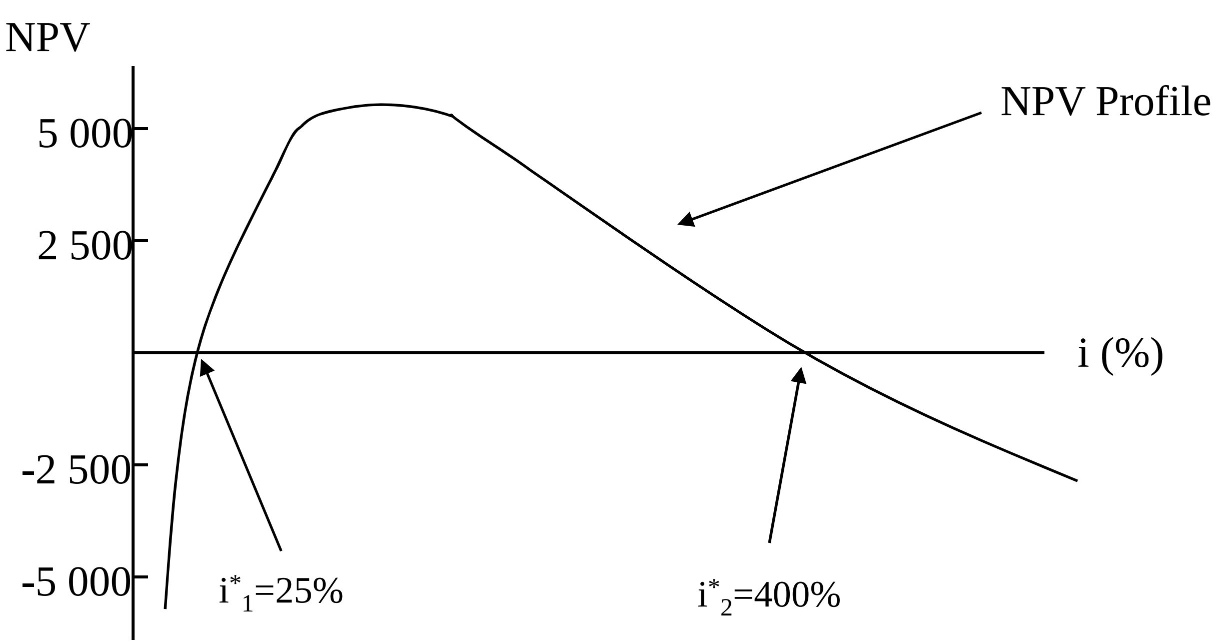
IRR: Increased Oil Production Project

- e.g. Increased oil production leads to earlier exhaustion of supply.
- Invest $8 000 in new oil pump.

EOY	With Old Pump	With New Pump	Δ
0	0	–8 000	–8 000
1	50 000	100 000	50 000
2	50 000	0	–50 000

$$\text{NPV} = -8\ 000 + \frac{50\ 000}{1+i} - \frac{50\ 000}{(1+i)^2}$$

NB. two sign changes => up to 2 roots



IRR: Increased Oil Production Project

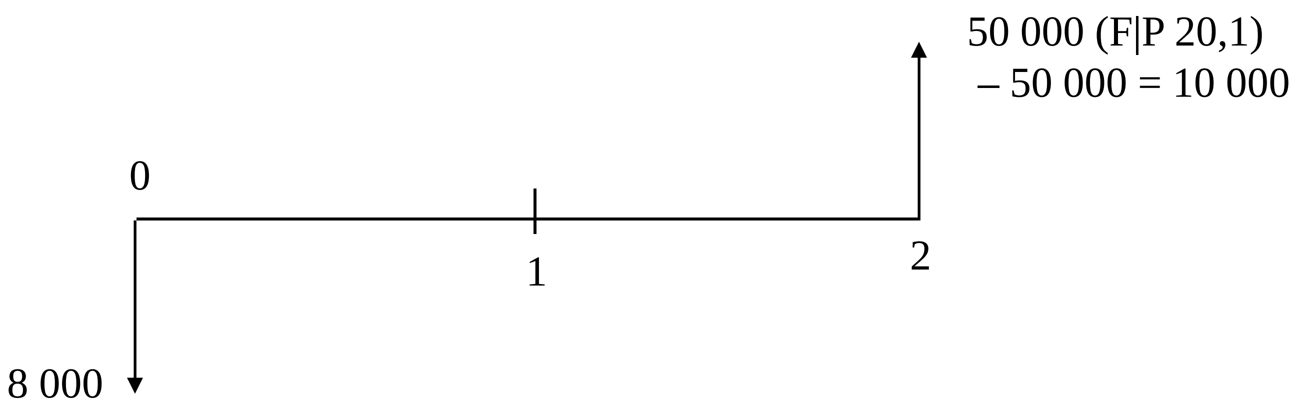
IRR on \$8 000 investment

= 25%, 400%?

What is the true benefit of the \$8 000 investment?

The investor receives \$50 000 one year ahead of time.

Assume this \$50 000 can be invested at EOY 1 at 20%.



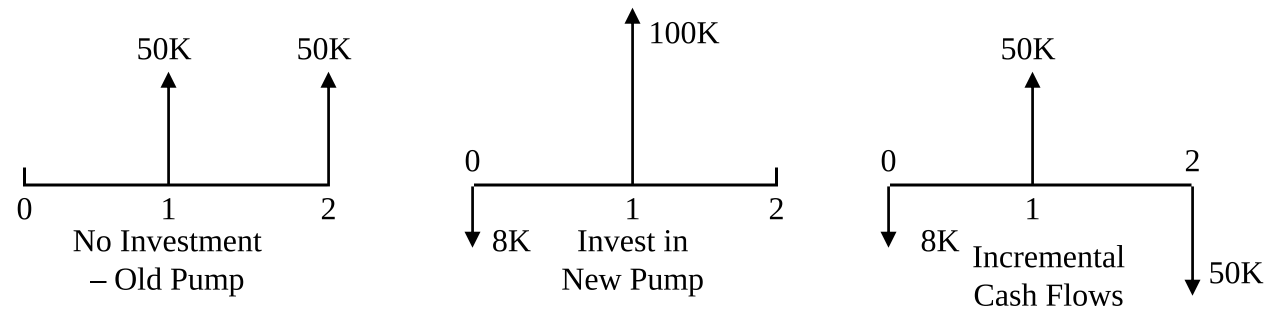
IRR: $0 = -8\,000 + 10\,000\,(P|F\ i^*,2)$

$$\boxed{\Rightarrow\ i^* = 11.8\%}$$

Assuming a reinvestment rate of 20%.

IRR: Increased Oil Production Project

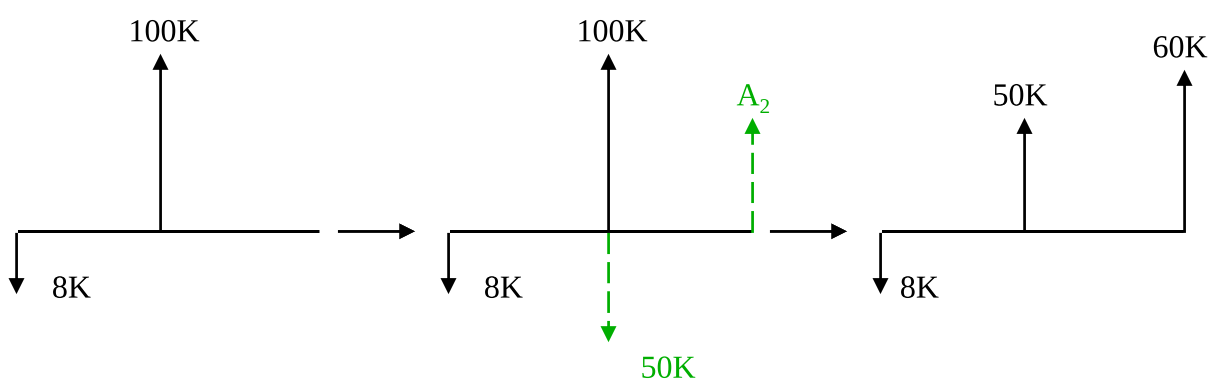
- New pump leads to earlier exhaustion of supply



$$\text{NPV} = -8\ 000 + \frac{50\ 000}{1+i} - \frac{50\ 000}{(1+i)^2}$$

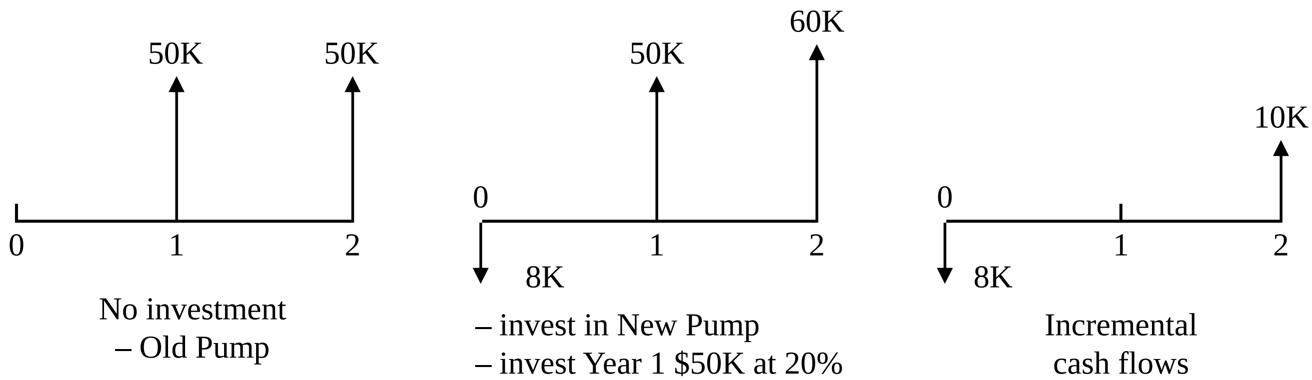
=> $i^*_1 = 25\%$, $i^*_2 = 400\%$ (Multiple IRR Solutions)

- True benefit of investment - \$50K one year earlier
- Assume this \$50K can be invested at 20%



$$A_2 = 50\ 000\ (F|P\ 20,1) = 50\ 000\ (1.2) = 60\ 000$$

IRR: Increased Oil Production Project



IRR: $0 = -8000 + 10000\ (P|F\ i^*,2)$

$$\boxed{\Rightarrow i^* = 11.8\%}$$

IRR Summary & Caveats

- IRR will always give same accept/reject recommendations
  i.e. if $i^* > MARR$, then $NPV > 0$
- However, IRR and NPV can rank projects differently
- This is a problem if:
  - projects are mutually exclusive
  - the capital budget is limited
- Use incremental IRR approach in these cases (or NPV)

External Rate of Return

$$\sum_{t=0}^{n} R_t (1+r_t)^{n-t} = \sum_{t=0}^{n} C_t (1+i^{'})^{n-t}$$

where:

$$A_t = \text{net cash flow in period } t$$

$$C_t = \begin{cases} A_t & \text{if } A_t < 0 \\ 0 & \text{otherwise} \end{cases}$$

$$R_t = \begin{cases} \mathrm{A_t} & \text{if } A_t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

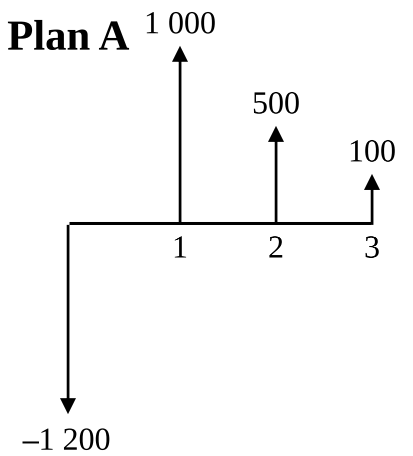
$$r_t = \begin{array}{l} \text{reinvestment rate for funds} \\ \text{recovered in period } t \end{array}$$

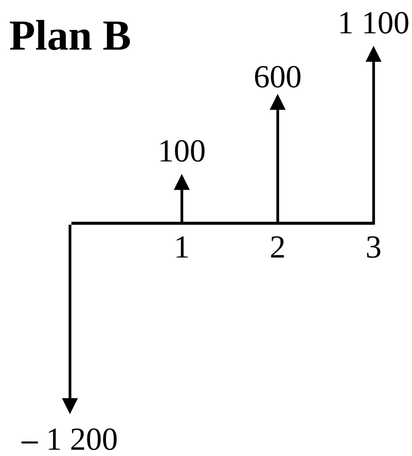
$$i^{'} = \text{external rate of return}$$

ERR avoids:

1. Multiple root problem
2. Reinvestment problem (r_t is usually the MARR)
3. Trial and error solution for i^*.

External Rate of Return Example





ERR: MARR = 5% = r_t (Reinvestment Rate)

Plan A 1000 (F|P 5,2) + 500 (F|P 5,1) + 100

$$= 1200\,(1 + i')^3 \qquad i' = \text{ERR}_A$$

$$\Rightarrow i' = \sqrt[3]{\frac{1728}{1200}} - 1 \qquad i'_A \approx 13\%$$

Plan B 100 (F|P 5,2) + 600 (F|P 5,1) + 1100

$$= 1200\,(1 + i')^3 \qquad i' = \text{ERR}_B$$

$$\Rightarrow i' = \sqrt[3]{\frac{1840}{1200}} - 1 \qquad \boxed{i'_B \approx 15\%}$$

<u>Alternative Chosen with an MARR = 5%:</u>

NPV	IRR	**ERR**
B	A	**B**

The Comparison of Alternatives

<u>Step 6</u>

Rank/Compare the Investment Alternatives

- determine which alternative is the best using the company's measure of merit
- choose the alternative with:
  - the highest value for the NPV, AW, FW methods
  - the smallest payback period
  - the highest rate of return for IRR and ERR methods
  - the largest savings investment ratio

<u>Step 7</u>

Perform the Supplementary Analyses

- sensitivity analysis – how does the outcome vary as the values of the parameters change
- estimates of cash flows, the planning period and discount rates are subject to error

<u>Step 8</u>

Select the Preferred Alternative

- analysis so far has concentrated on the economic factors

  "Find the most economical alternative"
- the final decision may also be based on intangible factors

Alternatives with No Positive Cash Flows

- cost reduction program
- compliance with legislation
  - pollution control
- mandatory service
  - find minimum cost alternative

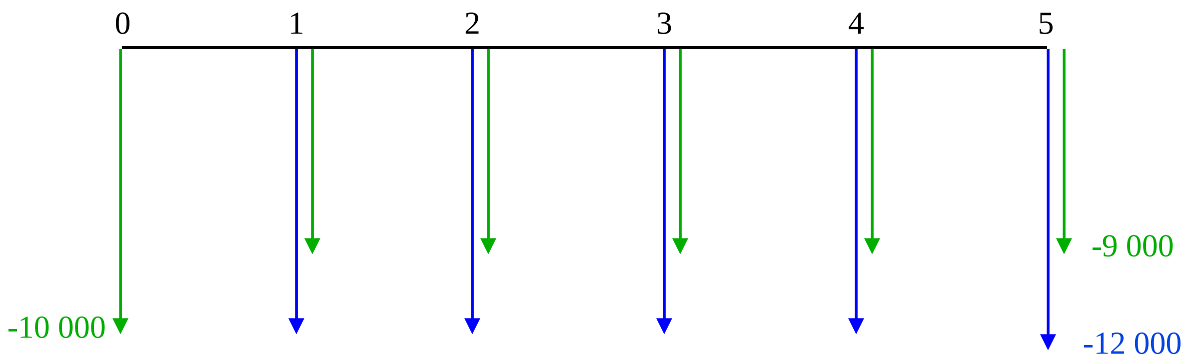
e.g. Cost Reduction Program

1. "Do Nothing" Alternative
   - continue with current level of cost
   - usually no investment required
2. "Invest in Lower Cost" Alternative
   - reduce on-going costs
3. "Invest in Higher Cost" Alternative
   - reduce on-going costs even further

- use incremental approach to determine level of investment

No Positive Cash Flows

MARR = 10%



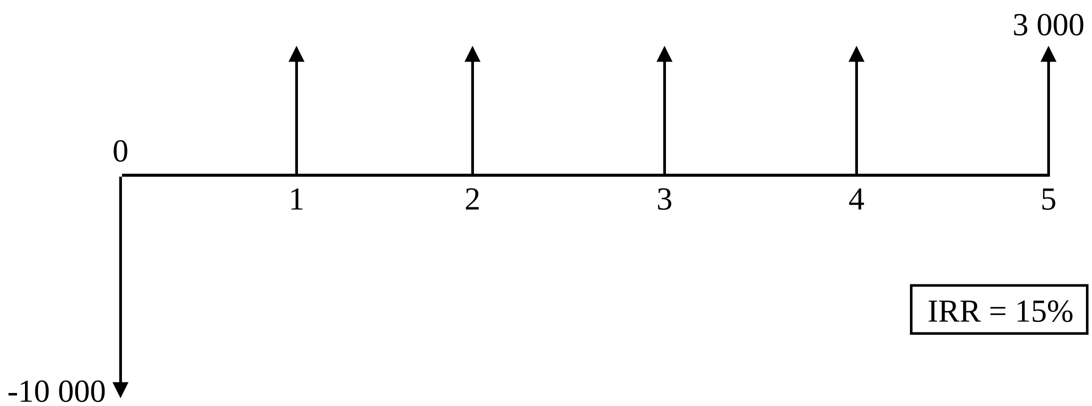
1. "Do Nothing" Alternative

   PV = –12 000 (P|A 10,5) = –$45 490

2. "$10 000" Alternative

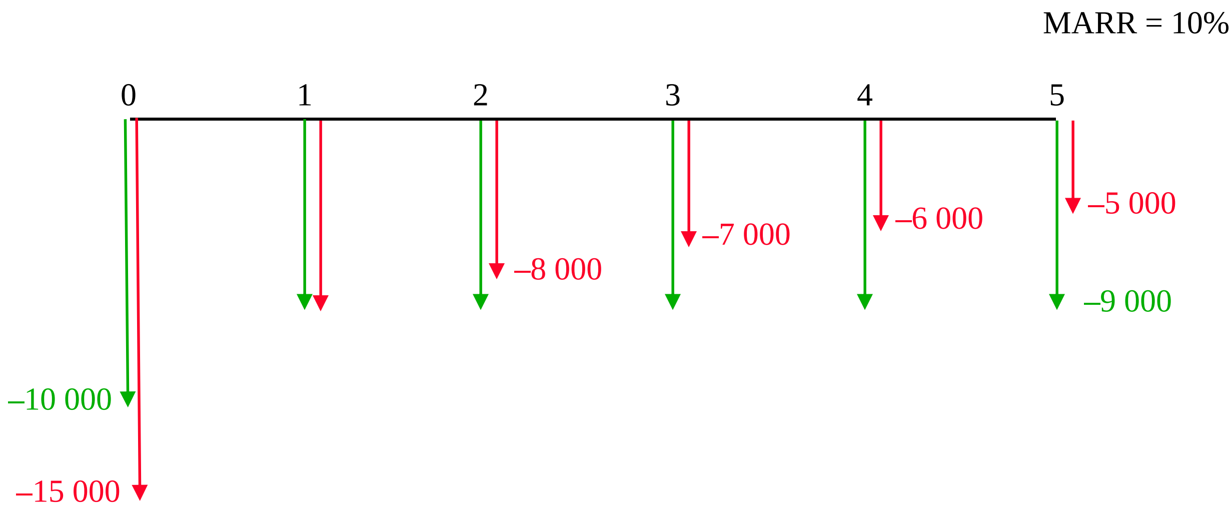
   Use an incremental approach.

PV_{2-1} = –10 000 + 3 000 (P|A 10,5)
= $1 372 > 0



IRR = 15%

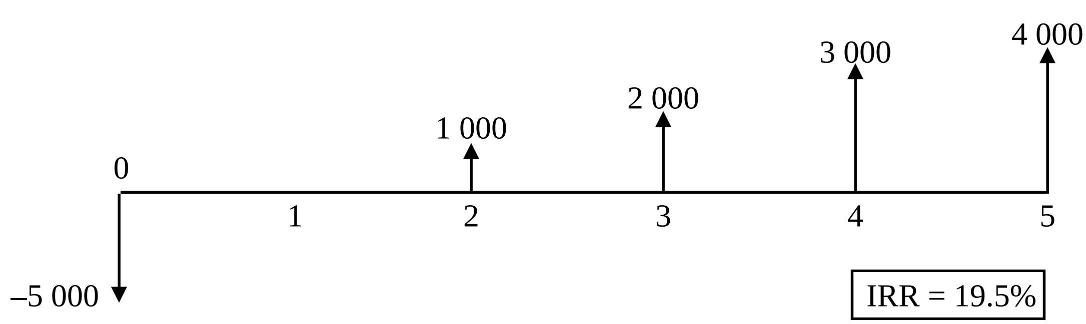
No Positive Cash Flows



3. “$15 000” Alternative

Use an incremental approach.

$$PV_{3\text{-}2} = -5\,000 + (P|G\ 10{,}5)\ 1\,000$$
$$= \$1\,862 > 0$$



Therefore choose Alternative 3.

The first $10 000 has an IRR of 15.0%.

The next $5 000 has an IRR of 19.5%.

Replacement Analysis

- frequently occurring problem
- reasons for replacing existing assets
  - replacement due to obsolescence
  - replacement due to deterioration
  - replacement due to inadequacy
- **DEFENDER** – asset currently in service
  – current value of the defender is its market value
- **CHALLENGER** – possibly more economical alternative
  – first cost includes all costs to make it operational
- replacement decisions are important to the firm

"get rid of that junk"

"we need the latest model"

- improper replacements can be a serious drain on the capital budget
- replacement analysis addresses the question of when to *retire* an asset
- unrecoverable past costs are *sunk costs* and are not relevant in economic studies of the future

Replacement due to Obsolescence

- dramatic operating-cost reductions of the challenger often exceed the capital-cost advantage of a defender
- improvements usually result in higher first costs for the challenger and reduce the salvage value of the defender
- use the current market value of the defender, not the book value in the accounting records

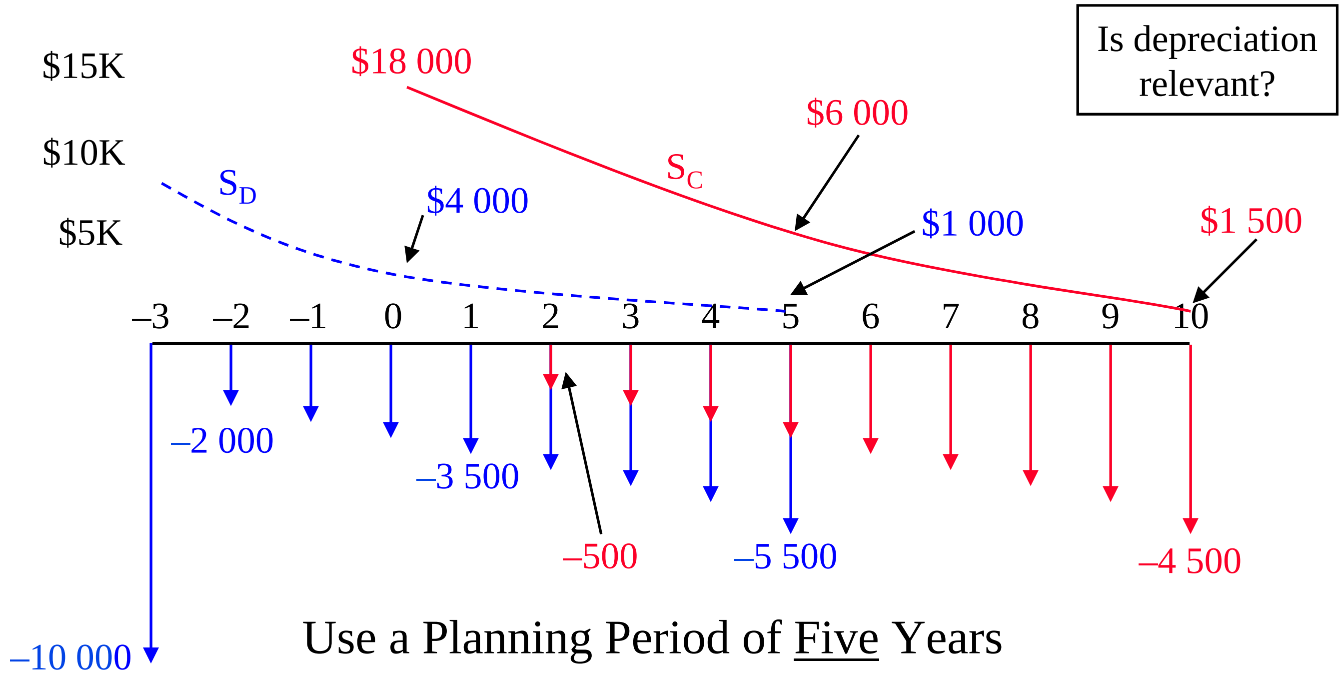
DEFENDER ALTERNATIVE:

- purchased 3 years ago for $10 000 (sunk cost)
- remaining asset economic life is 5 more years
- salvage value at that time will be $1 000
- current market value of the used defender is $4 000
- O&M (operating and maintenance) costs are expected to increase each year

CHALLENGER ALTERNATIVE:

- challenger can be purchased today for $18 000
- very low O&M costs first year due to warranty
- lower O&M costs than defender throughout asset life
- salvage value schedule must be estimated based on experience with this class of assets
- anticipated economic life of the challenger is 10 years

Replacement due to Obsolescence



Use a Planning Period of <u>Five</u> Years

	DEFENDER:	CHALLENGER:	
EOY	A_{Dt}	A_{Ct}	$\Delta = A_{Ct} - A_{Dt}$
0	0	–18 000+4 000	–14 000
1	–3 500	0	3 500
2	–4 000	– 500	3 500
3	–4 500	–1 000	3 500
4	–5 000	–1 500	3 500
5	–5 500+1 000	–2 000+6 000	8 500

$$\Delta \text{ NPV} = -14\,000 + 3\,500\,(P|A\ 10,4) + 8\,500\,(P|F\ 10,5)$$

$$= \$2\,372 > 0$$

$\therefore$ Replacement is the preferred option.

Replacement due to Obsolescence

- replacement analysis suggested CHALLENGER should be implemented now
- what if the CHALLENGER is also old technology and new technology will be available soon?
- should the CHALLENGER be implemented now or should the DEFENDER continue to be used until the new technology is available?

CHALLENGER 2 ALTERNATIVE:

- available five years from now
- lower first cost of $15 000
- lower O&M in Years 6-10 due to the asset being new at that time
- higher salvage value in Year 10

Advantages of CHALLENGER 2:

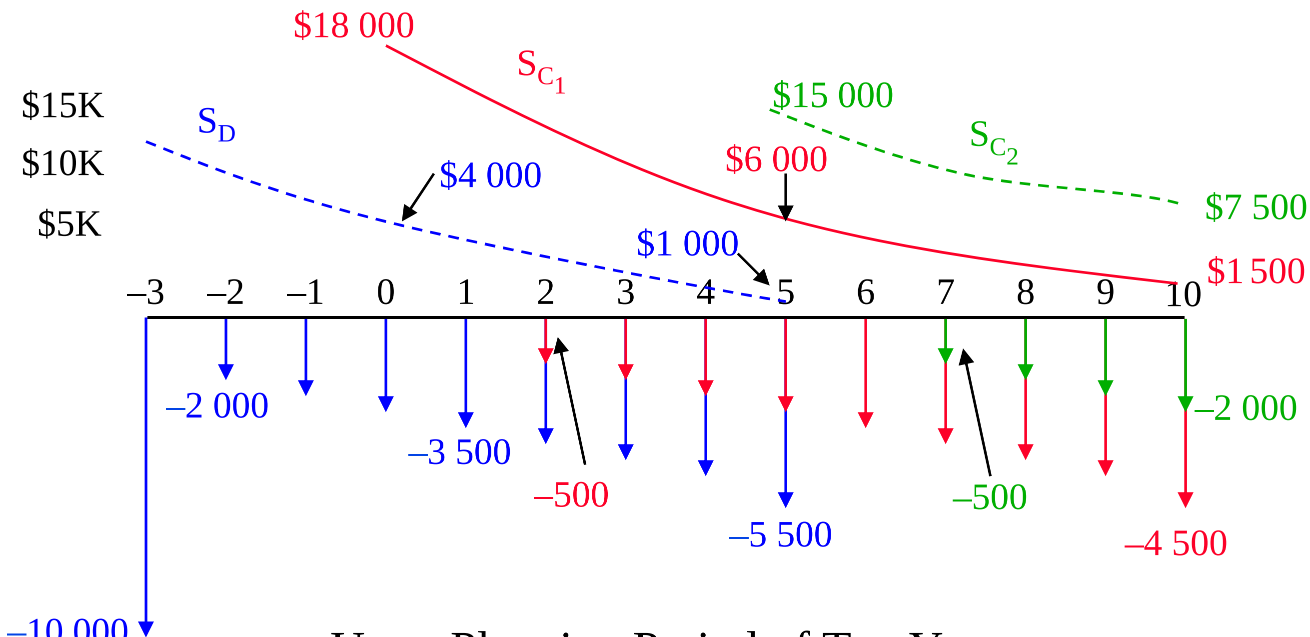
- defer capital budget expenditure for five years
- lower first cost and higher salvage value
- lower O&M costs in Years 6-10

Disadvantages of CHALLENGER 2:

- higher O&M costs during Years 1-5
- defender salvage value very low in Year 5

Replacement Due to Obsolescence

Challenger 2 Alternative



Use a Planning Period of <u>Ten</u> Years

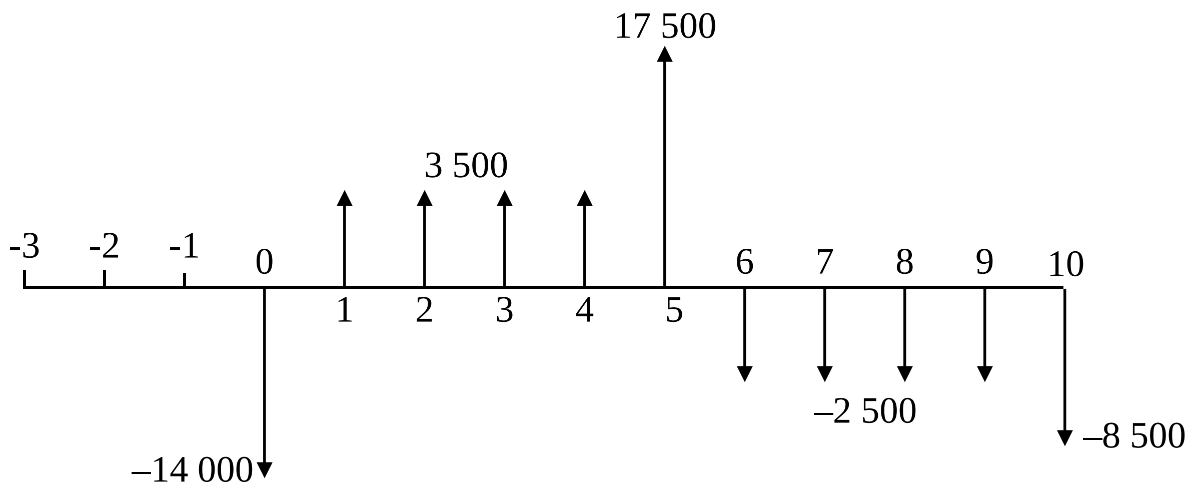
DEFENDER + CHALLENGER 2:

CHALLENGER 1:

EOY	$A_{D+C_2 t}$	$A_{C_1 t}$	$\Delta = A_{C_1 t} - A_{D+C_2 t}$
0	0	–18 000+4 000	–14 000
1	–3 500	0	3 500
2	–4 000	– 500	3 500
3	–4 500	–1 000	3 500
4	–5 000	–1 500	3 500
5	–5 500+1 000–15 000	–2 000	3 500+14 000
6	0	–2 500	–2 500
7	– 500	–3 000	–2 500
8	–1 000	–3 500	–2 500
9	–1 500	–4 000	–2 500
10	–2 000+7 500	–4 500+1 500	–2 500–6 000

Replacement due to Obsolescence

CHALLENGER 2 ALTERNATIVE



C_1 versus $D+C_2$

$$\Delta\ NPV|_{t=0} = -14\,000 + 3\,500\,(P|A\ 10,4) + 17\,500\,(P|F\ 10,5) - 2\,500\,(P|A\ 10,4)\,(P|F\ 10,5) - 8\,500\,(P|F\ 10,10)$$

$$\boxed{= -\$231 \quad < \quad 0}$$

$\therefore$ $D + C_2$ is preferred alternative.

The economic analysis suggests:

1. Do not replace the Defender now.
2. Use the Defender for five more years.
3. Replace the Defender with the Challenger 2 Asset in five years.

Replacement due to Obsolescence

CHALLENGER 1 ALTERNATIVE

NPV = –$231

Economic analysis indicates replacement (Challenger 1) should **not** be made at EOY 0.

What are the equivalent annual savings of deferring the replacement until EOY 5?

231 (A|P 10,10) = $38 per year.

Are these small annual savings worth the risk?

1. The defender is old and will have a higher probability of failure late in its life.
2. Should the company give up the immediate/short-run savings (payback evaluation is a measure of risk)?
3. How accurate are the estimates of future savings?

Note that the best option for the 10-year period is Challenger 1 at EOY 0 and then Challenger 2 at EOY 5. This is $2 376 better than D + C_2, an amount equivalent to that found in the five-year evaluation.

Replacement due to Deterioration

- replacement is often necessary due to
  - excessive operating costs
  - increased maintenance costs
  - higher reject rates
- as costs rise, a replacement study is warranted
- the challenger will not be acquired until its equivalent annual cost is lower than next year's cost for the defender
- what is the optimum replacement period?
- must verify that replacement is, in fact, necessary

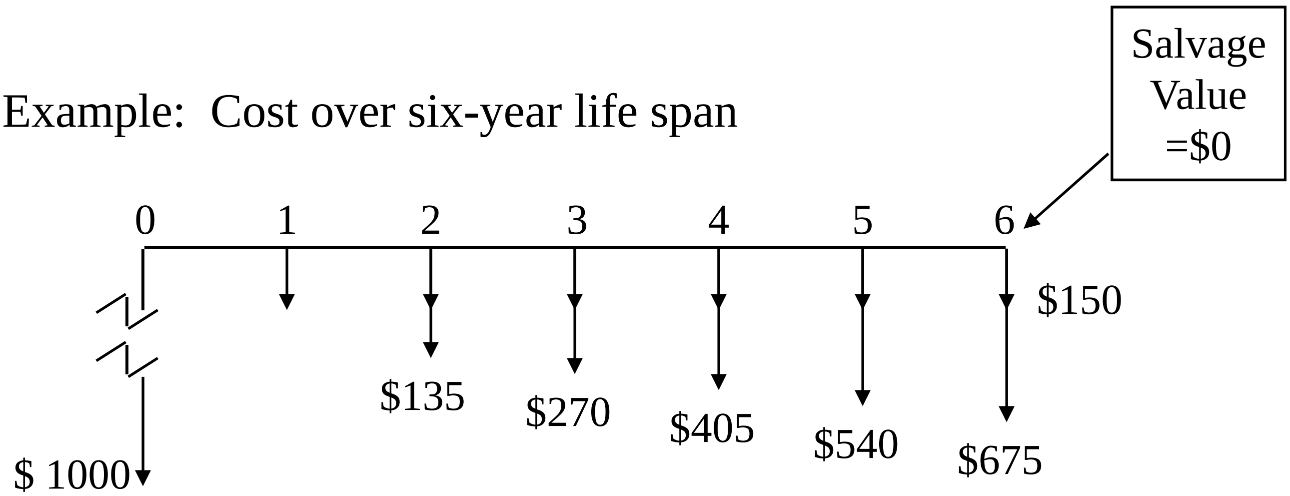
e.g. Equipment Optimum Replacement Interval

First cost	\$1 000
Salvage value	negligible
First years O & M	\$150
Annual increase in O & M	\$135
MARR	8%

The optimum replacement interval minimizes the equivalent uniform annual costs (EUAC).

Replacement Due to Deterioration

Example: Cost over six-year life span



$$
\begin{aligned}
NPV &= -1\,000 - 150\,(P|A\ 8\%,6) - 135\,(P|G\ 8\%,6) \\
&= -1\,000 - 150(4.6229) - 135\,(10.5233) \\
&= -3\,114
\end{aligned}
$$

What is the equivalent uniform annual cost?

$$
\begin{aligned}
EUAC &= -NPV\,(A|P\ 8\%,6) \\
&= 3\,114\,(0.2163) = \$674 \text{ per year}
\end{aligned}
$$

General case: *n*-year life span

$$EUAC(n) = [1\,000 + 150(P|A\ 8\%,n) + 135(P|G\ 8\%,n)]\,(A|P\ 8\%,n)$$

Replacement Due to Deterioration

$$EUAC = [1\,000 + 150\,(P|A\ 8\%,n) + 135\,\underline{(P|G\ 8\%,n)}]\ \underline{(A|P\ 8\%,n)}$$

but $\quad (P|A\ i,n)\ (A|P\ i,n) = 1$

and $\quad (P|G\ i,n)\ (A|P\ i,n) = (A|G\ i,n).$

$$\therefore\ EUAC = \underbrace{1\,000\,(A|P\ 8\%,n)}_{\text{Capital Recovery Cost (Salvage = 0)}} + \underbrace{150 + 135\,(A|G\ 8\%,n)}_{\text{Annual Operating and Maintenance Costs}}$$

In general

$$= \underbrace{P\,(A|P\ i,n)}_{\text{Acquisition Costs}} - \underbrace{S_n\,(A|F\ i,n)}_{\text{Salvage Value}} + (\text{O\&M Costs})$$

Capital Recovery Cost

Note that S_n (salvage value) is decreasing function of *n*. The older the asset, generally the lower its salvage value.

Optimal Replacement Interval

- that value of n that minimizes the EUAC

$$EUAC(n) = \underbrace{1\,000\,(A|P\ 8\%,n)}_{\substack{\text{Capital Recovery Cost} \\ \text{Salvage Value} = 0}} + \underbrace{150 + 135\,(A|G\ 8\%, n)}_{\text{O\&M Costs}}$$

n	1 000(A|P 8%,n)	(A|G 8%,n)	O&M Costs	EUAC
1	1 080	0.0000	150	1 230
2	561	0.4808	215	776
3	388	0.9487	278	666
4	**302**	**1.4040**	**340**	**641**
5	250	1.8465	399	650
6	216	2.2763	457	674
7	192	2.6937	514	706
8	174	3.0985	568	742
9	160	3.4910	621	781
10	149	3.8713	673	822

- n = 4 minimizes the cost of owning and operating this equipment

Minimum annual cost is $641.

- note that in this example the EUAC is not very sensitive to changes in and around the optimal life span

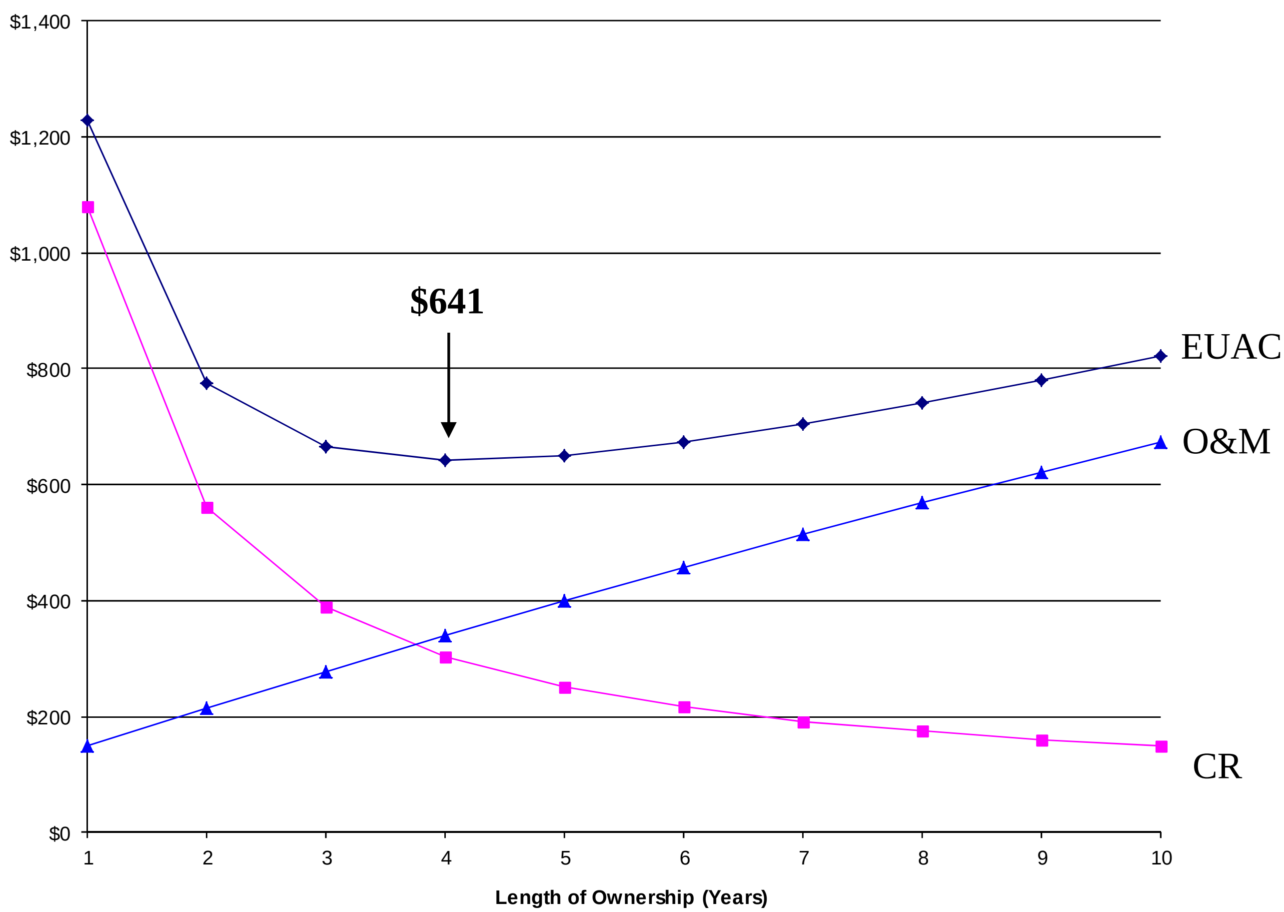
Optimal Replacement Interval

EUAC = Capital Recovery Cost + Annual O&M Cost

$= P\,(A|P\ i,n) - S\,(A|F\ i,n) + (\text{O\&M Costs})$

For any year *n*

$$EUAC\,(n) = \underbrace{1000\,(A|P\ 8,n)}_{\text{CR Cost}} + \underbrace{150 + 135\,(A|G\ 8,n)}_{\text{O\&M Costs}}$$



Replacement due to Inadequacy

- when the current operating conditions change, an older asset can lack the required capacity
- often a similar asset can be purchased to augment the capacity of the old asset allowing the value of the old asset to be retained within the firm

DEFENDER – existing asset plus supplement

CHALLENGER – new asset that can satisfy entire requirement

Computer System Replacement

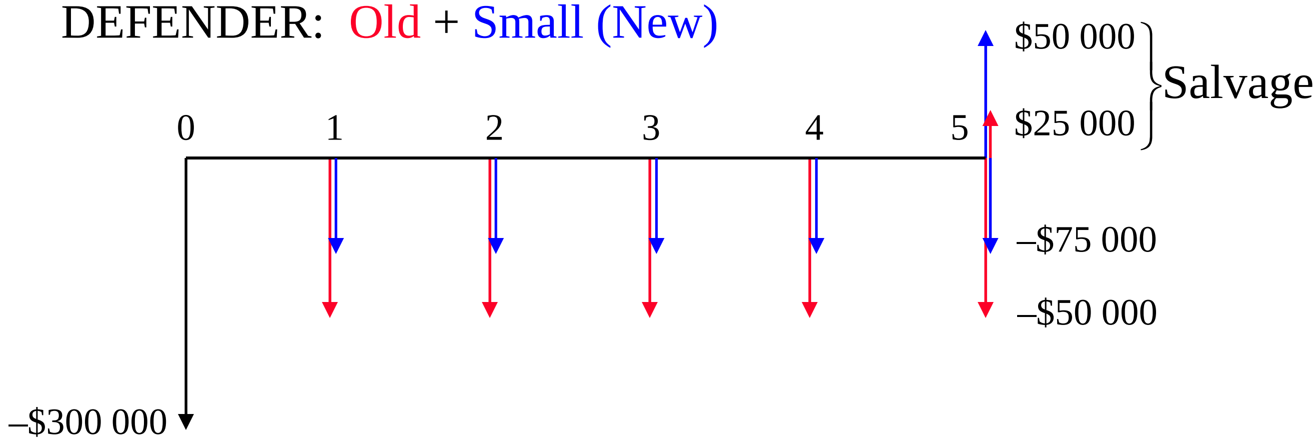
	Current Value	O&M	Salvage Value
DEFENDER:			
• existing system	$250 000	$75 000	$ 25 000
• small new system	$300 000	$50 000	$ 50 000
CHALLENGER:			
• new system	$600 000	$80 000	$250 000

- planning period 5 years
- MARR 30%

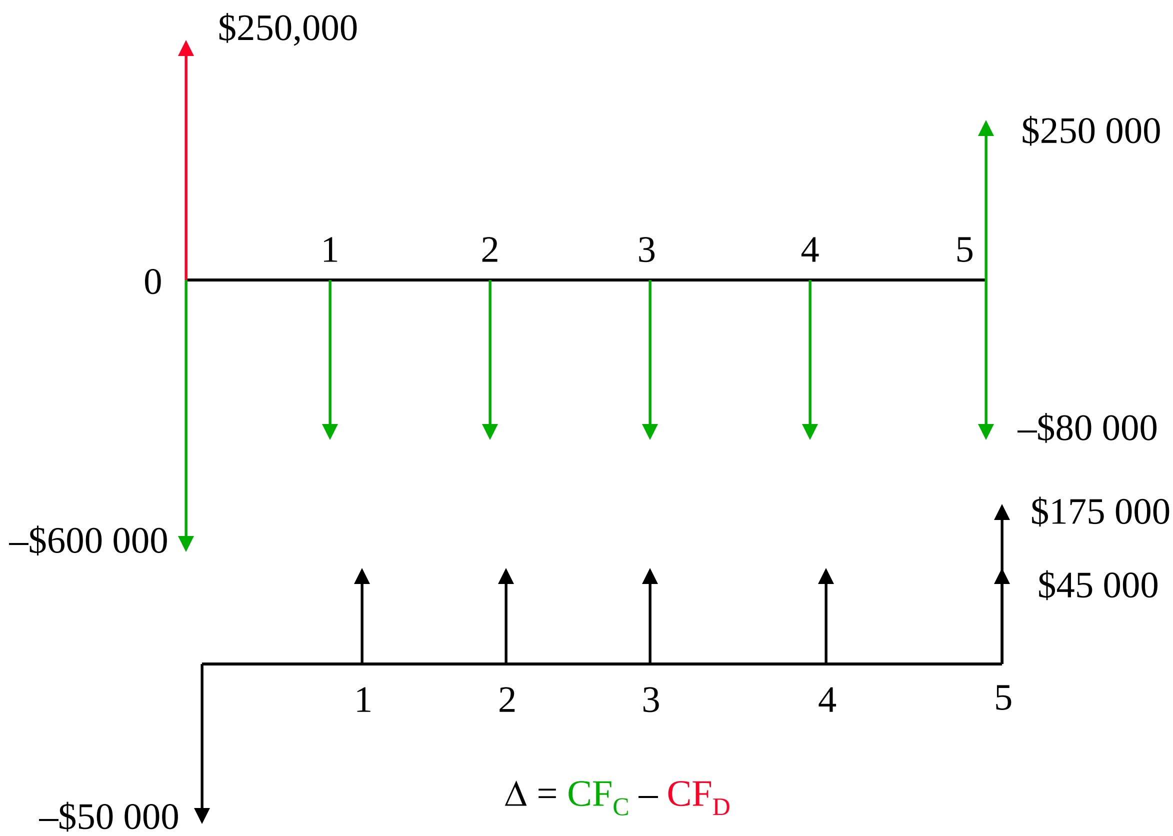
Replacement due to Inadequacy

Computer System Replacement

DEFENDER: Old + Small (New)



CHALLENGER: Large (New)



$\Delta = CF_C - CF_D$

Replacement due to Inadequacy

Computer System Replacement

EOY	CF (keep old)	CF (replace old)	Δ
0	–300 000	–600 000 + 250 000	– 50 000
1–5	–75 000 – 50 000	–80 000	+ 45 000
5	+25 000 + 50 000	+250 000	+175 000

NPV (keep) = –300 000 – 125 000(P|A30,5) + 75 000(P|F30,5)
= –584 247

NPV (replace)= –350 000 – 80 000(P|A30,5) + 250 000(P|F30,5)
= –477 513

NPV (replace) > NPV (keep) $\therefore$ replace computer

Incremental method: CF (replace) – CF (keep)

NPV (Δ) = –50 000 + 45 000 (P|A30,5) + 175 000 (P|F30,5)
= 106 733

$\therefore$ replace computer