ComPO: Preference Alignment via Comparison Oracles

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LLM Alignment from Human Preference Feedback

Input prompt:

I am working on **optimization**.
Please introduce Adam to me.

Output A:

Who is Adam?

Output B:

Adam is a popular optimizer widely applied to stochastic optimization.

Output C:

Adam is a first-order optimizer that blends momentum with per-parameter adaptive learning rates: it keeps exponential moving averages of the gradients (mean) and their squares (variance), applies bias correction for early steps…

Human Preference Feedback: C ≻ B ≻ A

A – Factual mistake

B – Correct, but too higher-level

C – Correct, detailed and informative

preferred:

detailed & concise, honest & harmless, informative & helpful, clarity & structured, respectful & accurate, User-centered

non-preferred:

misaligned with prompt, harmful info, verbose & lengthy, superficial & unclear, unsafe & dishonest, User-disobeyed

RLHF: Reinforcement Learning from Human Feedback

Goal: encourage preferred output & discourage non-preferred output

Preference Dataset

$\mathcal{D}(x, y^+, y^-)$

Preference pairs:

x: prompt

y^+: preferred response

y^-: non-preferred response

Reward Model

$r_\phi(x, y)$

Reward model r_ϕ:

Trained to provide:

- high reward for preferred one
- low reward for non-preferred response

Reinforcement Learning

$\pi^*(y \mid x)$

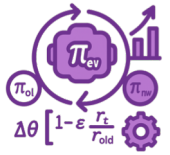
Optimal policy π^*:

Trained to:

- Maximize reward $r_\phi(x, y)$
- Policy guided rollout towards human preferred response

Stage 1
Human Annotation

Stage 2
Training Reward Model



Stage 3
PPO Policy Update

Obstacles:

(i) Resource-intensive online training: reward & policy model training

(ii) Heuristics raised from training due to inaccurate reward provision

Can we reduce it into an offline training, with a more deterministic objective/policy update?

DPO: Direct Preference Optimization

DPO loss: a reparameterization from RLHF objective

$$\mathcal{L}_{\text{DPO}}(\pi_\theta; \pi_{\text{ref}}) = \mathbb{E}_{(x,y^+,y^-)\sim\mathcal{D}}\left[\log \sigma\left(\beta \log \frac{\pi_\theta(y^+|x)}{\pi_{\text{ref}}(y^+|x)} - \beta \log \frac{\pi_\theta(y^-|x)}{\pi_{\text{ref}}(y^-|x)}\right)\right]$$

Likelihood margin:
We want larger margin between each winning/losing pair

$\pi(y^+|x) = -100, \quad \pi(y^-|x) = -5$

Large margin $\Longrightarrow$ Stronger signal

$\pi(y^+|x) = -100, \quad \pi(y^-|x) = -99$

Smaller margin $\Longrightarrow$ Weaker/noisy signal
DPO cannot utilize it, simply discarded...

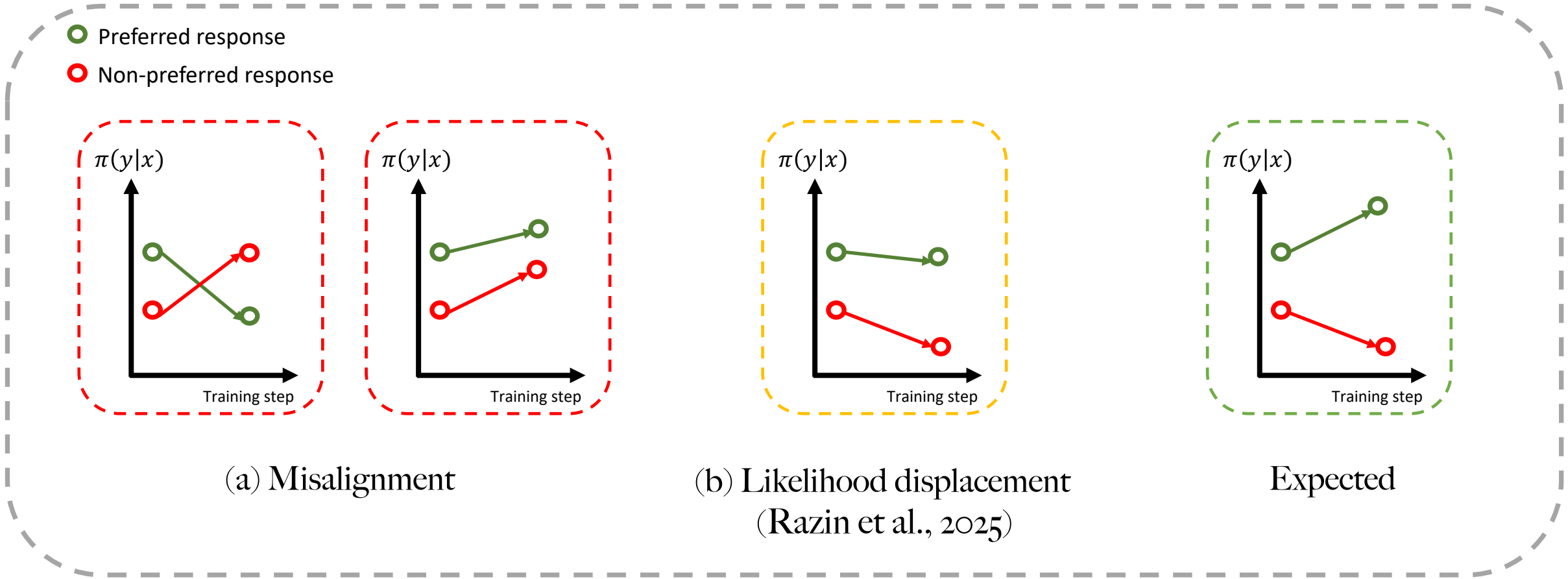
Can we extract useful information from these noisy preference pairs?

ComPO: Zeroth-order, Comparison-based method

DPO: Extract preference info via external **margin difference**

$\Longrightarrow$ failed when such difference is numerically small, especially under noisy pairs

ComPO: Extract preference info through internal **contrastive relation**



(a) Misalignment

(b) Likelihood displacement (Razin et al., 2025)

Expected

Comparison Oracles

Given that $f : \mathbb{R}^d \to \mathbb{R}$ is a function where neither its function value nor its gradient is accessible, we define a pairwise comparison oracle $\mathcal{C}_f$ in its simplest form as follows,

Definition 2.1 *We call* $\mathcal{C}_f(\theta, \theta') : \mathbb{R}^d \times \mathbb{R}^d \to \{+1, -1\}$ *a comparison oracle for function* f *if*

$$\mathcal{C}_f(\theta, \theta') = \begin{cases} -1, & \text{if } f(\theta') < f(\theta), \\ +1, & \text{otherwise.} \end{cases}$$

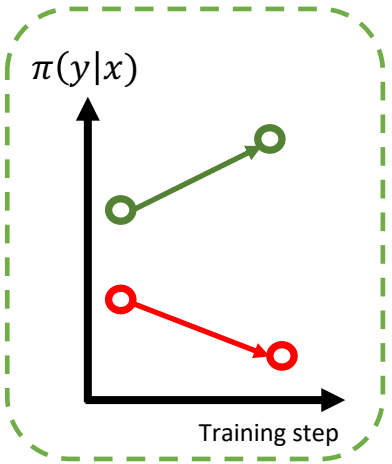
Goal: minimizing f, but only get access to implicit comparison between two sets of model parameter θ

tl;dr "**-1 returned: θ' is better than θ for f**"

Preference Comparison Oracles

Definition 3.1 *We say* $\mathcal{C}_\pi(\theta, \theta') : \mathbb{R}^d \times \mathbb{R}^d \to \{+1, -1\}$ *a preference comparison oracle for the model* π_θ *and a pair of preference data* $(\mathbf{x}, \mathbf{y}^+, \mathbf{y}^-)$ *from the offline dataset* $\mathcal{D}$ *if*

$$\mathcal{C}_\pi(\theta, \theta') = \begin{cases} -1, & \text{if } \pi_{\theta'}(\mathbf{y}^+|\mathbf{x}) > \pi_\theta(\mathbf{y}^+|\mathbf{x}) \text{ and } \pi_{\theta'}(\mathbf{y}^-|\mathbf{x}) < \pi_\theta(\mathbf{y}^-|\mathbf{x}) \text{ for } (\mathbf{x}, \mathbf{y}^+, \mathbf{y}^-) \in \mathcal{D}, \\ +1, & \text{otherwise.} \end{cases}$$



One-bit Compressed Approximation (Cai et al., 2022):

$$\mathcal{C}_f(\theta, \theta + r\mathbf{z}_i) \approx \text{sign}(f(\theta + r\mathbf{z}_i) - f(\theta)) \approx \text{sign}(\mathbf{z}_i^\top \nabla f(\theta))$$

↓ Random perturbation z within radius r

Gradient with Sparse Structure: Define oracle feedback $y_i = C_f(\theta,\ \theta + r\mathbf{z}_i)$, we have gradient

$$\hat{\mathbf{g}} = \underset{\|\mathbf{g}\|_1 \le \sqrt{s}, \|\mathbf{g}\| \le 1}{\text{argmax}} \sum_{i=1}^{m} y_i \mathbf{z}_i^\top \mathbf{g}$$

→ Choose g to align with the direction from random perturbed vectors & oracle feedback; think as a dot product

↓ **Zeroth-order gives noisy gradient make it sparse to reduce noise**

Billion-level Parameter Space

$$\hat{\mathbf{g}} = \underset{\|\mathbf{g}\|_1 \le \sqrt{s}, \|\mathbf{g}\| \le 1}{\operatorname{argmax}} \sum_{i=1}^{m} y_i \mathbf{z}_i^\top \mathbf{g}$$

Prior works solved within the dimensionality of 500, but what about LLMs, with billion parameters?

$$\hat{\mathbf{g}} = \underset{\|\mathbf{g}\|_1 \le \sqrt{s}, \|\mathbf{g}\| \le 1}{\operatorname{argmax}} \sum_{i=1}^{m} y_i \mathbf{z}_i^\top \mathbf{g}$$

Step 1: Sparsity simplification
Temporarily **eliminate** the sparsity norm constraint

$$\hat{\mathbf{g}}^0 = \frac{\sum_{i=1}^{m} y_i \mathbf{z}_i}{\| \sum_{i=1}^{m} y_i \mathbf{z}_i \|}$$

Step 2: Derive simplified gradient
Retrieve with closed form

Step 3: Apply gradient-entry threshold λ_g
Clip noisy gradient entry with lower magnitude, bringing sparsity

Step 4: Dynamic stepwise adjustment + threshold
Filtering out gradient with less successful oracle feedbacks

Bridging Optimization Theory towards LLM Training

Algorithm 2 Comparison-Based Preference Alignment (Practical Scheme)

1: **Input**: initial parameter $\theta_1 = [\bar{\theta}_1; \theta_t^{\mathrm{o}}] \in \mathbb{R}^d$, scaling for stepsize $\gamma > 0$, sampling radius $r > 0$, querying number $m \geq 1$, clipping thresholds $\lambda_g, \lambda > 0$, and iteration number $T \geq 1$.
2: **for** $t = 1, 2, \ldots, T$ **do**
3: Draw m i.i.d. samples uniformly from a unit sphere in $\mathbb{R}^{d^{\mathrm{o}}}$, i.e., $\{\mathbf{z}_i\}_{1 \leq i \leq m}$.
4: Compute $y_i = \mathcal{C}_\pi([\bar{\theta}_1; \theta_t^{\mathrm{o}}], [\bar{\theta}_1; \theta_t^{\mathrm{o}} + r\mathbf{z}_i])$ for $i = 1, 2, \ldots, m$.
5: Compute $\hat{\mathbf{g}}_t^{\mathrm{o}} = \frac{\sum_{i=1}^m y_i \mathbf{z}_i}{\| \sum_{i=1}^m y_i \mathbf{z}_i \|}$ and clip $\hat{\mathbf{g}}_t^{\mathrm{o}}$ by zeroing out the entries whose magnitude is less than λ_g.
6: Compute $\theta_{t+1}^{\mathrm{o}} = \theta_t^{\mathrm{o}} - \frac{\gamma |\{i : y_i = -1\}|}{m} \hat{\mathbf{g}}_t^{\mathrm{o}}$ if $\frac{|\{i : y_i = -1\}|}{m} > \lambda$ and $\theta_{t+1}^{\mathrm{o}} = \theta_t^{\mathrm{o}}$ otherwise.

Gradient entry threshold

Stepsize threshold (oracle feedback)

ComPO is trained on the noisy pairs that DPO cannot effectively utilize, i.e., with

$$\| \log \pi_{\mathrm{ref}}(\mathbf{y}^+ | \mathbf{x}) - \log \pi_{\mathrm{ref}}(\mathbf{y}^- | \mathbf{x}) \| \leq \delta,$$

* $\delta = 3$ is used in actual implementation, with running ComPO on the first 100 noisy pairs

Directly augmenting DPO

Table 1: Evaluation results on AlpacaEval 2, Arena-Hard, and MT-Bench under four model setups. LC and WR denote length-controlled win rate and win rate, respectively. Turn-1 and Turn-2 represent the scores to the answers from the first and follow-up questions in multi-turn dialogue. Here, we run 5 trials for $\text{DPO}_{\text{clean}}$+ComPO and present the best trail performance.

Method	Mistral-Base-7B						Mistral-Instruct-7B					
	AlpacaEval 2		Arena-Hard	MT-Bench			AlpacaEval 2		Arena-Hard	MT-Bench		
	LC (%)	WR (%)	WR (%)	Turn-1	Turn-2	Avg.	LC (%)	WR (%)	WR (%)	Turn-1	Turn-2	Avg.
DPO	9.71	6.27	2.9	6.20	**5.38**	**5.79**	24.14	16.71	**14.4**	6.28	5.42	5.86
$\text{DPO}_{\text{clean}}$	9.41	6.52	3.0	6.18	5.22	5.70	23.89	16.15	14.2	6.11	5.34	5.73
$\text{DPO}_{\text{clean}}$+ComPO	**11.66**	**6.55**	**3.2**	**6.22**	5.32	5.77	**26.17**	**18.32**	10.5	**7.78**	**7.63**	**7.69**
Method	**Llama-3-Base-8B**						**Llama-3-Instruct-8B**					
	AlpacaEval 2		Arena-Hard	MT-Bench			AlpacaEval 2		Arena-Hard	MT-Bench		
	LC (%)	WR (%)	WR (%)	Turn-1	Turn-2	Avg.	LC (%)	WR (%)	WR (%)	Turn-1	Turn-2	Avg.
DPO	4.14	10.43	**12.1**	6.61	5.85	6.23	32.59	31.99	22.9	8.30	7.55	7.93
$\text{DPO}_{\text{clean}}$	4.28	9.81	12.0	**6.64**	6.01	6.33	32.92	32.42	22.9	8.26	7.63	7.94
$\text{DPO}_{\text{clean}}$+ComPO	**5.39**	**10.93**	**12.1**	6.60	**6.28**	**6.44**	**35.79**	**35.03**	**23.1**	**8.39**	**7.71**	**8.05**

LC: Length-controlled winning rate against base model (GPT 4-Turbo), judged by GPT 4
WR: Winning rate (without length control) against base model (GPT 4-Turbo), judged by GPT 4

$\text{DPO}_{\text{clean}}$: Running DPO while excluding those noisy pairs

Augmenting DPO variants – SimPO (Meng et al., 2024)

Table 2: Direct augmentation results on SimPO over different models and benchmarks.

Model	Method	AlpacaEval 2		Arena-Hard	MT-Bench		
		LC (%)	WR (%)	WR (%)	Turn-1	Turn-2	Avg.
Mistral-Instruct-7B	SimPO	40.22	41.18	20.8	**7.94**	7.31	7.62
	SimPO + ComPO	**42.27**	**43.17**	**22.0**	7.83	**7.46**	**7.64**
Llama-3-Instruct-8B	SimPO	48.71	43.66	36.3	7.91	7.42	7.66
	SimPO + ComPO	**49.53**	**45.03**	**37.3**	**7.94**	**7.45**	**7.70**
Gemma-2-it-9B	SimPO	60.36	55.59	**61.1**	**9.07**	8.47	8.77
	SimPO + ComPO	**62.42**	**57.20**	**61.1**	8.99	**8.58**	**8.79**

* SimPO still relies on margin-difference style loss, but removed the reference model and added length normalization

Resolving Likelihood Displacement

Table 3: The log-likelihood for preferred and dispreferred responses for 3 independent trials with $\gamma \in \{0.1, 1\}$ and the default values for all of other parameters. Each cell gives a pair of log-likelihood for preferred and dispreferred responses $(\log \pi_\theta(\mathbf{y}^+|\mathbf{x}), \log \pi_\theta(\mathbf{y}^-|\mathbf{x}))$ after one trail of training. The results are indeed different since the perturbations $\{\mathbf{z}_i\}_{1 \le i \le m}$ are different for Trial 1, Trial 2 and Trial 3. However, we find that the log-likelihood for preferred response increase and the log-likelihood for dispreferred response decrease.

Llama-3-Instruct-8B $(\log \pi_\theta(\mathbf{y}^+\|\mathbf{x}), \log \pi_\theta(\mathbf{y}^-\|\mathbf{x})) = (-46.761, -47.410)$			
γ	Trial 1	Trial 2	Trial 3
0.1	$(-46.744, -47.411)$	$(-46.760, -47.411)$	$(-46.759, -47.410)$
1	$(-46.728, -47.520)$	$(-46.743, -47.525)$	$(-46.753, -47.517)$
Gemma-2-it-9B $(\log \pi_\theta(\mathbf{y}^+\|\mathbf{x}), \log \pi_\theta(\mathbf{y}^-\|\mathbf{x})) = (-133.122, -134.557)$			
γ	Trial 1	Trial 2	Trial 3
0.1	$(-133.122, -134.557)$	$(-133.122, -134.557)$	$(-133.121, -134.557)$
1	$(-133.059, -134.562)$	$(-133.122, -134.564)$	$(-133.112, -134.565)$

Memory + Computational Efficiency

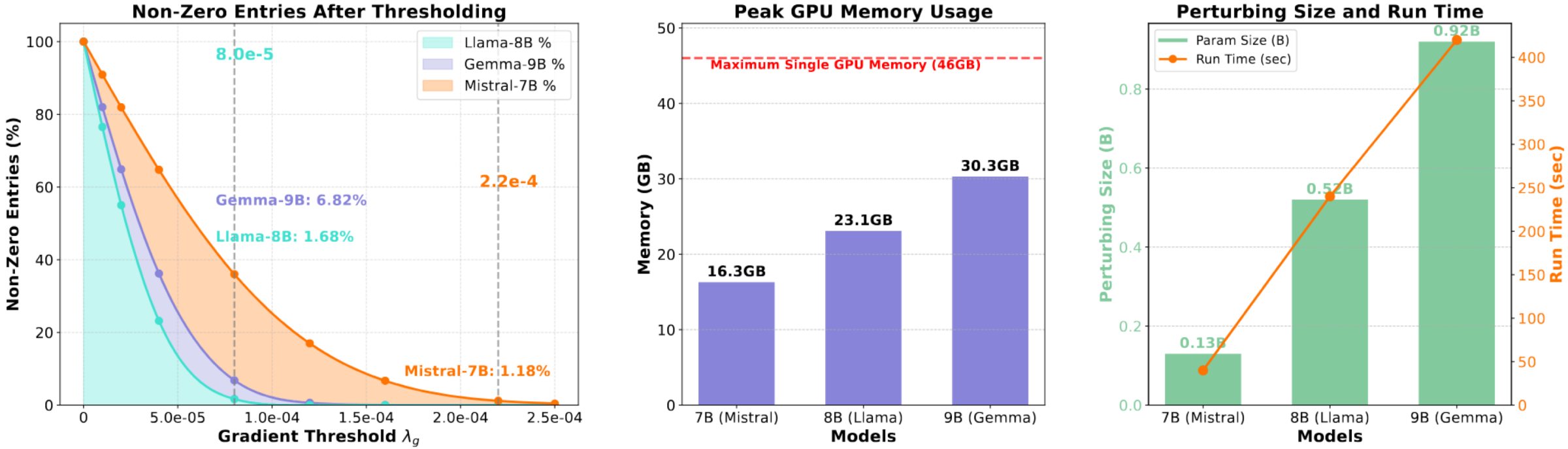


Figure 1: (Left) Percentage of non-zero entries in the final gradient across different gradient entry threshold λ_g; (Middle) Peak GPU memory usage across three models used in all experiments; (Right) Size of parameter space (output layer) in the comparison oracle perturbations and the run time for completing 600 perturbations using 30 NVIDIA A40 GPUs are shown.

* Running DPO and SimPO on Llama-8B-Instruct requires 77GB and 69GB, respectively, while ComPO only requires 23GB

Scalability: hyperparameter

1. Number of random perturbation in each iteration:

Table 4: Effect of the number of perturbations m on model performance. We report WR and LC on AlpacaEval 2; each entry includes the results of mean performance and standard deviation over 5 consecutive runs; the best run performance is shown in the parentheses.

Perturbation (m)	**800**	**1600**	**3300**	**5400**
AlpacaEval 2-WR %	17.32 ± 0.86 (17.94)	17.50 ± 0.65 (18.32)	19.21 ± 0.58 (**20.25**)	19.69 ± 0.36 (20.07)
AlpacaEval 2-LC %	24.72 ± 1.02 (25.12)	25.02 ± 0.91 (26.17)	25.91 ± 0.95 (27.14)	26.49 ± 0.81 (**27.20**)

2. Number of layer perturbed in LLM:

Table 5: Multi-layer perturbation improves performance. We report WR and LC on AlpacaEval 2 and WR on Arena-Hard; entries are mean $\pm$ std over 5 runs, with the best run in parentheses.

Layers perturbed (# params)	**AlpacaEval 2-WR %**	**AlpacaEval 2-LC %**	**Arena-Hard (GPT 4.1)-WR %**
1 (0.13B)	17.50 ± 0.65 (18.32)	25.02 ± 0.91 (26.17)	10.80 ± 0.21 (11.0)
3 (0.25B)	18.19 ± 0.81 (19.38)	26.00 ± 0.89 (27.09)	11.26 ± 0.36 (11.7)

3. Number of noisy pairs used in training:

Table 7: Results on scaling the number of noisy preference pairs used in the training.

Number of noisy pairs	AlpacaEval 2-WR %	AlpacaEval 2-LC %	Arena-Hard (GPT 4.1)-WR %
100	19.21 ± 0.58 (20.25)	25.91 ± 0.95 (27.14)	11.02 ± 0.13 (11.2)
300	20.07 ± 0.99 (21.35)	26.28 ± 0.81 (27.59)	11.76 ± 0.30 (12.1)

4. Ablation on gradient threshold λ_g:

Table 6: Effect of gradient threshold λ_g (transposed). Results are WR and LC on AlpacaEval 2; entries are mean $\pm$ std over 5 runs, with the best run in parentheses.

λ_g	0	4×10^{-5}	1.8×10^{-4}	2.2×10^{-4}	2.5×10^{-4}
Percentage of gradient entries updated	100%	63%	6%	1%	0.15%
AlpacaEval 2-WR %	15.72 ± 0.77 (16.34)	16.02 ± 0.69 (16.69)	19.02 ± 0.62 (20.15)	19.21 ± 0.58 (20.25)	16.10 ± 0.11 (16.21)
AlpacaEval 2-LC %	23.42 ± 1.03 (24.28)	24.01 ± 0.91 (25.10)	26.06 ± 0.81 (27.27)	25.91 ± 0.95 (27.14)	23.82 ± 0.23 (24.00)

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AlpacaEval 2-LC %	23.42 ± 1.03 (24.28)	24.01 ± 0.91 (25.10)	26.06 ± 0.81 (27.27)	25.91 ± 0.95 (27.14)	23.82 ± 0.23 (24.00)

Q&A – Thanks for joining!

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